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BOOK REVIEW

Book review

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Macello C. Borba & Mónica E. Villarreal. [Preface by Ubiratan D'Ambrosio. Afterword by Ole Skovsmose]. *Humans with Media and the Reorganization of Mathematical Thinking: Information and Communication Technologies, Modeling, Visualization and Experimentation.* Springer, NY, 2005, 229 pp, RRP October 2006: € 62.95 [hard cover] € 32.95 [soft cover], ISBN 0-387-24263-5

Borba and Villarreal view technology as having evolved with human development, rejecting attitudes that are technologically deterministic or which might otherwise engender a sense of disempowerment. They believe that access to technology is a right of, and a necessity for, citizenship. Drawing on over a decade of work in their research collective, and firmly grounded in data generated by and conclusions arrived at from their qualitative research, they illustrate a broad spectrum of possibilities for humans to work with technological media in mathematics education.

Although new information technologies are giving greater and greater access, particularly to people living and working in remote areas, the authors do not valorise these technologies, accepting that alternative artefacts have a rightful place. Their intention is for humans – in this case, mathematics teachers and students, including teacher education students, supported by researchers – to work together with each other and with technology in order to enable learners, from an early age in particular, to participate in *their* worlds. This means that the focus should be on the worldviews and the future perspectives of the learners rather than those of teachers or textbook authors. Consistent with their philosophy of empowering the learners, in pedagogical situations they advocate giving as much responsibility for their own learning as possible to students. Accordingly, from a mathematical perspective, they advocate a strategic role for the modelling and solution of problems seen as important by the students.

Although the book is situated in Brazil, with contributions from Argentina and Denmark, the material is of relevance to anyone with access to electronic technologies, no matter how modest. In addressing the problems of distance and limited financial support available for education in Brazil, the book illustrates what might be achieved under less than ideal

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economic and geographic circumstances. The increased access to communication offered by information technologies has important implications, not only for social justice, but for distance education internationally. At the same time, it opens up new possibilities for research when multiple, even simultaneous, discussions may be automatically recorded for later analysis. Although it is recognised by people who have considered the prospects for the world's youth from a global perspective – including Ole Skovsmose in the *Afterword* – that the situation regarding access to technology in Brazil is relatively privileged by world standards, developments in electronic communications are moving so quickly that what might exclude learners and their teachers today may be overcome, to some extent at least, at some time in the future. Witness the spread of mobile- or cell-phones in Africa, for example. In England, these phones are being used as mediating artefacts to teach adults literacy and numeracy skills.

Chapter 1 opens with the disarming and intriguing title: *Why another book about technology and mathematics education?* This suggests a recognition that technology is but one of the latest educational fads and all that might have been said has already been published in books and journals, or on the internet. In my experience, the theme appears to be very popular with policy makers as they attempt to show themselves abreast of the times, searching for that elusive solution that will raise their particular country's international standing and supposedly save money at the same time. On the other hand, people working in the mathematics teaching profession range across the spectrum from early adopters to laggards – most with a genuinely held belief that their particular stance will be of most benefit to their own students.

Acknowledging that technological interfaces (including pencil-and-paper) have transformed education since time immemorial and will continue to do so in unpredictable ways, Borba and Villarreal recognise the importance of the need to co-ordinate multiple representations made available through computer software (e.g., everyday language, symbolic language, tables, and graphs). Nowadays, even body movements can be coordinated with these standard representations. They claim that the book addresses epistemological issues that arise when these different technologies, which have become part of knowing, are associated with human beings. Based on the premise that technology alone is unlikely to bring about change in education, they propose that the book offers a range of pedagogies resonant with new technologies, *once* it is decided that technology is relevant for education.

Recognising that teachers who are under duress to incorporate technology into mathematics curriculum and pedagogy may try to minimise its impact on the status quo, 'domesticating' it to merely perform traditional tasks in a shorter time with more visual elegance, Borba and Villarreal attempt to counter this conceptualisation. As implied by the title, the overall goal of the book is to overcome the dichotomy between humans and technology that underlies many of the difficulties that the mathematics education community as a whole has with the implementation of technology in schools – and other learning spaces, I would add – in ways that are *not* domesticated.

Chapter 2, *Information technology, reorganization of thinking and humans-with-media*, focuses on historical and philosophical aspects of the interaction between humans and technology. It draws heavily on a paper by the Russian psychologist, Oleg Tikhomirov, written in the 1970s. Based on the perspective of a mediating role for the computer comparable to the Vygotskian perspective on the role of language, it discusses how computers can affect human cognition, with implications for education. Borba and Villarreal also draw on the 1993 work of Lévy who opposed the notion of a dichotomy between humans and technology on the grounds that it may be disempowering if a sense of

impotence is generated within a new 'technological order'. Lévy regarded technological devices as an extension of memory, as with other more primitive technologies (or tools), but with qualitative differences in that it is now possible for linear reasoning to be challenged by other ways of thinking and other forms of communication, including instant communication. Accordingly, Borba and Villarreal's perspective is that "humans are constituted by technologies that transform and modify their reasoning and, at the same time, these humans are constantly transforming these technologies" (p. 22). Knowing becomes an endeavour of a collective of humans and things.

Chapter 3, *Modeling as a pedagogical approach: Resonance with new media*, opens with a very succinct discussion of the concept of resonance as the weaving together of epistemology, pedagogy, and technology. The authors draw on the research of others to distinguish between three types of problem solving: (a) as a context for application of previously learned mathematical skills; (b) as a skill and an end in itself; and (c) as an art form, following the work of Polya – not in its trivialised, algorithmic form, but as the heart of mathematical activity involving challenging, non-routine problems.

While recognising the didactical value of problem posing in mathematics education, Borba and Villarreal assert that problem posing can also have a socio-political value, and this is reflected in the Brazilian education guidelines which advocate taking into account regional factors and students' interests, encouraging and supporting students and teachers to participate in their local communities or larger contexts. They also emphasise that student choice of project topics allows them to bring part of their cultural background and their concerns for the future, thereby enabling the entry of political themes. This conception of problem choice can be traced back to the pedagogical work of the Brazilian educator Paulo Freire, which was further developed in the ethnomathematics of Ubiratan D'Ambrosio. The evolution of the Danish tradition of modelling in mathematics can also be traced back to these two educators. Addressing new knowledge production, Borba and Villarreal assert that, due to the increasingly integral role played by ICTs in contemporary life, "students should be exposed to these new technologies of intelligence so that the knowledge which is produced in schools and universities is not disconnected from the rest of society" (p. 56).

Borba and Villarreal make the observation that the problem solving/modelling movement developed concurrently with the development and incorporation of information and communication technologies [ICTs] into education. However they claim that, historically, the problem posing and solving movement did not stress the role of calculators or computers. On the other hand, the concept of modelling has accelerated with the advent of these technologies which enable exploration and simulation. They conclude that, as with technology, modelling should not suppress other practices but co-exist with them, yet changing their nature.

Having established the importance of the *humans-with-media* concept and the importance of modelling as a pedagogical approach consistent with new media, in Chapter 4 Borba and Villarreal argue for the importance of experimentation. Technology offers support as a memory aid assisting with multiple representations in mathematics education, as is the case in the field of experimental mathematics. Drawing on the work of C. S. Peirce on *abduction*, characterised by the logic of discovery, they relate this mode of inference to the use of the world wide web.

Chapter 5 is devoted to an investigation of visualisation as a way of reasoning in mathematics research and learning, in a two-way process between internal and external representations within Nemirovsky and Noble's concept of *lived-in space*, created in an ongoing process, with and without computers. Borba and Villarreal provide several arguments for the pedagogical value of visualisation in mathematics education. They

conclude that "the media used to communicate, represent and produce mathematical ideas conditions the type of mathematics that is made and the kind of thinking to be developed in those processes" (p. 96). In other words, there is an 'intershaping relationship' where technology is seen as shaping humans, and humans as shaping technology. It is an ongoing process, even if the media are not immediately available. This idea has resonance with the concept of *instrumental genesis* as elaborated by Trouche (2004), drawing on the work of Vérillon and Rabardel.

Chapter 6 offers a series of case studies to exemplify modelling and media in action. Modelling, viewed as a pedagogical approach, is transformed as different humans-with-media collectives produce knowledge. Borba and Villarreal argue that new media and new pedagogical approaches do not supplant other forms but serve to transform them. They observe that second-wave ICTs, marked by browsing the internet, are more in line with the video-clip culture of today – that is, among younger people in more developed countries of the world. One of the case studies involves the use of the internet in supplying data for a project on BSE – commonly known as 'mad cow disease'. Based on the data, supported and challenged by their teacher, students were eventually able to find a logistic model for the outbreak of new cases between 1987 and 2001. Although the role of the internet here was not 'mathematical' in essence, it did enable the project to be developed in a real-world, interdisciplinary context. Nowadays, there is a burgeoning range of mathematical internet sites, including history of mathematics sites (see below), freely available to the public, and these may be able to shape to a larger degree the mathematics produced by students.

Chapter 7, *Experimentation, visualization and media in action*, presents another series of case studies based on classroom research, interviews with students, teaching experiments, and textbook analyses in order to illustrate the importance of human collectives thinking with media. The emphasis in this chapter is on the development of mathematical concepts. Borba and Villarreal conclude that "the experiences with computer technology, and the coordination of these experiences with other media, reorganizes thinking and transforms, in a recursive way, different humans-with-media collectives" (p. 167).

In chapter 8, *Mathematics and mathematics education online*, Borba and Villarreal observe that the traditional dichotomy of face-to-face versus distance education is inappropriate. Rather, the question should be: which pedagogical approaches are more suited to the internet? Although this discussion is necessarily focused on the minority of teachers and students living in the more developed world, it may be that technology will evolve in a similar way to the spread of cell-phones even in locations where access to other technologies is very limited.

Borba and Villarreal also stress that dialogical relationships need to be embedded in a technological structure of distance education, and recommend a model which includes synchronous activities (e.g., chat room) as well as asynchronous ones (e.g., email, bulletin boards). Following from a careful, innovative analysis of a chat room, they coin the phrase *multi-logue* to describe intersecting dialogues where individuals may skip from one discussion to the other, and where instructors may engage in parallel dialogues with students. They claim that their analysis of four online courses illustrates the appropriateness of the construct humans-with-media to understand the production of knowledge in this environment and also provides a means for describing the types of changes occurring.

However, these new forms of communication generate new research problems, requiring different forms of analysis. Given that qualitative research emphasises the need to work in 'natural environments', Borba and Villarreal ask how might emerging experiences in time and space be deconstructed? However, they then query, reflexively, whether posing such questions is a reflection of the very problem of trying to adapt old research practices to new

learning spaces. In other words, just as some early forms of online education adopted a 'book-on-screen' approach, neglecting the potential of new technologies for the development of new pedagogies, the researchers wonder if their questions are not trying to force old research parameters inappropriately onto new and evolving real and virtual learning spaces constructed within non-linear timeframes.

In Chapter 9, *Methodology: An interface between epistemology and procedures*, Borba and Villarreal recognise that the research process is non-linear, and argue for the importance of the concept of integrated research, composed of a network of research actions. They stress the importance of coherence between the researcher's view of knowledge, research procedures, and pedagogy. At the same time they recognise the importance of clarifying these procedures for the reader – as is the case in this book. They also justify the somewhat unusual placement of this chapter near the end of the book. This of course does not mean that the reader is constrained to read the chapters in the printed order, and I for one did not.

The final chapter by Borba and Villarreal, Chapter 10, is entitled: *Political dimensions of Information and Communication Technology.* The authors assert that ICTs in schools become a political issue when they provoke changes in epistemology and pedagogy, such as those defended in this book, which help to face the challenges of full democratisation of ICTs. Their research also has the political agenda of giving voice to students and teachers, focusing on what they can do rather than on what they cannot do. Rather than just one mathematics, which excludes many, they advocate that new collectives of humans-with-media develop different mathematics. This proposition extends across all levels of mathematics education.

Closing the main text is an *Afterword* by Ole Skovsmose who observes that Borba and Villarreal draw together two major ideas: that cognition is a social undertaking rather than an individual one, and that cognition includes tools. He reviews the text from three perspectives: epistemological, educational, and socio-political. Firstly, he notes that both humans and media are plural rather than singular nouns. Borba and Villarreal suggest an epistemic collectivism, implying processes of social interaction embedded in interpretations of learning, coming to know, and knowledge. Tools (media) could be thought of as auxiliary to practice, but also essential, "as defining the practice, and therefore as defining both content and forms of knowledge" (p. 212). In this text the computer is established as an important epistemological category, according to Skovsmose.

Secondly, from an educational perspective, considering the possibilities for the organisation of learning, Skovsmose notes the importance of problem-based and project learning tasks. He stresses once more that "tools are intrinsic to the nature of coming-to-know, and Borba and Villarreal provide clear insight into the qualities which computers might bring to the learning of mathematics" (p. 213).

From the socio-political perspective, as noted above, Skovsmose considers the question of who has access to computers on a worldwide basis, observing that these are a privilege for the minority. He is critical of the prevailing discourses in mathematics education which take classrooms with computers for granted, whereas the reality for the majority of learning sites is far from any such discursively constructed prototype. He suggests that the computerisation of mathematics education might be establishing new forms of social exclusion and inclusion and reminds the reader that Borba and Villarreal did consider issues of inclusion and exclusion in their final chapter, raising questions of citizenship and equality. These terms were used by Borba and Villarreal in their assessment of their research trajectory over a decade in a third world country. Obviously there are countries or regions even worse off than Brazil, but they have made a strong statement about their efforts to do the best for students and teachers in their particular part of the world.

Skovsmose closes with three further aspects of the socio-political dimension. He cautions against naïve acceptance of offers of computers and software packages, and any blanket recommendations being made in this field. He wonders whether insights gained from research such as this study could also have application in computer-deprived learning environments. Finally, in terms of quality of learning, he wonders whether computers might support reflection and critique (e.g., to address the reliability of utilising mathematical tools for certain problems and tasks, to challenge an exaggerated trust in numbers or an ideology of certainty among students). These, he says, become important questions in relation to how a computer environment in mathematics education might support the development of a critical citizenship.

1 Strengths and weaknesses of the book

This is a refreshing book in that it adopts a critical perspective on past and current practices in relation to technologies in mathematics education, and similarly adopts a careful and critical perspective on the potential of future developments in this field. Some of its attractive features are that, in my opinion:

1. The book can be read in virtually any order.
2. It foregrounds the themes of epistemology, democracy, and social justice.
3. There are careful introductions to each chapter.
4. The authors generously acknowledge the contributions of their graduate students.

One of the limitations of the book is that although it includes research projects with mathematics and mathematics teacher education students, it does not address the needs of vocational students or adult learners outside of the school system. Given the global impetus for lifelong learning, there is an enormous potential for development in the direction of mathematics education for adults at work, at home, and in community settings, whether in large cities, regional centres, rural communities, or even remote locations. The book also overlooks the possible advantages of technology to support distance education for people of all ages who are unable to access formal education due to physical, geographical social, cultural, aging, and other constraints.

As noted above, in terms of formal institutional mathematics education, Borba and Villarreal suggest that real problem solving, modelling, and project work be at the core, and go so far as to recommend that students choose their own subjects for investigation. This assumes that mathematics educators have a certain autonomy, which may be the case at the university level, but is not necessarily the case in other sectors of education in many countries. My experience of vocational teaching suggests that the limited time available is too precious for this liberty. Rather, I would recommend that vocational mathematics instructors work in collaboration with the other vocational teaching staff jointly to propose a series of projects that maximise the potential for students to learn and to use non-trivial mathematics in work-related contexts. This does not exclude student input into their projects, which could be conducted either alone or in small groups, but would involve their dealing with real vocational situations – simulated where necessary for ethical, safety, financial, etc., reasons.

Depending on official course regulations, this project work would be given recognition through a substantial weighting in the assessment process. In the school sector, such possibilities depend of course upon the degree of regulation at the school and state or national levels. Nevertheless, mathematics teachers and curriculum developers may be

encouraged by this book to implement a more open, interdisciplinary approach within their regulated learning spaces. Admittedly, this is not an easy task in many parts of the world, but it is possible – in my experience – to work in the interstices to address the needs of vocational education students and their employers (e.g., FitzSimons 2000), and it is still possible in the tertiary education sector in Australia, for example. Even within tightly controlled school sectors, integrating materials which focus on the history of mathematics would provide an opening, especially as there a growing focus on the uses of technological artefacts for student research (e.g., The International Study Group on the relations between the History and Pedagogy of Mathematics website http://www.clab.edc.uoc.gr/hpm).

I had a sense of uneasiness over the use of chat room texts for analysis: What are the ethical implications? Could students withdraw part or all of their transcripts – as is the case with interview data, in Australia at least? It would have been reassuring to learn that students had been included as research partners in the projects, rather than just as research subjects.

The book is very readable, with assertions supported by extensive, carefully documented data, based on practical, situated teaching experiences mostly in Brazil, but also in Argentina and Denmark. I believe that Borba and Villarreal have succeeded in their quest to justify the notion of humans-with-media as an epistemic concept. Their second proposition, for the shift to mathematics education curriculum and pedagogy based on a modelling approach, is consistent with their arguments concerning the critical use of technology. However, the more radical proposal of student choice of research topic, even non-mathematically oriented, would be likely to encounter serious obstacles, especially in countries where regulation holds sway and the focus, explicitly or implicitly, is on national or international competitiveness in large-scale assessments.

The quality of the scholarship is high: the material is well-documented and generalisations are well supported. The authors, Borba and Villarreal, are enthusiastic about their subject material but cautious in their claims. Rather than using their research to generate a manifesto, they have openly discussed problems encountered, and tried to be critical of their own findings as well as the literature. As mentioned by Ubi D'Ambrosio in the *Preface*, they have taken a critical consideration of the past into account when making projections for the future as well as setting the agenda for new research problems.

Brief editorial criticisms are that the chapters are not numbered in the List of Contents, and there are irritating grammatical errors, such as including apostrophes in dates (e.g., 1980s) and other plural nouns.

Who might benefit by using this book? The mathematics education community in general – researchers, teachers, and teacher educators who are interested in reflecting on the teaching and learning of mathematics, socio-cultural and political aspects of mathematics in school and society, and philosophical issues regarding the role of technology in mathematics education. How might it be used? It could be of benefit to researchers and research students. Individual chapters could form the basis of post-graduate research colloquia or teacher professional development (formally or informally organised). It could also prompt policy makers to question their current approaches to the issues addressed. The book is highly relevant and timely for the field of mathematics education.

Representing the culmination of over ten years collaborative research work in Brazil, supported by colleagues internationally, this book deserves to be taken seriously. Borba and Villarreal's critical approach to the integration of technology into mathematics education sets it apart from much of the literature published over recent years. *Humans-with-media and the reorganization of mathematical thinking: Information and communication technologies, modeling, visualization and experimentation* should have a place in the

libraries of researchers and mathematics educators at all levels, across cultural and social boundaries.

References

FitzSimons, G. E. (2000). Lifelong learning: Practice and possibility in the pharmaceutical manufacturing industry. *Education & Training, 42*(3), 170–181.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematics Learning, 9*(3), 281–307.