Educ Stud Math (2012) 80:73–85
DOI 10.1007/s10649-011-9366-6

Hating school, loving mathematics: On the ideological function of critique and reform in mathematics education

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Published online: 17 December 2011

Abstract Students' engagement with fictions in the form of "word problems" plays an important role in classroom practice as well as in theories of mathematical learning. Drawing on the Dutch historian Johan Huizinga and the Austrian philosopher Robert Pfaller, I show that this activity can be seen as a form of *play* or *game*, where it is pretended that mathematics is useful in real life in a way that it is not. With Pfaller, I argue that play can take hold of the imagination of the players, infusing everyday life with meaning borrowed from the imagery of the play and that these effects are more powerful when the play is forced and takes an institutionalized form. I show that mathematics education does in fact have these characteristics, including sophisticated mechanisms for translating in-game performance (test scores) to real-life goods (grades and examinations). A central theme of the article is the perceived discrepancy between mathematics education as it is, and how it supposedly could and should be in light of the properties of mathematics. The analysis implies that this gap actually is an effect of play and thus an inherent property of mathematics education itself.

Keywords Play · Ideology · Reform · Althusser · Pfaller · Word problems

> To think mathematically affords a powerful means to understand and control one's social and physical reality. Yet despite some 12 or so years of compulsory mathematical education, most children in the developed world leave school with only a limited access to mathematical ideas. (Noss & Hoyles, 1996, p. 1, cf. pp. 253–257)

> Mathematics, then, has degenerated from the most intellectually educative subject of all into a mechanical cramming of rules actually wrecking to the intellect. (Petrini, 1905, p. 203)

> This disinclination of our youths towards Geometry cannot possibly originate in anything but an improper treatment of the subject. Else, it would be exceedingly

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singular for a science, dealing especially with the innate ideas of the human mind and teaching lucidity and reason in the observation of external objects, to be alien and unpleasant to man. (Bergius, 1868, p. 88)

1 The standard critique

The quotations above are examples of what I refer to in this article as the *standard critique* of mathematics education. The first characteristic feature of such critique is that it takes for granted what mathematical knowledge is, how it is formed, and how it can benefit its bearers. It often includes the idea that mathematical knowledge must be actively discovered or constructed by the learner herself, through work on meaningful, realistic, problems. The resulting knowledge is typically seen as beneficial in a very general way, related for example to intelligence, high morals, self confidence, and democracy.

The second characteristic of the standard critique is that these ideas of the proper method and goal of mathematics education are used as background for a contrastive description of mathematics education as it is actually practiced. It is typically described as narrow-mindedly mechanical, useless and boring, based on passive memorization of facts and algorithms. The contrast between the potentially beneficent mathematics and the deficient mathematics education functions rhetorically as a call for reform.

For instance, in the first quotation above, it is stated on the one hand that “to think mathematically” is a “powerful means to understand and control one’s social and physical reality”. Then, on the other hand, that most children do not get access to these means. Mathematics *could* give “most children in the developed world” means to control their “social and physical reality”, but in fact it does not, because mathematics education is not doing what it should. Similar stories are told in the second and third quotations.

This form of critique of mathematics education has been expressed in Europe at least since the end of the eighteenth century (Lundin, 2008, pp. 31–32). A recurring reference is Pestalozzi (1977) who, throughout the nineteenth and twentieth centuries, was claimed to have achieved the original break with older teaching methods focusing on rules and memorization (see, e.g., Bucht & Svensk, 1894, p. 2 and Setterberg, 1913, p. 3). In the nineteenthh century Germany, expressions of the standard critique can be found in countless books on the methodology of mathematics education, for instance Niemeyer (1805, p. x and pp. 180–183), Diesterweg (1838, Bd 2, pp. 147ff), and Grube (1865, “vorwort”), just to name three relatively influential authors (for more examples, see Hartmann, 1904, pp. 38–75).

The standard critique is also typical for the reform movement of the early twentieth century and fits much of the rhetoric of the new mathematics movement of the 1960s (e.g., Dienes, 1959, pp. 1–2 and Håstad, 1966). The general ideas of the *Realistic Mathematic Education* program of Hans Freudenthal (1973, 1978), as well as Guy Brousseau’s theory of “didactical situations” (1997) are, despite their differences in focus both very much in line with it, focusing on problem solving, student activity, and meaningfulness, in more or less explicit contrast with a past of mathematics education circling around rote learning of facts and algorithms. In my thesis, I present several examples of the standard critique from the current Swedish context, including research in mathematics education (Lundin, 2008, pp. 29–34).

A perplexing phenomenon is that even though the standard critique calls for reform and change, it seems rather to sustain the *status quo*. There are connections here to what Foucault (1991) writes about the prison system: “[…] the critique of the prison and its methods appeared very early on […] indeed, it was embodied in a number of formulations which—figures apart—are today repeated almost unchanged” (p. 267). There are also parallels to the history of psychiatry (Åman, 1976, p. 419). The history of mathematics education seems thus to be a part of a more general theme of "imaginary breaks" with the past (cf. Braun, 1979, p. 1; Brousseau, 2004, p. 4; Illich, 1972, pp. 4–5; Oelkers, 1996). The purpose of this article is to provide an interpretation of this state of affairs.

In this article, "mathematics education" should be understood in two different senses. On the one hand, it refers to the highly standardized, often compulsory teaching of children and youth, with the purpose of providing knowledge that is in some sense generally beneficial. This puts the learning of specific mathematical techniques as part of a vocational training program as well as studies preparing for work as an engineer or as a professional mathematician outside the scope of this article. On the other hand "mathematics education" refers to our *conception* of this social institution. The idea is that mathematics education in the first sense, as something *material*, as a set of standardized and stable practices, generates and sustains the conception we have of it.

Furthermore, mathematics education does this by reference to a corresponding conception of mathematics. Mathematics education is, I will argue, perceived "through the lens" of mathematics, but not only this. The very lens itself is as much a product of the materiality of mathematics education as the resulting image.

This means that, in this article, mathematics is not taken for granted, neither as an axiomatic system, as a language, nor as platonic ideas. It is rather seen as a word, used on the one hand to bring together and make sense of certain "mathematical" phenomena and activities, and on the other as the name of a special kind of knowledge ("mathematical knowledge") which purportedly makes it possible to understand and master these mathematical aspects of reality. This article can then be seen as an analysis of the mechanisms through which this mathematical world view is established as common sense and how it functions as an ideology.

2 The fictions of mathematics education

A "word problem" in mathematics education is a written problem which can supposedly be solved with mathematics (e.g., Verschaffel, Greer, & de Corte, 2000). Here I will focus on word problems, which in one way or another refer to mathematical contents, which are about something other than mathematics itself (cf. Dowling, 1998). While such word problems have a long history, it was not until the second half of the nineteenth century that they became paradigmatic for the teaching of elementary mathematics (Lundin, 2008). Today, they can be found in virtually any mathematics textbook or exam. Here is an example from the 2003 Program for International Student Assessment (PISA) study:

> A carpenter has 32 m of wood and wants to build an edging around a flowerbed. He considers the following designs of the edging […] Draw a circle around "Yes" or "No" for each design, to show if the edging can be built with 32 m of wood or not. (Skolverket, 2003, p. 11)

Four different designs are shown in the problem, one rectangular (6×10 m), one rhomboid-shaped (also 6×10 m) and two with other shapes, both of them symmetric and only consisting of right edges, giving them the same 32 m circumference as the rectangle. According to the assessment manual, the right answer is that all but the rhomboid-shaped edging can be built with 32 m of wood.

This problem exemplifies something that I contend is immensely common in mathematics education, namely the attempt to make problem-solving activities meaningful by relating them to some specific situation outside school, but not quite succeeding in doing so, or not even wanting to state the problem in such a way that it corresponds to how one actually would go about in the situation at hand (Lave, 1992, p. 78). Thorulf Palm discusses this phenomenon in his dissertation *The Realism of Mathematical School Tasks* (2002). In a summary review of the literature, he lists five arguments supporting the use of this kind of word problems. Three of these arguments are that they promote, respectively, *learning* of mathematics, learning of *how to use* mathematics and finally, an *understanding of the usefulness* of mathematics (pp. 8–9; cf. Verschaffel et al., 2000, pp. xi–xii).

Many critics have observed that the kind of word problems found in textbooks most often do not have the positive learning effect that is hoped for (a list of references is provided in Verschaffel et al., 2000, p. xv). This critique can take two different forms. On the one hand, there is critique that challenges the very idea of this kind of learning process, taking aim at all or some of the arguments listed by Palm (2002). Notable examples of such critique are contained in Wilson (1951) and Dowling (1998, 2010). On the other hand, there is a much more common critique that targets the *implementation* of the word problems, discussing issues such as authenticity, realism, and fidelity (e.g., the contributions to Verschaffel, Greer, & Van Dooren, 2009). Palm belongs to this group. He proposes a framework for creating tasks that are high fidelity simulations of reality which, he argues, would be better suited for promoting learning than the tasks more often found in textbooks and examinations today (2002, 2009).

I contend that there is an *inherent* difficulty with this idea of learning through meaningful and realistic problem solving. Mathematical knowledge is taken to be a useful tool for action and understanding in the world "out there", outside the school. On the other hand, engagement with reality *as it is*, is not believed to lead to the formation of mathematical knowledge. According to the idea of learning through problem solving, a setting must be designed which makes the mathematics that you use in the "real world" *present* for the student to engage with. Thus, an important aspect of learning mathematics is to become *aware* of the fact that mathematics is indeed a useful tool for action and understanding. Only then will mathematical knowledge be useful because only then is it possible to see how it can be used.

This is clearly articulated by Skovsmose (1994) who notes the necessity of making the mathematics which is "objectively there" visible for the students, who otherwise will not see it (p. 94). Similarly, Niss (2001) claims that there is a paradox of relevance for mathematics education—that only those who know mathematics understand its importance. I contend that this idea—of a simultaneous formation of *competence* to understand and master the world and the formation of a *perspective* which makes the world appear in such a way as to make this competence relevant—is peculiar to and characteristic of mathematics education.

What is being sought for is engagement with *compelling fictions* of use of mathematics. While working on word problems, students should in some sense believe themselves to be using mathematics to solve a real problem, even though it is at the same time apparent that there is no activity outside school to which this problem solving actually corresponds. This fictional use is supposed to amount to a transformation, involving both perspective and skill.

The educational process should, according to this idea, ideally consist of a sequence, F_1, F_2, … of increasingly demanding fictions (in the sense of being increasingly difficult to comprehend and master). In so far as these fictions are mathematically meaningful, realistic, and compelling, they are assumed to prepare the student to understand and master a field of applications outside school of which she may have no experience whatsoever. To put this philosophically, the doctrine of learning through realistic problem solving assumes that reality can only be approached as a kind of limit, in the mathematical sense, to a certain sequence of fictions. The world "as it is" falls outside the space of the educationally effective. You do not learn by "being there". On the other hand, the settings which *are* educationally effective, the increasingly more demanding word problems, the settings in which you only *imagine* the use of mathematics to be "real use", cannot be "out there", where the actual use of mathematics takes place. Although any "real" setting can be approximated to any degree of precision, it is only by *not* being "real" that it becomes educationally effective.

As the anthropologist Jean Lave puts it, "a word problem is one way to express beliefs about how everyday experience and mathematics should be related in order for math learning to take place effectively" (1992, p. 75). Lave state that word problems:

> reflect a theory of learning, an epistemological, cognitive and social account deeply embedded in Western thought of how knowing and understanding change and grow. […] It involves the belief that to know something requires that the learner be separated, or distanced, from the situated experience to be known; that the learner must abstract features of the experience, generalize about them and then transport them into a variety of novel situations in which they can be recognized to apply. (p. 76)

The number of word problems which students generally encounter in school is an impressive testimony to the currency of this theory of learning. To reiterate, the problem I see is *not* that mathematics education is "not realistic enough". The point is that the very *ideal* of mathematics education is a "realism" which should be understood as constituted in contrast to the "reality" which it emulates. Mathematics education does not *want* reality. If one just walks around in the world, one does not learn mathematics. Mathematics education wants a *realism* that in a very special regard is *different* from reality. This difference is assumed to be essential for the learning of mathematics to take place. Mathematics education is founded on a belief in the productive potential inherent in such a difference. It is founded on a belief in the possibility of designing settings in school which differ from reality in such a way that the engagement with *them* is a better preparation for understanding and mastering reality than engagement with reality itself.

3 The holy seriousness of play

The previous section suggests that the practice of mathematics education can be seen as an engagement *en masse* in pretending that work on "word problems" in some way corresponds to real use of mathematics. I argued that this is not only what students often do, but that dominating theories of learning actually *demand* that students participate in this pretending, as it is taken to be a precondition for effective learning of mathematics.

In this section, I will relate this activity to the concept of *the holy seriousness of play*, coined by the Dutch historian Johan Huizinga (1998). Drawing on the Austrian philosopher Robert Pfaller (2002), I will show how this play opens up at least three different possibilities of identification—of what in fact is going on, and at the same time of who we ourselves are, participating in this peculiar activity.

3.1 Immersion

The first possibility is to just go on and play the game. Students working in the mathematics classroom know very well that different rules apply there than in real life outside school: "They too know what a word problem is" (Lave, 1992, p. 77). At the same time though, students and teachers alike are obliged (according to the idea of learning through simulation) to act *as if* working on the tasks amounted to solving real problems. They are obliged to take them seriously and be interested in their solution. The activity is "seen through", as being clearly differentiated from everyday life. Nevertheless, the imaginary of the game infuses the activity with meaning. It is, in a sense, "believed in practice".

Pfaller talks about this particular kind of belief as "an illusion held in suspension through better knowledge" (2002, p. 44). The point is that we *keep believing* in the illusion because we intellectually "know" that we do not believe in it and thus think that we are not affected by it. The belief does not worry us, like we might worry that we are mistaken in some important matters, because in this case we know very well (or so we think) what we are doing.

Referring to Huizinga (1998), Pfaller explains that the very fact that the illusion is seen through does not diminish its grip of those engaging with it. On the contrary, as its seen-through character makes the contents of the belief in some sense invisible for its holders, this opens the way for a total immersion in it. Following Huizinga, Pfaller calls this total immersion *the holy seriousness of play.*

When a car does not start, it is not uncommon to say "Well, come on now!" Something similar can occur when one accidentally hits oneself with a hammer; one may shout at it. Since this may happen regardless of there being anyone else around, we obviously *act*, at these moments, as if things could listen to what we say. Not only that: Why would we talk to them if we did not also attribute to them the capacity to obey (in the case of the car) or react with guilt and shame (in the case of the hammer)? As we know very well that this is not the case, this is exactly a seen through illusion, held in suspension through a better knowledge, which on the surface would seem to dispel it. To get to the point of this example, we see through these illusions so well *that we do not even notice their existence.*

Nonetheless, we may feel an extreme urge to act in accordance with them, that is, to talk to the car and shout at the hammer as we hit ourselves on the thumb. This is, I claim, part of the story about the word problems of mathematics education. Taken together, they constitute a world—the world in which they make sense. We do not notice how thoroughly different this world is from the world of our practical life outside school because we know so well that it is not real. We "see through" it as we focus on what is more important: To get the problems solved, that is, to play the game (cf. Lave, 1992, p. 82).

Let me reconnect here with the problem from the 2003 PISA study that I presented above. The "see-through" part of this problem is that we immediately recognize that it is not really about carpentry. We "know" that it is not "real". On the other hand though, if we are in the noncritical immersion mode of belief which is the topic in this section, we *accept it* as a meaningful mathematical task, the purpose of which is the learning of mathematics. On the one hand, we do note the nonrealism of the task, but on the other, we do not consider the task to be nonsensical. We do not reject it, but go on, trying to solve it (or, if we are teachers, assessing the students' solutions). The point is that there is something here that we, to try to put it succinctly, *do not notice that we do not notice*, and this is the irrelevance of this task and the mathematics involved, for the actual practice of carpentry. The acceptance of this problem involves belief of the same kind as in talking to a car or a hammer.

An alternative would be to consider both of these actions—the fictional flowerbed constructions and the talking to dead objects—as thoroughly nonsensical, and I want to stress the reasonableness of this alternative. But in fact we do not recognize these situations as nonsensical, and this is because we *in a quite special sense* believe in the fictions, respectively, that the word problem in the PISA 2003 study has something to do with carpentry, and that cars and hammers understand what we say (or shout).

The concept of *the holy seriousness of play* provides a key to understanding the philosopher and historian Ivan Illich's observation that "Classroom attendance removes children from the everyday world of Western culture and plunges them into an environment far more primitive, magical and deadly serious" (1972, p. 32). The environment he talks about is contingent upon the perception of mathematics education as a sort of game and also on the fact that we are not quite aware of the fictitious character of the world in which we are immersed. We enjoy solving mathematical problems in the same way as one can enjoy scoring a goal in football, talking with a car or shouting at a hammer. It is an action *as if*; on the one hand not serious at all, on the other hand more serious than life itself.

3.2 Cynical distance

A second possibility is to actively distance ourselves from the activity in which we participate. If the first way of relating to the fictions of mathematics education is like looking at a fascinating object through a perfectly clear window, what I am referring to here corresponds to suddenly noticing the window, resulting in a total change in the focus of attention, from the object being watched to the setting in which the watching takes place. The difference I am concerned with has to do with articulation and consciousness. When talking to a hammer, we never notice that we act in accordance with a fiction. As soon as we have reached our goal, the release of tension resulting from our imaginary induction of shame, we forget about the whole thing (cf. Pfaller, 2002, p. 34). The ever-present complaint that mathematics education is unrealistic and irrelevant testifies to a quite different process. Those complaining (mostly students) are obviously conscious of the fact that the tasks are fictitious.

This way of *consciously seeing through* has been discussed at length by the German philosopher Peter Sloterdijk in terms of *a cynical reason* (1987, see also Žižek, 1989, pp. 29–33). It is connected to a feeling of powerlessness in the face of reality. The world, seen from this perspective, cannot be changed by a single individual. It is a "system", "society", or in this case "school", created and managed by idiots, in which we have been put and to which we have to adapt.

The cynical position does not pose any threat to the order which it sees through. It is a position of acceptance. An important point is that these two ways of taking part in the practices of mathematics education should not be seen as clearly separated. Rather, they coexist, as we sometimes just go about the series of tasks, "enjoying the game", while at other times we sense its pointlessness and instead enjoy, perhaps, that we at least understand that it is "just a stupid game" that we are obliged to play.

An important consequence of the concept of play is that the "idiots" who "believe in the system" and have supposedly created it accordingly, do not in fact exist. The concept entails that everybody sees through. Everybody "understands", everybody has access to the cynical stance. This means that even though some people just play along according to the rules most of the time, they do not "believe" in it, in the sense that they would defend the actual practices of mathematics education as proper realizations of the higher goal of promoting mathematical knowledge. It is much more likely that if they got an explicit question, they would distance themselves from any such claim.

The belief in accordance to which people nonetheless act, thus in a sense only exists in an embodied form interwoven with action. We believe, so to speak, with action, or in action. The Slovenian philosopher Slavoj Žižek talks about this phenomenon in terms of a "collective unconscious fantasy" guiding our actions (Žižek, 1989, p. 33). What this means is that there is a fundamental common understanding, among those participating in the practices of mathematics education, regarding what these practices mean, what they are "about" and their ultimate purpose. Even though there may be disagreement on whether or not any particular "word problem" is better or worse, no one would doubt that they are indeed "mathematical problems" and that work on the *right kind* of such problems is a proper way to "mathematical knowledge". While cynical distance entails a questioning of the actual practice of mathematics education, it does not entail any doubt in the mathematical world view from which mathematics education derives its purpose and meaning. The cynic takes mathematics for granted.

3.3 Faith

The third and final possibility of identification that I will discuss here is to take distance from the activity as it is but to have faith in its ultimate necessity. While the cynic does not doubt the importance of mathematics, it does not endorse it either. Characteristic for the faithful is a call for reform (Pfaller, 2002). It is thus the stance of the standard critique.

A substantial amount of evidence has been created, through research in mathematics education, testifying to the general inefficiency of the activity of solving word problems as a means of helping individuals master their everyday and professional life (e.g., the research referred to in Verschaffel et al., 2000, p. xv). This evidence could, most definitely, be taken to imply that this kind of mathematics education simply does not work. It is a certain kind of faith that makes this evidence appear, instead, as a call for increased efforts to *make it work*.

Drawing on Pfaller, I contend that what has happened here is that the imagery of the word problems has been transformed from *fiction* into *essence*. In the example with the car above, this transformation would correspond to the belief that cars *could* really understand what we say to them, were we only articulate enough, talking their language and so forth.

When we are immersed in play, we just "do it". We talk to the car, without bothering about the well-known fact that it does not really make much sense. When we take a cynical distance, we notice that we are forced to do nonsensical things—in the case of mathematics education, that we have to work on problems that are not realistic. The faithful finds a *reason why* the game is played, seemingly in reality itself, and at the same time identifies a corresponding explanation why it "does not work". In fact, however, this reason is a hypostatization of something suggested by the game itself. Car talk suggests that cars have a capacity to understand. The form of engagement with word problems, the centrality of this practice in mathematics education, not to mention the place of mathematics education in society at large, together quite strongly suggests that this practice *could* lead to generally beneficial knowledge.

An important but difficult point in Pfaller is that this kind of faith entails a mix-up between success and failure, joy and suffering (Pfaller, 2002, p. 227). Simplifying somewhat, the line of thinking is as follows: the worse mathematics education appears to be, the greater the suffering everybody has to endure (possibly including oneself!), the greater the gap between how mathematics education is and how it could be, the more majestic the task of reform, the greater the enticement of being its subject—the subject of reform. Faith entails the painful joy of being one who knows, who sees the sorry state of mathematics education in the light of all that it could be, and dutifully shoulders the burden of reform. One should note, with Pfaller, the thoroughly unhegelian nature of this analysis: instead of leading over to a new configuration, the force of contradiction here *sustains* status quo (Ibid., p. 126).

If this explains the enthusiasm with which research embraces the discourse of failure, another equally important question remains: How is the hypothesis of a working mathematics education established as plausible, in the face of so much evidence talking against it? In the next section, I will suggest an answer to this question by attending to the institutional structure of mathematics education, focusing on the fact that we are all obliged to participate in it and that in-game performance (test scores) is constantly and systematically translated into real-life goods (grades and examinations).

4 The ethics and morality of mathematics education

Ethics, for Spinoza, is a matter of good and bad action, the difference between which is exclusively constituted through *consequences* (Deleuze, 1988). Good actions have beneficial consequences, and vice versa. The consequences just follow, mechanically, from action. What is good and bad does not need attribution. It is both unnecessary and pointless to *claim*, inside such a system, that something is good or bad, since such statements would either (if correct) state the obvious, or (if wrong) be completely ineffectual. Being able to do good according to this kind of ethics is not a matter of values but of knowing the mechanics of the system.

Mathematics education quite clearly constitutes such a system through its measurements of mathematical knowledge. It establishes *mathematical knowledge* as good, by making such knowledge have beneficial consequences. Firstly, it should be obvious that these consequences are thoroughly *made*. Measurements, grades, and examinations have consequences only inside the system in which they play a central role. Secondly, it should be as obvious that opinions, thoughts and feelings towards this system do not affect its proper functioning. The system is in no sense "inner".

Despite its obvious constructedness, the education system of which mathematics education is a part is not perceived as arbitrary. Despite the fact that we "see through" it and know that it is not what it pretends to be, we know very well (so we think) *why it is there*, and why it must be there. We understand its purpose. I have highlighted this in terms of a collective unconscious fantasy. We have this fantasy, basically, that this system is not really "made" at all, or, if it is made, it is made as a kind of mirror of reality, so that the ethics that the system constitutes is actually, on the most fundamental level, a *natural* ethics. It may be, for instance, that the "wrong kind" of mathematical knowledge is measured, and that it is measured in the "wrong way". However, what is not doubted is that these measurements *should* and *could* measure *something*, which, by itself would manifest its importance, had the "made" system not been there.

One should note that what is valued in the fictional world of mathematics education is the capacity to *solve problems*. This capacity, so the story of this ethics goes, is at the same time the capacity to further the common good of society, contributing to science and technology, as well as to economic growth (cf. Boltanski & Thévenot, 2006). It is an ethics quite in accordance with the ideology of modern capitalism (as is, one could add, the ethics of Spinoza). On another level though, mathematics education is quite at odds with many cherished values of modernity, for instance the value of free choice.

Mathematical knowledge is, in a very fundamental way, not a matter of choice, neither regarding *what*, *how much*, *when*, nor *how* one is to learn. The assumption here is that, contrary to what is the case with for instance food and fashion, individuals are not competent to answer these questions in regard to mathematics. There needs to be a single curriculum, as well as a system of measures to control its implementation. In modern society, mathematics education is seen as such a serious matter (cf. the holy seriousness of play) that it cannot be left to the whims of youth and parents. Mathematics education is thus on a practical level safeguarded by administrative rules and regulation. The cynic's feeling of not having any choice but to "do it", despite understanding its pointlessness, is quite sensible since the obligation to participate in mathematics education is backed up by law.

While we are allowed to be critical towards mathematics education and even to publicly acknowledge the deficiency of our own mathematical understanding, we are, on the other hand, obliged to have faith in the *necessity* of mathematics education and in the *importance* of that which we do not understand. We are obliged to have faith in mathematics.

The importance for mathematics education of not only students and teachers, but also the general public, having the right idea of what mathematics is, can be explicitly read out of much research in mathematics education as well as in curricula. As Palm (2002) noted, a common argument for realism is exactly that it promotes a proper "understanding" of the "usefulness" of mathematics (a line of argumentation which can be traced back at least to the early nineteenth century, see Lundin, 2008, p. 219). The main purpose of the *Mathematics Delegation*, commissioned by the Swedish government in 2003, was to "enhance the status of mathematics" and to create "greater awareness of the value and practical significance of mathematics in the entire society" (Sverige. Matematikdelegationen, 2004, pp. 23, 28).

Faith in mathematics makes the ethical system of mathematics education seem *necessary*, regardless of how misdirected and destructive this system happens to be at the moment. In light of this faith, there is *no other way* than the way through mathematics. This is a parallel to the starting point of the book by Pfaller that I have drawn on in this paper. He asks how it is possible that the peoples of Western capitalistic societies not only accept, but actually welcome political measures that obviously (Pfaller contends) are against their own interests (2002, p. 9). The proposed answer is precisely that these measures are seen as *the only way*, and that people actually *take pride* in showing that they understand this. Inasmuch as this seems true of the ideological dynamics of the market, it certainly seems true regarding mathematics education. When people confess that they know very little of mathematics, they simultaneously take great pride in stating that they are nonetheless highly aware of its great importance. Hence, they support the *duty* of taking part in mathematics education (reformed, of course).

I will conclude this paper by relating this ethical machinery infused with moral obligation to the concept of ideology. An important starting point for the kind of thinking that I pursue here is the French philosopher Louis Althusser and his concepts of *interpellation* and *ideological state apparatuses* (2001, pp. 85–126). This paper can be seen as an attempt to work out what his theory can amount to in the particular—but crucially important—case of mathematics education. Althusser talks about ideology in terms of "materiality", and means that ideology is "materialized" in its institutionalized practices. He writes that:

> all ideology represents in its necessarily imaginary distortion not the existing relations of production (and the other relations that derive from them), but above all the (imaginary) relationship of individuals to the relations of production and the relations that derive from them. What is represented in ideology is therefore not the system of the real relations which govern the existence of individuals, but the imaginary relation of those individuals to the real relations in which they live. (Althusser, 2001, p. 164)

Bringing this line of thought to mathematics education, what it claims is that the "system"—how mathematics education *constitutes* an ethical system in which mathematical knowledge is "good"—is a materialization of a certain "imaginary relationship". In our case, this relationship is the idea that the relationship between man, nature and society is mediated by "mathematical knowledge", the image that "production" (among other things) involves "use" of mathematics. With Althusser, mathematics education can be seen as a "materialization" of this image, or fantasy, or myth, of the relationship between mathematics, man and nature. We live, so to speak, "in" it. It has been made material. The ethics it embodies is an ethics in the sense of Spinoza—it functions as a machine. Thus, one does actually meet mathematics every day and we do need mathematical knowledge to solve the problems of this everyday life, namely insofar as this everyday life takes place in the realms of mathematics education. Inside these realms, mathematical knowledge is indeed a precondition for reaching influential positions in society and it may very well be a precondition for self confidence. To understand the myth of mathematics education, it is of crucial importance to note that it is, in this sense, not only an "idea" existing in people's heads, but more importantly, and exactly according to Althusser's theory, a material reality.

Moving from Althusser to Žižek, we can get a better understanding of what it means that this materiality of mathematics education is perceived as derived from reality itself (as is characteristic for the faithful stance). I showed above that theories of learning through realistic problem solving try to identify a certain kind of *productive difference*, between real-life and the educational setting, which will promote mathematical learning. The difference sought for is intimately related to mathematics, the object which is to affect the learner. This object—which is supposed to be "ontologically" present in reality, but which needs the help of education to become, so to speak, an epistemological reality for the pupil —is something which is in reality "more than reality itself" in the sense of Žižek (e.g., 2009, p. 17). It emerges only as the result of a certain perspective; a certain way of seeing which I think can quite accurately be described as the "modern" perspective on reality. Modernity *sees* mathematics in the world, and mathematics is hence, following the terminology of Žižek, its *object petit a* (Lundin, 2010). The place of mathematics education in modern society testifies to the power this object has over the imagination of modernity (cf. Castoriadis, 1997, pp. 364–373). And the argument of this article is that this power results from the workings of mathematics education itself.

The proponents of the standard critique are thus in a sense more right than they could ever imagine when they argue that mathematics education is a central and necessary part of modern society. While mathematics may not be very useful as a means to understand and control the social and physical reality, the argument of this article shows that the very *attempt to make it useful* contributes in a fundamental way to the very constitution of the peculiarly modern reality in which we imagine such use to take place.

Mathematics education can be seen as a game of pretending to use mathematics in an imaginary world. The fact that it has been played by so many, so seriously, for such a long time, and is intimately connected to mechanisms that constantly and systematically transform in-game performance to real life goods, generates great confusion. It is as if mathematics education was a badly performed play. We immediately see that the actors are amateurs, that the script is bad and the directing poor. But we also identify, in this failed performance, a brilliant idea which would no less than change the world, were it only made real by a proper crew. We are caught in a vain struggle to bring about this perfect delivery which will teach men and women to "think mathematically" and thus finally give them means to understand and control the social and physical reality. It is a captivating vision for sure. The conclusion to draw from my analysis is that it can never be anything but a vision. This may seem disheartening, but that is not my intention. It is not a call to give up on the higher goals of the standard critique, self confidence, democracy, a more equal society and the like. I only want to show that the route via mathematical thinking, in which we currently invest so much, is a dead end and that we thus need to look for other ways forward.

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