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PRINCIPLED TEACHING FOR DEEP PROGRESS: IMPROVING
MATHEMATICAL LEARNING BEYOND METHODS
AND MATERIALS

ABSTRACT. This paper contributes to knowledge about principled action which makes a difference to learners' attainment. We report on the Improving Attainment in Mathematics Project,¹ a project focusing on low-attaining secondary students. The purpose of the project was to introduce innovations in practice through action research with 10 teachers over 2 years, and evaluate the effect on students' learning using national test scores, teachers' reports, non-routine tasks and other performance indicators. However, this is not a study which shows how certain methods lead to better results. While it was found that learning improved, the methods and strategies the teachers used were not always generalisable across the project, indeed some were contradictory. Continued searching led to the identification of common underlying principles of teaching which different teachers manifested in different ways. Overt methods were less important than the collection of beliefs and commitments which underpinned teachers' choices. There was, however, a convergence of practice around a focus on long-term development, the process of becoming a learner of mathematics, rather than short-term gains. In addition, we had to deal with some of the realities of authentic collaborative research with practitioners.

KEY WORDS: low attainment in mathematics, mathematical learning, mathematical progress, mathematical thinking, teachers' beliefs, teaching principles

1. INTRODUCTION

In this paper, we report on a project to improve attainment in mathematics among low-attaining secondary students. We start by describing the background of the project, and the theoretical context, placing it alongside key studies which address teaching reform for underachieving groups. We describe briefly an exploratory study of a few underachieving students in one class in which the seeds of this project were sown. We then outline the project and show how the realities of researching collegially with teachers can make it hard to carry out even a simple research design. To situate what follows, we briefly give some outcomes in terms of pupils' improved learning. The main aim of the paper is to report the practices of teachers involved in the project and the complexities of identifying what was common about the teaching which led to improved engagement and learning.

2. BACKGROUND CONTEXT

The field of enquiry for this paper is England where there is a strong tradition of segregating students according to past achievement in mathematics into 'sets.' Those with most problems in learning are grouped with others who also have problems. While the common feature of such groups is underachievement in mathematics, they typically contain a disproportionate number of those from disadvantaged social, cultural and racial groups and those whose first language is not English (Boaler et al., 2000; Ireson and Hallam, 2001; Secada, 1992). Our view, as the Project researchers and authors of this paper, is that 'improving the teaching and learning of particular students, in a wide range of educational settings, is a core problem of practice' for mathematics educators (Even and Ball, 2003; p. 142)

English curriculum aims since 1988 have included the development of numeracy in a broad sense: knowledge, skills, understanding and applicability. There are many similarities with reform curricula elsewhere in the world, and assessment systems have successfully embedded limited forms of enquiry and application into practice for nearly two decades. More recently, mathematics teaching in England has undergone nation-wide development based on giving teachers materials with which to structure their teaching at four levels: the curriculum, lesson sequences, individual lesson contents and the activities within those lessons. Teachers are advised to split lessons into three-parts: mental/oral starter, main activity and plenary review; to state the learning objectives of each lesson; to encourage discussion, sharing of methods, development of visual images, explicit correction of errors, development of technical competence alongside problem-solving² abilities, and several other features of known good practice (DfEE, 2001). This approach is supported by a networked training system and framed, through inspection and accountability structures, by political objectives in terms of test results. Appearances are that, to the extent that annual national test results have risen, this strategy has been successful in raising achievement in mathematics for many students and raising expectations of teachers (for a well-researched critique of this result see Brown et al., 2003).

Yet, taking national test results at face value, about 25% of students at 11 and 50% at 16 fail to achieve the target minimum level and, consequently, find themselves failures in mathematics. As early as age 11 this is a serious disadvantage, since those entering secondary school below the expected level are consigned to sets of students who are expected to focus on repeating earlier work to 'catch up.' It is extremely rare for such students to make their way out of such groups into those expecting to achieve higher grades (Boaler et al., 2000). In recognition of this, various

solutions have been offered to schools. For example, in 2001 all schools were provided with worksheets, plans and lesson ideas which repeated the same approaches recommended for earlier years and more recently a small collection of intervention lesson plans were provided targeting significant parts of the curriculum. As with all official pedagogic advice in England, a top-down model of inservice training and in-school dissemination, plus regular school inspection, is undertaken to ensure teachers are following the advised teaching methods, or are able to justify alternative ways of teaching.

During 2001 it became clear, through discussions with local schools, that many experienced teachers were dissatisfied with the materials provided for students who arrived in secondary school without having achieved the expected level in national tests. These teachers felt that their own approaches would achieve better results than an uncritical application of the given programme and teaching approaches. Some of them felt free enough within their schools to follow their own methods; others felt constrained by the inspection regime to follow the official advice against their own judgement. Some newer teachers felt that they could do better, but lacked knowledge about how to develop better practice. Funding was obtained from the Es-mee Fairbairn Foundation for the Improving Attainment in Mathematics Project (IAMP) to explore other or additional ways of teaching these target students.

Literature about mathematical achievement of low-attaining students often focuses on those with identifiable learning difficulties, such as dyslexia, or language differences (e.g. Ellerton and Clarkson, 1996), or on identifiable groups which tend to underachieve (e.g. Leder, 1995) or on individual learners with idiosyncratic methods or pathological understandings. In IAMP, classes would contain students from all these groups and ranges, but also others who are underachieving for a variety of social and educational reasons. Students would be effectively outside the national system of assessment and accreditation since their achievements are so low that they are seen as unlikely to be able to gain ground and meet political targets.

3. EVALUATIONS OF CHANGING TEACHING METHODS

Recently published evaluations of US curriculum projects and teaching schemes show broadly similar results: that those who use materials designed to promote mathematical thinking, exploration, problem solving, realistic mathematics, classroom discussion and practical application do at least as well in traditional tests as comparison groups, but also do much better in tasks which allow students to display more discursive skills which

cannot easily be tested (Carroll and Isaacs, 2003; Hiebert, 1999; Senk and Thompson, 2003). To some extent, all UK mathematics teaching includes some of these features, yet teachers of the target groups tend to make low assumptions about learners' capacity to benefit from such approaches and revert to more traditional step-by-step, simplified, procedural mathematics in trivial contexts (Boaler et al., 2000)

Most of these US studies treat students as homogenous, only in a few cases showing results pertaining to particular social and attainment groups (e.g. Mokros, 2003). Thus we do not generally know whether success is partly due to social background factors which enable students to achieve more when given more intellectual responsibility, nor how much underachievement is perhaps masked by the improved results of the majority. An important exception is the QUASAR project which involved a number of US middle schools in urban disadvantaged areas (Silver and Stein, 1996). This showed that methods of teaching which focused on problem-solving, discussion, choice and learners' ideas enabled students who typically underachieve for a variety of reasons connected with social deprivation to do far better both in traditional tests and in problem-solving tasks.

A few, but not all, of the published evaluations of US innovations make explicit that different teachers can have different effects when using the same innovative materials (e.g. Romberg and Shafer, 2003). A common critique of such studies is the lack of information about what went on in classrooms (Kilpatrick, 2003, p. 473) as teachers adapted their practice to fit new schemes, new goals, and in some cases new assessment models, although it has been known for many years that teachers do not necessarily act in ways envisaged by curriculum designers (Arsac et al., 1991; Boero et al., 1996; Griffiths and Howson, 1974; Niss, 1999). Indeed Ridgway et al. (2003) point out that

Simply importing curriculum materials into the classroom is not sufficient to implement change that can lead to improvement. Simply providing professional development is not sufficient alone. Simply providing leadership that supports reform is insufficient without materials, professional development, and accompanying resources (p. 217).

But materials, development and leadership can also be insufficient if what results is merely the training of behaviour. For example, teachers in the UK have been 'trained' through inservice sessions, supportive materials and use of accountability mechanisms to use more whole class discussion sequences in their lessons, yet these can be effective or ineffective (Ofsted, 2004).

A review of research suggests that it might be impossible to connect certain teacherly actions to particular kinds of learning (Koehler and Grouws,

1992). Indeed, studies of such aspects as teacher clarity, proportions of time spent on different parts of lessons and so on are inconclusive. What is common is the view that the more time spent on development of concepts, however this is done, the better the learning. More recently, evaluation of the implementation of the effects of imposed structures in England shows that factors such as opportunity to learn (what is taught and when) and time (how long is spent on maths) overwhelm all other factors, including how closely teachers are following the guidelines (Brown et al., 2003).

While implementing imposed structures, individual teachers will create very different lessons and learning environments including subverting the intentions behind the innovation; further, it may not be the imposed structures, but the quality of the mathematics teaching within those structures and the professional development and leadership which accompany the innovation which make the difference.

The TIMSS seven-nation comparative study shows that among the sample of videos from high-achieving countries there is a wide range of different typical classroom practices (Hiebert et al., 2003). Critics of such comparisons point out that different countries have different aims in mathematics teaching. For example, the UK has focused more on problem-solving and data-handling than technical accuracy whereas Hong Kong has focused more on procedural knowledge and competence, so different tests will show different strengths for different countries. However, when further analysis of the video lessons was carried out, there were common features which are masked by paying attention to superficial aspects such as whether teaching is whole-class or not, students work in groups or not, students discuss together or not, how the board is used, when homework is discussed and so on. The common features of successful countries appear to be more subtle, the most significant characteristic being the way that mathematical concepts are presented. The complexity of the concepts and methods is preserved, rather than simplified, in the ways teachers work.

4. PHOENIX PARK

In Boaler's (1997) study of two matched English schools with very different teaching styles and grouping strategies, students were all following the same national curriculum and the same national assessment structure. There were no particular schemes or externally-imposed methods in use in the more successful, mixed-ability, school; the less successful school used a published scheme and traditional setting. It was the teaching and teachers who were different, and the teaching and teachers who made a significant difference to results. Those who were teaching mixed-ability

groups using exploratory, open tasks helped their students achieve much better results than those using a more traditional, text-based approach. It was clear that the lowest attaining students were benefiting significantly from these approaches. The school Boaler called Phoenix Park was not unique in its methods, and in many ways lessons at Phoenix Park were not examples of the 'best' practice of such teaching. However, they showed that even with flawed management and practice, methods which focused on exploration and sense-making rather than techniques and exercises achieved better results than traditional methods rigorously applied.

The positive effects of practices similar to those described in Boaler³ suggested to us that radically different approaches to teaching and learning mathematics from those suggested by authorities would be more likely to enable 'failing' students to recover interest and achievement in mathematics. However, it must also be said that Phoenix Park students did not end up knowing a lot of mathematics. Their test results outshone those of the comparison school, selected because of its socio-economic similarity, but did not outshine those of schools with more advantaged social profiles. In addition, achievement of students was strong in aspects which required and yielded to situationally specific problem-solving, but, whereas this is a powerful skill to have, it does not provide the generality and abstraction which leads to (a) technical competence and (b) a foundation for higher mathematical study. Whether the confidence arising from advanced problem-solving skills enabled more advanced mathematical development later is still unclear.

5. MATHEMATICAL THINKING OF LOW-ATTAINING STUDENTS

In 1998, project was carried out in a school with a similar social profile to Phoenix Park. I (Watson) believed that an essential aspect of making progress in conventional school mathematics was to develop abilities beyond those required by elementary mathematical exploration and application. I set out to see if some low-achieving students in the early years of secondary school could display the thinking skills, generalisations and abstractions which characterise both pure esoteric⁴ mathematics and the artificial word 'problems' arising in textbooks and tests. To some extent this question was answered positively earlier in the Low Attainers in Mathematics Project (LAMP) (Ahmed, 1987) which showed that such students could indeed think mathematically when being taught by teachers who were undergoing professional development as participants in a learning community. I wanted to see if the same could be said of students with 'ordinary' teachers. Working as a teaching assistant for two lessons per week

for one term, I explored whether, in a naturalistic classroom setting with a non-specialist teacher, students could be prompted to show the ways of thinking which were, according to Krutetskii (1976), characteristic of gifted mathematicians such as: grasping formal structures, logic, generalisation, flexibility, and so on.

Students in this classroom did not discuss mathematics at more than the level of sharing their answers and methods through the teacher. They were not routinely offered tasks which required much more than technical competence or practical skills such as measuring, producing diagrams and bar charts, cutting and fitting shapes, and so on. This meant that there were very few opportunities in class to think in complex ways with mathematics, multi-stage tasks, or to make and use generalisations. As a teaching assistant I generally sat alongside students who were working on the task set by the usual teacher and asked them extra questions designed to see if they could shift beyond the current exercise. For example students who had been constructing their own names on a coordinate grid and noting the coordinates they used were asked how they could know, without drawing, whether four given coordinate pairs would produce a rectangle or not. They were expected to shift from seeing each coordinate pair as a separate label to looking at relationships between coordinate pairs, and express these in general. All students prompted in this way were able to do so.

Another example is that students who had filled in a worksheet with a sequence of subtractions from 100 were asked to look at pairs such as $100 - 37$ and $100 - 63$ and say what relationships they could see in the numbers. From this they constructed the general family of $a + b = c$; $c - a = b$; $c - b = a$. Again, all those who were directed to think about structure and generality were able to do so, although the original task was not very challenging.⁵

Through keeping systematic records of these interactions from notes made during the lessons I found that every student in the class except one⁶ could display some aspects of the characteristic ways of working well with mathematics described by Krutetskii, and within the class *all* aspects identified by Krutetskii were displayed at least once by at least one student (Watson, 2001).

This evidence confirms the findings reported from LAMP by Trickett and Sulke (1988) and goes further, because the teachers with whom they worked were involved in a project to develop precisely those characteristics of mathematical behaviour, whereas the teacher with whom I worked was not. The only encouragement to think more deeply was coming from my intermittent extra prompts, not from their regular teaching. Although Krutetskii's students were displaying these characteristics in a sustained way in advanced mathematics, and 'my' students (and those of LAMP)

were not, this evidence contradicts any temptation to ascribe low attainment to learners' inability to think in mathematically useful ways. In this classroom it was not true that students 'could not' think mathematically, only that they 'did not' and were rarely offered the opportunity to do so.

We are not attempting to make any generalisation from this classroom, but the study led to a way of thinking about low-attaining students in terms of their proficiencies rather than deficiencies (Sztajn, 2003; Watson, 2001). I hypothesised that identifying and using students' proficiencies, rather than focusing on remediating their deficiencies, might be a way to work effectively with such students, and that systematic application of positive and challenging strategies might lead to better learning.

6. IAMP: IMPROVING ATTAINMENT IN MATHEMATICS PROJECT

6.1. *The project as proposed*

In IAMP, we aimed to develop the work of LAMP by creating a team of teachers who wanted to work on students' mathematical thinking as a way to develop their achievement and interest in mathematics within the current curriculum and assessment regimes. It was conceived as an intervention project in which teachers would put into practice ideas which were different from the prevailing content-focused 'catch-up' regimes. The teachers would be seen as co-researchers, exercising and evaluating their own professional judgement in a supportive group. They would meet regularly in groups with the researchers and discuss progress, gradually trying out more and more strategies and reporting the results in ways compatible with their usual practice. It was proposed that 10 teachers be recruited, in two different geographical areas, who would create small action-research projects over 2 years in their classrooms, introducing and developing practices which focused on the development of mathematical thinking informed by theory, reading, research, input and discussion within the project group.

Recruitment was achieved by contacting mathematics departments of schools and asking for volunteers. The plan was to introduce them to teaching strategies such as those described in the work of LAMP and Phoenix Park; problem-solving heuristics such as those described by Polya (e.g. 1962); interactional strategies and task design such as those developed by Watson and Mason (1998). Teachers would select from these inputs, putting ideas systematically into practice and evaluating the effects on learning, while supported by the group and monitored by researchers. The hypothesis was that these methods would help students achieve more in tests than the national average for students starting at the same baseline, and that

teacher-specific performance indicators would also show improvement in aspects of mathematical behaviour relating to the choice of strategies. The data to be collected systematically would comprise audiotapes of group discussions, audiotapes of interviews with researchers, videos of lessons, pupils' work, pupils' tests, field notes of discussions and lessons and teachers' notes. These data would be analysed qualitatively by the researchers, using an ongoing grounded theory approach to build up descriptions of teachers' practices, while also developing quantitative and qualitative data about pupils' mathematical performance. The idea was to relate practices, including task-type and interactional strategies, to improvements in attainment.

6.2. The project as it happened

In practice, the project did not work out as planned because of the realities of practice and differences of definition and belief which had to be negotiated. This section describes the central realities and differences and how they were resolved. Explicitness about the realities of innovation is necessary to situate the results, but in this case the diversity of practice itself, rather than hindering the research, led to some important findings.

6.2.1. Realities of practice

In practice, nearly all the teachers who joined the project were already known to the researchers, and only just enough volunteered. The sample can only be described as self-selected and opportunistic. This was not a surprise, since the expectation that teachers would commit themselves to acting in contrast to current orthodoxy was bold, and there had to be a measure of trust on their part. They were all teaching lower secondary mathematics sets in which at least half the class were achieving below the government standards for entry to secondary and the others were only barely achieving. In most of the classes, all students were underachieving to some extent.

We soon found that not all the teachers were able to take a fully systematic approach to action research. For example, several were working in highly pressured contexts and were physically unable to generate or collate data of any non-ephemeral kind, although they were making changes to practice and thinking about the consequences. The support of the researchers had to be flexible to take account of personal circumstances. One teacher queried the expectation to change her practice at all, and claimed to have no intention of changing, but her usual mode of teaching was to make constant small changes and evaluate their effectiveness which fitted well with our aims. Her personal goal was to analyse her practice and become

more articulate about the effectiveness of what she did. A few teachers agreed to use strategies offered in the project in gradual ways, fitting them into their usual practice, but not everyone chose the same strategies, and some rejected what others found most useful. As these different expectations became clear we still kept teachers in the project, because we were interested in what they *would* do to try to improve and evaluate learning.

Different teachers provided different quantities and kinds of data, and worked to different rhythms in their action research cycles. The researchers had to ensure that there was a common minimum of action and data for each teacher, and often the termly visit from the researcher and project meetings (two per term) were the only prompts for reflection, evaluation and re-planning. It became clear that the audio-recordings of meetings, discussions and observers' notes of lessons would provide the desired minimum of common data, so we had information about attitudes, beliefs, ideas, typical practices and some specific practices from every teacher, as well as teachers' reports and anecdotes about the progress of their pupils. From some teachers we had considerably more data than this.

All data were transcribed and analysed using a 'grounded theory' approach. The data were generously coded and categorised, then reflectively compared and recategorised, several times by researchers. Emergent findings were fed back regularly to the whole team, including the teachers, and this provided not only validation of interpretations but also the basis for planning the generation of more data about particular issues which had emerged, either through discussion or through future observations of action. In this way there was constant analysis, comparison, triangulation, new action, data generation, and regular inspection of differences and gaps. By about the middle of the second year we recognised that nothing new was emerging in terms of knowledge about teaching beliefs, priorities and decisions, except a gradual shift towards long-termism (described below) which we would continue to monitor. When we prepared papers and reports of the Project, teachers were again invited to read and criticise the analyses and interpretations so that, in the end, we were sure that we had fairly represented them and not theorised the findings in ways which were alien to their perceptions. Many were happy for their real names to be used and, since some of what we had to report contradicted 'official' views, this indicated to us that our interpretations were valid.

6.2.2. Meanings of mathematical thinking

As described above, there was little agreement about particular teaching strategies and task-types, partly due to a lack of agreement on a definition of 'mathematical thinking'. Many early project meetings were spent discussing this, not to reach agreement but to generate the range of different

meanings the phrase had for them, and through this sharing to present each of them with possible new meanings. It was agreed that students should be given the opportunity and encouragement to ‘think hard’ about mathematics, but the question of whether there were particular ways of thinking which were unique to mathematics at this fairly elementary level was never resolved. Some teachers referred to the categories of ‘using and applying mathematics’ in the National Curriculum (DfE, 1995); others wanted specifically mathematical ways of seeing and reasoning to be described. One teacher took this further and generated an e-mail discussion which resulted in a special issue of *Mathematics Teaching* (Pitt, 2002). We did not see this discussion as a weakness of the project at all; any imposed definition would not have resulted in uniform understandings, since, as we mentioned above, teachers are likely to interpret imposed ideas to fit their own existing beliefs and practices. It was not a shared understanding of ‘mathematical thinking’ which drew these teachers together, but a shared sense that particular kinds of engagement with mathematics would be more beneficial than others. The focus of the early part of the project became to identify what was seen as improvement and what was common about the kinds of engagement seen as beneficial.

6.2.3. *Definitions of improvement*

Improvement meant different things to different teachers. The target students were in general so demoralized by their previous mathematics experiences that behaviour, participation, self-belief and mathematical knowledge were all legitimate foci for change. There was eventual agreement that *any* of the following criteria might characterise ‘doing better’:

- being more active in lessons, for example by participating in discussion, asking and answering questions, volunteering for tasks, offering their own methods
- being more willing to share ideas with others: teachers, peers, whole class
- showing more interest in mathematics, for example by doing more homework, working on extended tasks, commenting positively in evaluation tasks
- being more willing and able to tackle routine, non-routine and unfamiliar tasks
- looking for and expecting to find coherence in tasks; expecting mathematics to make sense
- doing better than expected, or than comparison groups, on certain types of question in national and in-house tests
- showing improvements in behaviour and attendance

6.2.4. *Different attitudes to monitoring improvement*

It had been intended to use national test performance data to contribute to the evaluation of students' progress. However, during the Project several teachers refused to use these (or any) tests, even where they were assumed to be obligatory, because they felt the experience would be disheartening for learners. We could not, therefore, develop quantitative measures for progress in all cases and had to 'make do' with partial data.

7. DATA ANALYSIS

7.1. *National tests*

Three teachers provided data which compared their Year 7 test results to those from comparison classes or cohorts within the same school, chosen by them and taught by other teachers. These were not always parallel classes in the sense of having similar distributions of level scores at entry to secondary school. Because of the number of possible variables, and the small group sizes, and the lack of information about other teachers, we had to be careful not to draw hasty conclusions from the raw data, which showed that project students outperformed comparison groups. We wished to compare the performance of project and comparison classes of different types of test question. Questions were categorized according to the demands they would make on learners. Some types were too infrequent to use statistically, the remainder were as follows:

Mental and written calculations (M)

Thinking: transforming, applying, combining, interpreting methods (T)

M + T

T + remembered facts

Each question was scored so that a complete correct answer would be worth 1 mark, partly correct answers were worth 0.5, and incorrect or omitted answers worth 0. This rescoring from the official mark scheme was necessary to adjust for the fact that 'thinking' questions typically, officially, attracted more marks. Scores were averaged for each class and each question-type, thus accounting for different class sizes by making average question-score the unit of analysis, rather than individual student scores. At this stage, project classes outperformed comparison groups in each school, for each question-type, except in one case. However, we had not yet taken account of our lack of baseline data and had to trust the teachers to have chosen appropriate comparisons. For their purposes, this data was adequate because they *wanted* to be compared to these particular groups who had followed a more procedural, coverage-based, approach to the curriculum, using official guidelines. Indeed, the teachers who did not provide

data were very happy with an *ad hoc* approach to comparison, home-made tests, and their own reports of improved participation and enhanced classroom experience for teachers and students. The lack of matched groups and knowledge of the comparison cohort had to be taken into account when analysing the results further. Our hypothesis was that the project classes might have a different spread of achievement across questions of different types than the comparison classes. We therefore had to adjust the overall scores to make them comparable. We did this by scaling the comparison group question scores to make the overall average scores equal within schools, so that ratios of question-type scores within project and comparison groups were maintained. This allowed us to compare question types within each school on, as it were, a level playing field. Questions were ranked by performance and ranks were averaged for each question-type. Using a Mann–Whitney test on differences in average rank (Hayes, 1974: p. 778) we found that the comparison group performances on T questions were significantly lower at a 10% confidence level than performance on other question-types. In the project groups, performance between different question-types did not differ significantly. However, when project and comparison groups were compared using a Wilcoxon Signed Rank Test (Hayes, 1974, p. 180), project groups did significantly better on T-type questions at a 5% confidence level, but not significantly differently on other question types.

Even with the difficulties of working with this realistic sample data, we can nevertheless say that:

In questions which focused on mathematical thinking, such as requiring application of techniques and knowledge in an unusual way, or multi-stage reasoning, students in the tested Project groups did *significantly better than those in the comparison groups*.

It looked as if project teachers were making a difference to their students' ability to engage with non-routine mathematics questions in tests. This statistical data was backed up by teachers' reports from other groups, and data from other classes for which we did not have comparison groups, and we have no reason to suppose that other groups would have been any different. Though it was thin, this data convinced us that we were seeing the same kinds of effect as reported by Senk and Thompson (2003). However, in this study we could not yet say that it was associated with particular kinds of task or teaching strategy. The suggestion that the tested groups were doing better on 'thinking' questions led to the creation of a new evaluation tool which we hoped more teachers would agree to use.

7.2. *Exploration tasks*

To get more knowledge about whether the teaching was improving mathematical thinking in general, and not just in test questions, we asked all teachers to introduce pre-and post-tests in the second year of the project. They agreed to do use exploration tasks in a typical lesson context, except that teachers minimised the help they would usually give with interpretation, suggesting strategies, and so on. Information was gathered about the way teachers set and managed the task. Students' work was assessed using criteria typically used in the UK for high-stakes assessment of such work.⁷ For this analysis, each student's work was compared individually and differences between their performance in the pre- and post-task described both qualitatively and in a crude qualitative manner. Taking teacher input into account, it was found that, apart from two individuals, *all* students for whom we had both tasks showed improvement in their willingness to work on the task (evidenced by time spent on task and/or the amount of written work done), and produced more ideas, more lines of enquiry and were prepared to take more risks in the post-test, such as trying out different ways to look at the task, stating conjectures, making generalizations, and offering justifications. All qualitative descriptions of their work, apart from two students out of the total cohort,⁸ showed the use of more complex mathematics and mathematical argument; crude counting of ideas, lines of enquiry, examples given, types of examples given, statements and justifications showed every student except two at least doubling these frequencies and many doing excessively more than that. Achievement in these qualities was therefore significantly stronger at the end of the year.

7.3. *Summary of results*

This means that the students who might usually be expected to become more disaffected and reluctant to work, after being classed as 'failing' on entry to secondary school, were instead becoming more willing to work and more engaged mathematically. They showed significant improvement in the use of thinking skills to tackle unfamiliar tasks, or tasks involving some complex organization and adapted knowledge. In addition, teachers reported more enthusiasm among students. One class who had started the year by noisily rejecting the pre-test task asked for the post-test to extend over several lessons. This result cannot be dismissed as an effect of teaching which focused on extended exploratory tasks, because most of the teachers did not do this. We have not yet said anything about the nature of the teaching apart from the fact that teachers decided for themselves how to work to improve mathematical thinking, rather than procedural knowledge and coverage.

8. LOOKING FOR COMMON PRACTICES

These results repeat what has been found in several other studies in which students are taught mathematics through a focus on thinking and understanding rather than focusing solely on coverage of content. However, previous studies with similar results have not necessarily focused on the lowest achieving students, and they have tended to use given methods, approaches or materials. In this project there were no common methods or materials; some teachers used ideas from the readings we had given and discussed, others devised their own approaches alone or with others. We were unable to, indeed we did not intend to, impose certain methods on the teachers, whatever our own beliefs about effectiveness. The project teachers had joined to develop *their* practice, not to adopt ours. Some practices they used would have comfortably fitted into a typical 'reform' classroom; some would have comfortably fitted into a classroom in which silent textbook work was the norm. It was clear that any explanation of how these teachers had managed to be successful would be found at a deeper level than the observable actions and norms, and not through examination of the prevailing external goals and culture. The identification of such underlying features took most of the second year of the project. The focus shifted from finding methods which lead to improved attainment for low-achieving students to finding what is common, if anything, among teachers who make a difference.

8.1. *A common view of mathematical actions*

Eventually we constructed, from data, with teachers, a set of actions which would show that learners are thinking about mathematics. We also recognised that when similar activity occurred without being prompted this could be taken to be strong evidence of mathematical thought. The closest we came to shared teaching strategies was that most teachers were, by the end of the project, encouraging most of these kinds of mathematical activity in most lessons and hence, in some but not all lessons and cases, had made changes to their teaching:

Choosing appropriate techniques	Generating own enquiry
Contributing examples	Predicting problems
Describing connections with prior knowledge	Giving reasons
Finding underlying similarities or differences	Working on extended tasks over time
Generalising structure from diagrams or examples	Creating and sharing own methods

Identifying what can be changed	Using prior knowledge
Making something more difficult	Dealing with unfamiliar problems
Making comparisons	Changing their minds
Posing own questions	Initiating their own mathematics

Shared discussion encouraged teachers to prompt the use of these activities, and incidents of unprompted use were recorded by the researchers and teachers where possible. Within the limitations of collecting classroom data we claim that there was a shift from prompted to unprompted use as ways of working on mathematics offered first by teachers were later adopted by learners. Our evidence for this is that teachers would regularly tell us about such incidents and observers sometimes saw instances of unprompted evidence. However, rigorous data collection eluded us because of the limitations of the project and teachers' lives.

8.2. *Contrasts in practice*

We drew the conclusion that, even within a small study with an emphasis on sharing practices, superficial descriptions of methods, and even of tasks, do not capture the essence of excellent teaching. Moreover, we had data of such depth and variety that we were able to embark on a lengthy grounded analysis to find out what *was* there, lurking beneath the surface differences.

When teachers were questioned about their practices, intentions and actions it was clear that their reasons for action, both in action and on reflection, were complex. We were unable to distinguish between teachers by discerning distinct relationships within their distinct sets of intentions and actions. When we compared teachers to each other we saw their similarities and differences as a braid, so that for any aspect of teaching they might be apart in intention but coincident in action, or apart in action and coincident in intention. For example, it was not the case that two teachers whose view of mathematics was different would then necessarily diverge further when discussing challenging tasks; nor was it the case that two teachers who saw individual responsibility as important would necessarily act in similar ways when a student failed to show responsibility. Two teachers whose questioning styles appeared to be different might see themselves as making similar mathematical challenges to students. In another comparison, one teacher would repeat everything said by a student in whole class sessions, desiring that everyone should hear; another would not repeat at all, desiring that everyone should listen. More trivially, one teacher would make a big issue about students who forgot to bring pencils to class, since becoming a better learner includes planning to have the right tools; another would

quietly lend a pencil without comment, wanting to remove all obstacles to mathematical engagement. We saw these practices as constituting a braid in which there are many similarly wiggly strands within an overall common direction, rather than regarding every difference as a sign of overall divergence.

The complex relationship between beliefs and practices (Thompson, 1984, p. 124) is made more complex in this study, because it turned out that the teachers' stated beliefs, or truth claims, about education, students and mathematics were similar, and the improvement of their students' achievement was similar, but the teachers' practices were different at observable, superficial levels. However, we are not making a claim at this stage that beliefs are more important than action.

8.3. *Common principles beneath superficial differences*

When we analysed the data further we found underlying influences so powerful and so much at odds with usual practices for low-achieving students that we made sure we presented these back to the teachers several times for affirmation (Watson, 2003). These teachers articulated shared beliefs that all students could learn mathematics, that mathematics is intrinsically interesting, and that it is the teacher's job to support learner's approaches to mathematics as it is, with all its complexities. They rejected fixed notions of ability and learning style; they rejected the notion that mathematics has to be clothed as something else to be interesting; they rejected artificial simplification to bring a reduced form of mathematics to the learners. The worth of all students dominated their views. These beliefs emerged as overarching principles which guided their decisions and actions, namely:

- all have a right of access to a broad mathematics curriculum;
- all students should develop their reasoning and thinking in and through mathematics;
- mathematics can be a source of self-esteem;
- students have to become mathematical learners;
- adolescents have to develop the ability to exercise rights and responsibilities of citizenship through mathematics;
- teachers have to take account of reality:

We elaborate these principles below:

8.3.1. *Right to access mathematics*

While many adolescents appear to have obstacles to engagement in mathematics, all students have the right to, and are capable of, full engagement with the subject. Outstanding features of mathematics which make

it interesting, and which make learning easier, are the inter-connections between different topics and representations, and the relationships between and within mathematical structures.

This common belief was, however, manifested in different ways. For example, the flexibility and transformation required to do hard arithmetic mentally informs algebraic generalisation, so one teacher used a lot of mental work. Another teacher would ensure students always were able to choose the technology for any calculation, as her priority was for them to make efficient choices from all that is available. Neither teacher was right or wrong (although the students of the first teacher did better in mental arithmetic tests); each was working on valuable mathematical skills. Analysis of the nature of the tasks, the resulting activities, the language and symbols used, and the classroom norms showed that the learning opportunities offered to learners in these two groups were different, so the mathematics they would learn would be different (Towers and Davis, 2002).

8.3.2. *Development of reasoning and thinking*

All students are entitled to learn mathematics in ways which develop thinking and confidence in problem-solving. Mathematics offers, as well as numeracy skills, logical reasoning, discussion and argument about abstract ideas, and an arena for careful analysis, categorisation, generalisation. It also offers experience in solving unfamiliar problems, perhaps by looking for familiar structures and bringing knowledge and thought to bear on them.

For some teachers, the development of thinking was pursued through engagement with incompletely defined problems; for others it was pursued through mental challenge, such as having to keep several things in mind at once.

8.3.3. *Maths is a source of self-esteem*

For a variety of reasons, success in mathematics can be a source of self-esteem for students. Some may do better in mathematics than in other subjects, particularly if their literacy is also weak. The feeling that they are progressing in a high-status subject can be valuable, particularly if they failed to make progress in their previous schools. But more than this, a positive experience of mathematics can empower them mentally.

In the Project, self-esteem was developed through:

- responding to, using and generating students' own questions;
- fostering awareness of learning, such as using practice exercises for self-assessment;
- offering challenge and support instead of simplifying the work;
- enabling students to take risks, such as creating their own hypotheses;
- developing 'togetherness' in classrooms when working on mathematics.

8.3.4. *Rights and responsibilities as citizens*

All students are entitled to have access to the mathematics necessary to function in society, beyond minimal functioning. They need to be able to solve mathematical problems, and solve problems mathematically, with an awareness of number, space and probability, if they are to be good employees and citizens.

8.3.5. *Identity as mathematics learners*

Learners can see their goal as to learn, or to finish tasks, or to fit in. Some cannot see how to fit in so they appear to choose not to fit in; some see silence and inactivity as safe ways to fit in. In this Project the goal of teachers was to help learners have the goal to learn. They believed that self-regulated learning could be ‘taught’ or ‘caught’ (Pape et al., 2003). For many students, engagement in mathematics became its own reward. We saw this as evidence of a shift in their identity in relation to the subject.

8.3.6. *Taking account of reality*

While improvement of mathematical thinking and self-esteem are appropriate goals for educationists, students also need to be achieving in ways recognised by the outside world so ultimately test results are important. However, Project teachers came to believe that these were best improved through developing underlying attitudes and ways of thinking alongside mathematical knowledge so that future mathematical learning would be easier.

8.4. *Principled action: Convergent views about time*

We saw it as unlikely that these teachers were radically different from other teachers, so why, then, were these teachers acting in ways which contradicted ‘usual’ practices? Their commitment to the project, the opportunity to talk with and learn from each other and the researchers, and the effects of these in terms of supporting changes in, or unorthodox, practices seemed to be releasing and/or supporting fundamental humane attitudes to low-attaining students.

Our analysis led to an understanding that what the project participants had in common were principles for practice, and that we shared a goal that we describe as ‘deep progress’ (Watson et al., 2003).

Deep progress means that students:

- learn more mathematics,
- get better at learning mathematics,
- feel better about themselves as mathematics students.

Sometimes feeling better follows from learning more; sometimes students have to feel better before they learn more; sometimes students have to redevelop good learning habits before they can learn more. Ideally, students will make progress in all three aspects.

How teachers translated principles into action varied, as we have said, but there was some convergence. They all recognized that principles generated a need to:

- establish working habits which may have been lost through disaffection and low expectations
- provide tasks which generate concentration and participation, taking the view that 'doesn't concentrate' is not the same as 'cannot concentrate'
- develop routines of meaningful interaction
- choose how to react to correct and incorrect answers
- give students time to think and learn
- work explicitly or implicitly on memory
- use visualisation
- relate students' writing and learning
- help students be aware of progress
- give a range of choice
- be explicit about connections and differences in mathematics
- offer, retain and deal with mathematical complexity
- develop extended work on mathematics

All these areas were discussed in the team. What was common was the *will* of teachers to make deliberate, justifiable, decisions about their actions in these areas, in ways which were always focused on deep progress. For example, the development of working habits was seen by one teacher as behavioural training whereas, for another, personal responsibility was seen as the way to achieve them. However, there was, as the Project progressed, growing agreement about a perceived difference between short-termism and long-termism. Project teachers saw short-termism operating in general in the education system, typified by:

- prescribed curriculum coverage to be completed before the next test;
- students expected to display good work habits immediately and punished if they do not;
- moving rapidly from one task to another if concentration wanes;
- a focus on 'finishing work', either in class or at home.

Teachers and learners in the Project eventually worked to a different timescale. Short-termism was gradually abandoned and long-termism was put in its place.

- Longer was spent on each topic than was recommended nationally, but content coverage was still important. Coverage without understanding and memory was seen as pointless – understanding and memory need time to develop.
- Longer was spent on establishing good work habits, which might mean undoing previously developed habits. If this took substantial amounts of lesson time, it was worth it.
- Longer was spent on thinking and on particular tasks, to establish participation, reasoning, understanding and a sense of connectedness.
- The focus was on learning as much as possible, rather than finishing tasks.

All teachers found themselves working to sustain students' interest in a topic over time, not varying the task and topic frequently, but encouraging deeper thought of the kinds described earlier. This often worked against usual practice of providing students who found it hard to concentrate with frequent changes and new tasks, or changing topic frequently to avoid boredom. Sustaining work on one topic over a period of time was seen to promote deep progress, awareness of progress and hence self-esteem through being a good learner of mathematics. Improved concentration and participation enabled tasks to be extended, because learners were actively engaged in thinking about them. The relationship between these aspects was seen to be complex and symbiotic; one does not guarantee another. There was no such thing as 'a guaranteed extendable task'. The extension was created by the class, the task, the teacher, the questioning and prompting, and the learning over time.

Ways in which this shared principle was achieved in action included modelling for students how they might themselves develop ongoing self-questioning. For instance:

- When students were asked to create their own examples, they were encouraged to make up really hard ones which might present new challenges to work on. Some students who had been making up their own scalars for enlargement activities were then asked to explore what would happen if they used negative scalars.
- All hypotheses were followed with the question 'why?'
- Changes could be made to a situation, allowing students to apply what they have learnt so far; for example, having found out that when two lines cross you get two pairs of equal angles, which are supplementary, students were then given a third line and asked to find out about the angles obtained if two of the lines are parallel.

- Reflection on worked examples which students have completed, asking them to say which were easy and which were hard, and why, made whole exercises the focus of study rather than the technique on its own.
- Offering repetition for familiarity, but with significant variations each time, achieves fluency through accumulated experiences and gives students a sense of the range of possibilities in a topic. Some students had completed some work on ratios of two quantities successfully. They were then offered similar work (demonstrations, practical tasks, exercises) on ratios of three quantities. This helped them become more fluent with two, and also gain more understanding about ratio. Some then asked if they could do four!
- ‘Learn something new’ and ‘learn as much as you can’ became stated goals for lessons.

9. CONCLUSION

Although the outcome of the project is that pupils did better than expected, or better than comparison groups, or better in terms of progress in attitude, willingness and ability to engage with mathematics, we know that this kind of result can be found elsewhere. For example, the principles were identified through careful research of the teachers’ intentions and actions are very similar to the ‘dimensions and core features of classrooms’ related to projects reported by Hiebert and others (Hiebert et al. 1997). The more significant result is the finding that these teachers who were *free to innovate for themselves* were able to improve the attainment, engagement and mathematical thinking of low-achieving students *through action which arises from their own shared principles* in supported practice, experiment, discussion and evaluation. Their actions did not follow from a particular set of strategies, rather from the creation and adoption of strategies followed the intended aims of action based on principles. It is especially significant that these teachers were able to do this within a complex structure of imposed curricula, schemes and recommended lesson-types which they adapted or resisted to different degrees.

In this paper we have described what lies behind some successful mathematics teaching which contributed to better achievement for over 250 secondary low-attaining students over 2 years. This teaching was not dependent on materials, nor on a published scheme, nor on particular teaching methods or tasks. Rather, it turned out to be based on common principles, the most universal of these being the creation of space and time for learning through extended thinking time and extended tasks. Furthermore, the results show that giving even low-attaining, demoralised students more choice, freedom, challenge, responsibility and time enables them to

gain recognizable, testable skills. This does not decry projects in which low-attaining students are helped by direct instruction, training or mastery methods, rather it shows that the Project principles work at least as well, and indeed better, than principles geared towards short-term performance. We conclude that the search for a 'holy grail' of successful methods, organisations and structures for improving mathematics is a misguided quest:

- (i) It may be the search which is important, not the grail: relative success may be due to collegial discussion and reflection rather than the innovative activity.
- (ii) If we find a grail, we shall all see it differently: implementation of new ideas varies according to a wide range of teacher and institutional variables.
- (iii) We may be looking in the wrong place, and for the wrong thing: methods, organisations, structures and tasks may not be as important as principles and their supported manifestations.

It would be informative to re-visit the students in a few years' time to see if there were lasting effects on their achievement, or whether the gains in confidence, ways of learning and knowledge are dissipated in future classes with future teachers.

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NOTES

1. Funded by the Esmee Fairbairn Foundation (Grant numbers EDU 01-1415 and 02-1424). The views expressed in this paper are those of the authors and not of the funding body.
2. In this paper, 'problem-solving' refers to the solution of complex, possibly incompletely-defined, problems with mathematical or everyday contexts, for which several operations may have to be identified and tested, or other methods of solution devised. It does not refer to textbook word problems unless they are of this type.
3. See Ollerton and Watson (2001) for detailed descriptions of these classrooms.
4. The term 'esoteric' is acquired from Cooper and Dunne (1998) to mean mathematics which is not embedded in a realistic or pseudo-everyday context.

5. This research made use of a collection of generic prompts which have been published in Watson and Mason (1998).
6. The one for whom this was, perhaps, doubtful had a very weak short-term memory and could not even recall the result of a calculation he had done moments before.
7. The actual criteria used were those developed by the Association of Teachers of Mathematics for 100% coursework General Certificate of Secondary Education (the high-stakes 16+ UK examination) in 1988–1994. These provide a highly detailed description of mathematical activity used for port-folio assessment.
8. By this time we had 8 teachers still in the project from whom we had a little more than 100 pre- and post tests. Attrition and erratic attendance were a big problem in these classes.

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