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Gestures and conceptual integration in mathematical talk

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Abstract Spontaneous gesture produced in conjunction with speech is considered as both a source of data about mathematical thinking, and as an integral modality in communication and cognition. The analysis draws on a corpus of more than 200 gestures collected during 3 h of interviews with prospective elementary school teachers on the topic of fractions. The analysis examines how gestures express meaning, utilizing the framework of cognitive linguistics to argue that gestures are both composed of, and provide inputs to, conceptual blends for mathematical ideas, and a standard typology drawn from gesture studies is extended to address the function of gestures within mathematics more appropriately.

Keywords Conceptual blends · Discourse · Embodiment · Fractions · Gesture · Metaphor

1 Introduction

Researchers into the communication and construction of meaning, including linguists, anthropologists, cognitive scientists and psychologists, have in recent years turned their attention to the phenomenon of spontaneous gesture associated with speech (e.g., Goldin-Meadow 2003; Kendon 1997; Kita 2003; McNeill 1992, 2000, 2005). Mathematics educators are also concerned with the construction and communication of meaning; yet only in recent years has research in mathematics education begun to include gesture and bodily movement as either potential sources of information on how we think about mathematics, or as contributors to mathematical thinking and communication itself (for example, Nemirovsky, Tierney, & Wright 1998; Núñez 2006; Radford 2003; further references are below). When mathematics is seen as an embodied, socially constructed human product, physical gesture is neither epiphenomenal to cognition nor

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irrelevant to communication. Instead, gesture constitutes a particular modality of embodied cognition, and, along with oral speech, written inscriptions, drawings and graphing, it can serve as a window on how learners think and talk about mathematics.

The purpose of this study was to examine the ways that spontaneous physical gesture, as one modality, is used in communicating about mathematical ideas and problem solving. A concurrent goal was to collect a corpus of gestures related to one mathematical topic, fractions. The topic of fractions was selected because it is important in the elementary school curriculum, yet understanding fractions and learning how to operate with them can be problematic for both children and adults (*cf.*, Behr, Harel, Post, & Lesh 1992; Mack 1990). Another concurrent goal was to begin to develop an analytic framework appropriate to understanding gesture and other modalities within the domain of mathematics, a discourse domain in which words, symbols, images and bodily motion are used in thinking and communicating about highly structured yet often abstract topics.

2 Theoretical framework

The research takes place within the theoretical framework of embodied cognition (Varela, Thompson, & Rosch 1991), and utilizes the tools of cognitive linguistics and gesture studies (Fauconnier and Turner 2002; McNeill 1992, 2000, 2005). From embodied cognition, the research draws its fundamental theoretical commitment, namely, that human thought, including mathematical thinking, is embodied at multiple levels: through imagery, bodily motion and gesture, and through the experience of living, with specific biological capabilities, in a world with particular physical properties. The overriding concern of this framework, and of this specific study, is in how students utilize their own embodied capabilities to construct understandings that can be communicated within a community that shares a common biological and experiential heritage.

Cognitive linguistics examines language not as a static entity linking arbitrary symbols to their assigned meanings, but instead as a dynamic construction that reflects a series of unconscious mental mappings based on our experiences and existing understandings (Lee 2001). An important mechanism within this framework is conceptual integration (or blending) of mental spaces. As defined by Fauconnier and Turner, "Conceptual integration…connects input spaces, projects selectively to a blended space, and develops emergent structure" (Fauconnier and Turner 2002, p. 89). For example, the notion of the "number line" is a blend that draws from two input spaces: our knowledge of numbers (initially, whole numbers, and later, rational and real numbers), and our imagery and knowledge of the geometric entity called a "line." The blend draws certain elements and relationships from each of the input spaces, and creates a new entity, the "number line," that has properties not found in either of the input spaces. The basic elements of this blend are illustrated in Fig. 1.

Conceptual integration can be seen as a general mechanism that encompasses more specific mappings such as conceptual metaphor; the latter have been used in the analysis of mathematical ideas ranging from arithmetic to calculus (e.g., Bazzini 1991; Lakoff and Núñez 2000; Nuñéz, Edwards, & Matos 1999; Pimm 1981; Presmeg 1992, 1997). In this paper, the theory of conceptual integration will be used to

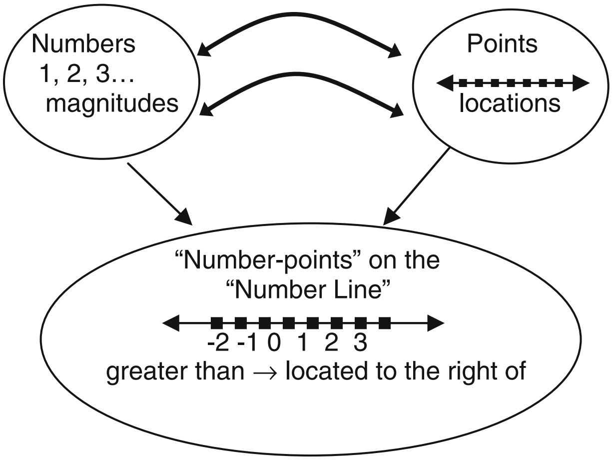

Fig. 1 Conceptual blend for "number line"

analyze both speech and gesture, in order to describe adult students' thinking about fractions.

"Gesture" here is defined as spontaneous "movements of the arms and hands... closely synchronized with the flow of speech" (McNeill 1992, p. 11). The question of what a gesture means in a particular context, as produced by a particular speaker, is clearly an important one when utilizing gesture in the analysis of ideas. Parrill and Sweetser (2004) define the meaning of a gesture as, "the relationship between how the hands move in producing a gesture, and whatever mental representation underlies it, as inferred both from the gesture and the accompanying speech" (p. 197). Clearly, the researcher has no direct access to "whatever mental representation" underlies a gesture, but must use the linguistic, social, and cultural contexts, as well as the activity in which the speaker is engaged, in order to construct a plausible interpretation of a gesture. Gestures can be analyzed as conceptual blends, and this analytic framework will be used to analyze the gestures found in the current study. In analyzing a gesture, however, instead of both inputs to the blend being abstract conceptual spaces, the blend that produces a gesture draws on one's knowledge of one's immediate physical environment, that is, on the affordances offered by the hands, arms, body and the surrounding objects and physical space. This mental space is referred to as "Real Space" (Liddell 2000, as cited in Parrill and Sweetser 2004, p. 201).

3 Related research

Previous research on gesture and mathematics has examined a variety of mathematical tasks, including conservation of volume (Alibali, Kita, & Young 2000; Church and Goldin-Meadow 1986); learning to count (Alibali and diRusso 1999; Graham 1999); solving simple equations (Goldin-Meadow 2003; Perry, Church, & Goldin-Meadow 1988); classroom communication (Goldin-Meadow and Singer 2003; Goldin-Meadow, Kim, & Singer 1999); motion and graphing (Nemirovsky, this issue; Nemirovsky et al. 1998;

Radford, Demers, Guzmán, & Cerulli 2003; Robutti 2006); undergraduate mathematics (Núñez 2006; Smith 2003), and collaborative problem solving (Reynolds & Reeve 2002).

This research both highlights the role of the body in mathematical thinking and learning (Nemirovsky, this issue; Nemirovsky et al. 1998; Núñez 2006; Robutti 2006), and reinforces the findings of research on gesture in other settings. For example, it has been found that gesture and speech can "package" complementary forms of information, and can be utilized by the speaker to support his or her thinking and problem-solving (Arzarello 2006, this issue; Goldin-Meadow 2003; Kita 2000; McNeill 2005; Radford 2003, this issue). In several studies, learners are able to express their understanding of a new concept through gesture before they are able to express it in speech, and a "mismatch" or non-redundancy between the information expressed through gesture versus speech can be an indicator of "readiness to learn" the new concept (Alibali et al. 2000; Church and Goldin-Meadow 1986; Goldin-Meadow 2003). The current research will examine specific gestures as evidence for the ways that the participants conceptualize mathematical notions related to fractions.

4 Methodology

4.1 Participants

The participants in the study were twelve volunteers from a required course for prospective elementary school teachers taught by the author. The participants were all women, approximately 20 years of age, and they received a small amount of extra credit for their participation. This group of participants was selected in order to extend the study of gesture and mathematics to adult learners, and because it was anticipated that the students might still be in the process of constructing a full understanding of fractions (Mack 1990). Hence, their existing, possibly partial understanding of this topic might be displayed both through speech and gesture, as well as written inscriptions. The topic of fractions had not yet been addressed in the course, so the participants were drawing on their prior school (and out-of-school) experiences with fractions in answering the questions.

4.2 Procedure

The participants were interviewed in pairs by the author in sessions lasting approximately 30 min. The sessions were videotaped, and began with the interviewer asking each of the participants a set of questions designed to elicit their memories of learning fractions and their current ways of talking and thinking about this topic. The questions included the following:

- How were you first introduced to the idea of fractions?
- Do you remember anything that was particularly difficult about learning fractions?
- Have you used fractions in everyday life or in other classes?

The pairs of students were then asked to solve five written problems, each on a separate sheet of paper, consisting of simple comparison, addition, subtraction, multiplication and division of fractions and mixed numbers. They worked on the problems together, and were then asked to explain their solutions. In concluding the interview, the researcher asked:

- How would you define a fraction?
- How would you introduce fractions to children?

4.3 Data analysis

The gestures were initially classified using a scheme created by psychologist David McNeill. The most important types, or dimensions, distinguished by McNeill are: *iconic gestures*, which "bear a close formal relationship to the semantic content of speech" (in other words, which visually resemble their concrete referents); *metaphoric gestures*, where "the pictorial content presents an abstract idea rather than a concrete object or event" (McNeill 1992, p.14); beat, a repetitive gesture that "indexes the word or phrase it accompanies as being significant" (op. cit., p. 15), and *deixis*, a "pointing movement [that] selects a part of the gesture space" (op. cit., p. 80). The gestures produced during the interviews were recorded and categorized in terms of which type or dimension was most salient; the speaker and the concurrent speech were also recorded for each gesture.

5 Results

5.1 Numerical summary of results

A corpus of 251 gestures was collected across the six interview sessions; there was a total of 172 min of videotaped data. The corpus does not include a large number of deictics (pointing to the paper) that occurred while the students were solving the written problems. The students produced 235 gestures while answering the interviewer's questions; an additional 16 non-deictic gestures were observed while the students worked on the problems. The average rate of gesture production while the students were talking with the interviewer was 5.9 gestures per minute. The total number of gestures produced by individual participants during an interview session ranged from 6 to 84 (again, excluding deictic gestures directed at the paper containing their written problem solving). Table 1 presents the breakdown of the gestures in terms of which of McNeill's dimensions was most salient.

McNeill developed these dimensions from a corpus of 790 gestures collected from participants who were asked to watch an animated cartoon and then describe it. It was expected that an interview about mathematics would elicit fewer iconic gestures and more metaphoric ones than in McNeill's corpus, and this expectation was upheld: in the "cartoon" narratives, 41% of the gestures were primarily iconic and only 2% primarily

Table 1 Types of gestures by most salient dimension

	Iconicity	Beat	Metaphoricity	Deixis	None/other	Number
Frequency	58	60	66	13	54	251
Percentage	23	24	26	5	22	

metaphoric, as compared with 23% and 26%, respectively, for the fraction interviews (McNeill 2005, p. 42).

A total of 81 out of the 251 gestures (32%) referred to fractions, parts of fractions, or operations with fractions. Of these 81 gestures, 40% were primarily metaphoric and 35% primarily iconic. In order to analyze the gestures associated with fractions, we will first examine the ways that fractions are commonly defined and understood.

5.2 Gestures and the conceptualization of fractions

The concept of a fraction can be defined in several ways, depending on the context and audience. A basic definition from a text for prospective elementary school teacher states: "The *fraction m/n* of an object is the amount obtained by dividing the object into *n* equal parts and taking *m* of these parts" (Jensen 2003, p. 91). Fractions are also defined as expressions for the quotient of two quantities. From another perspective, a fraction can also be considered simply as, "a form for writing a number, a notational system, a symbol, two numbers written with a bar between them" (Lamon 1999, p. 27).

In the study, participants primarily talked about fractions as a part of a whole, but also as a symbol that can be manipulated or operated on. They also frequently described the use of tangible objects, that is, mathematics manipulative and realia, in learning about fractions (12% of the gestures were about such objects). Each of these views of fractions will be analyzed using conceptual integration theory; in addition, an episode in which a student discussed her confusion about equivalent fractions will be examined in detail.

5.3 Fraction as part of a whole: an iconic gesture for "cutting"

As noted above, a basic definition of fraction refers to dividing a whole into equal sized pieces. The participants in this study often expressed this notion of a fraction in a literal way; gestures referring to "cutting," "splitting" and "slices" often accompanied participants' speech. There were thirteen instances of such gestures, comprising 22% of the iconic gestures, and nearly 5% of the entire corpus.

An example of such a gesture is shown in Fig. 2 and Video Clip 1, in which student LR describes how her teacher introduced fractions by cutting a "pie." LR's "cross-cutting" gesture sequence (in Fig. 2a) clearly displays the process of dividing an imaginary pie or

Fig. 2 An iconic gesture for "cutting". In transcripts, *slash* indicates a speech pause, *asterisk* denotes a self-interruption, and *ellipses* represent omitted segments of speech. **a** "… like cutting the pie in like // pieces …" **b** "… and then she cut in like eighths"

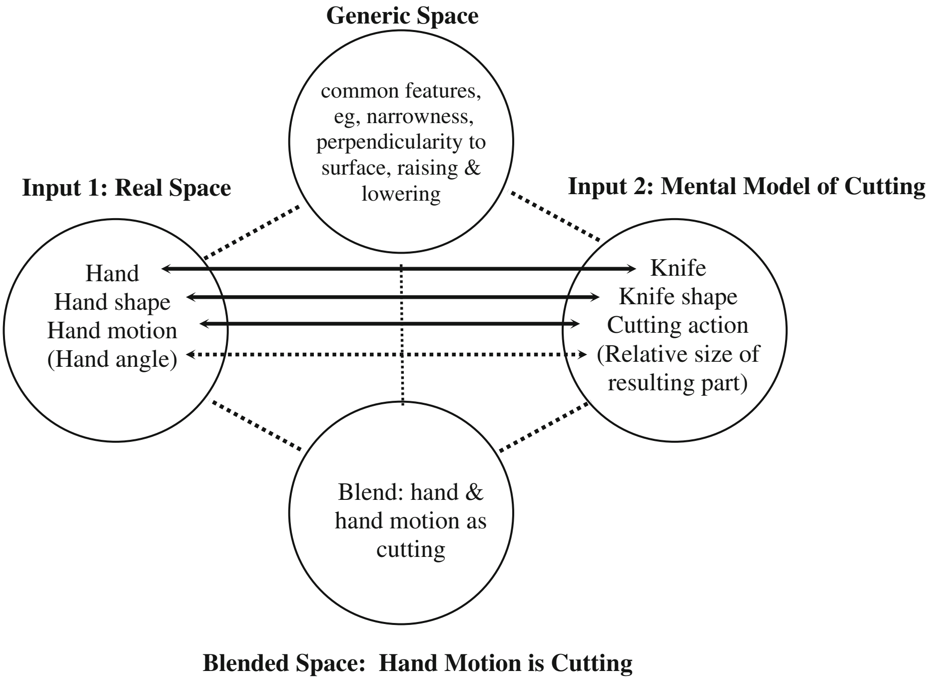

Fig. 3 Conceptual blend for the iconic gesture of "cutting"

circle into halves, and then fourths. During the second gesture sequence (Fig. 2b), her "cutting" hand movement is similar, but this time she turns her hand clockwise only 45°, to make an "eighth" slice, and then turns counter clockwise 90° to show a second "one-eighth" slice on the left side of the "pie."

Given a shared cultural background with the speaker, it is easy to interpret this gesture as referring to the action of carefully cutting an imaginary pie. Yet it bears asking: how do we

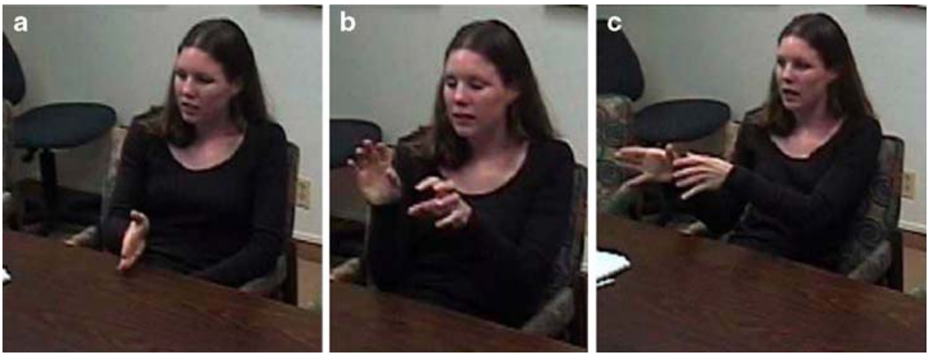

Fig. 4 JL describing a difficult fraction concept. **a** "it can split and split …" **b** "and still be,* this is" **c** "the same as this"

make this interpretation? From the perspective of cognitive linguistics, this occurs through a blend of two conceptual spaces: one containing our knowledge and control of our hands and arms, and one containing our conceptualization of the act of cutting. Figure 3 illustrates the conceptual blend that gives rise to this gesture. The two inputs are shown on the left and right sides of the diagram. At the top, the “generic space” refers to elements that the two spaces have in common; these commonalities allow our minds to construct the blend, shown in the bottom circle. In this case, the generic space includes such features as the perpendicularity of both the hand and the knife to the surface of the table, the fact that both are narrow relative to their lengths, and that both can be moved up and down. In utilizing the affordances of her hand and arm to highlight these commonalities, LR evokes a conceptual blend that allows an interlocutor to “see” her hand as a knife being used to cut or slice something.

Although the blend for this iconic gesture is straightforward, it is notable that most participants’ gestures for “cutting” or “splitting” were not as precise as LR’s in the spacing or orientation of the “cuts” (an example is shown in Fig. 4a). An analysis in terms of conceptual integration can take account of these variations. In Fig. 2b, the 45° angle that LR made with her hand was a meaningful part of the Real Space input, resulting in a blended space in which the amount “one-eighth” was embodied visually and concretely. These “optional” visual elements, “Hand angle” and “Relative size of resulting part,” are shown in parentheses in the blending diagram, in order to indicate that they are not found in all gestures for cutting.

5.4 Comparing equivalent fractions

The next example presents a gesture sequence showing three of McNeill’s types or dimensions: iconicity, metaphoricity, and deixis, in the context of comparing equivalent fractions. In this segment of discourse (see Video Clip 2 and Fig. 4), the participant, JL, is responding to a question about which aspects of learning fractions were difficult for her:

> JL: “whole/concept of how you just, it can **split and split**[a] and **still be,**[b] this is the **same** as **this**[c], and you know that’s really confusing to me.”

JL’s speech, on its own, is not straightforward to interpret, but one possible reading is that she found it confusing that, given an initial fraction (represented with a manipulative or drawing), one can “split” it to generate a series of equivalent fractions (for example, splitting a rectangle that represents one-half in half again, to make two-quarters, and then four-eighths). Yet, in spite of this “splitting” (which might be expected to create something smaller), each equivalent fraction has the same value as the original one.

JL’s first gesture, shown in Fig. 4a, is iconic; she makes an iterated motion toward the right, holding her hand perpendicular to the table, while saying “split and split.” Note that her cuts are parallel, rather than intersecting, and lack the precision of those shown in Fig. 2. Her third gesture, shown in Fig. 4b, is primarily deictic; JL uses the space in front and to the side of her to locate, and hence distinguish, between two referents in her speech (“this” and “**this**”, referring to two equivalent fractions). However, the form of the gesture, with palms perpendicular to the table and facing each other, fingers slightly curled, suggest that the gesture was not only deictic but also metaphoric. This form is consistent with holding a physical object between the hands, which suggests a conceptualization of a fraction as an object. Although it cannot be pursued here, gesture analysis may shed further light on the pervasive metaphor in which numbers, procedures and other mathematical "entities" are treated as "objects" by students and mathematicians alike (cf., Radford 2002; Sfard 1991). In the current context, it is important for learners to understand that a fraction can symbolize a rational number, that is, the equivalence class of all fractions representing the same value, as well as being a number-object itself. It is this dual identity that seems to have confused JL in the sequence above: she wonders how you can "split and split" a fraction (seen as an object), and yet end up with the "same" thing (the rational number represented by the fraction).

5.4.1 A metaphoric gesture for comparing: "this is the same as this"

Within this gesture sequence is an interesting example of a metaphoric gesture. In JL's second gesture, shown in Fig. 4b, the right and left hands are next to each other, palms facing outward at an angle, with the fingers slightly spread and hooked. She alternates lifting and dropping each hand as she says the phrase, "and **still be**,* this is the same as **this**." In this case, the gesture slightly precedes the co-expressive speech, which is the phrase "is the same as." In this gesture, the referent is not a tangible object or concrete action; rather, it is an abstract relationship, a comparison asserting "sameness" between two entities. For this reason, McNeill would categorize this gesture as primarily metaphoric.

Parrill and Sweetser (2004) propose that even metaphoric gestures have an iconic aspect, in that, by means of the hand shapes and motions, they invoke some visual or concrete situation, entity or action. This concrete entity or situation is not arbitrary; rather, it is selected (usually unconsciously) because it provides specific elements and an inferential structure that help support our understanding of the abstraction expressed through the gesture. Thus, a metaphoric gesture involves a sequence of two conceptual mappings, an iconic one between Real Space and the visual/concrete situation (as conceptualized by the speaker), and a second between this conceptual space (the source of the metaphor) and the intended abstract meaning (the target; Parrill and Sweetser 2004).

Table 2 shows this double mapping for JL's "comparing" gesture (Fig. 4b). In the first mapping, each of the two hands maps to one of the two physical objects being compared. The form taken by each hand may represent either the object itself, or the act of holding the objects. The coordinated motion of the two hands and arms results in first one hand/object being made salient (lifted), then the other (first object is dropped while the second is lifted). This alternating up and down motion is repeated, drawing attention first to one object and then another.

Table 2 A double mapping for JL's metaphoric gesture for *"the same as"*

Iconic mapping Real space (Alternating up and down gesture)	Metaphoric mapping Source (Making salient visually)	 Target (Comparing)
Left hand	A physical object	An abstract object (e.g., a fraction)
Right hand	Another physical object	Another abstract object (e.g., a fraction)
Moving a hand up	Making an object salient visually	Considering one abstract object (e.g., the value of a fraction)
Alternating moving each hand up and down	Making one and then another object salient visually	Comparing two abstract objects (e.g., the values of two equivalent fractions)

In Table 2, the up-and-down gesture forms a Real Space input to an iconic mapping between the motion of the hands and the displaying of two physical objects (first and second columns). The alternating display of two objects, in turn, forms the source domain for a metaphoric conceptual mapping in which the speaker is talking about comparing two abstract entities, in this case, two fractions (second and third columns).

By performing (and perceiving) this double mapping, the participants in the discourse are able to express (and "see") the idea of comparing two fractions *through* the physical movements of JL's hands and arms. It is this double mapping from Real Space to a concrete mental space, and from this concrete mental space to an abstract mental space that, according to Parrill and Sweetser (2004), constitutes metaphoricity in a gesture.

5.5 Iconic gestures and tangible objects

Considering the corpus as a whole, the proportion of iconic gestures was nearly as high as the proportion of metaphoric ones (23% compared to 26%). One clear source for iconic gestures were the responses given to questions asking the participants how they were first introduced to the idea of fractions and how they might introduce fractions to children. Eleven out of twelve participants mentioned tangible materials in their responses to these questions, referring to either purposely-designed mathematics manipulatives, or realia (pies, pizza, etc.). A total of 30 gestures (12% of the corpus) accompanied speech referring to tangible materials. In half of these cases, the speech referred to the objects themselves (e.g., "eight pieces" "hands-on things"), and in half to physical actions with these materials (e.g., "splitting up pictures," "one portion colored in"). Figure 5 shows three examples of such gestures, along with the accompanying speech.

In one sense, these gestures can be seen as simple imagistic illustrations of students' experiences with hands-on materials in learning about fractions. However, the gestures, and their physical referents, may play a more important role in students' learning and thinking. Mathematics manipulatives, by design or selection, afford only certain kinds of actions and configurations for the user; they offer, "stable representations of constraints" (Hutchins 2005, p. 1555). These physical objects thus serve as what Hutchins calls material anchors for conceptual blends. Hutchins described material anchors as follows: "If conceptual elements are mapped onto a material pattern in such a way that the perceived relationships among the material elements are taken as proxies (consciously or unconsciously) for relationships among conceptual elements, then the material pattern is acting as a material

Fig. 5 Gestures iconic to tangible materials. **a** "just a stick or a rod" **b** "take the pie pieces out" **c** "hands-on things"

anchor" (ibid., p. 1562). Although a novice learner may not initially "see" the mathematics "embodied" in a manipulative, his or her actions are bounded and guided by the material pattern of the object. One hypothesis is that it is this material pattern (and the associated action) that is re-embodied later in the form of gesture.

As an example, in Fig. 5a, the participant, LR, spreads her hands, palms facing in open C-shapes, as she says, "I think we used, like, just **a stick or a rod**." She goes on to say, "and then dividing it **again and again**," accompanied with a sequence of cutting gestures. This "stick or rod" might be a rod composed of Unifix cubes stuck together, or any other tangible object that could be repeatedly divided. In this case, the "material pattern" she has in mind permits the mapping of an important aspect of the concept of fraction, the idea of repeated division (as in physical separation). In Fig. 5b, AT says she would "take the **pie pieces out**," referring, in all likelihood, to a set of plastic fraction circles made of different sized pieces; this artifact highlights another aspect of understanding fractions, the part-whole schema. Thus, in each case, the tangible object referred to through both speech and gesture has provided a material anchor for the conceptual blend representing the students' current thinking about fractions.

5.6 Fraction as a notation: gestures for mathematical inscriptions

As noted above, one way to conceptualize a fraction is as a particular kind of notation or symbolic expression. This conceptualization of fraction was apparent in gestures that might, on first glance, be considered iconic. Recall that in McNeill's definition, iconic gestures resemble "a concrete object or event" (McNeill 1992, p. 14). However, only 34 of the 58 iconic gestures in the fraction talk literally referred to concrete objects, actions or events. The remaining 24 iconic gestures (10% of the corpus) were dynamic gestural depictions of written mathematics expressions or procedures, what I have called "algorithms in the air" (Edwards 2003). These gestures are iconic in a sense, since they do refer to a material artifact, act or "event" (the written inscription of an algorithm). Yet, the students were not talking about the form or shape of these symbols (in the way that a person who talked about a spiral stairway might gesture in a circle). Instead, the students used iconic gestures to communicate about abstract or general mathematical objects or processes (fractions, addition, etc.), using gesture to evoke the symbolic expressions that are used to represent the mathematics.

Fig. 6 Iconic–symbolic gestures. **a** "you put one under the other …" **b** "… don't you cross them?"

Thus, these kinds of gestures display a particular type of iconicity: rather than referring to a concrete object in and of itself, the gesture refers to a symbolic, written inscription, which in turn represents a specific mathematical entity or procedure. This type of reference has been called a "chain of signification" (cf., Cobb, Gravemeijer, Yackel, McClain, & Whitenack 1997; Presmeg 2006; Walkerdine 1988), and is very common in mathematical discourse. In a chain of signification, a sign-signifier pair (for example, a written inscription for a mathematical procedure, and the procedure itself) becomes bound so closely that they function as a signified for a new sign (in this case, the gesture corresponding to the action of writing the inscription).

In order to distinguish this kind of iconic gesture from the kind originally identified by McNeill (which refers to concrete actions or events, rather than abstractions), I have proposed calling the first type of gesture "iconic–physical" and the second type "iconic–symbolic" (Edwards 2003). Examples of iconic–symbolic gestures from the fraction study are shown in Fig. 6.

Figure 6a is a snapshot from a long sequence of very precise gestures in which the student, MB, "acts out" a vertically organized written algorithm for the addition of fractions. In Fig. 6b, both participants are referring to cross-multiplication (which they have confused with the actual algorithm for multiplying fractions). Note that this image shows two different ways to enact the idea of multiplying the numerator of one "fraction" (ratio) with the denominator of another: on the left, GG uses her index finger to create the figure of an "X" in the air, while on the right, KG utilized the angle of her hand to "connect" the two multiplicands.

From the perspective of conceptual integration, iconic–symbolic gestures arise from another double mapping, similar in some ways to that found in metaphoric gestures. An example of this type of mapping is analyzed in Table 3.

In this mapping, the affordances of the hands and physical space in front of the speaker allow her to produce a gesture evoking elements of a written inscription, and relations between its parts, which in turn refers to the mathematical procedure itself (cross multiplication).

6 Discussion

The overall goal of the study was to collect and analyze a corpus of gestures related to one mathematical topic, fractions, both to understand how gestures are used in communication

Table 3 A double mapping for KG's iconic–symbolic gesture

Iconic mapping	Representational (Symbolic) Mapping	
Real space	Symbols (Sign)	Referent (Signifier)
Angle of left hand	$\frac{\square}{\square} \searrow \frac{\square}{\square}$	Association between two elements (specifically, multiplication)
Area near top of left hand	$\frac{\boxtimes}{\square}\ \frac{\square}{\square}$	Numerator of one fraction or ratio
Area near bottom of left hand	$\frac{\square}{\square}\ \frac{\square}{\boxtimes}$	Denominator of second fraction or ratio

about mathematics, and to develop an analytic framework appropriate to understanding gestures produced within this specific domain of human cognition. Mathematics is one of a small number of human activities in which what we are talking about is primarily abstract. The data from this study indicate that, even for an elementary topic such as fractions, the abstract nature of mathematics was made evident through the high proportion of gestures showing metaphoricity in the corpus. Yet the participants also produced an almost equal number of gestures showing iconicity. This iconicity was of two types. One type referred to concrete objects or processes, often related to tangible materials utilized in early instruction about fractions. In the other type of iconic gesture, which was given the label "iconic–symbolic," participants' gestures re-enacted the physical process of writing out a mathematical procedure, or referred to visual locations and elements of mathematical symbols. This latter kind of gesture highlights the importance of the symbolic form in these students' thinking about mathematics, and the way that symbolization can form a "chain" of meanings in this domain.

These findings are significant from an educational perspective in several ways. First, the analytic framework of conceptual integration offers a more detailed understanding of how learners construct their understandings of mathematical concepts, and gestures are one modality through which people express what they are thinking. Yet, in mathematics, the source of these gestures may be even more significant. The use of tangible objects, for example, mathematics manipulatives, is widespread in elementary education. One perspective on these materials is that they somehow "represent" the abstract mathematics that we want children to learn. However, since the concepts are not yet understood by the children, it is hard to see how even carefully designed manipulatives can "represent" or "convey" the ideas. The notion of manipulatives as material anchor is more productive; it is through the affordances and constraints of the objects, and the actions allowed by them, that the new understandings are scaffolded (in conjunction with the careful guidance and language of the teacher). Virtually all of the participants in the current study produced gestures that evoked hands-on manipulatives or realia, and many of these gestures were very precise with regard to the structural details of the objects as they corresponded to aspects of fraction understanding.

A similar level of detail was found in the participants' iconic–symbolic gestures, whether reproducing, step-by-step, the written algorithm for adding fractions or connecting the numerator of one fraction with the denominator of another. These gestures are, again, evidence of how students think about fractions, once he/she have learned how to manipulate them symbolically. In general, teachers' understanding of their students' thinking can only be enhanced by attending to the rich modality of gesture, and by considering carefully the sources of these gestures and the conceptual mappings that produce them.

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