Educ Stud Math (2012) 79:127–147
DOI 10.1007/s10649-011-9333-2

Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice

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Published online: 23 June 2011

Abstract There is broad acceptance that mathematics teachers' beliefs about the nature of mathematics influence the ways in which they teach the subject. It is also recognised that mathematics as practised in typical school classrooms is different from the mathematical activity of mathematicians. This paper presents case studies of two secondary mathematics teachers, one experienced and the other relatively new to teaching, and considers their beliefs about the nature of mathematics, as a discipline and as a school subject. Possible origins and future developments of the structures of their belief systems are discussed along with implications of such structures for their practice. It is suggested that beliefs about mathematics can usefully be considered in terms of a matrix that accommodates the possibility of differing views of school mathematics and the discipline.

Keywords Teacher beliefs · Nature of mathematics · School mathematics · Belief systems

1 Introduction

Research aimed at describing and characterising the beliefs of teachers concerning the nature of mathematics, as well as theoretical analyses of the same, have been based on the assumption that teachers' ideas of what mathematics is will influence the way in which they teach the subject (Skemp, 1978; Sullivan & Mousley, 2001). Hersh (1986, p.13, cited by Thompson, 1992) summarised the situation as follows:

> One's conception of what mathematics *is* affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it …The issue, then, is not, "What is the best way to teach?" but, "What is mathematics really all about?"

Since mathematics is what mathematicians do and create, answering Hersh's essential question demands a consideration of the mathematical activity of mathematicians; activity

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that has been contrasted with that which typically occurs in school mathematics classrooms (Burton, 2002). Such a difference suggests that teachers' views of the nature of mathematics may differ from mathematicians' views of the discipline. Alternatively, teachers may view mathematics as a discipline in similar ways to mathematicians but regard the school subject differently, and it is this possibility that is explored here. The paper begins with a brief comparison of the discipline of mathematics and school mathematics followed by a discussion of existing understandings of teachers' beliefs about the discipline and ways in which these beliefs are generally accepted as impacting teaching. A refined version of the two-way matrix presented by Beswick (2009) showing how various combinations of beliefs about the discipline and the school subject might logically be expected to impact practice is then described. Two cases, one of which is an expanded version of that reported in Beswick (2009), are examined in terms of the matrix. Finally, implications for teacher education and professional learning are discussed. The specific research questions, addressed from both theoretical and empirical perspectives, are:

1. Is it possible for mathematics teachers to hold differing views of mathematics as a discipline and mathematics as a school subject?
2. How might such disparities arise?
3. What implications for practice might such disparities have?

It should be noted that beliefs are taken to be indistinguishable from knowledge[1] and hence beliefs about mathematical content and pedagogy are inclusive of that which is more usually described as mathematics knowledge for teaching (most notably by Ball and colleagues, e.g. Ball, Thames and Phelps 2008). Consistent with this, beliefs are referred to as constructed in the same sense that knowledge is constructed.

2 School mathematics and mathematics as a discipline

Ernest (1998), cited in Burton (2002), suggested that there are differences between mathematics classrooms and the work of research mathematicians in relation to (a) whether knowledge is created or existing knowledge is learned, (b) who selects the problems to be worked on, (c) the time frames over which problems are worked on and (d) the purpose of the learning (for personal achievement or to add to public knowledge). From a constructivist viewpoint, such as taken in this paper and adopted by Burton (2002), learning is inherently creative because it results in the construction of knowledge that is new in the context of the learner concerned. In typical classrooms, however, mathematics problems are selected by the teacher and solvable in matter a of minutes, whereas mathematicians have considerable autonomy about the problems on which they work, typically struggle with a problem for extended periods, and do so with a view to publishing new mathematical results.

Knoll, Ernest and Morgan (2004) described contrasts between the activity of pure mathematicians and school classroom mathematical activity as "sharp" and worked from the assumption that this should not be the case. In describing mathematical research, they drew attention to its creativity and the use of strategies such as the search for examples and counter-examples, cases and constraints, patterns and systems of rules, the use of justification and proof and the framing of problems. These are all things that could and arguably should be part of school mathematics. According to Burton (2002), desirable

[1] See Beswick (2011) for a detailed argument for the equivalence of beliefs and knowledge.

commonalities between features of the practice of mathematicians and school mathematics include the notion that both contexts constitute communities of practice in which the identity and agency of the participants relates to their ability to engage in mathematical enquiry, and in which collaborative work is the norm. In addition, she cited the common search for connectivities among mathematical ideas, an appreciation of mathematical aesthetics and the role of intuition in mathematical work.

One difference that appears beyond reconciliation relates to the purpose of mathematical work in classrooms compared with that of mathematicians and, hence, the nature of acceptable warrants of knowledge in each context. Ernest (1999) pointed out that the community of mathematicians requires warrants of new mathematical knowledge, whereas in education, although learners may be required to justify their mathematical ideas, ultimately teachers require evidence that the student has in fact constructed the desired knowledge (which itself is not contentious in this context). In essence, mathematicians assess mathematics but educators assess learners.

In spite of this, it appears that the differences between school mathematics and the discipline could be greatly reduced by an increased emphasis on the use of practices associated with research mathematics in school mathematics classrooms, and there appears to be consensus that this is a worthy goal. In Burton's (2002, p. 171) words it could lead to the "creation of an aware citizenry able to appreciate the joys of mathematics, as well as its usefulness".

3 Teacher beliefs about the nature of mathematics and their relationship to practice

If school mathematics and mathematicians' mathematics are to be reconciled then teachers must have an appreciation of the nature of mathematics that is akin to that of mathematicians. This was the premise that underpinned the project reported by Knoll et al. (2004). Ernest (1989) described three categories of teacher beliefs about the nature of mathematics that have been widely adopted and used (e.g. Beswick, 2005, 2009). The first is the Instrumentalist view that sees mathematics as, "an accumulation of facts, skills and rules to be used in the pursuance of some external end." (Ernest, 1989, p. 250) According to this view of mathematics the various topics that comprise the discipline are unrelated. The second category is the Platonist view in which mathematics is seen as a static body of unified, pre-existing knowledge awaiting discovery. In this view the structure of mathematical knowledge and the interconnections between various topics are of fundamental importance. Ernest's (1989) third category is the problem solving view in which mathematics is regarded as a dynamic and creative human invention; a process, rather than a product (Ernest, 1989), and the view that best reflects relatively recent changes in the way that mathematicians view their discipline (Cooney & Shealy, 1997).

Beswick (2005) summarised connections among Ernest's (1989) categories of beliefs about the nature of mathematics, an adaptation of the corresponding categories that he proposed for beliefs about mathematics learning, and Van Zoest, Jones and Thornton's (1994) categories relating to mathematics teaching. These connections are summarised in Table 1, in which beliefs in the same row are considered theoretically consistent, and those in the same column have been regarded by some researchers as constituting a continuum. Beswick (2005), like the authors on whose work she drew, acknowledged that individual teachers are unlikely to have beliefs that fit neatly in a single category. In addition, beliefs related to specific aspects of the particular context in which a teacher is working, for example about specific students' interests and abilities, can also influence which of their

Table 1 Categories of teacher beliefs

Beliefs about the nature of mathematics (Ernest, 1989)	Beliefs about mathematics teaching (Van Zoest et al. 1994)	Beliefs about mathematics learning (Ernest, 1989)
Instrumentalist	Content focussed with an emphasis on performance	Skill mastery, passive reception of knowledge
Platonist	Content focussed with an emphasis on understanding	Active construction of understanding
Problem solving	Learner focussed	Autonomous exploration of own interests

From Beswick (2005, p. 40)

other beliefs are most influential in terms of shaping their practice in that context (e.g. Beswick, 2004). It is possible, therefore, for a teacher with beliefs that fall into more than one category to teach consistently with one view in one context and the other in a different context.

The point just made is related to the more general observation that theoretical consistency does not guarantee consistency among the beliefs of individual teachers. Indeed, there are many examples in the literature of teachers, usually in the early stages of their careers, teaching in ways that are apparently not consistent with their beliefs about teaching (e.g. Frykholm, 1999; Sosniak, Ethington & Varelas, 1991). Such discrepancies can be explained readily in terms of the different contexts in which beliefs are articulated and then observed (e.g. Beswick, 2003) and there is a growing consensus that teachers are indeed sensible in the ways in which they espouse and enact their beliefs (Leatham, 2006). Indeed participants in Liljedahl's (2008) study suggested more than a dozen reasonable ways in which apparent contradictions between pre-service elementary teachers' espoused beliefs, intended practice, and actual practice could be explained.

Speer (2005) argued from a theoretical perspective that reported discrepancies between the beliefs that teachers profess and those that are inferred from their actions are likely to be artefacts of the research methods employed. She argued that all beliefs are to some degree inferred (from either the teacher's actions or words) and hence the dichotomy is false. Apparent conflicts can also result from a lack of shared understanding resulting in teachers and researchers meaning and understanding different things from the same words. In addition, Speer (2005) asserted that data from which beliefs are inferred should be obtained in conjunction with data about practice if the aim is to link the two.

Speer's (2005) arguments are consistent with the methodological imperatives articulated by Schoenfeld (2003). Like Speer (2005), he stressed the need to consider practice if one wants to link beliefs to it. Furthermore, he argued that meanings and interpretations should be made carefully with attention to the development of a shared vocabulary, that the unit of analysis must be chosen appropriately to allow beliefs that are sufficiently specific to explain particular behaviours to be identified, that alternative explanations should be considered and that explanations of links between beliefs and practices should focus on mechanisms rather than correlations. Speer (2008) provided an excellent example of research that adheres to these principles at a fine grain size, connecting collections of beliefs with specific teaching practices.

Green's (1971) description of belief systems provides theoretical insight into how apparently contradictory beliefs may arise. Of particular relevance here is the notion of clustering of beliefs. Disjoint belief clusters are likely to develop when the relevant beliefs are formed in different contexts (place or time) and may be contradictory since beliefs in separate clusters are not, in the normal course of events, juxtaposed to highlight their inconsistency (Green, 1971). It is at least theoretically possible, therefore, for teachers to hold beliefs about the nature of the discipline of mathematics in isolation from their beliefs about the school subject. From a theoretical perspective, therefore, research question 1 can be answered in the affirmative. That is, it is possible for teachers to hold differing beliefs about mathematics as a school subject and as a discipline, and clustering provides a metaphorical picture of how the relevant part of such an individual's belief system might be structured. In Section 3.1 two examples from the literature that suggest possible catalysts for the construction of such belief clusters are considered.

From a methodological point of view, in the study reported here Schoenfeld's (2003) tenets were observed as far as was possible. Specifically, it was important that the teachers were able to view and confirm both the data and inferences about their beliefs that were made from it. The aim of the study was to look at beliefs that influenced practice quite broadly and hence evidence of beliefs likely to be influential at this scale was sought. The teachers were not prompted to articulate beliefs about school mathematics as distinct from the discipline of mathematics. Finally, theoretical considerations are offered to explain apparent connections between the teachers' beliefs about mathematics and their likely and observed practices.

3.1 Examples from the literature

With the notable exception of Thompson's (1984) seminal study of the beliefs of secondary mathematics teachers that was a major influence on the formation of Ernest's (1989) categories of beliefs about the nature of mathematics, relatively little attention has been paid to teachers' beliefs about the nature of mathematics, and there are even fewer reports of teachers of mathematics holding different views about the discipline and school mathematics, and apparently none involving secondary teachers. This could be a consequence of the lack of research in the area or because such inconsistencies are rare and it is perhaps reasonable to assume that the phenomenon would be less common among secondary mathematics teachers since they are commonly required to have studied mathematics to a high level—Moreira and David (2008) suggested a mathematics major is a usual requirement. However, given the worsening shortage of mathematics teachers in many countries this is likely to become less and less the case. In the study from which the cases reported here are drawn eight of the 25 teachers had studied mathematics to third year university level and, of these, just three claimed to have majored in mathematics. Following are two instances from the literature, both involved primary pre-service teachers.

In the first of these, Mewborn (2000) used Green's (1971) ideas about beliefs systems to describe the beliefs and practice of a pre-service primary teacher, Carrie, progressing through her course. According to Mewborn (2000), Carrie began with a fully integrated set of beliefs about students, teaching and learning, but held negative beliefs about mathematics and was consequently unsure of how she could teach that subject effectively. As a result of working with an experienced teacher, Carrie realised that mathematics could indeed be taught in accordance with her existing beliefs about students, teaching and learning and hence was able to adopt mathematics teaching practices that were consistent with them. Mewborn (2000) argued that Carrie altered her beliefs about school mathematics throughout the course as a result of her developing practice, but acknowledged that her beliefs about mathematics as a discipline did not appear to change. However, the continuing isolation of Carrie's beliefs about the discipline of mathematics was not raised as potentially problematic.

The second example does not relate directly to beliefs about the nature of mathematics, but illustrates how teachers can create clusters in their belief systems in order to preserve existing belief structures and thereby avoid the upheaval inherent in accommodating new and conflicting ideas. In her study of pre-service primary teachers, Schuck (1999) found that many held beliefs about the importance of making mathematics enjoyable, but did not believe that having sound personal knowledge of mathematics was important to their ability to teach it well. Indeed, many believed the converse to be true (Schuck, 1999). Schuck (1999) attributed this to the personal experiences of these teachers with learning mathematics that had resulted in their feeling insecure with regard to their own mathematical ability. Constructing the belief that mathematical ability is not requisite for effective mathematics teaching allowed them to maintain belief in themselves as potentially effective teachers (Schuck, 1999). The teachers Schuck (1999) described may have taught mathematics in ways superficially consistent with a problem solving view of the discipline but with motivations that had nothing to do with an appreciation of its aesthetic appeal or understanding of what a mathematician might mean by doing mathematics.

3.2 Theoretically possible implications of disparate beliefs about mathematics as a school subject and a discipline

We have seen that school mathematics, as it is typically experienced by students, is different from the mathematical activity of mathematicians, and the notion of clustering allows that it is at least theoretically possible for teachers to hold differing beliefs about the nature of mathematics depending upon whether they are considering it as a discipline or as a school subject. In addition, there is some (limited) empirical evidence that teachers can indeed construct separate belief clusters about school mathematics and the discipline in order to teach the subject in a manner consistent with their beliefs about mathematics teaching. Table 2 represents a refinement of the matrix presented by Beswick (2009). It proposes theoretically reasonable implications for teaching of each possible combination of Ernest's (1989) three categories of beliefs about the nature of mathematics in relation to mathematics as a school subject and as a discipline. The descriptions of practice contained in the matrix cells are intended to be consistent with beliefs about mathematics teaching and learning corresponding to the undifferentiated views of the nature of mathematics shown in Table 1.

It is also assumed that, although in the context of their classrooms teachers' beliefs about school mathematics are likely to be more influential than their beliefs about the discipline, the latter constitute an underlying rationale for practice. The cells thus describe possible, not necessarily consciously held, motivations for teaching in ways that are consistent with the beliefs about school mathematics that the teacher holds. They represent a theoretically reasonable answer to research question 3 concerning the effects on practice of possible disjunctions between beliefs about the discipline and the school subject. The elements of Table 2—beliefs about the nature of school mathematics, beliefs about the nature of the discipline of mathematics and mathematics teaching practice—are used to structure the presentation of the study results in the following sections. These form the basis of empirical contributions towards answers to the three research questions.

4 The study

The two cases reported here are drawn from eight that were part of a larger study, other aspects of which have been reported elsewhere (e.g. Beswick, 2005, 2007). The study

Table 2 Impacts on practice of combinations of beliefs about the discipline and school mathematics

		Beliefs about the nature of (the discipline of) mathematics		
		Instrumentalist	Platonist	Problem solving
Beliefs about the nature of (school) mathematics	Instrumentalist	School mathematics is about learning basic skills that students will need in everyday life.	School mathematics is about learning basic skills that will allow understanding of higher level more interesting mathematics later.	Mathematics can be creative but you need to have a set of basic skills first. Mathematical creativity is not for school.
	Platonist	School mathematics is a body of hierarchical interconnected knowledge that needs to be learned so that it can be applied to practical situations.	School mathematics is part of a body of hierarchical interconnected knowledge understanding of which forms the basis on which some will learn higher level mathematics.	School mathematics is a body of hierarchical interconnected knowledge understanding of which will enable the gifted few eventually to be mathematically creative.
	Problem solving	Learner/process focus is aimed at motivating students to learn the skills they need in everyday life.	Learner/process focus is aimed at motivating students so that they come to understand more of the body of hierarchical interconnected knowledge that is mathematics.	Learner/process focus is aimed at helping students to appreciate mathematics as a powerful and creative process.

aimed to uncover beliefs of secondary mathematics teacher about mathematics teaching, mathematics learning and the nature of the discipline that affected their teaching of mathematics. For each teacher a set of between four and eight central beliefs that appeared most powerfully to influence their mathematics teaching were inferred from survey, interview and observational (where available) data. In all but one case no differences between beliefs about the school subject and the discipline of mathematics were evident. This is, if they could be placed in the matrix shown in Table 2 they would be positioned in cells in the diagonal running from top left to bottom right. Three were difficult to place because beliefs about mathematics (as either a school subject or a discipline) were less influential than beliefs about other matters. For example, one teacher's practice was most strongly influenced by beliefs about students and their abilities, and the need for the teacher to maintain classroom order. It should be noted that the teachers were not prompted to distinguish between mathematics as a school subject and as a discipline and so when such a distinction arose it can more confidently be viewed as a central feature of that teacher's relevant beliefs. The cases reported here include the one teacher whose beliefs about mathematics as a school subject and as a discipline were clearly distinct. This case was described by Beswick (2009). The other involved a relatively inexperienced teacher whose beliefs about mathematics teaching appeared to be a in a state of flux influenced by as yet unreconciled beliefs about the discipline of mathematics. She appeared to be positioned on the un-conflicted axis of Table 2 but there were signs that her changing belief structure might move her to a neighbouring cell.

4.1 Instruments

Data concerning teachers' beliefs about the nature of mathematics were collected using a survey requiring responses on a five-point Likert scale, to 26 items, taken from similar instruments devised by Howard, Perry and Lindsay (1997) and Van Zoest et al. (1994), relating to beliefs about mathematics, its teaching and its learning, an audio-taped semi-structured interview of approximately 1 h duration, and, where possible, classroom observations. The interview asked teachers to: reflect upon their own experiences of learning mathematics; describe an ideal mathematics classroom and compare this with the reality of their own mathematics classes and respond to 12 statements about the nature of mathematics and 12 statements about the teaching and learning of mathematics, based upon the findings of Thompson's (1984) case studies of secondary mathematics teachers. The 12 statements regarding the nature of mathematics comprised four each corresponding to Ernest's (1989) three views of mathematics, and the statements relating to the teaching and learning of mathematics were representative of corresponding views of mathematics teaching and learning shown in Table 1. Six of the eight teachers in the larger study were also observed teaching—one declined to be observed and the other, the more experienced teacher considered in this study, was absent from school during the observation period as a result of personal issues. Nevertheless, classroom observation data for the other teacher whose case is described here are presented.

4.2 Procedure

The teachers completed the beliefs survey during the first few weeks of the school year. The eight teachers to be interviewed were selected from teachers who on their surveys indicated their willingness to participate in this aspect of the research. They were chosen to represent as broad a range of survey results as possible. The interviews were conducted 7 months after the administration of the surveys and observations of approximately 12 lessons for each teacher were made in the subsequent 2 months. Surveys and interview transcripts were examined for evidence of the teachers' beliefs about mathematics, mathematics teaching and mathematics learning and particularly for such beliefs evident from more than one data source or for apparent contradictions among beliefs either within or between data sources. All of the data for each of the teachers were considered together and analysed with a view to indentifying relevant beliefs. Finally, inferences concerning their most centrally held beliefs made from the analysis were provided to the teachers for comment and/or correction and ultimately approval as described by Guba and Lincoln (1989).

In the following sections, the cases of Sally and Jennifer are presented. Each begins with details of their current teaching along with background information that emerged from their interviews about experiences that seemed to have been influential in shaping their beliefs.

5 The case of Sally

Sally had been teaching secondary mathematics for 18 years. She had studied tertiary mathematics for 3 years and had since completed a Master of Education (M. Ed) degree. Some years earlier she had spent 3 years as the district Senior Curriculum Officer (SEO; Mathematics), for the Education Department. Sally was currently teaching a Grade 7 (12–13 year olds) mathematics class and a combined Grade 9 and 10 (14–16 year olds) class, in which the students were studying non-compulsory advanced mathematics courses. Sally was a senior teacher with responsibilities including provision of school leadership in mathematics.

Sally had been influenced by the reform agenda in mathematics, and particularly by the 3 years that she had spent as a district SEO. During this time she had been involved in producing state guidelines for teaching mathematics from Kindergarten to Grade 8. She described how she had been required to present workshops for both primary and secondary teachers, and that this had provided both the stimulus and the opportunity to think about new ideas in mathematics education. She contrasted this with the lot of classroom teachers who face constant urgent demands that make it far more difficult to focus on broader issues.

5.1 Sally's beliefs about the nature of (the discipline of) mathematics

Sally's responses to the beliefs survey suggested that she held beliefs consistent with a problem solving orientation to mathematics and there were no apparent contradictions among her responses. For example, she strongly agreed or agreed with items such as:

> Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking.
> Justifying the mathematical statements that a person makes is an extremely important part of mathematics.

Consistent with this, she strongly disagreed or disagreed with such items as:

> Mathematics is computation.

In discussing the origins of mathematical content, she was comfortable with the sciences and other practical needs as sources of mathematics, but with regard to mathematics being self-generating she acknowledged that it could be so, but said, "I'm not really sure why, …

it's a bit like a brain exercise for some people", and also spoke in the third person when she agreed that some people enjoy mathematics, saying, "They enjoy it for itself".

Sally indicated that she suspected that the results of mathematics were not tentative but rather in most cases they were "sort of law". She agreed with the statements, "Mathematical content is coherent. Its topics are interrelated and logically connected within an organisational structure or skeleton", and "Mathematics is an organised and logical system of symbols and procedures that explain ideas present in the physical world", adding that it was the logic of mathematics that appealed to her.

5.2 Sally's beliefs about the nature of (school) mathematics

When asked what sprung to mind in response to the word "mathematics", Sally answered in terms of the strands of the national curriculum, describing this as much broader than that which had been her own and the general view of mathematics in earlier times. She also believed that mathematics was now, "much more exciting and certainly less boring and academic" than it previously had been. When asked about her own experiences of learning mathematics in primary and secondary school Sally recalled working from textbooks and hence that maths was "something you did in a book" but that she nevertheless enjoyed because she was "good at it". Sally made no other references to her own experiences as a mathematics learner.

When responding to statements about the nature of mathematics, Sally frequently answered in terms of school mathematics and had difficulty in considering mathematics as a discipline that extended beyond the school context. This was so even when she was specifically prompted to consider the discipline as a whole and not just mathematics taught in school. For example, in response to the statement, "The content of mathematics is "cut and dried". Mathematics offers few opportunities for creative work", she said:

> … I disagree quite strongly I think … I don't think that the content of maths is cut and dried. I think a lot of the professional development that's gone on in the last 10 years … I think has opened up huge opportunities for creative work …

She similarly equated the discipline of mathematics with school mathematics in agreeing that, "Mathematics is an exact discipline, free from ambiguity and conflicting interpretations". In addition, her description, in this context, of the shift in emphasis from answers alone to the processes that produce the answers, that she believed had occurred in mathematics education suggested that she interpreted the statement to mean that "Mathematics is an exacting discipline …" she said:

> Yes, I think generally, a lot of mathematics is an exact discipline and free from ambiguity and conflicting interpretation, but I think we're moving away from that too.

In responding to the statement, "Mathematics is a challenging, rigorous and abstract discipline whose study provides the opportunity for a wide spectrum of high-level mental activity", Sally agreed and again related it to school mathematics, and particularly to what she believed was a shift from pure to applied mathematics that had and was still occurring in that context. Sally saw this change in emphasis as being most apparent in primary schools, less so in secondary schools, particularly in more senior classes, and not at all evident in courses designed for senior students such as her Grade 9 and 10 class. For students not likely to study tertiary level mathematics, she regarded mathematics as now making fewer demands in terms of high-level mental activity than it had in the past, because it was more "applied".

When asked to respond to the statement, "Mathematical content is fixed and predetermined, as it is dictated by ideas present in the physical world" Sally again answered in terms of the school curriculum. She said:

> … I don't believe that it is fixed and predetermined; there is always this pressure from you know, TASSAB [the state Assessment Board] or somebody … trying to reinforce that message that it is fixed and predetermined. But I suppose that that's very specific sort of content type of stuff, but … I suppose that generally the ideas are present in the physical world. It's just that we're changing the way that perhaps we look at them, I think.

A similar focus on the content of school mathematics was evident in her response to the statement, "Mathematics is continuously expanding its content and undergoing changes to accommodate new developments". She said:

> … I think that maths has, has become much more than just number and that it, you know, people can see the relevance of things like chance and data … the efforts that whoever it is that are making to revise what's actually happening in Grades 9 and 10, 11 and 12 too, to make some sorts of changes in terms of the content and to accommodate things that people believe are important.

5.3 Sally's beliefs about mathematics teaching and learning

Sally's survey responses concerning the teaching and learning of mathematics were consistent with those about the nature of mathematics, reflecting a Learner Focussed/ Autonomous Exploration of Own Interests orientation. She strongly agreed that:

> It is important for children to be given opportunities to reflect on and evaluate their own mathematical understanding.

and agreed with items such as:

> Ignoring the mathematical ideas that children generate themselves can seriously limit their learning.
> Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics.

Sally described an ideal mathematics classroom as one in which the students were engaged and motivated, and involved in practical problem solving investigations that had originated from their own interests and questions. They would be working either as individuals or in groups and perhaps for extended periods of time. The class would encompass a range of ability levels and the tasks in which they were engaged would be accessible to all. They would have access to computers, and their activities would be characterised by making and testing hypotheses, describing patterns and discussing their ideas.

Sally was consistent in her disapproval of teaching procedures without meaning, and on several occasions likened the use of rote learned algorithms to the performance of a trick. She also believed that, "Students should not be satisfied with just carrying out mathematical procedures; they should seek to understand the logic behind such procedures". Sally's responses also revealed a firm belief in the importance of recognising that there could be many appropriate ways in which to solve a mathematical problem and that the teacher should neither prevent students from using alternative methods that were meaningful for them, nor convey the idea that a particular way is necessarily the only way.

Sally described content arising from the students' interests as more likely to "make an impact" and believed that "some of the really worthwhile things happen" as a result of the teacher digressing in response to a student's question. Consistent with this, Sally also agreed that, "The teacher should appeal to students' intuition and experiences when presenting material in order to make it meaningful", describing this as an important way of engaging them in mathematics. In this context she also said that while it was not always possible for the teacher to know why particular content was included in the curriculum, it was very helpful if they did.

Although Sally expressed quite student-centred views of mathematics teaching, she did not believe that the teacher's role was insignificant or passive. In response to "The teacher should encourage the students to guess and conjecture and should allow them to reason things on their own rather than show them how to reach a solution or answer. The teacher must act in a supporting role", she said:

> I think that the teacher has a fairly important role, to facilitate, or suggest to them, or guide them in terms of reaching a solution or an answer, … I think that's much more time efficient. I think you have to have a balance and so, yes, I do think that part is really important, but if you can't just be so open-ended that they never get there …

Similarly, in response to, "It is the teacher's responsibility to direct and control all instructional activities including the classroom discourse. To this end, she must have a clear plan for the development of the lesson", she expressed qualified agreement, stressing that "control" suggested rather a "narrow path", whereas she saw lessons potentially following a "broader path". Sally believed that the teacher should indeed have a plan for the development of the lesson, but should also be flexible enough to take it in a different direction.

6 The case of Jennifer

Jennifer had been teaching for 2.5 years, having completed a 2-year Bachelor of Teaching. Her initial degree was in Commerce during which she had studied mathematics for 2 years. She was teaching a Grade 7 class for Mathematics, English, Studies of Society and Environment, and Science, and Mathematics classes in Grades 9 and 10.

In her interview Jennifer described her enjoyment of mathematics throughout all of her schooling. In primary school she had enjoyed the positive feedback in the form of praise and stickers that followed from speed, accuracy and neatness. At that stage of her schooling mathematics was "getting the right answer". In the context of her secondary school experience, Jennifer referred to liking mathematics because it had unambiguously right answers, and "was easy and straight forward". Jennifer particularly enjoyed studying the subject at an advanced level in Grades 9 and 10.

Jennifer's experience of studying mathematics at university was somewhat different from her earlier experiences. In this context she distinguished between mathematics that she saw as very applied and other topics, that seemed, to her, less so, like matrices. She described the statistics she learned as "fascinating" and expressed particular liking for a subject called Quantitative Decision Making, which she described as:

> … great because that was all applied, all things like game theory, decision making, … and that was just like more logical thinking to me, like logical reasoning, and I just loved that.

At the time of the study Jennifer was also a participant in a professional learning project aimed at improving the mathematics attainment of indigenous secondary students. The project promoted the use of open-ended problem solving tasks and included an expectation that participants report on the outcomes of trialling such tasks with their students (Callingham & Griffin, 2001).

There was no apparent distinction between Jennifer's beliefs about the nature of mathematics as a school subject and as a discipline and so these data are presented in a single section.

6.1 Jennifer's beliefs about the nature of mathematics

Jennifer's survey did not provide a clear indication of her view of the nature of mathematics. She agreed with both:

> Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking.

and,

> Mathematics is computation.

Jennifer found it difficult to explain what she believed mathematics was, but eventually described it as experimentation. When asked to respond to the statement, "The results of mathematics are tentative; subject to revision in the light of new evidence" she said:

> Yes, yes they are. I think that hypotheses are put out there and then especially with the use of computers because numbers, we can test numbers now that are bigger and bigger.

Consistent with this she also agreed with the statement, "Mathematics is continuously expanding its content and undergoing changes to accommodate new developments". Jennifer also agreed that, "Mathematics is a challenging, rigorous and abstract discipline whose study provides the opportunity for a wide spectrum of high-level mental activity" and was adamant that the content of mathematics is not "cut and dried" and that it does indeed provide opportunities for creative work.

When discussing the origins of mathematics Jennifer was undecided as to whether or not "Mathematical ideas exist independently of human ability to discover them". In the end she seemed to conclude that mathematics is pre-existent but that people construct their understandings of it over time. In her words:

> Well I think that mathematics evolves over time, but, so maths is there, like the zero was there a long time before it was actually discovered, so I think yes, it is there, the idea, but then it's constructed by humans, so that we can use it.

Her reference to mathematics evolving is consistent with her other statements regarding the changing and expanding nature of mathematical content, while her virtually unconscious linking of usefulness to the purpose of humans' mathematical constructions is consonant with her clear preference, expressed in relation to her tertiary studies, for applied mathematics. Jennifer also conceded that some mathematics comes from within mathematics itself: "… I guess there's maths in it, like maths just for the beauty of maths". However she found it easier to agree that mathematical content originates from "the needs of the sciences and other practical needs".

Some of Jennifer's responses to statements about the structure of content in mathematics were somewhat contradictory. For example, in response to the statement, "Mathematical content is coherent. Its topics are interrelated and logically connected within an organisational structure or skeleton", she said, "No, they're no, they're very all over the place". Later when questioned about what she meant by this she explained how she must have misinterpreted the statement originally:

> … oh yes, no I do agree with that. How did I interpret it before? … well I think that organised structure might have put me off because I thought that might be the way that the lessons were, like an organised structure of lessons.

She also disagreed that "Mathematics is an exact discipline, free from ambiguity and conflicting interpretations" and "Certainty is an inherent quality in mathematical activity. The procedures and methods used in mathematics guarantee right answers". Nevertheless, she agreed that "Mathematics is an organised and logical system of symbols and procedures that explain ideas present in the physical world". It is probable, given the nature of the tertiary mathematics that she described, that Jennifer had in mind probabilistic domains of mathematics such as statistics and game theory when responding to the statements about certainty and ambiguity in mathematics. These were aspects of mathematics that she had not encountered before university and she seems to have incorporated them into her existing belief system by delineating a category of mathematics that she characterised as applied.

6.2 Jennifer's beliefs about mathematics teaching and learning

Jennifer's survey responses suggested that she was yet to decide whether a traditional approach or a more inquiry-based teaching approach was most effective in terms of students' mathematics learning. For example, she agreed or strongly agreed with each of the following:

> Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics,
> Children always benefit by discussing their solutions to mathematical problems with each other,
> A vital task for the teacher is motivating children to resolve their own mathematical problems.
> Allowing a child to struggle with a mathematical problem, even a little tension, can be necessary for learning to occur.

but also agreed that:

> Listening carefully to the teacher explain a mathematics lesson is the most effective way to learn mathematics.
> Telling children the answer is an effective way of facilitating their mathematics learning.

When asked to describe an ideal mathematics classroom Jennifer spoke at some length about the professional development project that she was involved in, that encouraged teachers to use rich, group problem solving tasks in their mathematics teaching. She described implementing the approach with her Grade 9 class and spoke about the way that she encouraged students to explain their reasoning both to her and to other students. Jennifer described these lessons as her "favourites" even though they were exhausting.

Consistent with her clarified view of mathematics as coherent and its topics as logically connected, Jennifer described an important part of the teacher's role as follows:

> … I think that it's really important to have the links, the links, clear … so that if the kids understand one thing then you need to make the links to the other things, like to the new material very clear.

Similarly she agreed with the statement, "The teacher's explanations should assist students to 'see' the relationships between the new topic and those already studied" and described this as "very important".

Jennifer expressed firm belief in the importance of appealing to students' interests and intuition; listening to and capitalising on their suggestions; and encouraging them to reason, conjecture and solve problems on their own. She regarded this as more important than adhering to a predetermined plan for the lesson. She also agreed that "students should not be satisfied with just carrying out mathematical procedures (but) should seek to understand the logic behind such procedures". In addition, teachers should "probe for potential misconceptions in the students by using carefully chosen examples and non-examples" and she stated that, "… there are lots of different ways for getting the right answer".

In spite of this, Jennifer also expressed agreement with the statement, "Students learn mainly by attentively watching the teacher demonstrate procedures and methods for performing mathematical tasks and by practising those procedures". This contradiction was also evident from her description of the non-problem solving lessons that she taught:

> I have a real mixture, like some lessons we have automatic response which is just … , which is just like, that's taken back to primary school and I, and when I was at primary school I hated it because it used to put me under too much pressure, but the kids now, they, they really like that because I take down the marks, you know and, and stuff and it seems really formal to them and they really enjoy that, and they call out their marks in front of all their peers and they do all the terrible stuff, but really when they're doing the test they're going (unclear whispering) like they're talking to the person next to them, and they think that they're being really sneaky about it, … once they've got to the point of the algebraic manipulation, then we get a bit more traditional …

6.3 Jennifer's classroom practice

The pace of all Jennifer's lessons was slow and measured and all were whole class and teacher directed. Jennifer was always the source of the answers, either writing them on the board, or walking around with the answer sheet.

In the first Grade 7 lesson observed, Jennifer asked the students to recall a marching activity that they had done outside in a previous lesson which she related to rectangular arrays. She then proceeded to draw rectangles on the whiteboard and divide them into various numbers of rows and columns and then use these to talk about fraction addition and subtraction. The students copied Jennifer's board work, using grid paper to assist with drawing. For example, having drawn a rectangle divided into four rows and five columns Jennifer asked the students to write down sums that could be answered using it. When the students responded with examples involving various numbers of twentieths she challenged them to find the sum of 1/4 and 1/5. The next problem required students to construct a rectangle that showed both halves and thirds. Troy volunteered to draw his rectangle on the board. He drew a 3×4 rectangle. Jennifer used a coloured marker to show how it could be divided into halves and thirds, then asked if anyone had drawn a different rectangle. Nathan volunteered to draw his, a 2×3 rectangle. Jennifer instructed the class to draw Nathan's rectangle and to use it to find 1/2+1/3. The class were then asked to draw a rectangle to show quarters and sixths and to use it to find 1/4+1/6.

In subsequent lessons Jennifer began by talking through several examples of adding fractions on the board using neatly pre-drawn rectangles, and shading each fraction before arriving at the final answer. After several examples the students were set to work on a short list of similar questions written on the board. After working on them for some time Jennifer would go through the answers with the whole class. Jennifer seemed interested in how the students were learning and particularly in whether the rectangle drawing was helping them to understand fractions.

Jennifer's Grade 9 class were studying trigonometry. Each lesson began with work from the whiteboard where triangles had been very neatly pre-drawn. Often the students were required to copy examples into their books. Trigonometry was defined as relating to right triangles, and the sine, cosine and tangent ratios were introduced in the one lesson with a brief statement that for a given angle, each is the same regardless of the size of the triangle, and then definitions were written on the board for students to copy. The mnemonic, SOHCAHTOA, had been written in large letters on a strip of card above the whiteboard and students were instructed on its use for remembering the definitions of the three ratios. Instructions for using a calculator and solving problems were very clear but devoid of explanation of why various steps should be performed. The students worked through a series of exercises from pages copied from a text. The exercises were graded, dealing with one ratio at a time and starting with questions in which the unknown was the numerator of a fraction and then the denominator. In each case the students were required to copy model examples from the whiteboard and then to practice the procedure.

Jennifer appeared more comfortable teaching her younger students. For example she used prescribed seating arrangements for her Grade 7 students that she changed fortnightly, and even though she justified this in terms of the benefits of working with a variety of people and avoiding the distractions of friendship groups, she allowed the Grade 9 students to choose where and with whom they sat. With respect to the Grade 7 situation she observed that, "They accept it, they accept anything".

7 Discussion

In the following sections each of the cases is discussed in turn.

7.1 Sally

Unlike the cases reported in the literature (Mewborn, 2000; Schuck, 1999), Sally was an experienced mathematics teacher with a relatively strong mathematics background albeit not a mathematics major. She appeared to have a problem solving view of school mathematics but, to the extent that she could conceive of the broader discipline, she seemed to have Platonist view of it. A teacher such as Sally could teach in ways consistent with a problem solving orientation according to Table 1 but not because this is the way in which she viewed the discipline. Rather, it appeared that she had constructed beliefs about school mathematics as something separate from mathematics as a discipline and it was this that allowed her to teach in ways consistent with a problem solving orientation.

Sally's case suggests that teachers may adopt an approach to mathematics teaching that appears to be logically consistent with a problem solving view of mathematics (Ernest

1989) without having such beliefs. Sally's beliefs about the school mathematics and the discipline appear to intersect in the central cell of the bottom row of Table 2. Sally consistently focussed on developments in school mathematics that had occurred since her entry into the profession and it was her experience in the profession that appeared to have had the greatest influence on her beliefs. Nevertheless, her own experiences of learning mathematics, perhaps reflected in her early teaching practice, appeared to form a backdrop against which more recent developments were contrasted. It was these changes, rather than her thinking about the nature of mathematics, that appeared to dominate her thinking. There was, therefore, little indication of the source of Sally's beliefs about the discipline, nor evidence that they had changed. One could speculate that the divergence in her beliefs about the discipline and school mathematics had its origin in her school years much like the students described by Schoenfeld (1989). He described senior secondary students who had adopted the rhetoric of mathematics as a logical and creative discipline while simultaneously claiming it involved mainly memorising. Schoenfeld accounted for the discrepancy by suggesting that in one case the students were referring to a discipline that they had heard about but not yet experienced and in the other they were describing their experience of mathematics at school.

7.2 Jennifer

Jennifer's view of mathematics was most closely aligned with Platonism with aspects of a problem solving view confined to the discovery of mathematics that was already "there" but previously not known. Aspects of an Instrumentalist view were also evident in her emphasis on the usefulness of mathematics, her preference for aspects she perceived as applied, and its origins in human needs.

Jennifer was much less experienced and hence more similar than Sally to the pre-service teachers described by Mewborn (2000) and Schuck (1999). She appeared to be struggling to reconcile her predominantly Platonist beliefs about the nature of mathematics with a desire to teach mathematics consistently with a problem solving perspective. There was no evidence that she had formed beliefs about school mathematics that were distinct from those she held about the discipline but in wanting to embrace a problem solving approach to her teaching, a tension was established that could conceivably lead to such a separation. Such a move to accommodate new experiences and beliefs would be similar to that which she appeared to have made in relation to applied and pure mathematics while at university.

It seemed that Jennifer had given intellectual assent to newer, student-centred approaches to teaching mathematics and was making genuine attempts to incorporate these into her teaching, however, she was still strongly influenced by her own experiences as a student and the predominantly Platonist beliefs about mathematics that she had constructed in these contexts. Given that she was still participating in the professional learning project that was promoting a problem solving approach to teaching it is likely that she had not yet had time to work through the implications of these ideas for her teaching, nor to develop sufficient resources to apply such an approach across her teaching. This is consistent with the observation that, although she was striving to teach fractions meaningfully, Jennifer's repertoire appeared not to extend to meaningful ways to approach trigonometry. Although she had trialled problems provided by the professional learning project with her Grade 9 class, it is also possible that she was less confident about trying new approaches of her own with older, potentially less compliant, students.

Jennifer seems to be positioned in the central cell with most of her teaching consistent with a Platonist view of both the discipline and school mathematics. Her struggle to use a problem solving approach can be seen as an effort to move to the bottom row of the table. Reconciliation could be achieved in at least three possible ways: (1) by abandoning this effort, (2) by adopting a belief structure similar to Sally's or (3) by radically reconceptualising her beliefs about mathematics so that she comes to see the discipline from a problem solving perspective (see the bottom right cell of Table 2). The last of these options is arguably most desirable but unlikely in the absence of further experiences of mathematics, beyond her teaching, that help her to reconstruct her existing beliefs. The professional learning to which she referred appears to have been effective in inspiring her to try different approaches to teaching but not to have addressed her beliefs about what mathematics is; hence her conflict.

8 Implications and conclusion

This section begins by briefly providing answers to the research questions followed by a discussion of broader implications of the study for professional learning.

8.1 Research question 1: Is it possible for mathematics teachers to hold differing views of mathematics as a discipline and mathematics as a school subject?

Sally's case provides evidence that suggests that experienced and mathematically well-qualified secondary mathematics teachers, in addition to inexperienced pre-service primary teachers with limited mathematical knowledge such as described by Mewborn (2000), can hold differing views of mathematics as a school subject and as a discipline. Just how common such a disjunction is, particularly among teachers who are as mathematically qualified as Sally is impossible to judge from this study. It is, however, evident that studying mathematics to third year university level does not guarantee that beliefs about the nature of discipline developed in that context will be influential in a classroom context.

8.2 Research question 2: How might such disparities arise?

In Sally's case it appears that her experiences since joining the teaching profession constituted the major influence on her beliefs. In particular, her role as SEO (Mathematics) required her to reflect on mathematics teaching and afforded opportunities for her to be immersed in the latest thinking about appropriate mathematics pedagogy. These experiences were all in the context of the school mathematics curriculum and nothing in her professional experience appeared to have prompted her to revisit her views about mathematics as a discipline. In Green's (1971) terms, these beliefs had not been integrated with her newer beliefs about mathematics as a school subject. In addition, Sally's beliefs about the discipline of mathematics appeared to be not at all central in her belief system concerning mathematics teaching and, hence, not readily evoked when asked to consider mathematics in this sense. This is not to say, however, that they were not influential in more subtle ways such as suggested by Table 2.

Jennifer was in the initial stages of her teaching career and hence at a stage described by Frykholm (1999, p. 102) as representing a "window of opportunity" during which she was reassessing her early experience and beliefs in relation to the classrooms she had encountered and the professional learning opportunities that had been presented. Jennifer appeared to be experiencing tension between her beliefs about mathematics, formed as a result of her experiences at primary and secondary school and university, and her evolving beliefs about how the subject is best taught. How one possible mechanism by which this tension could be resolved would be for her to construct separate clusters of beliefs about mathematics as a discipline and mathematics as a school subject, resulting in a belief structure similar to Sally's that would allow her to maintain her existing beliefs yet adopt a teaching approach consistent with a different set of beliefs about the nature of mathematics. Alternatively, she could fundamentally change her beliefs about mathematics as a discipline from a largely Platonic orientation to a problem solving view. In the absence of any impetus or assistance from her practice or from professional learning to engage in this reconstructive activity it appears to be a more difficult and less likely possibility.

8.3 Research question 3: What implications for practice might any such disparities have?

Rather than teachers' beliefs about the nature of mathematics as a discipline necessarily influencing their teaching in theoretically consistent ways it appears that, where there is a difference, these beliefs interact with those they hold about school mathematics to influence their beliefs about teaching and learning. Of course if there is no difference between a teacher's beliefs about the school subject and discipline then greater consistency with practice could be expected (bearing in mind other possible reasons for apparent inconsistencies (e.g. Beswick, 2003; Liljedahl, 2008)). The cells in Table 2 suggest ways in which distinct beliefs about the discipline of mathematics and school mathematics might reasonably interact to influence beliefs about teaching and learning mathematics. The cases presented here relate to two of the nine cells and, hence, this study has provided only partial evidence of the empirical validity of these conjectures.

8.4 Implications for professional learning

One possible interpretation of the cases presented here is that university study does not necessarily help prospective teachers to develop a contemporary understanding of the nature of their discipline. Jennifer's university experience was more recent than Sally's and there is evidence that she had acquired some understanding of contemporary developments but that this had been added on to her existing beliefs constructed from earlier schooling experiences rather than transforming her view. It is impossible to know whether or not Sally experienced a struggle similar to Jennifer's either during her university studies or the early part of her teaching career, but many years in the profession had focussed her attention firmly on mathematics as a school subject removed from the broader context of the discipline. It was experiences in the profession, and particularly her years as an SEO (Mathematics), that seemed most influential.

Much attention has been paid in the mathematics teacher professional learning literature to teachers' content and pedagogical knowledge with the latter a particular focus for secondary teachers with relatively strong backgrounds in mathematics. This study suggests that more attention needs to be paid to the beliefs about the nature of mathematics that the teachers have constructed as a result of the cumulative experience of learning mathematics in primary and secondary school, and university, and, for experienced teachers, from years of involvement in the profession. Experiences that cause teachers (including both pre-service and experienced teachers) consciously to juxtapose conflicting beliefs held in isolated clusters (Green, 1971) about mathematics as a discipline, mathematics as a school subject, and the ways in which they are being encouraged to teach it, and that can assist them to reconceptualise the discipline of mathematics may assist some teachers to reconcile their various beliefs and to develop practice that is founded on deep understandings of what "mathematics is really all about" (Hersh, 1986, p.13, cited by Thompson, 1992).

Further research is necessary to confirm or contradict these ideas, but recognition that at least some teachers have different beliefs about the nature of school and mathematicians' mathematics may go some way to explaining apparent inconsistencies among teachers' beliefs about mathematics and its teaching and learning and the apparent instability of beginning teachers' commitments to contemporary mathematics teaching (e.g. Ball, 1990). More importantly, it points to a crucial and largely missing element in current professional learning efforts, that of focussing on the views of mathematics that underpin the teaching approaches being recommended, and highlights early career experience as a key time during which teachers could be better supported to work through conflicts in their belief systems and to undertake the onerous work of significant re-conceptualisation.

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