Early Childhood Educ J (2016) 44:255–262
DOI 10.1007/s10643-015-0707-4

Investigating Children's Abilities to Count and Make Quantitative Comparisons

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Published online: 1 May 2015

Abstract This study was designed to investigate children's abilities to count and make quantitative comparisons. In addition, this study utilized reasoning questions (i.e., how did you know?). Thirty-four preschoolers, mean age 4.5 years old, participated in the study. According to the results, 89 % of the children (n = 30) were able to do rote counting and 70 % (n = 24) were able to do rational counting. When children were asked how they knew how many objects were in a set, 30 responded that they used a counting strategy. Sixty-five percent of children (n = 22) answered "zero" when no block was given and 21 children answered "nothing" when they were asked what zero meant to them. About quantitative comparisons, 65 % of children (n = 22) answered correctly when they were asked more and less questions.

Keywords Counting · Quantitative reasoning · Zero concept · Numeric reasoning · Early number concepts

Introduction

Children possess tremendous knowledge of mathematics and bring it to the classroom as they enter schools. However, this prior knowledge is often disregarded when assessing children's mathematics knowledge and skills (Lee 2014). Assessment focuses on evaluating what children can answer in the form of school mathematics without considering their reasoning for answering with correct or incorrect answers. Children sometimes answer correctly or incorrectly, but the more important aspect of the assessment is to know their reasoning behind their answers for why and how they come up with the answers.

In order to teach young children mathematics in a developmentally appropriate manner, it is important for teachers of young children to know and to assess what children really know about mathematics [Copley 2009; Lee 2014; National Association for the Education of Young Children (NAEYC) and National Council of Teachers of Mathematics (NCTM) 2010; NCTM 2001]. Knowing what children know helps teachers develop and implement math lessons by scaffolding from what children know to what they need to know (Vygotsky 1978).

There are several assessment and screening tools available in early childhood education to evaluate children's mathematics skills and knowledge, including the Woodcock-Johnson test (Woodcock et al. 2001), the Bracken Basic Concept Scale (Bracken 1998), and the Test of Early Mathematics Ability (Ginsburg and Baroody 2003). These tools demonstrate an acceptable level of reliability and validity and have been utilized to measure children's mathematics knowledge and skills in certain contents and topics associated with numbers and geometry (Clements et al. 2008). However, these instruments fail to assess children's reasoning though it is essential to know in order to help children further promote their math knowledge and skills. Existing assessments commonly aim to assess children's knowledge of formal and school mathematics and to score or label whether children answer correctly or to screen children's ability and skills. These

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assessments provide limited information to teachers of young children as they plan to promote children's knowledge and skills of mathematics. To overcome these limitations, Clements et al. (2008) developed a comprehensive measure based on the Rasch model to assess children's early math performance considering their math developmental trajectories. However, this instrument is still limited to provide children's reasoning behind their answers since previous assessment tools have been designed to mainly assess children's knowledge and skills without considering their reasoning or reasoning strategies.

This study aims to address this gap by asking children reasoning questions (i.e., how did you know?). In order to adequately assess children's early knowledge and skills of mathematics, it is critically important to know their reasoning or reasoning strategies instead of only assessing whether they can count or correctly answer the questions.

Review of Related Literature

Children's Early Numeracy Skills

Over the last two decades, researchers have accumulated a wealth of evidence showing that from birth to age 5, young children develop an understanding of mathematic concepts, including informal ideas of more and less, taking away, shape, size, location, time, pattern, and position (Baroody et al. 2006; Clements and Sarama 2009; Lee 2014; Lee et al. 2009, 2015). These concepts are surprisingly broad, complex, and sometimes sophisticated. It is such a fundamental and pervasive feature of the child's cognition that it is hard to see how children could function without it. Burnett and Farkas (2009) argue that all children, regardless of background and culture, are endowed with instinctive abilities including not only number, but also other mathematics contents including basic geometry.

According to the Preschool Early Numeracy Skills Test results, numeracy skills are essential math skills in early childhood and are represented best by the three highly related but distinct factors of numbering, relations, and arithmetic operations (Purpura and Lonigan 2013). Young children generally develop a common pattern of learning number and arithmetic. For example, young children might learn to verbally count without correct sequence; they will say "strings" of counting words, which might not follow the conventional sequence, something like "One, two, three, four, five, six, ten, seven, nine." With more exposure to numbers, children will become familiar with number words and be able to verbally say numbers in the correct sequence. Understanding these common patterns of growth as well as developmental trajectories in math helps teachers comprehend children's mathematical developmental level, set up instructional goals, and implement math lessons in a developmentally appropriate manner (Clements and Sarama 2009; Lee 2014).

Young children's thinking is relatively concrete (Constance 1989; Piaget 1952). They are able to see that this set of objects has more items than that one, and they can add 3 toy dogs to 4 toy dogs to get the sum. Yet in other ways, young children's thinking is very abstract. They know that adding always makes more and subtracting less. They have abstract ideas about counting objects, including one-to-one correspondence (one and only one number word should be assigned to each object) and the abstraction principle (any discrete objects can be counted). Young children also learn other kinds of mathematical language, like the names of numbers and words for comparing quantity (e.g., "more," "less").

Therefore, when assessing young children's numeric knowledge and skills, it is important to include reasoning questions as these are valid in authentically measuring children's thought processes in terms of math ability. Assessment tools such as the Test of Mathematic Ability Third Edition [(TEMA-3) Ginsburg and Baroody 2003], Bracken Basic Concept Skills (Bracken 1998), and High/Scope Preschool Child Observation Record (High Score Educational Research Foundation 2003) are some of the instruments which assess children's numeric knowledge and skills without asking about children's reasoning. This study aims to explore children's abilities to count and make quantitative comparisons with reasoning questions.

Variability in Children's Numeric Reasoning

Appropriately assessing young children's mathematics knowledge is critical in understanding their mathematics learning and development. Nevertheless, there is some variability in children's numeric reasoning. The next section explains some of this variability.

Competencies

Children's minds are not simple. On the one hand, from an early age they seem to understand basic ideas of addition and subtraction (Copley 2009) and spatial relations (Clements and Sarama 2009; Lee et al. 2015). They can spontaneously develop various methods of calculation, like counting on from the larger number (given 9 and 2, the child counts, "nine…ten, eleven;" Purpura and Lonigan 2013). At the same time, children display certain kinds of mathematic incompetence, as for example when they have difficulty understanding that the number of objects remains the same even when they are merely shifted around (Piaget 1952) or when they fail to realize that an odd looking triangle (for example, an extremely elongated, non-rightangle, "skinny" triangle) is as legitimate a triangle as one with three sides the same length (Clements and Sarama 2009).

Mathematical language

The importance of mathematical language is underscored by the fact that the amount of teachers' math-related talk is significantly related to the growth of preschoolers' conventional mathematical knowledge over the school year (Klibanoff et al. 2006). Language is clearly deeply embedded in mathematics learning and teaching. Often, children's verbal expressions offer only a glimpse of what they know and think. Numerous researchers have reported that children's mathematic ability in understanding numerical concepts begins early in life (Ginsburg et al. 2008; Purpura and Lonigan 2013). Knowledge gaps appear in large part due to the lack of connection between children's informal and intuitive knowledge and school mathematics. This is especially detrimental when this informal knowledge is poorly developed. Young children bring impressive informal mathematical strengths to the classroom. What children know and how well they think and respond to problem-solving situations may vary depending on home backgrounds, and thus culture might interact with assessment to produce "differential validity" of the assessment tools.

To learn about what is hidden in children's minds and their mathematical reasoning, teachers need to engage in effective clinical interviewing (Ginsburg et al. 1983). In this study, we aim to investigate children's numeric reasoning utilizing a clinical interview. We focused on children's early number concepts and their numeric reasoning by measuring their abilities of rote (or verbal) counting, rational counting, cardinality rule, "zero" concept, and quantitative reasoning (more and less concept).

The study is guided by operational definitions as follows:

- Rote Counting: Counting using number words in the correct order;
- Rational Counting: Counting number words in the correct order and saying the correct number as objects are counted;
- Cardinality Rule: The last number counted is the number of items in the set.

The findings may provide fundamental empirical evidence of the need for assessment instruments to assess children's developmental mathematics abilities.

Methods

Participants

Participants in this study were 34 preschoolers (mean age 4.5 years), 16 boys and 18 girls. Among 34 participants, 22 children were from low-income families (i.e., qualified for reduced-cost or free lunch). Thirty-two children spoke English only and two were identified as bilingual (i.e., speaking both English and Spanish). Five children were enrolled in a private religious preschool and 29 were enrolled in three public pre-K schools.

Data Source

This study utilized a clinical interview method to assess children's abilities to count and make quantitative comparisons by assessing their abilities in rote counting, rational counting with the cardinality rule, "zero" concept, and quantity comparison between two sets of blocks.

During the interview, reasoning questions were utilized to investigate children's thoughts on each of their responses. Clinical interview is identified as a very effective method of gathering information on children's mathematical thinking (Ginsburg et al. 1983; Ginsburg 2009; Labinowicz 1985; Merrifield and Pearn 1999). Children were interviewed individually for approximately 12–15 min. The researchers used five questions to assess children's abilities to count and make quantitative comparisons. Five questions were generated based on existing assessment tools on counting and comparisons [e.g., Bracken's School Readiness Assessment on Math, Research-Based Early Maths Assessment (REMA), core competencies of counting and quantitative comparisons by Weiland et al. 2012] including Common Core State Standards on counting and cardinality rules. The researchers specifically focused on counting and comparisons since these two areas have been identified as essential skills for preschoolers and core competencies in early math (CCSS, n.d.; NCTM 2001; Weiland et al. 2012). The questions include: (1) Can you count? (2) How many blocks do you have? How did you know? (3) Can you show me 5 blocks? How did you know there were 5? (4) (No block is given to the child.) How many blocks do you have? If a child says, "zero," the following question was asked: What do you mean by "zero"? and (5) (Give the child five blocks and interviewer keeps six blocks.) Who has more/less blocks? How did you know?

Results

Data from the interviews were analyzed descriptively to assess children's abilities to count and make quantitative comparisons with reasoning. The analyses were categorized into four core competencies:

- Rote/verbal counting
- Rational counting with the cardinality rule
- Concept of zero
- More and less

Rote/Verbal Counting

As Table 1 shows, 30 of the children showed some level of rote/verbal counting ability. Thirty children showed rote counting ability which included 22 children who were able to count 1–10 without errors and five children who were able to count 1–5 without errors. However, four children did not show rote counting ability by skipping some numbers when counting (e.g., one child skipped 4 when she was counting from 1 to 5 and another child skipped 3 but counted 1–6). Children who skipped numbers or were unable to rote count successfully were given a second chance to count in order to eliminate any unintentional errors. In Table 1, data of children's counting from 1 to 5 represent those who were able to do so without any errors.

Rational Counting with Cardinality Rule

When children were given blocks based on their rote/verbal counting ability (e.g., children who were able to count from 1 to 5 were given 5 blocks; children who were able to count from 1 to 10 were given 9 blocks), they were asked to tell how many blocks they had. As shown in Table 2, 24 children showed rational counting ability as well as cardinality rule. Six children skipped numbers when they were counting blocks. Four children counted correctly but skipped the object (block) when they were counting. When children were asked how they knew how many blocks they had, 30 children responded that they counted and four children looked at the interviewer and counted the blocks again without responding.

Table 1 Participants' percentage of score of rote counting

Level	Percentage (*n*)
Skipping number	12 % (4)
Rote counting 1–5	24 % (8)
Rote counting 1–10	65 % (22)
Total (*N*)	100 % (34)

Table 2 Participants' percentage of rational counting with cardinality rule

Levels	Percentage (*n*)
Skipping number(s)	18 % (6)
Skipping object(s)	12 % (4)
Rational counting with the cardinality rule	70 % (24)
Total (*N*)	100 % (34)

Concept of Zero

When no block was given to a child, she/he was asked to tell how many blocks he/she had. As shown in Table 3, 12 children answered "zero." When the interviewer asked them what "zero" meant, 21 children answered "nothing;" one child counted backwards from 10 to zero (Fig. 1). The following presents the interview transcript of this child.

More and Less Concept

Children were given five blocks and the interviewer had six blocks. Children were asked both questions: Who has more blocks? and Who has fewer blocks? Twenty-two children answered correctly on both more and less questions, while twelve children answered incorrectly. Figure 2 gives sample interview transcriptions. Child 1 answered correctly and Child 2, 3, 4, and 5 answered incorrectly.

Discussion

As the study results show, there are variances in children's responses while assessing their counting ability regardless of children's family income level. When children are asked to count, skipping a number is a common mistake. Which number(s) children often skip and the reasons need to be further investigated to better assist them as they rote count. An important finding of this study is related to rational counting with the cardinality rule. Typically, when children are assessed, they are asked to show a certain number of objects. If a child fails to show the correct number, it is considered "incorrect" without considering why the child is unable to answer correctly. This question (e.g., "show me five blocks") itself involves several mathematical concepts such as early number concept (e.g., classification, sorting, comparison, etc.), one-to-one correspondence, rational counting, and the cardinality rule (knowing how many are in a set; Lee 2014). Children are unable to show rational counting ability if they misunderstand one of these mathematics concepts. Reasoning questions are essential in assessing what pre/early number concepts children are

Table 3 Participants' percentage of "zero" concept

When no block was given to a child, the child answered	Percentage (*n*)
Zero	65 % (22)
Nothing	18 % (6)
Don't know	12 % (4)
Unable to answer	6 % (2)
Total (*N*)	100 % (34)

Fig. 1 Interview transcription on "zero" concept

Interviewer: What does "zero" mean?
Child: That's hmm…
(The child paused and showed her fingers. She started to count from 10 showing her fingers. She put each finger down as she counted and counted slowly).
Ten, nine, eight, seven, six, five, four, three, two, one, zero.
Interviewer: What does "zero" mean?
Child: You have to count.
(The child started to count her fingers from 10 to zero again.)

missing since being able to assess which pre/early number concepts the child does not possess is critically important to know in order to help the child master that particular math/number concept.

As the study results show, children showed variance in responding to "how many" questions: 18 % of children skipped numbers and 12 % of children skipped objects. Learning how to count for the first time is considered a type of social knowledge, which requires children to learn from a social agent or medium (Copley 2009). Providing children opportunities to practice counting on an everyday basis is an important part of teaching them to be familiar with numbers and to be able to count. Effective ways to teach children to practice rote/verbal counting include integrating or utilizing counting songs, reading counting books to them, exposing them to numbers in the classroom settings (e.g., number capacity for each play center), or using number-associated words in daily life (NAEYC and NCTM 2010). For example, a teacher might say, "Three children can play in our block center. Raise your hand if you want to play in the block center." Once children raised their hands, the teacher would count aloud or ask children to count the number together. Helping children practice counting in their daily life is an effective way to expose them to numbers. In addition, some children might have one-to-one correspondence skills. Teachers of young children can point to one child at a time when counting.

In terms of assessment, it is necessary to assess variances of children's counting ability and to pair reasoning skills with rational counting. Without assessing what children know, there is always a gap in teaching and learning in early mathematics. When assessing children's mathematics performance or screening their mathematics proficiency, it is critical to assess what children actually know, including their reasoning skills.

The majority of children answered "zero" when no block was given to them. This study further investigated the response of "nothing." The concept of "zero" is very important since it is the fundamental concept of place value. In early elementary years, children have difficulty in understanding place value (i.e., each place has its own value). The common mistake is from children's misunderstanding of "zero" as "nothing." For example, children might see "0" as meaning the same thing in the numbers 102 and 120. In 102, zero refers to the absence of a 10 s place, while in 120, zero means the absence of the 1 s place. This is an important finding (i.e., that some of the children thought that "zero" was the same meaning as "nothing") which provides a critical implication for teaching practice. When introducing children to the concept of "zero," it is necessary to systematically help children understand "zero" as an absence of something. This will help children as they learn about place value in upper elementary grades. In terms of research, children's perception of "zero" needs to be further investigated to determine how this perception is related with their later concept of place value.

About 30 % of the children in the study confused the terms "more" and "less." The "more and less" concept is a type of social knowledge which children learn from social agents (teachers, guardians, other media, etc.). Those who correctly answered on "more and less" tasks utilized counting strategies to determine the quantity. This finding implicates that it is necessary to provide children with more concrete opportunities to compare in their daily lives to help them become more familiar with comparing quantities.

In conclusion, when assessing children about mathematics, it is critically important to ask the child reasoning questions (e.g., Why do you think so? How did you get it?). This would provide educators with rich information about what children

Fig. 2 Interview transcriptions on "more and less" concept

Child 1
(Child was given 5 blocks and the interviewer had 6 blocks.) *Interviewer: Who does have more blocks?* *Child 1: Hmm… (wait for about 30 seconds and pointed interviewer's blocks)* *Interviewer: Who has less?* *Child 1: (Pointing himself) Me.* *Interviewer: How did you know?* *Child 1: I counted.*
Child 2
(Child was given 5 blocks and the interviewer had 6 blocks.) *Interviewer: Who does have more blocks?* *Child 2: Hmm… (Child shrugged her shoulders and kept quiet for about 30 seconds. She started to count.) One, two, three, four, five. (The child counted her blocks and stopped. She started to count the interviewer's blocks). One, two, three, four, five, six. This has one more (pointing at the interviewer's blocks).* *Interviewer: Who does have less?* *Child 2: (The child didn't answer but started to count her blocks.) One, two, three, four, five. (She pointed to the interviewer's blocks and started to count by pointing to each block.) One, two, three, four, five, six.*
Child 3
(Child was given 6 blocks and the interviewer had 5 blocks.) *Interviewer: Who does have more blocks?* *Child 3: (The child counted his blocks.) One, two, three, four, five, six. (The child started to count the interviewer's blocks.) One, two, three, four, five. (The child didn't answer the question).* *Interviewer: Who does have less?* *Child 3: (The child counted his blocks again) One, two, three, four, five, six. (She counted the interviewer's blocks again) One, two, three, four, five. (The child looked at the interviewer.)* *Interviewer: Who do you think has more blocks?* *Child 3: (The child counted her blocks and the interviewer's blocks again.)*
Child 4 *(Child 4 is a bilingual child and communicates in proficient Spanish and limited English.)*
(Child was given 5 blocks and the interviewer had 6 blocks.) *Interviewer: Who does have more blocks?* *Child 4: (Pointing to himself.)* *Interviewer: How did you know?* *Child 4: (He counted his blocks.) One, two, three, four, five. (He counted the interviewer's blocks.) One, two, three, four, five, six.* *Interviewer: Who does have less?* *Child 4: (Points to the interviewer.)* *Interviewer: How did you know?* *Child 4: (Child counted his blocks again.) One, two, three, four, five. (Child counted the interviewer's blocks.) One, two three, four, five, six.*
Child 5
(Child was given 5 blocks and the interviewer had 6 blocks.) *Interviewer: Who does have more blocks?* *Child 5: Me* *Interviewer: How did you know?* *Child 5: I counted.* *Interviewer: Who does have less?* *Child 5: You* *Interviewer: How did you know?* *Child 5: I counted.*

know and why they think in a particular way. This would ultimately help teachers to meaningfully implement math lessons by scaffolding what children know and what children need to know (Lee et al. 2003; Lee 2014; Vygotsky 1978).

This exploratory research paper calls attention to the need for assessment tools to evaluate mathematical reasoning (e.g., numeric reasoning, algebraic reasoning, geometric reasoning, measurement reasoning, etc.) instead of focusing on outcome-based correct answers reflecting school mathematics. Specifically, it is necessary for assessors or researchers to closely examine each question in existing assessment instruments to determine whether one question/item involves more than one math concept, determine whether the item evaluates what children know about mathematics, and break the question into several levels if necessary. Instead of asking children, "show me five blocks," an assessor needs to break this question into several items since this question involves several math concepts. For example, to be able to accurately assess children's counting in this question, it is necessary to ask children to count to assess whether they are able to both rote count and do one-to-one correspondence. Finally, it is also necessary to assess whether children are able to give an answer which involves the cardinality rule (knowing that the last number represents the quantity of a set).

In this study, children's demographic backgrounds such as family social economic status (SES) and language was not considered as a factor in their abilities to count and make quantitative comparisons. As in many other areas, preschool children of lower SES generally perform more poorly on simple mathematical tasks than do their more privileged peers (Lee et al. 2008). Lower SES children show less proficiency in mathematics than do their middle class peers, particularly when metacognition is required. In addition, for children whose first language is not English, math could pose a challenge (Lee et al. 2011). Therefore, it is recommended for future researchers to consider how these two factors influence children's abilities to count and make quantitative comparisons.

References

Baroody, A. J., Lai, M., & Mix, K. S. (2006). The development of young children's early number and operation sense and its implications for early childhood education. In B. Spodek & O. N. Saracho (Eds.), *Handbook of research on the education of young children* (2nd ed., pp. 135–152). Mahwah, NJ: Lawrence Erlbaum Associates.

Bracken, B. (1998). *Bracken basic concept scale-revised*. San Antonio, TX: Psychological Corporation, Harcourt Brace and Company.

Burnett, K., & Farkas, G. (2009). Poverty and family structure effects on children's mathematic achievement: Estimates from random and fixed effects models. *Social Science Journal, 46*, 297–318.

Clements, D. H., & Sarama, J. (2009). *Learning trajectories in early mathematics—Sequences of acquisition and teaching*. Encyclopedia on Early Childhood Development. Retrieved October 14, 2014 from http://www.eyeonkids.ca/docs/files/numeracy.pdf

Clements, D., Sarama, J., & Liu, H. (2008). Development of a measure of early mathematics achievement using Rasch model: The research-based early math assessment. *Educational Psychology, 28*(4), 457–482.

Constance, K. (1989). *Numbers in preschool and kindergarten: Educational implication of Piaget's theory*. Washington, DC: National Association of Education for Young Children.

Copley, J. (2009). *The young child and mathematics* (2nd ed.). Washington DC: National Association of Education for Young Children.

Ginsburg, H. (2009). The challenge of formative assessment in mathematics education: Children's minds, teacher's minds. *Human Development, 52*(1), 109–118.

Ginsburg, H., & Baroody, A. (2003). *Test of early mathematics ability*. Austin, TX: Pro-ed.

Ginsburg, H., Kossan, N., Schwartz, R., & Swanson, D. (1983). Protocol methods in research on mathematical thinking. In H. P. Ginsburg (Ed.), *The development of mathematical thinking*. New York, NY: Academic Press.

Ginsburg, H., Lee, J. S., & Boyd, S. (2008). Mathematics education for young children: What it is and how to promote it. *Society of Child Research, 22*(1), 3–22.

High Scope Educational Research Foundation. (2003). *High/scope preschool child observation record*. Ypsilanti, MI: High Scope Educational Research Foundation.

Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology, 42*(1), 59–69.

Labinowicz, E. (1985). *Learning from children*. Menlo Park, CA: Addison Wesley.

Lee, J. (2014). Is children's informal knowledge of mathematics important? Rethinking assessment of children's knowledge of mathematics. *Contemporary Issues in Early Childhood, 15*(3), 293–294.

Lee, J., Autry, M., Fox, J., & Williams, C. (2008). Investigating children's mathematics readiness. *Journal of Research in Childhood Education*, *22*(3), 316–328.

Lee, J., Collins, D., & Winkelman, L. (2015). Connecting 2D and 3D: drafting blueprints, building, and playing with blocks. *Young Children, 70*(1), 32–35.

Lee, J., Lee, Y., & Amaro-Jimenez, C. (2011). Teaching English language learners (ELLs) mathematics in early childhood. *Childhood Education, 87*(4), 253–260.

Lee, J., Lee, J. O., & Collins, D. (2009). Enhancing children's spatial sense using tangrams. *Childhood Education, 86*(2), 92–94.

Lee, J., Meadows, M., & Lee, J. O. (2003). *What causes teachers to implement high-quality mathematics education more frequently: Focusing on teachers' pedagogical content knowledge*. Retrieved from ERIC (ED 472327).

Merrifield, M., & Pearn, C. (1999). Mathematics intervention. In Early Years of Schooling Branch (Eds.), *Targeting excellence: Continuing the journey* (pp. 62–70). Melbourne.

National Association for the Education of Young Children & National Council of Teachers of Mathematics. (2010). *Early childhood mathematics: Promoting good beginnings*. Retrieved from http://www.naeyc.org/files/naeyc/file/positions/psmath.pdf

National Council of Teachers of Mathematics. (2001). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Piaget, J. (1952). *The origin of intelligence in children*. New York, NY: International University Press.

Purpura, D. J., & Lonigan, C. J. (2013). Informal numeracy skills: The structure and relations among numbering, relations, and arithmetic operations in preschool. *American Educational Research Journal, 50*(1), 178–209.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Weiland, C., Wolfe, C. B., Hurwitz, M. D., Clements, D. H., Sarama, J. H., & Yoshikawa, H. (2012). Early mathematics assessment: Validation of the short form of a prekindergarten and kindergarten mathematics measure. *Educational Psychology, 32*(3), 311–333.

Woodcock, R., McGrew, K., & Mather, N. (2001). *Woodcock–Johnson III Tests of achievement*. Itasca, IL: Riverside Publishing Company.