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Differentially positioned language games: ethnomathematics from a philosophical perspective

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Abstract This paper discusses a new philosophical perspective for ethnomathematics which articulates Ludwig Wittgenstein's and Michel Foucault's theoretical notions. It is conceived as a theoretical toolbox which allows the analysis of, on the one hand, the mathematical language games of different forms of life and their family resemblances and, on the other hand, the Eurocentric discourses of academic and school mathematics and their effects of truth. Based on fieldwork done in rural forms of life in the south of Brazil, examples of the use of this perspective are presented. The paper analyzes language games of those different forms of life and the school mathematics discipline, highlighting the complex network of learning and powers that makes *other* mathematics than that known as *the* mathematics be positioned "in a void" in school curricula.

Keywords Ethnomathematics · Wittgenstein's later work · Michel Foucault's work · Mathematics education

1 Introduction[1]

> (…) I would like to recount a little story so beautiful I fear it may well be true. It encompasses all the constraints of discourse, those limiting its powers, those controlling its chance appearances and those which select from among speaking subjects. At the beginning of the seventeenth century, the Shogun heard tell of European superiority in navigation, commerce, politics and the military arts, and that this was due to their knowledge of mathematics. He wanted to obtain this precious knowledge. When

[1]Some of the ideas presented in this introduction and in other sections of the paper are part of a paper previously published in Portuguese (Knijnik, 2008).

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someone told him of an English sailor possessed of this marvelous discourse, he summoned him to his palace and kept him there. The Shogun took lessons from the mariner in private and familiarized himself with mathematics, after which he retained power and lived to a very old age. It was not until the nineteenth century that there were Japanese mathematicians. But that is not the end of the anecdote, for it has its European aspect as well. The story has it that the English sailor, Will Adams, was a carpenter and an autodidact. Having worked in a shipyard he had learnt geometry. (Foucault, 2010, p. 225).

Foucault's story describes a carpenter who had "a precious knowledge," a knowledge that placed Europeans in a position of superiority. The carpenter was an autodidact who was kept by the Shogun in his palace to allow the Shogun to learn mathematics. The story is also about an inaugural tie between the cultures of two very distant places which, in times past, had never communicated with each other. Will Adams is considered to be the first Englishman to have lived in Japan. In his youth, he was an apprentice of a well-known sailor, from whom he learned the art of building ships.[2] In one of four letters still preserved, dated 1611, Will Adams wrote:

So in process of four or five years the emperor called me, as divers times he had done before. So one time above the rest he would have me to make him a small ship. I answered that I was no carpenter and had no knowledge thereof. "Well, do your endeavor," saith he; "if it be not good, it is no matter." Wherefore at his command I built him a ship of the burden of eighty tons or thereabout; which ship being made in all respects as our manner is, he coming aboard to see it, liked it very well; by which means I came in favor with him, so that I came often in his presence, who from time to time gave me presents, and at length a yearly stipend to live upon, much about seventy ducats by the year with two pounds of rice a day daily. Now being in such grace and favor by reason I learned him some points of geometry and understanding of the art of mathematics with other things. (Tappan, 1914, p. 329).

To transmit to the Shogun, "some points of geometry and understanding of the art of mathematics" made the sailor the Shogun's hostage, but also made the Emperor a captive of the sailor, so that mathematics would allow the Shogun "to retain power and to live to a very old age." To the carpenter's mathematics—a mathematics of the "shipyard floor"—a very special place was provided, that of "divine knowledge—blessed by God" (Tappan, 1914, p. 330). The knowledge became sacred and was implicated in the perpetuation of power. From the cracks of the rough floor of a carpenter's shipyard to the heights of abstract knowledge led an ascending path. It was mathematics that emerged from the "dirt of the social practices" of the world of labor, where it had been stranger, "out of place," but returned to its "'fair' and 'convenient'" place, guided by the "dream of purity" (Bauman, 1998) that characterizes the transcendental mathematics conceived by modernity.

What if we were to think about this ascending purification process from another perspective? What if we could, inspired by Wittgenstein's teachings, think not about the existence of *a single* mathematics—the one that Lizcano (2010) identifies as a form of life of the

[2] The chapter "The coming of Will Adams to Japan" starts with the following editor's note: "Will Adams was the first Englishman to make his home in Japan. His knowledge of shipbuilding made him so useful to the emperor that, although he was treated with honors and liberality, he was not allowed to leave the country. The Japanese of the street in Yedo which was named for him still hold an annual celebration in his memory. The letter from which the following extracts are taken—with modernized spelling—was written in 1611. It begins with his departure from the coast of Peru." (Tappan, 1914, p. 325).

"European tribe" (and which with "its purity and order" Foucault recalls having given superiority to the Europeans—in terms of navigation, commerce, politics and military art)—but about many *different* mathematics that among themselves would not maintain a necessary epistemological subordination (since from the sociological point of view, it would be naïve if we did not consider such subordination) to the Eurocentric one in which we were schooled?

These are the central questions which led me to conceive the ethnomathematical perspective discussed in this paper. It is grounded in the ethnomathematical field, which leads me to start my discussion by looking back to the 1970s, when ethnomathematics was established. Of great importance is the work of Ascher and Ascher (1986), D'Ambrosio (1990), Gerdes (1991), and Zaslavsky (1973) at the beginning of this field. Since then these authors [and other ethnomathematicians who came later, like Powell and Frankenstein (1997) and Presmeg (1998)] have shown the relevance of considering culture at the heart of the learning and teaching processes of mathematics. They admitted the existence of different ethnomathematics, such as those practiced by indigenous peoples, by adult workers in diverse contexts, by peasants and so on.[3] During that period, ethnomathematicians using ethnographic procedures have developed empirical studies in diverse cultures, aiming to show the diversity of the mathematical practices of those cultures (Knijnik, 2007; Barton, 1996; Bishop, 1988). But time has passed, and new questions have arrived.

Introducing *power* into the ethnomathematical discussions avoided a naïve understanding of the mathematical diversity. Making power explicit in ethnomathematics could allow us to analyze how the politics of knowledge operates in schooling processes and, in particular, in the mathematics curricula. What is at stake here is to consider school mathematics not as a set of fixed subjects whose higher level of abstraction would allow students to cope with the multiple dimensions of their lives or, in the words of Wittgenstein (2004), in the multiple forms of life to which they belong, but as an arena marked by struggles for the imposition of meaning. This imposition is produced by the *double violence* as conceived by Bourdieu (2003): first, there was an imposition of one culture over others—in this case, European culture over other Western cultures, then the second violence happened: that imposition was forgotten. So European mathematical thinking imposed itself on other Western ways of mathematizing. In particular, school mathematics inherited this sort of imposition, and in the end, the mathematical subjects which are taught in educational institutions are "naturalized" (Knijnik & Wanderer, 2006). They are taken as the only possible knowledge worthy to be included in the school mathematical curriculum.

As Silva (1992) says:

> Contrary of what the liberal vision makes us believe, neither knowledge in general, nor school knowledge, constitute absolute products of an unceasing, disinterested process of seeking the truth. […] The asymmetric relationships between conflicting classes and groups act to enhance the value of a given kind of knowledge and devalue that of others, to include the cultural traditions of the dominant groups and classes among the kinds of knowledge that are worthy and valid to be transmitted and to exclude the cultural traditions of subordinated classes and groups. (p. 80, my translation)

But to introduce power in the ethnomathematics discussions was not enough. It was also necessary to examine, from a philosophical viewpoint, some of the statements that were

[3] From his first works onwards, D'Ambrosio highlighted that what we call mathematics is a specific ethnomathematics—the one practiced by mathematicians at academic institutions.

taken for granted in this field of mathematics education, for example the statement that assumes the existence of many ethnomathematics.

These topics constitute the background of the ethnomathematical perspective discussed in this paper. Going further in the effort to work as an ethnomathematician, I propose to consider an ethnomathematical perspective conceived as a theoretical toolbox (in the Deleuzian sense[4]). This toolbox allows the analysis of the mathematical language games of different forms of life and their family resemblances, as well as the Eurocentric[5] discourses of academic and school mathematics and their effects of truth. In fact, this toolbox itself is built based on theoretical tools coming from two different philosophies. The first is Ludwig Wittgenstein's later work, which is set out in his book *Philosophical Investigations*. From this period of Wittgenstein's theorizations come the notions of "forms of life," "language games," "use," and "family resemblances." The second set of theoretical tools that give support to my ethnomathematical perspective comes from Michel Foucault's work. Specifically Foucauldian notions such as "discourse," "power," "resistance," and "counter-conduct" are used. In summary, the paper discusses an ethnomathematical perspective built on theoretical tools from Foucault and Wittgenstein. The discussion allows me to show how this perspective supports some of the claims of the ethnomathematics field, such as those mentioned above. In the next two sections, I discuss Wittgenstein's and Foucault's theoretical tools which are used to describe the ethnomathematical perspective presented here.

2 The ethnomathematical perspective and Wittgenstein's later work

One of the key points to be tackled in the field of ethnomathematics is the need to build a philosophical rationale that can allow us to admit the existence of different mathematics or, as formulated by D'Ambrosio, different ethnomathematics. Diverse studies (e.g., Knijnik, 2007; Vilela, 2007; Wanderer, 2007) have shown that Wittgenstein's positions during his period of maturity (whose main source of reference is *Philosophical Investigations*) are productive in arguing about the nonexistence of a unique mathematics, the one that is usually named as "the" mathematics, with its grammar marked by formalism and abstraction (Knijnik, 2007). Firstly in this section, I briefly present Wittgenstein's later key notions which, as discussed elsewhere (Knijnik, 2007), differ from what Wittgenstein (1994) proposed in the *Tractatus Logico-Philosophicus*. In his later work, when he sought to answer the question "What is language?" he asserted that "We must not ask what language is, but rather how it functions" (Condé, 1998, p. 86). When operating this theoretical displacement, Wittgenstein moved to "the non-metaphysical descriptions of our linguistic practices" (Peters, 2002, p. 2). He indicated that it is no longer possible to talk simply in a language, but only in languages, that is, "a huge variety of uses, a plurality of functions or roles that we could see as language games" (Condé, 1998, p. 86). Therefore the meaning of a

[4] Deleuze argues that "a theory is exactly like a box of tools. It has nothing to do with the signifier. It must be useful. It must function. And not for itself. (…). We don't revise a theory, but construct new ones (…). A theory does not totalize; it is an instrument for multiplication and it also multiplies itself" (Bouchard, 1977, p. 208).

[5] Here it is important to mention that to characterize academic discourses as Eurocentric means, in this context, to highlight the hegemonic mathematical discourse produced in Europe, and its cultural and social imposition in countries like Brazil since the colonization process started in the sixteenth century. To maintain the use of this adjective for school mathematics discourses reinforces the understanding of the "strong" family resemblances those discourses have with academic ones in Western society (Giongo & Knijnik, 2010).

word emerges as we use it in different situations, and the same expression, in different contexts, will mean different things. As Wittgenstein pointed out, “the meaning of a word is its use in the language” (Wittgenstein, 2004, p. 20). This implies that “the nature of linguistic meaning becomes fully embedded in human action and human life” (Hanna, 2010, p. 13).

In his later work, Wittgenstein abandons any essentialist concept of language. Indeed, if the meaning of a word is determined by the way we use it, *use* can be understood as something closely tied to practice and not “as the expression of a metaphysical category” (Condé, 2004, p. 48). As Gerrard (1991, p. 130) said, “if meaning is a function of practice, then there is no room for any determinations of meaning that are not part of our practice.”

Thus Wittgenstein’s theoretical mature work and the work of some of his interpreters (for example Glock, 1996 and Condé, 1998, 2004) allow the inference that language games and the rules that constitute them are strongly affected by the way we use language. This means that language games should be understood as immersed in a form of life, strongly amalgamated with nonlinguistic practices. In aphorism PI#23[6] Wittgenstein is clear about this point: “Here the term ‘language game’ is meant to bring into prominence the fact that the speaking of language is part of an activity, or of a forma of life.” And it is also necessary to highlight that “a form of life is a culture or social formation, the totality of communal activities into which language-games are embedded” (Glock, 1996, p. 125). Indeed, since meaning is given by use, meaning can change with every use we make of words. “What we do is to bring words back from their metaphysical to their everyday use (PI# 116), so we need friction. Back to the rough ground” (PI#107).

These ideas lead us to the notion of form of life as “the intertwining of culture, world-view and language” (Glock, 1996, p. 124). In this intertwining the meanings we give to words are mediated by *rules* that are conceived in our social practices. A set of such rules constitutes a *grammar*. Hanna (2010, p. 13) argues that, differing from his position in the *Tractatus*, in Wittgenstein’s later work, he considers that “logic is not essentially separate from the original phenomenon of meaningful language itself, and is essentially normative, that is, logic is fully embedded in the all-encompassing rational human constructive activity we call language.” Based on this argument, Hanna justifies why, in the Wittgensteinian perspective, this logic is called “grammar” (p. 13).

In fact, this notion of grammar is very fruitful in enhancing ethnomathematical thinking. It allows the analysis of modern rationality, because it “guides” the interactions between different language games (Condé, 2004, p. 170). Underlining the emphasis on learning to use the rules of a grammar, Condé writes that Wittgenstein means “grammar and language games as a rationality that is forged from the social practices in a form of life and that is no longer based on ultimate principles” (Condé, 2004, p. 29). Here comes a key point: when one abandons the idea of a single, natural, reason-producing structure, it is possible to understand rationality as an “invention,” a “construct” (Condé, 2004, p. 29). It is this “construction” that will enable language to articulate itself inside a form of life and establish which rationality will indicate to us what we should accept.

Based on this argument, we can admit that there is more than a single rationality, which means that different grammars—different logics—can coexist even inside the same form of life. So it is possible to admit that modern rationality—and the mathematics that gives support to it—may not be the only rationality of our epoch: other ways of reasoning can coexist in a same form of life. But here comes a question central to the ethnomathematical perspective discussed in this paper: How can one recognize that these other ways of

[6] In this paper, aphorisms from Wittgenstein’s book *Philosophical Investigations* will be expressed by PI# followed by the number addressed by the author to the aphorism.

reasoning can be identified with "other mathematics"? It is precisely the Wittgensteinian notion of "family resemblances" that helps us to answer the question. In fact, using it, we can argue that there are language games of non-Western school forms of life which are "mathematical" because we identify family resemblances between them and the language games we were schooled in the West. This is the criterion to be used to decide on the ways language games are "mathematical" or not. In summary, we have achieved a theoretical justification for why we can consider social practices of "the others" as mathematical practices.

Based on the above justification, I can identify as mathematical some practices of the forms of life of peasants belonging to the Brazilian landless movement. For example, as mentioned elsewhere (Knijnik, Wanderer, & Oliveira, 2005), in the Brazilian landless peasant forms of life that we studied, a peasant explained that, when estimating the total value of what he would spend to purchase inputs for production, he rounded figures "upwards," ignoring the cents, since he did not want "to be shamed and be short of money when time comes to pay." However, if the situation involved the sale of the product, the strategy used was precisely the opposite: the rounding was done "downwards," because "I did not want to fool myself and think that I would have more [money] than I really had." As we can observe, these language games have family resemblances to those of Western school forms of life, in which there is a fixed rule determining how to round up and round down numbers.

But it is important to highlight that when pointing out that two language games have family resemblances, this does not mean there is an identity between the games. It emphasizes that they have similar characteristics, as members of a family have. Moreover, we can identify interconnections between different language games belonging to the same form of life or to different ones.

In short, Wittgenstein in his later work, by denying the existence of a universal language, enables us to question the notion of a universal mathematical language. This allows us to argue, from the philosophical viewpoint, about the existence of different mathematical language games—such as Western school mathematics and the Brazilian landless peasant mathematics (Knijnik, 2007; Knijnik & Wanderer, 2010), which have family resemblances.

In fact, fieldwork done in the southernmost state of Brazil showed that the landless peasants of that form of life practice two different language games when they need to calculate the area of the surface to be cultivated—in their own Portuguese words, "cubar a terra." For example, if the land surface to be measured is a quadrilateral, the first language game consists in adding up its four sides. Then this sum is divided by four. So the quadrilateral is transformed into a square with sides corresponding to a quarter of the quadrilateral perimeter. The final step of this language game is to calculate the area of that square. The result of the whole process is used as the area of the quadrilateral land surface. It is obvious that when the land to be measured is a square the result of this language game coincides with the one practiced in the school form of life. However, when the land surface is an irregular quadrilateral, the peasants obtain a result that is greater than the result from the school mathematics language game.

The second language game of "cubar a terra" of a quadrilateral observed in the fieldwork consists of the following steps. First, the pairs of opposite sides of the quadrilateral are added together. Next, each of these sums is divided by two. It is easily seen that at this point, the quadrilateral is being transformed into a rectangle. The last step is to calculate the area of this rectangle using the same rule as the school mathematics language game.

This second language game presents peculiarities that must be highlighted. Its final result is equal to (when the land is a rectangle) or greater than the result achieved by school mathematics language games such as, for example, the one that uses the "Heron formula." It is also interesting to observe that one of the rules of this second peasants' language game is changed when the land has a triangular shape. As explained by one of the landless teachers: "If the land is like a triangle, they [the men] take the base and there [pointing out to the opposite triangle vertex] they put zero." It is clear that in this case the peasants identify the triangle with a quadrilateral one of whose sides has length zero. Then they apply the rules of the language game described above.[7]

Here it is worth mentioning the argument of Ernest (1991) about the uses of ethnomathematics in our school teaching practices. For him, "there is a conflict between the location of mathematics in the world of the student's experience, and the need to teach theoretical mathematics to provide the powerful thinking tools of abstract mathematics. [...] There is no way to avoid these conflicts" (p. 214). The pertinence of such an argument is highlighted when using Wittgenstein's later positions: the meanings assigned to the language games practiced in forms of life outside school cannot be automatically transferred to the school form of life. To move from one form of life to another does not guarantee the permanence of the meaning, which shows the complexity of this kind of operation (Knijnik & Duarte, 2010). Therefore, it is relevant to take Ernest's arguments into account if we do not want to trivialize an ethnomathematical perspective and take for granted what is, in fact, much more complex.

Let us now look at the ideas developed in *Philosophical Investigations* (Wittgenstein, 2004), briefly presented in this section, to ascribe new meanings to the mathematics of the English mariner Will Adams. In the European version of the story, Adams had been an apprentice at a shipyard and was autodidactic. Following Wittgenstein's positions we are therefore led to think that Adams had learned a mathematics constituted by language games whose rules were strongly involved in the life culture(s) of the shipyard carpenters of the time, marked by the rationality(ies) of that(those) form(s) of life, expressed through their own grammar. However, the story does not refer to any specificity that might correspond to this mathematics, whose use enabled the Shogun to satisfy his wish to see "*a ship being made in all respects as our manner is*," as one reads in the excerpt of Will Adams' letter quoted above. This silence may be read as indicating that the "precious knowledge" the Shogun wanted to appropriate was the mathematics of the "European tribe." Was it not this mathematics—whose language games worked with rules marked by rigor, formalism and abstraction—that granted superiority to its "indigenous people"? A mathematics that was marked by very different uses from those of the form(s) of life of carpenters working in English dockyards.

3 The ethnomathematical perspective and Foucault's contribution

This section focuses on the contribution of Foucault's thinking to the ethnomathematical perspective presented this paper. Nevertheless initially we must make explicit that such articulation has theoretical consistency as well as being useful in enhancing the discussions of the ethnomathematics field. In fact, Wittgenstein's later and Foucault's non-essentialist positions and, in particular, the convergent meaning ascribed by them to language and the theoretical closeness of Wittgenstein's notion of language games and Foucault's notion of discursive practices enable us to articulate elements of their thinking. The work of Jorgensen (2007) presents a fruitful example of the theoretical articulation of both philosophers.

[7] Examples of other mathematical language games different from those that constitute school mathematics are given elsewhere (Knijnik, 2007; Giongo & Knijnik, 2010).

I claim that it is relevant to introduce this articulation into the ethnomathematical perspective analyzed here. It allows me to highlight that not only can we admit the existence of "other" mathematics (an assertion based on Wittgenstein's later ideas), but that also we can examine how these diverse mathematics function in the social sphere, i.e., how power works to position them differently in the social world. It also allows us to study how people manage to overcome the effects of truth that position one of these mathematics—the Eurocentric mathematics—as the rule by which the others are measured and, in the end, hierarchized, i.e., devalued. These tasks can be achieved by questioning the Eurocentric discourses of academic and school mathematics and their effects of truth.

Foucault's specific contribution to the conception of the ethnomathematical perspective is based on the notions of *discourse*, *truth*, *power*, *regime of truth, resistance*, *counter-conduct*, and genealogy. The French philosopher considers discourse "as practices that systematically form the objects of which they speak" (Foucault, 2002, p. 49) and not as a "mere intersection of things and words: an obscure web of things, a manifest, visible, colored chain of words" (p. 48). Thus, discourse is seen not as a mere juxtaposition of signs that would express a direct and transparent connection between the meaning and the significant. It is seen in its positivity, in what it makes emerge as an event. In particular, the discourse of mathematics education is seen as connected to a "group of statements that belong to a single system of formation" (p. 107).

One of the key points of Foucault's theorizations refers to the relationship he establishes between power and truth:

> Truth isn't outside power… it is produced only by virtue of multiple forms of constraint. … Each society has its regime of truth, its 'general politics' of truth; that is, the types of discourse which it accepts and makes function as true, the mechanisms and instances which enable one to distinguish true and false statements, the means by which each is sanctioned… the status of those who are charged with saying what counts as true. (Foucault, 1994, p. 316)

Based on this non-metaphysical understanding of truth and its ties to power, the philosopher characterizes a "regime" of truth. In fact, he argues that truth "is linked in a circular relation with systems of power that produce and sustain it, and to effects of power which it induces and which extend it—a 'regime' of truth" (p. 317).

In summary, we can say that Foucault is mainly interested in the processes of "truthralization," i.e., in how, at a given time, the system of production, regulation, distribution, circulation, and operation of statements works. Here we find the usefulness of Foucauldian theorizations for the ethnomathematical perspective discussed in this paper. As previously mentioned, one of the purposes of such a theoretical toolbox is to allow the analysis of the Eurocentric discourses of academic and school mathematics and their effects of truth. Here Foucault's voice is clearly heard. The meanings ascribed to *discourse* and *effects of truth* are considered in the framework of his theorizations, and their meanings address to *power* and to *regime of truth* (as mentioned above in this paper). In summary, Foucault's thinking has been productive for our ethnomathematical perspective, in particular for the analysis of the statements that constitute the school mathematics discourses[8] which circulate and are taken as truths in the school mathematics education practiced in the south of Brazil.

Foucault's notion of genealogy is also relevant to the ethnomathematics perspective. As discussed elsewhere (Knijnik & Wanderer, 2010), the French philosopher uses Nietzsche's concept of effective history, which "deals with events in terms of their most unique

[8] As discussed in Giongo and Knijnik (2010), the language games that shape school mathematics discourse have strong family resemblances to those that constitute academic mathematics discourse.

characteristics, their most acute manifestations. […] The forces operating in history are not controlled by destiny or regulative mechanisms, but respond to haphazard conflicts" (Foucault, 1977, p. 154). Foucault considers that "it is necessary to master history so as to turn it to genealogical uses, that is, strictly anti-Platonic purposes. Only then will the historical sense free itself from the demands of a suprahistorical history" (p. 160).

Based on Foucault's notion of genealogy, we can discuss the uses—in the past and in the present—of units of land measurement in Brazil. In the mid-nineteenth century, in the northeast region of the country, there was an event to which mainstream historical studies have given little (or no) attention: the *Quebra-Quilos Revolt* (in Portuguese the *Revolta do Quebra-Quilos* or, literally, the "revolt of the kilogram-breaker"). At that time this Brazilian region faced a serious economic crisis, caused by the drop in cotton and sugar prices on the international market. In a context of economic crisis, social conflicts, and high taxation, in 1862, the government officially replaced the system of weights and measures in force in the country by the French metric system. These were the *conditions of possibility* (as conceived by Foucault) for the emergence of the Quebra-Quilos Revolt, which occurred mainly in the backlands of the states of Paraíba, Pernambuco, Alagoas, and Rio Grande do Norte. The Quebra-Quilos Revolt was a popular movement organized by the poor. It did not have the support of the dominant groups, and the participants did not have any social or economic prestige. Their actions consisted in occupying cities, especially on market days when there were more people around, and in inciting the population to break or damage the new weights and to destroy the documents of the municipal councils and the archives of the notary public. The government violently repressed this popular resistance movement. The prisoners it took were treated as prisoners of war, and the French metric system became the standard measuring system in the country and also internationally (Santos, 2005).

However in Brazil the imposition of the French metric system did not only lead to the Quebra-Quilos Revolt. Fieldwork done in the southernmost state of the country shows that "local" units—like the *colonia*, *leguas*, and *tamina*—were used after that imposition and are still used in peasant communities (Oliveira, 2011). The research was performed at a village notary public's office, and there, property documents were found in which lengths and areas were described with those "local" units. To these uses we can ascribe not Foucault's meaning of resistance, as in the Quebra-Quilos Revolt, but a movement of *counter-conduct*, as described by the philosopher in his course "Security, Territory and Population." There Foucault argues that in its beginning, the verb "to govern" was attached to "the art of conducting somebody" and also to its converse, "to conduct oneself differently from that conduction," which means to exercise "revolts of conduct," or counter-conduct. Binkley (2009, p. 76) explains Foucault's understanding of counter-conducts, saying that they:

> are distinguished from economic revolts against power […] by their emphasis on the government of the self as the stake of revolt, and the specific rejection, through inversion and reversal, of the precise ways in which one is told that one should govern oneself. Counter-conducts emerge from within the specific logics of a given mode of conduct, inverting the series that runs from the macro-level technologies of rule to the specific ethical practices by which individuals rule themselves.

Therefore it can be said that counter-conduct movements embrace changes of attitude towards issues of power and effects of truth, which implies that we can consider the Brazilian land measurement language games,[9] in which specific units (different from those

[9] Here I am referring to the three different land measurement language games practiced by landless peasants in the southernmost state of Brazil, which were discussed in Knijnik (2007) and in this paper.

of the metric system) were/are used (referred to above), as peasant counter-conduct practices.

Brazilian land measurement language games have family resemblances to those practiced by Western school mathematics. The work of Oliveira (2011) and Santos (2005) also highlights this topic. When compared with the mainstream language games taught in educational institutions, we can observe that their results are nonexact, which many have seen as errors. But here Foucault's words prevail: "perhaps there are no errors in the strict sense of the term, for error can only emerge and be identified within a well-defined process" (Foucault, 2010, p. 223). For instance, as shown in Knijnik (2006), it can be considered that practicing language games to measure the land, when examined in the contingency of the landless peasant form of life with which they are associated, does not present any error "in the strict sense."

Because they are useful for decision making in agricultural practices and, moreover, are easy to use, the peasants do not disqualify their regional, local knowledge—their subjugated knowledge, as Foucault would say—that, paraphrasing Rivera (2004) in her analysis of the process of subjection with regard to medicine, I would say "are opposed, in a game of parallelisms and marginalities, to the knowledge of the mathematicians of institutionalized mathematics."

Certainly the peasant language games are inexact, if compared to those of school mathematics. But, as Wittgenstein said (PI# 88), inexact does not mean unusable. In his own words:

> 'Inexact' is really a reproach, and 'exact' is praise. And that is to say that what is inexact attains its goal less perfectly than what is more exact. Thus the point here is what we call 'the goal.' Am I inexact when I do not give our distance from the sun to the nearest foot, or tell a joiner the width of a table to the nearest thousandth of an inch? No single ideal of exactness has been laid down.

4 Final words: Shogun, Will Adams, and school mathematics education

I now return to Foucault's epigraph. Besides the meanings ascribed to the Shogun's little story as presented so far, the epigraph can be analyzed from other angles. Let us hear the words with which Foucault introduces this "little story so beautiful I fear it may well be true": "It encompasses all the constraints of discourse, those limiting its powers, those controlling its chance appearances and those which select from among speaking subjects" (Foucault, 2010, p. 225). What do these words suggest? What ideas do they mobilize?

In order to answer these questions, it is initially necessary to place, albeit briefly, the text from which the epigraph was taken in the ensemble of the philosopher's works, and then to present the analysis performed by Foucault (2010) in *The Discourse of Language*,[10] specifically in the part of his argument directly connected to the words in the epigraph.

In *The Discourse of Language*—the paper that corresponds to the lecture delivered at the Collège de France on December 2, 1970—the philosopher again takes up the discussion begun in *The Archaeology of Knowledge* (Foucault, 2010). In this paper, however, Foucault operates a theoretical displacement, introducing a discussion about power. On "ask[ing]

[10] The editor of the book (Foucault, 2010) explains in a footnote that the original French text was named "*L'ordre du Discours.*" The English translation was by Rupert Sawyer and was first published in Social Science Information, April 1971, pp 7–30.

himself about the conditions of possibility of the discourse in its materiality as an enunciative event" (Diaz, 2005, p. 77), the philosopher takes into account the hypothesis that "in every society the production of discourse is at once controlled, selected, organized and redistributed according to a certain number of procedures, whose role is to avert its powers and its dangers, to cope with chance events, to evade its ponderous, awesome materiality" (Foucault, 2010, p. 216). The philosopher argues that such procedures can be characterized as external rules—"to some extent, active on the exterior […], they concern that part of discourse which deals with power and desire" (p. 220). But the procedures can also be characterized as internal rules—"where discourse exercises its own control rules concerned with the principles of classification, ordering and distribution" (p. 220) and those that "limit the exchange and communication of the discourses and that determine its social appropriation" (Castro, 2004, p. 231).

The internal rules of the control of discourse involve "the mastery of another dimension of discourse: that of events and chance" (Foucault, 2010, p. 220). The philosopher will include in this group three elements: the commentary, the author, and the disciplines. The commentary indicates the existence throughout society of a lag between primary and secondary texts, "between the texts that can be said, and texts that say what has already been said, [which] limits the discursive possibilities, imposing the primary texts as a limit" (Castro, 1995, p. 231). It "gives us the opportunity to say something other than the text itself which is uttered and, in some ways, finalized" (Foucault, 2010, p. 221). The author functions "as the unifying principle in a particular group of writings or statements, lying at the origins of their significance, as the seat of their coherence" (p. 221). It is a procedure that, throughout history and in different contexts, has taken on different functions. Finally, the third element: the disciplines—whose organization "is just as much opposed to the commentary principle as it is to that of the author" (p. 222).

Let us take a closer look at the last internal procedure of discourse control. What does Foucault say about the disciplines that could be useful for thinking in a renewed way about the discipline of school mathematics? For the philosopher, the discipline constitutes "a sort of anonymous system freely available to whoever wishes, or whoever is able to make use of them, without there being any question of their meaning or their validity being from whoever happened to invent them" (p. 222). Therefore the discipline opposes the principle of the author. It also opposes the principle of commentary, since, in a discipline, "what is supposed at the point of departure is not some meaning which must be rediscovered, nor an identity to be reiterated" (p. 223).

However, the production of propositions that have not yet been formulated needs to fulfill certain requirements. The philosopher explains this point: "A discipline is not the sum total of all the truths that may be uttered concerning something; it is not even the total of all that may be accepted, by virtue of some principle of coherence and systematization, concerning some given fact or proposition" (p. 223). If, for Foucault, "medicine does not consist of all that may be truly said about disease" (p. 223), we might think of extending this position to school mathematics and, paraphrasing the philosopher, state that school mathematics cannot be defined by the sum of all truths that concern the language games involving quantifications (such as, for instance, calculating the areas of surfaces).

Thus we would say that school mathematics does not gather all the language games that involve calculating areas. It expels "out of its margins" games such as those of peasant mathematics (Knijnik, 2007), with their specific rules which are different from the formal and abstract rules that shape the grammar of (academic) mathematics. In *The Discourse of Language*, Foucault (2010) examines yet another ensemble of procedures "that limit the exchange and communication of discourses and that determine their social appropriation"

(Castro, 2004, p. 94). This includes, for instance, the ritual, which "defines the qualifications required of the speaker (…); it lays down gestures to be made, circumstances and the whole range of signs that must accompany discourse" (Foucault, 2010, p. 225). It also includes the educational system—"a political means of maintaining or of modifying the appropriation of discourse, with the knowledge and the powers it carries with it" (p. 227).

It is precisely when the philosopher discusses this third group of rules serving to control discourse that the philosopher narrates the little story with which this paper began. Now we have a better understanding of the rarefaction rule formulated by Foucault (p. 224): "It is more a question of determining the conditions under which it may be employed, of imposing a certain number of rules upon those individuals who employ it, thus denying access to everyone else." We have a better understanding of the reasons that led him to say that all coercions of discourse are expressed "in a single figure" that we can identify as being the Shogun. The Shogun's powers were limited due to his lack of mathematical knowledge, "the precious knowledge" of the "European tribe" that made that tribe superior to the rest. The perpetuation of his position as emperor meant holding the European Will Adams, who possessed this "precious knowledge," so that the emperor, and only the emperor, would have access to the secrets of the "marvelous discourse" of mathematics. Foucault, ironically, questions the idea that this narrative could be read as indicating that "to the monopolistic, secret knowledge of oriental tyranny, Europe opposed the universal communication of knowledge and the infinitely free exchange of discourse" (p. 225).

Indeed, as the history of Western science—and in particular the history of Western mathematics—shows, the communication and exchange of knowledge, over time, have functioned through subjection procedures such as those listed by the philosopher. One of the most exhaustively mentioned examples in the literature is that of the Pythagorean school (Chassot, 2011, p. 140) whose mode of functioning can be identified with a "fellowship of discourse" in the meaning ascribed by Foucault to this expression. There, discourse was preserved or reproduced, "but in order that it should circulate within a closed community, according to strict regulations, without those in possession being dispossessed by this very distribution" (Foucault, 2010, p. 225). As the philosopher clearly highlights, fellowships of discourse such as those that existed in the past cannot be found today.

But it must be acknowledged nevertheless that, despite the configuration of the current academic world (in which the use of new technologies has made it easier to circulate what is produced), "we still find secret-appropriation and non-interchangeability at work" (Foucault, 2010, p. 226). Bourdieu (2003), in a different theoretical framework from Foucault, also argued that the struggles for the symbolic capital that characterize the scientific field—which is particularly appropriate for the field of mathematics—as well as the interests that are at stake in the production of science, with, among others, its economic and political impositions (and implications), show the "nonpurity" of the scientific field and constraints of all kinds, which ultimately constitute "secrets" that operate coercively in the circulation and dissemination of science.

What about the field of mathematics education? The processes of teaching and learning mathematics at different places and in different cultures show how the communication of mathematics knowledge operates on the "school floor": we clearly identified coercive procedures that constrain the circulation of the discourses there. Maybe we could think that in the field of mathematics education, there is something similar to "a doctrinal group," as conceived by Foucault. Now the movement would be in the opposite direction to that of the society of the discourse, because what would move mathematics education teachers and researchers would be the broadest possible inclusion of all in "their secrets"—which, for this very reason, would no longer be considered as such. Above all teachers and researchers want to disseminate their discourses, and to impose "their" truths, on the greatest possible number of "faithful."

The above discussion helps understand Foucault's question in greater depth.

> What is an educational system, after all, if not a ritualization of the word; if not a qualification of some fixing roles of speakers; if not the constitution of a (diffuse) doctrinal group; if not a distribution and an appropriation of discourse, with all its learning and its powers? (p. 227)

Was it not this learning and these powers that will ultimately make "other" mathematics than the mathematics that we know as "the" mathematics be positioned "in a void" (p. 224)? Was it not this learning and these powers that ultimately brought together the autodidactic English sailor Will Adams, "who possessed the secret of these 'marvelous discourses'" and the Shogun, so that "in private" he learned mathematics and "retained power"? Is it not learning and power that move school mathematics education?

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