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Working like real mathematicians: developing prospective teachers' awareness of mathematical creativity through generating new concepts

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Abstract This paper describes the experience of a group of 17 prospective mathematics teachers who were engaged in a series of activities aimed at developing their awareness of creativity in mathematics. This experience was initiated on the basis of ideas proposed by the participants regarding ways creativity of school students might be developed. Over a period of 6 weeks, they were engaged in inventing geometrical concepts and in the examination of their properties. The prospective teachers' reflections upon the process they underwent indicate that they developed awareness of various aspects of creativity while deepening their mathematical and didactical knowledge.

Keywords Creativity in mathematics · Prospective mathematics teachers

1 Introduction

It is widely agreed that mathematics students of all levels should be exposed to thinking creatively and flexibly about mathematical concepts and ideas (e.g., NCTM, 2000). To that end, teachers must be able to design and implement learning environments that support the development of mathematical creativity. Unfortunately, working with in-service mathematics teachers reveals that although most of them express appreciation for developing creativity, many teachers do not possess the abilities needed to integrate activities that foster students' creativity, mostly due to lack of prior experience or proper college preparation (Shriki, 2005). Various studies point to the fact that teachers tend to teach the way they were taught in school (e.g., Pehkonen, 1997). Consequently, there is a need for purposeful preparation in order to develop teachers' ability to implement innovative teaching approaches that encourage creativity.

One of the things I attempt to do within the mathematics Methods course I teach to prospective teachers (PTs) is to develop their awareness of creativity in mathematics. This

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is done through self experience in various environments that encourage and support the development of such consciousness.

In this paper, following a brief literature review, I present and then analyze the experience of a group of PTs whom I engaged in a series of activities designed to develop their awareness of the complexities and the multifaceted nature of mathematical creativity. This series of activities included attempts to generate new geometrical concepts and discover their properties.

2 Literature review

In his book *A Whole New Mind*, Daniel Pink (2005) argues that we are at the forefront of a new age in which skills such as creativity, intuition, and the ability to associate apparently unrelated objects and events into something new and different are becoming increasingly important. In this section, I relate to some issues that are associated with creativity—its disputed definition, its appropriate place within the educational system, and its assessment.

2.1 The nature of creativity in general and in mathematics in particular

Many researchers have struggled to define the characteristics and meaning of "creativity". A review of the literature about creativity in general and mathematical creativity in particular reveals various definitions, characterizations, interpretations, and approaches to this question. In his examination of the research literature that has attempted to define creativity, Mann (2006) shows the lack of an accepted definition and presents more than 100 contemporary definitions. Due to the complex nature of creativity, most of these extant definitions are vague or elusive (Sriraman, 2005).

In what follows, I distinguish between definitions of creativity that relate to the final product and those that relate to the creative process. Further aspects concerning the creative process will be considered in Section 4.

Creativity in terms of the product When referring to creativity, many researchers are concerned primarily with the final product. For instance, creativity involves the use of common cognitive processes to result in original and unusual products (Weisberg, 1993); creativity is the ability to produce unexpected, original, and useful work (Sternberg & Lubart, 2000); creativity is the production of a novel or remarkable response to a given problem (Torrance, 1974); creativity is the production of an outcome that is both novel and useful, as defined within a certain social context (Plucker & Beghetto, 2004). More specifically in the context of mathematics, creativity of students is defined as having "an unusual ability to generate novel and useful solutions to simulated or real applied problems using mathematical modeling" (Chamberlin & Moon, 2005, p. 38). The authors suggest that mathematical creativity is revealed when a nonstandard solution is created for solving a problem that may be solved using a standard algorithm. Sriraman (2009), however, argues that in the context of creativity in mathematics, "the results of creative work may not always have implications that are "useful" in term of applicability in the real world...it is sufficient to define creativity as the ability to produce novel or original work" (pp. 14–15).

Many of these definitions of creativity are relevant only with respect to adults who are considered as being at the forefront of a certain domain (Chamberlin & Moon, 2005). Consequently, creativity in mathematics is often regarded as the exclusive domain of professional mathematicians (Sriraman, 2005). The question is therefore whether creativity means purely the discovery of an original result, or perhaps a student's discovery of a known result or an innovative mathematical strategy can also constitute creativity. Since defining creativity merely in terms of novelty and usefulness is not practical for the identification and development of the creative thinking processes of school students, some researchers distinguish between professional- and school-level creativity (e.g., Sriraman, 2005). Eminent mathematicians such as Jacques Hadamard (1945) and George Polya (1954) argued that the only difference between the work of a mathematician and that of a student is their degree. In other words, each operates at his or her respective level, and we should recognize that students are also capable of generating a creative product.

Creativity in terms of the process Some researchers refer to creativity using expressions that relate to cognitive abilities such as aptitude, approach, and knowledge (Sternberg & Lubart, 1996), or to conceptual thinking abilities that involve flexibility, fluency, and originality, as well as engaging in nonalgorithmic decision-making (Ervynck, 1991).

Although several researchers perceive creativity as a general ability (e.g., Beghetto & Kaufman, 2009), others view it as stemming from the interactions between a specific domain and an individual (e.g., Csikszentmihalyi, 1999). Neumann's (2007) literature review points to similarities across disciplines with respect to the mechanism and psychology of creativity as well as personality traits that include "a high valuation of esthetics, a broad range of interests, an attraction to complexity, and the ability to deal with conflicting information" (p. 202). This is in opposition to the perception that being creative in mathematics is not the same as being creative in other disciplines due to each domain's specificity (Milgram & Livne, 2005).

2.2 Creativity and education, conditions for nurturing creativity

Although most teachers would agree that it is important to develop students' creativity, the literature indicates that creativity is normally not encouraged at schools (Sriraman, 2005). Unlike professional mathematicians, who are often engaged in problems that are full of uncertainty, at school level, most curricular and pedagogical approaches hardly ever offer students this open-ended view of mathematics, avoiding opportunities to engage students in ill-posed or open-ended problems, thus preventing them from any extended period of independent work on these types of problems. When students are requested to apply several methods systematically in order to solve a problem, they do not have to employ any creative strategies or thoughts (Haylock, 1987). In order to encourage the development of mathematical creativity, teachers first and foremost "need to be prepared to recognize, cultivate, and draw out creative potential of all students" (Beghetto & Kaufman, 2009, p. 41). For that purpose, teachers should engage students in creative exploration, without being limited to rule-based applications that prevent recognizing the essence of the problem to be solved (Mann, 2006), and encourage their students to take intellectual risks for sharing their exceptional mathematical insights (Sriraman, 2009). Students should be provided with problems whose solutions are not immediately known and in which they must therefore use more than merely routines and algorithmic processes (Haylock, 1987). In such cases, through teachers' support and guidance, students have the opportunity to integrate their fragments of knowledge to form a coherent view of the situation and employ various processes that at first glance appear to be unrelated (NCTM, 2000). Students should also be provided with opportunities to design and answer their own problems (Mann, 2006). Refraining from development of creativity in the classroom conveys the impression that mathematics is merely a set of skills and rules to memorize, and in doing so, many students' natural curiosity and enthusiasm for mathematics might vanish.

Teachers should encourage mathematically creative students to share their ideas and insights (Sriraman, 2005). Examining the works of creative individuals, it appears that some of their breakthroughs depended on collaboration and social support (John-Steiner, 2000), and exploring employees in a large, creative organization reveals the importance of being stimulated by interaction with colleagues (Neumann, 2007). Such interaction might support the development of creativity since thoughts are often inspired by talking to people and exchanging ideas. In a study that was designed in order to gain an insight into the nature of creativity, Sriraman (2009) interviewed five prominent mathematicians. One of his main findings relates to the fact that "all the mathematicians acknowledged the role of social interaction in general as an important aspect that stimulated creative work" (p. 20). Furthermore, communicating about mathematics, discussing mathematical ideas, sharing thoughts, and justifying reasoning to classmates can enable students to reflect on learning as well as to organize and consolidate their thinking about mathematics (NCTM, 2000).

2.3 Assessment of creativity

The significance of creativity in school mathematics is often minimized because it is not formally assessed on standardized tests, which are designed to measure mathematical learning (Chamberlin & Moon, 2005). The problem with relating to students' works as "creative" is rooted in the definition of creativity as a useful, novel, or unique product (Beghetto & Kaufman, 2009). Referring to this problem, the researchers raise questions such as: "What level of novelty and meaningfulness is necessary for something or someone to be considered creative?... Does student discovery of a hitherto known result also constitute creativity?" (p. 40). Although according to the traditional view of creativity students' works would not be considered creative, the researchers argue that students' self discovery may still be considered creative if we examine the issue of creativity from a personal point of view, namely, whether the students' discoveries were new for them.

The assessment of creativity also relates to self evaluation. Specifically, although it is important that teachers challenge students with assignments that require mathematical creativity in order to be able to assess their creativity, it is no less important that teachers develop their students' ability to assess their own creativity. Teachers should discuss and negotiate with their students issues that concern the evaluation of their own thinking processes as well as the products themselves. Such self evaluation in itself requires creativity and will, in turn, enable students to refine their solutions in successive iterations (Chamberlin & Moon, 2005).

Nevertheless, it should be noted that I could not find specific recommendations or guidelines aimed at providing teachers with an assessment tool for evaluating students' creativity, with respect neither to the process nor the product.

In summary, given the complex nature of creativity and its meaning, it is plausible to assume that teachers' ability to initiate learning environments that nurture students' mathematical creativity necessitates purposeful support and guidance. This guidance should facilitate the PTs in consolidating their view and beliefs about the various aspects that relate to creativity because "what teachers believe is a significant determiner of what gets taught, how it gets taught, and what gets learned in the classroom" (Wilson & Cooney, 2002, p. 128). It is therefore suggested that teacher education programs should develop PTs' awareness of the various issues associated with creativity and provide them with appropriate tools for fostering their future students' creativity (NCTM, 2000).

3 Developing prospective teachers' awareness of mathematical creativity: the case of one prospective teachers group

In this section, I present data concerning the course of a study aimed at enabling a group of 17 PTs, studying toward a B.A. degree in mathematics and earning a teaching certificate for middle- and high-school mathematics, to work like "real mathematicians" and gain new insights into the meaning and complexities of mathematical creativity. The Methods course in which the study took place and the methodology used for data collection and analysis are also presented.

3.1 The study

As in other mathematics Methods courses I have been conducting, this course started with a discussion on the image of a "good teacher" (for elaboration, see Section 3.2) and continued as a three-phase process over 6 weeks (Shriki, 2008). In the first phase, the PTs discussed the issue of creativity and were then asked to invent and name a new geometrical concept. In the second phase, they presented and discussed their ideas in class, and in the final phase, they worked on discovering some of the properties of the new defined concept and formulating relevant theorems, and again presented their product and discussed their ideas.

Given that the ability of teachers to be reflective is linked to their ability to connect thoughts and actions (Wilson & Cooney, 2002), to become aware of viewpoints underlying their mathematical performance in terms of what it means to solve problems and to reason (NCTM, 2000), and to develop new ways of understanding what it means to engage in and teach mathematics (Simon, 1994), throughout the process, the PTs reflected on various aspects of creativity.

In this paper, the focus is mainly on the development of the PTs' awareness of creativity in mathematics, in terms of the creative process and product, along with partial findings that concern their development of mathematical and didactical knowledge.

3.2 The setting: mathematics methods course

The objective of this two-semester second year course is to introduce students to didactical methods and approaches for teaching middle-school geometry (first semester) and algebra (second semester). The research was carried out during the first semester only. This course is the PTs' first mathematics Methods course and is taken in the second year of their studies (out of four). In the previous year, besides learning in basic mathematics courses, the PTs attended a course in which they learned the basic principles of designing a lesson, experienced peer teaching, and were instructed on how to reflect on their experience through following specific guidelines and instructions. The discussed Methods course follows the Israeli Ministry of Education guidelines for middle-school mathematics teaching that emphasizes comprehension of mathematical processes and mathematical–logical thinking by teaching students practices such as making assumptions and arriving at conclusions and generalizations.

Because these guidelines include neither recommendations for specific teaching approaches nor teacher education suggestions, teacher educators are free to create their own. Consequently, I chose to plan the Methods course according to the rationale, principles, and recommendations of the NCTM Standards (2000). In this program, the PTs are provided with opportunities to engage in mathematical tasks designed to help the PTs rethink their conceptions of what mathematics is, how it is learned, and how it should be taught. These tasks include an inquiry approach to learning, implementation of the “what if not?” strategy (Brown & Walter, 1990) for problem posing and solving (Lavy & Shriki, 2007), computerized settings, project-based learning (Lavy & Shriki, 2006), discussing students’ misconceptions and how to manage them, and more.

While working on these tasks, the PTs are encouraged to make assumptions, arrive at conclusions and generalizations, and discuss pedagogical and mathematical issues that spontaneously emerge from their experience. Throughout the process, the PTs are asked to reflect (both orally and in writing) on their experiences. Specific guidelines for reflection are provided for each task. For example: How did you decide which problem to pose? What were your indecisions while solving the problem? What are the new things you have learned during your experience? And so forth.

During the first lesson of the Methods course, I ask the PTs to produce a list describing their perception of the image of a good teacher. Most PTs usually refer to affective aspects such as being patient and humane, cognitive aspects such as the ability to solve every mathematical problem, and didactical aspects such as knowing how to explain things well and encouraging students to ask questions (Lavy & Shriki, 2008). Subsequent to the production of the written lists, all of the PTs’ statements are written on the board. We then analyze the statements, clarify their meaning, and delve deeper into it in an attempt to establish a common language for the course. This discussion is tape-recorded. The PTs’ lists are kept until the final lesson of the course, when the process and discussion are repeated. After producing new lists, the PTs are given back their initial lists and are asked to identify differences between the two lists and reflect on the learning experiences that affected the changes in their perceptions. Finally, we discuss their reflections and suggest implications for school teaching. I use the gained data for implementing modifications in the content and approaches employed in the course.

3.3 Data collection and analysis methods

3.3.1 Data sources

In order to examine the development of the PTs’ awareness of creativity in mathematics, three types of data were collected for analysis: (1) classroom audiotapes of the PTs’ statements during the sessions in which we discussed the issue of creativity; (2) the PTs’ mathematical work, in which they were engaged in inventing a new geometrical concept and discovering its properties; (3) the PTs’ written records of their thoughts, as they emerged during their experience, their trials and insights, conflicts and confusions, and their reflection on their entire experience, and on the new things they had learned or discovered about themselves.

3.3.2 Data analysis

Analysis was ongoing, and it continuously informed the data-gathering process. Namely, in each phase of the study, new issues were introduced in class, based on the analysis of the data that were gained in the previous phase. Following analytic induction (Taylor & Bogdan, 1998), the entire corpus of data was reviewed and analyzed to identify themes and patterns and generate initial assertions regarding the focal points that emerged. These themes were compared to theories that concern creativity in general and creativity in mathematics. The analysis revealed four main themes: (1) development of the PTs’ awareness of creativity in mathematics, (2) development of mathematical knowledge, (3) development of didactical knowledge, and (d) development of a new perception as to the nature of mathematics.

3.4 Session 1: discussing creativity

Although my experience with teaching the Methods course has indicated that PTs relate to creativity only toward the end of the course —as the course has introduced them to various innovative learning environments (see Section 3.2)—in one class, creativity was discussed in session 1, as PTs were discussing their ideas of the good teacher. Below is a verbatim report of selected sections of the classroom discussion[1]. (T denotes teacher, and PT denotes prospective teacher):

T: *You mentioned that a 'good teacher' is one who is concerned with developing his or her students' creativity. What do you mean by 'creativity'?*

PT_1: *I think it is the ability to solve non-routine problems.*

PT_2: *Wait, I don't fully understand your question. Do you mean 'creativity' in general or just 'creativity in mathematics'?*

T: *This is a very good question. What do you think—is there any difference between them?*

PT_2: *Sure! I don't think that every creative painter or musician is also a creative mathematician, and vice versa.*

I suggested limiting the discussion to "creativity in mathematics" and asked them what they considered as creativity in mathematics:

PT_3: *Providing original proofs to standard problems.*

PT_4: *Raising and implementing imaginative ideas.*

PT_5: *Inventing new theorems and theories.*

PT_6: *To be a genius!*

At this point, several PTs started talking aloud, expressing their reservation about defining creativity in mathematics as belonging only to geniuses. Then one of them said:

PT_7: *Genius is something that is innate. Creativity is something you can develop with proper guidance.*

The PTs then shared their past memories as school students, brought up examples from lessons in which substitute teachers presented them with nonroutine problems where the best students were not necessarily the first to solve the problem. The PTs maintained that because these problems did not require algorithmic thinking but rather creative thinking, other students were able to solve them.

We continued the discussion, examining issues such as: Why is it important to be mathematically creative? Do you consider yourself to be mathematically creative? How can you recognize a mathematically creative student? How can you determine whether

[1] The quotes were carefully translated from Hebrew, trying to maintain their intentions. Due to space limitations, I chose to include only excerpts that have a direct relevancy to the main issue of this paper.

a certain product is an outcome of creativity? Finally I asked the PTs to suggest ideas for promoting the mathematical creativity of students. After a few seconds of silence, one of them said:

PT_9: *It's very difficult to answer this question, because we didn't experience it at school when we were students ourselves, or even here at this college. I guess our teachers had no idea either….*
(Humming of approval...)

PT_5: *I think we should let our students work like real mathematicians do.*

T: *What do real mathematicians do?*

PT_5: *Build theories.*

T: *How do they build theories?*

PT_{10}: *They formulate concepts, axioms, and theorems, and base their theory upon them.*

T: *That's interesting. Let's try to work like real mathematicians do. Let's try to formulate new concepts, raise conjectures, and prove or refute them.*

PT_{11}: *What do you mean? We are not Pythagoras…*

T: *And what do you mean by that?*

PT_{11}: *I don't believe students are the ones who have to discover new concepts, theorems or regularities. All these can be found in the books. The students have to acquire knowledge. Part of this process of acquiring new knowledge is expressed by the students' ability to prove known theorems. Not to invent or discover them.*

T: *O.K., let's focus on geometry. Can you think of a new geometrical concept, one you had never heard of before?*

PT_{11}: *I believe mathematicians have already invented everything they could think of…Who are we to challenge them?...*
(Laughter... Humming of agreement...)

This discussion sheds light on two issues particularly relevant to how mathematics is learned and should be taught, as perceived by the PTs. The first pertains to the nature of mathematics as a closed or open domain and the second to the nature of creativity.

The nature of mathematics and its teaching Many PTs view mathematics as a closed domain. They assume that all possible concepts have already been invented by mathematicians, and mathematicians are the only ones capable of contributing to the domain. Given this approach to mathematics, it is plausible that as teachers these PTs will refrain from encouraging their students to invent new concepts and theorems and generate new ideas. If, however, mathematics is perceived as an open and flexible domain, teachers might be motivated to challenge their students and inspire them to contribute to this field. Moreover, being inexperienced and having no opportunity to develop their own creativity (Haylock, 1987; Mann, 2006; Sriraman, 2005), the PTs assume only professional mathematicians can be creative. Thus, it may be conjectured that by exploring the world of mathematics and developing awareness of creativity, young teachers will be able to encourage their students to develop their own creativity (Pehkonen, 1997).

The nature of creativity Drawing on the PTs' discussion, I infer that for them creativity is marked by three indicators. First, and in line with earlier findings (e.g., Chamberlin & Moon, 2005; Plucker & Beghetto, 2004; Sternberg & Lubart, 2000; Torrance, 1974; Weisberg, 1993), they regard creativity in terms of the final product: providing original proofs to standard problems, raising and implementing imaginative ideas, and inventing new theorems and theories. Second, and in line with other results (e.g., Csikszentmihalyi, 1999; Milgram & Livne, 2005), they refer to creativity as domain specific rather than as a general ability. Finally, they distinguish between being creative and being a genius and view creativity as developing through proper guidance such as solving nonroutine problems. Though they are unaware of work of mathematics education researchers, their observations are in line with Hong and Aqui's (2004) distinction between the academically gifted and the creatively talented and Chamberlin and Moon's (2005) literature review indicating that mathematically gifted students sometimes prefer speed over creativity.

The above conversation inspired me to take a creative position myself and revise the planning of the course. So I continued:

T: *Your assignment for our next meeting is to think of a new geometrical concept, based upon the concepts you are already familiar with.*

The PTs were given a week for generating a new geometrical concept. Given the benefits of sharing ideas with classmates (Sriraman, 2005; Neumann, 2007), observing each other's perspectives and methods, learning how to understand and evaluate the thinking of others, and how to build on those ideas (NCTM, 2000), the PTs were encouraged to work in pairs. The PTs were requested to documents all their thoughts, their trials, failures and hesitations, and the new insights they had gained through their experience. They were also asked to reflect on their entire experience, and on the new things they had learned or discovered about themselves and about teaching and learning.

3.5 Session 2: introducing the generated concepts

At the beginning of the following session, I was curious to hear the PTs' impressions and the insight they gained during the week. It was apparent that the PTs invested time and effort in finding an attractive or sophisticated name for the invented concept (for example, Hexastar, polycurve, Bi-Triangle, 4^{square}, and Chickenfig). All of the defined concepts comprised basic figures such as triangles, quadrilaterals, and circles, and each definition was accompanied by at least one prototypical illustration. In Fig. 1, there are three examples of the definitions suggested by the PTs (before refining them in class) and the illustrations that were scanned from the PTs' works. I chose to present these examples because the final products that were based on these definitions were richer and more comprehensive than the others (see Section 3.5).

Three PTs chose to work by themselves, and the other 14 worked in pairs. Subsequent to presenting their generated concepts, the PTs were encouraged to share their written reflections. The following three quotes refer to the main themes that emerged during the class session:

PT_{11}: *In opposition to what I said last week, I now believe that students have the ability to invent new concepts.... When you gave us the assignment I was a little frustrated, because I didn't know where to begin. I took a textbook and examined some of the concepts in the book. I noticed that they all follow the*

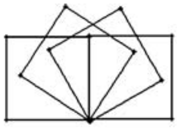
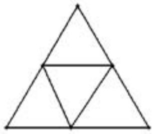
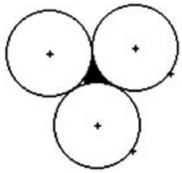
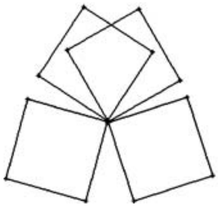
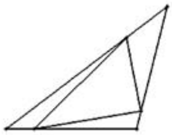
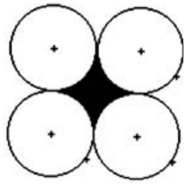
Definition	4^{square} is a figure comprised of 4 congruent squares having at least one common vertex	Bi-Triangle is a figure comprised of two triangles, where one is inscribed in the other	Polycurve is a shape obtained between tangent congruent circles
Illustrations			
			

Fig. 1 Examples of definitions and their illustrations taken from the PTs' works

same procedure—the definition is presented using formal words, then there is a drawing, and finally some of the attributes of the concept are described, followed by proofs. I felt, however, that it would be easier for me to start with sketches, to chose one, and then to define it, although this was not the case in the textbooks. After several trials I arrived at a drawing which looked nice and interesting. I began to wonder how to define it. It wasn't as easy as I thought it would be. I had to break the drawing up into its components, and think about how they relate to each other.... I believe this what you meant by creativity...

PT_5: *I also started with a drawing, but I used dynamic computer software. I begun with a quadrilateral, and tried to build various other figures on its sides. I dragged the figures and rotated them. While doing so I began to think that I would never be able to define the figure I was drawing. Something in the 'dynamism' of the figure bothered me. Shouldn't it be static? Are there several figures on the screen or merely one figure in various positions?...Yes, I believe I was thinking creatively, because the entire process was new to me, and I was the one who asked the questions as well as the one who answered them. There was no one to guide me.*

PT_7: *I don't know if what I did can be considered to be creative thinking, because we didn't actually reach any agreement as to its meaning. But there is no doubt that I was thinking differently. When I started out, I was curious to see whether it was possible to accomplish the assignment you gave us, because it seemed strange at first.... So perhaps students must be motivated by some unusual external stimulus in order to begin thinking differently.*

Although only one week had passed between the two sessions, it appears that the PTs' new experience with the unfamiliar activity motivated them to challenge their views regarding issues particularly relevant to teaching, learning, and creativity, and it also contributed to the development of their mathematical knowledge.

The nature of creativity It is evident that instead of referring mainly to the product, in line with previous works on creativity (e.g., Ervynck, 1991; Sternberg & Lubart, 1996), the PTs primarily referred to their thinking processes and to the manner in which they approached the assignment, considering them as creative thinking. They characterized it as "thinking differently", regulating their own course of thinking, posing problems, asking questions, and searching for proper answers.

In keeping with Cunningham (2004) who showed that when students, rather than the teacher, formulate new problems, a sense of ownership is fostered, it appears that indeed the PTs began to develop self confidence regarding their ability to create in mathematics, expressing a sense of ownership of their product. This in turn resulted in a high level of engagement and curiosity as well as enthusiasm toward the process of learning mathematics.

Development of mathematical knowledge The new experience contributed to the development of the PTs' mathematical knowledge, especially with respect to the meaning of definitions, the nature of mathematical objects (should they be static, dynamic, or perhaps both), and how the components of figures relate to each other.

The PTs mentioned the key role that images played in their process of thinking. All of them started out by drawing a figure, altering it, and when satisfied with their product, trying to define the final figure. Further research is needed to determine the role of drawings as they directed the PTs' thinking processes and their connection to the nature of the assignment.

Mathematics teaching The assignment inspired the PTs to start reflecting on their future role as teachers. In line with Mann (2006) and Sriraman (2005), they regard their role as motivating their students by providing them with some sort of stimulus to begin thinking differently.

The PTs' next assignment was to find as many properties as they could for their new defined concept and to formulate relevant theorems. The PTs were advised to use dynamic computer software in order to facilitate their work and increase the likelihood of arriving at interesting conjectures.

Being aware of the fact that overemphasizing the importance of providing formal proofs for conjectures prevents the raising of what could be regarded as "wild conjectures" (Lavy & Shriki, 2007), the PTs were instructed not to worry about their inability to prove some of their conjectures.

The PTs were given 5 weeks to complete this assignment. As in the previous phase, they were asked to continue documenting their thoughts and course of work. They were also asked to suggest possible implementation of this approach in class, relating to its advantaged and disadvantages, the role of the teacher in such a learning environment, and so forth. In addition, the PTs were also requested to reflect on their experience and its contribution to their professional development as future teachers, with deliberate attention to creativity in mathematics. They were also encouraged to continue consulting each other and exchanging ideas, especially those who chose to work on their own rather than in pairs. During these 5 weeks, we did not discuss the assignment or the subject of creativity in class; instead, we discussed the van Hiele theory and analyzed the school mathematics curriculum in the context of the theory; we identified aids for developing students' intuition toward geometrical figures, their properties, and interconnections (for example, using paper folding and constructions of matches and straws); and we also started to discuss students' misconceptions in geometry, their underlying causes, and how to manage them.

3.6 Session 3: presenting the properties of the generated concepts

A total of ten presentations were made during three whole sessions. While presenting their products, the PTs described their course of work, demonstrated their trials, shared their failures and points of confusion, and exhibited their findings. The PTs were encouraged to ask for their classmates' advice in cases where they felt they were "stuck", and some of them were provided with interesting ideas for developing their products.

Most of the theorems they formulated referred to special lines (in particular "heights", "medians", and "diagonals", as redefined for the new concept) of the figure they chose to define, to its circumcircle or inscribed circles, and to relationships between areas and perimeters. The following are some examples taken from the PTs' works (for definitions, see Fig. 1).

The pair of PTs who invented the "4^{square}" discovered seven theorems that relate to the concept. One of them relates to the fact that corresponding vertices of the squares lie on the same circle (see Fig. 2a, scanned from the PTs' work), and they found their radii. This pair of PTs also discovered that any two adjacent squares constitute a kite (ADJG in Fig. 2b). Designating ∠DAE as α and the length of the square's side as a, they found that the area of the kite depends on α in the following manner: $S_{\text{ADJG}} = \frac{a^2 \cdot \sin\left(45 - \frac{1}{2}\alpha\right)}{\sin\left(45 + \frac{1}{2}\alpha\right)}$. They also presented a graph that describes the relation between α and the kite's area.

The pair of PTs who invented "Bi-Triangle" discovered six theorems, three of which related to what they termed as "regular Bi-Triangle", where the two triangles are equilateral. They discovered that in this case the ratio between the areas of the two circumcircle equals the square ratio of the measures of the triangles' sides and that triangles BED, CFE, and ADF are congruent, regardless of the position of the internal triangle (see Fig. 2c). The pair of PTs who worked on the "Polycurve" discovered six theorems. One of the theorems relates to the relation between the radius of each circle, the number of circles, and the defined area, using regular polygons (see Fig. 2d).

Not all of the PTs' conjectures were proven or refuted prior to the presentation. Given the limited time available for the presentations, we agreed to postpone this to a later date.

As can be seen, the PTs did not go too far with their mathematical discoveries. To me, however, the PTs' reflections on the process were no less important than their products since reading their reflections enabled me to learn about the process they experienced, about the change they had undergone, and about the new perceptions they had developed. The following are a few excerpts taken from their written reflections. These excerpts were chosen because they represent most of the PTs' utterances (see Table 1) and capture the various aspects that were associated with the entire process.

PT_{12}: *I didn't take an active part in either of the first two lessons in which we discussed the issue of creativity. I don't want you to interpret my silence as a lack of interest. I just had no idea what to say. This is the first time that I have undergone such an experience. You gave us an opportunity to see the beauty of*

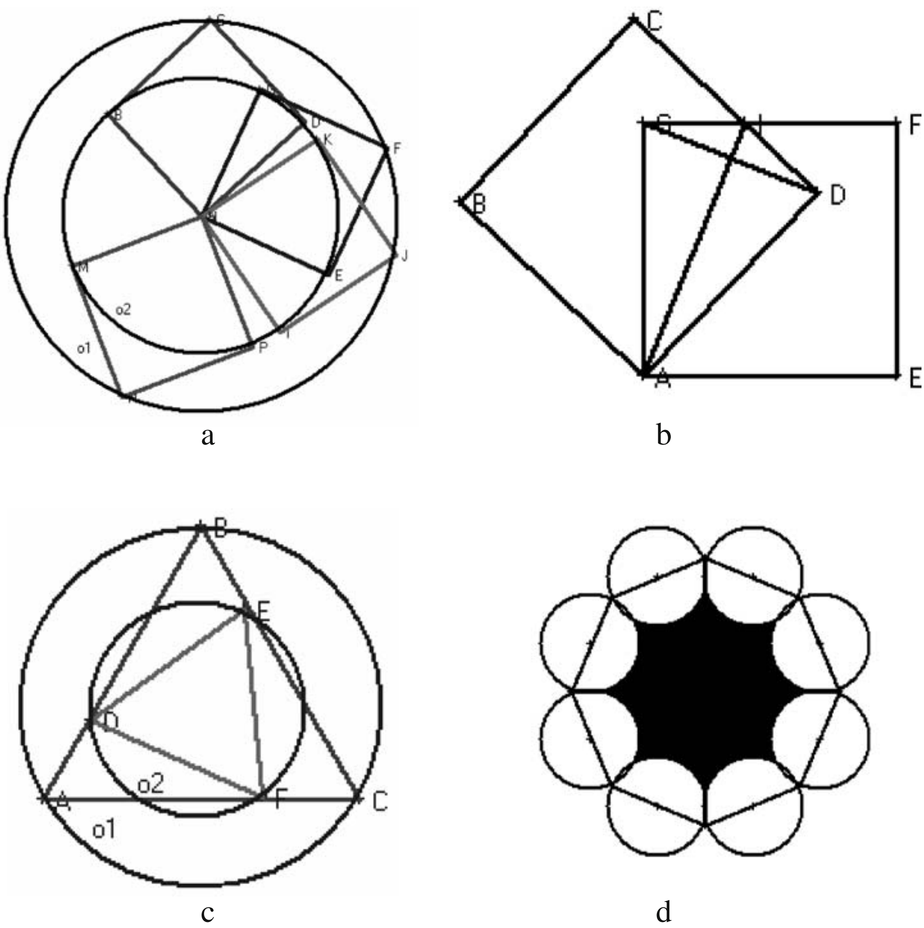


Fig. 2 Examples taken from the PTs' works

mathematics and to develop our ability to do something new, different, and unusual. In that sense, it was really a creative doing…

PT_7: *I think that working creatively is concerned with fun. For me the entire process was enjoyable and exciting, and this is what motivated me. Isn't that what creativity is about? I think it is. Enjoy while you create and love your creations. These are the feelings I want my students to have...*

PT_9: *I feel like I am in the middle of the process, and I need more time in order to complete the assignment. I still have many things to explore and discover about my "QUADZOID". I know I will keep trying to prove my conjecture, although I have already submitted my work. I will do it not for the grade, but for myself. I am really curious to know whether it holds for all QUADZOIDs...and curiosity is very much like creativity. People cannot be creative unless they are curious about what they do.*

PT_8: *When we first discussed the importance of developing our future students' creativity, I thought to myself "Yes, it is important. But what does it really*

Table 1 The PTs' evolvement of perceptions regarding creativity, mathematics, and the teaching of mathematics

The nature of creativity and mathematical creativity			The nature of mathematics and its teaching		
First session	Second session	Third session	First session	Second session	Third session
Emphasis on the final product	Emphasis on the process: asking questions, searching for answers (16); posing problems (15); thinking differently (14); new/unfamiliar (12); regulating the course of thinking/self guidance (10); motivated by curiosity (8)	Emphasis on affective aspects: The creative process is accompanied/motivated by positive emotions like fun and enjoyment (11); excitement (9); curiosity (8); self-fulfillment (8)	Mathematics as a closed domain	Students are able to formulate new concepts and problems (11)	Mathematics is full of beauty/patterns/ surprises (13)
Creativity as domain-specific		The process: new/unfamiliar/ different (15); necessitates taking risks (8)	Only mathematicians are able to invent mathematics	Students can be creative in mathematics (11)	Students can produce new/different/unusual things in mathematics (mathematics as an open domain) (14)
Creativity can be developed through proper guidance		The product: new for oneself (12); a result of one's own thinking (10); sense of ownership of the product (9)	Teachers should provide problems, and the students should solve them	Teachers should motivate students to pose their own problems(10)/to think differently (9)	Teachers should inspire students to develop their own creativity/ stimulate students' thinking (14)

mean? How can we do that?"... It seemed to me like a vague discussion... Now I wonder - was I creative? Can my work be characterized as being creative? I thought about it a lot, and I realize that I don't really need a clear definition of creativity. I enjoyed the process, it was new to me, and therefore I believe that my product is a creative product. Just because it is my own work, a product that is a result of my way of thinking. It was truly working like real mathematicians!

PT_{11}: *I remember being frustrated during the first lesson. I believe it was because I didn't have a clue as to where to begin.... So at that point I could chose between giving up and picking up the glove, taking the risk of daring and perhaps failing. Actually, I had no choice but to follow the latter option.... Once you decide to take such a risk, however, nothing can stand in your way. This is what creativity is about...*

The above PTs' reflections indicate that they developed awareness of the meaning of being creative in mathematics. Their new perceptions can be divided into two main aspects. The first concerns the "doing" itself as well as the creative product and the second relates to affective characteristics. It should be noted, however, that four PTs did not share these insights (see Section 5).

The nature of the creative doing and the product The PTs emphasized the fact that they did something new, different, and unusual. It should be noted that they referred to the doing itself as unusual and not to their product; in other words, they highlighted the process rather than the product. In fact, most of the PTs did not relate to their product in an evaluative manner but mostly felt pride of their ability to produce their own creation. The fact that their product was new to them was enough for the PTs to consider it as a creative product, a result of their own creative thinking. This view is in line with the arguments presented by Beghetto and Kaufman (2009) and Sriraman (2009).

Affective characteristic of creativity The PTs' reflections were manifested by the use of emotional expressions such as fun, enjoyment, and excitement. Curiosity was perceived by the PTs as being essential for stimulating creativity and motivating the creative doing. In line with Goldin (2008) who considers emotional feelings to be fundamentally important in mathematical learning, it appears that the emotions that surfaced during the work on the assignment and the PTs' self-awareness of their emotions were the driving forces behind their actions. In turn, "some aspects of the structure of mathematics, as a disciplined way of generating knowledge and as a traditional school subject, can raise high affective barriers to students' curiosity and inventiveness." (Goldin, 2008, p. 1). Indeed, research on the role of emotion in the context of creativity (e.g., Fredrickson & Branigan, 2001) suggests that positive emotions such as joy encourage individuals to engage persistently in their environment and partake in its associated activities. Such emotions also support the broadening of one's available repertoire of cognition and actions, thus enhancing creativity.

3.7 Summary of the results

Table 1 summarizes the main issues that were discussed by the PTs during the three phases of the study, namely, creativity in mathematics, the nature of mathematics, and the teaching of mathematics. The table includes only issues that were referred by at least half of the PTs. In the parentheses appears the number of PTs who related to each specific issue. Numbers are not included for the first session since no reflection was carried out during that lesson.

As can be seen from Table 1, at the beginning of the process, while concentrating mainly on the creative product, the PTs connected their perceptions with the image of mathematics as a closed domain, where only mathematicians can be productive. In this sense, the teachers' role is to provide students with "final products". In the second session, subsequent to their initial experience in generating new concepts, they emphasized the creative process and its characteristics. Their new insights inspired their perceptions regarding the role of the teachers as motivating the creative process and developing the ability of students to "generate" mathematics. Finally, by the end of the process, the emphasis is also on the positive emotions that accompany the creative process, and at the same time mathematics is perceived as an open domain to which everyone can contribute and feel a sense of ownership of the product.

It should be noted that the PTs referred to the development of their mathematical knowledge mainly in the second session, when they had to invent a new concept (see Section 3.4). Examination of the PTs' products reveals that they were indeed based on fundamental geometrical knowledge. Therefore it can be maintained that the PTs mainly developed their "meta-mathematical" knowledge regarding the meaning of mathematical definition.

4 Discussion

Reid and Petocz (2004) recognize that "it is a fairly difficult exercise to discern what is meant by the term 'creativity', or to decide what may be interpreted as a 'creative' object, or to describe the cognitive traits that characterize a 'creative' person" (p. 46). Furthermore, the question is what actually has to be creative—the person, the idea, or the object? Given such complexity and vagueness, Reid and Petocz believe that an attempt to foster creativity in learning may be equally difficult. It is, however, as they believe, important to examine the discussion on creativity and see which aspects are important to teacher educators.

Considering these issues, and believing that teachers should view the creativity of students as inherent in learning (Beghetto & Kaufman, 2009), I looked back at the process the PTs experienced and asked myself whether they managed to develop awareness of mathematical creativity and of its multifaceted nature and in what ways the learning environment was supportive. Since the experience was very short and long-term possible impacts have not yet been inspected, the examination of the process is based on the PTs' documentations, products, quotes, and reflections, as well as on the characteristics of the learning environment.

4.1 The learning environment

Creativity is difficult to develop if one is limited to rule-based applications and does not recognize the essence of the problem to be solved (Mann, 2006). According to Mann, many teachers emphasize algorithms, speed, and accuracy rather than working toward developing their students' creative applications. Failing to encourage creativity in mathematics classrooms implies missing an opportunity to develop the students' mathematical understanding fully. Indeed, the framework of the learning environment of the Methods course enabled the PTs to work freely, providing the PTs the opportunity to pose their own problems (Haylock, 1987); there was no "correct" solution or correct answer, and the assignments were designed to enable the PTs to adopt a self-directed and self-assessed form of learning, and they were not forced to follow any specific sequence of rules, algorithms, or instruction. Such learning contributes to the development of awareness of creativity in mathematics (Chamberlin & Moon, 2005). The PTs' engagement in the process generated internal motivation, interest, and curiosity, and as research shows, the greater the inner motivation, the greater the plausibility of creative discoveries (e.g., Starko, 2001). Finally, the entire process was accompanied by reflection, which, as conjectured by Sriraman (2009), supports the development of awareness of mathematical creativity.

4.2 Developing awareness of the nature of creativity

As mentioned in Section 2, creativity can be examined either through the lens of the value of the product or through the lens of the process. In the current study, the PTs reflected mainly on the process.

The product Various researchers believe that creativity is a process that results in an outcome that must be universally recognized. I believe, however, that in order to develop creativity in mathematics, the outcome must first be appreciated by the learners themselves. Clearly, in our case, the PTs' products are of no value to the community of the mathematicians. The important point is that they were given the opportunity to work "like mathematicians" and, as they claim, to develop curiosity toward the possibility of generalizing their findings and to refer proudly to their products in terms of "unusual", "different", "new for me", and "a result of my own way of thinking".

The process and associated affective characteristics The PTs were emotionally involved in the process and mentioned fun, enjoyment, excitement, and curiosity as portraying their involvement in the process. The PTs also emphasized the importance of readiness to take risks and its role in creative doing. Enjoyment, curiosity, and taking risks are all associated with the development of creativity, as elaborated in the following paragraph.

Enjoyment is essential for capturing students' interest, developing their talent (Csikszentmihalyi, Rathunde & Whalen, 1993), and motivating them to develop their creativity (Starko, 2001). The PTs' reflections indicate that enjoyment and creation are interwoven and stimulate each other. The effect of positive emotions on improvement of performance and creative problem solving is well grounded in the neuropsychological literature (e.g., Ashby, Isen & Turken, 1999). Unfortunately, it does not always happen that enjoyment is achieved by students in learning mathematics, as raised by Mann (2006) who found that many prospective elementary school teachers describe their memorable childhood experience in school mathematics as unpleasant.

Curiosity and creativity are frequently used as synonyms by teachers when describing students, although each term bears its own characteristics in the context of teaching and learning (Hensley, 2004). Curiosity can be defined in terms of what it produces, namely, exploration (Perry, 2001), since open-ended learning experiences, in which the goal is not a single correct answer, foster an environment of inquiry, which in turn produces curiosity (Church & Ravid, 2003). Curiosity emerges from the desire of students to know "why" (Hensley, 2004) and in return enables them to create new knowledge from personal meaning.

The readiness to take risks is related to emotional aspects as well. Fear of risk might suppress curiosity, creativity, and the willingness to explore new things (Perry, 2001), and as history shows, intellectually courageous moves often lead prominent mathematicians to their findings (Movshovitz-Hadar, 2008). Educational researchers (e.g., Silver, 1997) discuss the need to encourage students to take risks while engaging in problem-solving activities. Taking risk means, among other things, not being intimidated by making mistakes and is therefore essential in order to develop creativity (Mann, 2006). Although accuracy in mathematics is important, strict emphasis on accuracy when assessing a child's conceptual understanding of mathematics discourages risk taking in applying one's knowledge and creativity in the development of original applications when solving a problem.

In summary, it can be concluded that the learning environment, as well as the nature of the assignments, supported and promoted the development of the PTs' awareness of mathematical creativity and its multifaceted nature. The PTs generated their own definition for a creative product as well as identified some of the affective characteristics that are associated with the creative process. The new experience was also beneficial in terms of developing the PTs' mathematical knowledge, particularly with respect to the meaning of definitions, and to their insights regarding their role as future teachers in supporting the development of their students' creativity. The PTs are however only at the beginning of a long and exciting journey during, which they will have to consolidate their views regarding creativity in mathematics, how to nurture it, assess the processes and products of their future students, and more.

5 Final remarks

1. Not all PTs enjoyed the process. Four expressed their reservations about the idea of inventing new concepts, considering it to be useless. Analysis of these PTs' reflections reveals that their reservations about the process can be attributed to two main reasons: (1) high standards of self-judgment in examining the products ("In any case, what I will invent or discover will be worthless") and an inability to moderate their expectations from themselves ("I would appreciate myself much more if I would solve a difficult problem than if I would just invent some new concept") and (2) a fear of taking risks, which is essentially an inherent part of working within an environment that does not impose strict rules or procedures and does not necessarily guarantee immediate success ("Even if I will discover something, there is no guarantee that it would be something significant...It might be that I will not be able to prove my findings"). Further research is needed to determine whether these are indeed prohibitions of creativity in mathematics, and if so, whether they can be attributed to personal characteristics or to circumstances imposed by the nature of the assignments.
2. Although developing the PTs' ability to examine the value of a product is essentially part of the broader process of developing awareness of creativity, I chose, at this initial phase of the course, not to discuss with the PTs the value of their products. I believe that they have to acquire additional pedagogical knowledge and some other experiences in order to be able to evaluate their products and compare them with other products that result from other types of experiences. Although the PTs did not express any disappointment when they realized that their products were about to be evaluated only several months later, it would be interesting, however, to repeat the study, this time allowing the PTs to develop criteria for evaluating and assessing their own products and those of their peers. It is plausible to assume that such experience will further develop their awareness of creativity.
3. The PTs' statements voiced at the beginning of the process imply their initial perception regarding the manner in which mathematics is taught and learned in

school, namely, rote learning for the purpose of knowledge acquisition. This view stems from their own previous experience as school students and is originated in what Lortie (1975) describes as "apprenticeship of observation", during which the PTs consolidate their world view regarding teaching and learning. The described experience opened a window to the possibility of teaching differently. In order to implement changes in instruction, teacher education programs need to be changed (Mann, 2006). Understanding the role of creativity in mathematics is an important first step; however, it may be conjectured that changes will only be effective if creativity in mathematics is allowed to be part of the entire educational experience. Specifically, if developing creativity in mathematics is one of the goals, teacher education programs may need to be designed so as to uproot conservative approaches while assimilating others. As research shows, if PTs experience teaching methods that are different from the ones they themselves experienced as students, is it more likely that they will implement them later on in their work as school teachers (Beswick, 2005).

4. Although the field of creativity had been studied extensively and in depth, many questions are still open. With respect to teacher education programs, such questions might refer to aspects of the mechanisms that might encourage and drive the creative doing, possible obstacles that stand in the way of developing PTs' awareness of creativity and ways of overcoming these obstacles, various issues that relate to the pedagogical knowledge required for being capable of nurturing students' creativity in mathematics, issues that concern the evaluation of a product and the process, and many others.

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