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First Graders' Understanding of Measurement

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Abstract This study focused on the meaning of measurement to a group of 16 first grade students. A university professor and the teacher of the students partnered together using qualitative analysis of field notes, student interviews, and student work samples gathered from September through May of a school year. Findings indicate students' knowledge of measurement including transitivity, unit iteration, conservation of number and length, and social knowledge of measurement terms and tools increased over the year. Researchers identified six themes of students' measurement understanding including that children's literature played a motivating role in student-initiated measurement activities. Recommendations call for first grade measurement activities focused on what it means to measure rather than on how to measure. Researchers caution that educators using mathematics curriculum and assessment should not assume that primary grade students understand conservation and unit iteration.

Keywords Measurement · Constructivism · Children's understanding · Mathematics · First grade

Introduction

Measurement is an education objective from kindergarten through the elementary years. Understanding measurement is difficult for young children as evidenced by their low scores on state mandated achievement tests (Kamii & Housman, 2000). One reason for this difficulty may be due to the way in which measurement has been traditionally taught as empirical procedures in how to use measurement tools, such as rulers, with little focus on children's ability to reason about measurement problems (Kamii & Clark, 1997; Clements & Bright, 2003). The purpose of this study was to analyze what measurement means to first grade students. Results of this study and similar studies may then help teachers rethink the way measurement should be taught.

This study was informed by constructivist theory and research on children and measurement (Kamii & Clark, 1997) as well as by hermeneutic phenomenological theory focusing on the importance of children's lived experiences (van Manen 1990). Children's lived experiences refer to the everyday events in children's lives, some of which may involve measurement. The researchers, a public school first grade teacher and a university professor, worked in partnership to study what measurement meant to children in the first grade teacher's classroom.

Researchers used qualitative, descriptive methods to gather data in a naturalistic setting of a first grade classroom (Anderson & Dousis, 2006; Leedy & Ormrod, 2005). The research questions of interest to the researchers were:

1. How does children's measurement knowledge at the beginning of the academic year compare to what they know at the end of the academic year?
2. In what ways do children express their knowledge of measurement in both teacher-initiated and student-initiated measurement activities?

These questions evolved from the first grade teacher's concerns about the difficulties young children at her school

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experience in their understanding of measurement; state mandated measurement objectives; low scores in measurement on state mandated achievement tests; and a desire to improve teaching to better facilitate students' understanding of measurement.

Method

The setting for the study was a first grade classroom of 16 students, located in a public elementary school situated in a middle-to-upper class suburban community in a Midwestern state in the United States. Of the 16 students in the classroom, six were females and 10 were males; 14 students were Caucasian and of varied European ancestry, one was Middle Eastern, and one was Chinese. Students were 7–8 years old with a mean age at the beginning of the study of 7 years 9 months. The two female researchers were of Caucasian Anglo-European background. Each researcher had over 20 years teaching experience including teaching first grade.

Procedures

Students were interviewed once in September after school had begun and again with the same interview questions in May as the school year was ending. The classroom teacher facilitated and engaged students in measurement activities on a weekly basis. Examples of teacher-initiated activities included students' estimations of length of their long jumps; use of non-standard measurement tools to measure distance from the classroom to the office; estimations of circumference and weight of various sized pumpkins; and measurement of plant growth with both non-standard and standard measurement tools. (A detailed list of activities is available from the first author.) The classroom teacher and university professor took field notes of students' measurement activities during teacher-initiated activities and student-initiated activities including student-initiated comments about measurement. The university professor visited the classroom each week from September through May for 1–2 h each visit to help facilitate measurement activities, observe, and record children's activities and comments.

Data Collection

Data were collected over a 9 month period from September through May. Data sources consisted of: two identical student interviews on measurement conducted first in September and then again in May; field notes taken during observations of measurement activities; and students' work samples including their writing and drawings made during measurement activities.

Interviews

Researchers conducted individual interviews with students of their ideas about measurement in a small, quiet room down the hall from the classroom that took about 45 min per student to complete. Both researchers analyzed both sets of beginning and end of year interview data. Interview tasks, selected from previous research, assessed the abilities involved in understanding measurement: conservation of number and length, transitivity, and unit iteration. Interview tasks also included a 14 item set of questions developed by the researchers about social knowledge of measurement terms and tools (available from the first author). The interview tasks on transitivity, unit iteration, and conservation were patterned after those described in the research of Piaget, Inhelder, and Szeminska (1960) and Kamii and Clark (1997). Students were assessed for demonstration of an understanding of conservation, transitivity, and unit iteration. The interview in this study used the three sticks task, the tower task, and the inverted T task to assess transitivity and unit iteration, two cognitive abilities necessary in measurement. The interview also included two Piagetian tasks to assess conservation of number and length, prerequisites to understanding measurement. The researchers used the established protocols for administering and analyzing responses found in Piaget et al. (1960) (tower task), Kamii and Clark (1997) (inverted T task), Piaget and Szeminska (1965) (three sticks task) and Inhelder, Sinclair, and Bovet (1974) (conservation tasks).

Kamii and Clark (1997) define transitivity as, "...the ability to deduce a relationship from two or more other relationships of equality or inequality" (pp. 117–118). In this study, students were asked to deduce the length relationship of a visible shorter stick to a hidden longer stick in terms of a third visible stick they knew that was longer than the visibly shorter stick but shorter than the hidden longest stick. Unit iteration is the ability to think of using a smaller unit (block) to determine the length of a larger unit (tower of blocks) by repeating the smaller unit along the length of the larger unit (Kamii & Clark, 1997). This understanding involves deriving a part-to-whole relationship such as that of using an inch to calculate a foot (12 in). Conservation of number involves the ability to understand that the number of items in an array remains constant even though the items may be rearranged in a different array (Inhelder et al. 1974). Conservation of length is the ability to understand that two sticks of equal length remain equal even though their position in relation to one another may be changed (Inhelder et al. 1974).

Data Analysis

Researchers scored interview responses for evidence of conservation, transitivity, unit iteration, and knowledge of measurement terms and tools. Any disagreements between researchers' judgments were resolved by review and discussion of the data until agreement was reached. Resolutions were necessary for less than three percent of the scores. Transitivity, unit iteration, and conservation were categorized as demonstrated, partially demonstrated, or not demonstrated using the protocols established in the research literature (Piaget et al., 1960, tower task; Kamii & Clark, 1997, inverted T task; Piaget & Szeminska, 1965, three sticks task; and Inhelder et al., 1974, conservation tasks). Responses to the fourteen items of the social knowledge of measurement terms and tools section of the interview were scored using a scoring rubric with 0 as the lowest possible score and 41 as the highest. Actual mean scores were 24 (in September) and 34 (in May) indicating that students' knowledge of the names of measurement tools (such as ruler) and what the tools were used for was evident at the beginning of the study and grew during the year.

Researchers conducted multiple readings of the field notes and student work samples individually searching for patterns and themes of measurement (Marshall & Rossman, 2006). Researchers then deliberated on identified patterns and themes until agreement was reached on six major themes of students' understanding of measurement.

Findings

Interviews

The findings derived from interviews concerning transitivity, unit iteration, conservation, and social knowledge of measurement terms and tools are presented in Table 1 for both the September and May interviews.

The findings presented in Table 1 indicate growth in all areas from the beginning to the end of the school year. The findings are similar to those of Piaget et al. and Kamii and Clark who found that transitivity develops before unit iteration. At both the beginning and end of the school year, more students in this study demonstrated transitivity than unit iteration. There were no cases in which a student showed unit iteration without transitivity.

Of some concern is the finding on conservation of number and length. While most students in this study demonstrated conservation of both number (14) and length (9) and showed growth in both areas across the school year, what is striking is that at the end of first grade, there remained two students who did not conserve number and seven who did not conserve length. Many primary mathematics programs require conservation of number and length as a prerequisite to understanding and solving mathematics problems. Educators should not assume that all children entering second grade have constructed conservation of number and length. Children who do not conserve may encounter difficulties when confronted with mathematics programs and tests that require an understanding of number relationships, identity, and reversibility in thinking.

Table 1 Number of students (N = 16) who demonstrated transitive reasoning, unit iteration, conservation, and social knowledge at the beginning (September) and end (May) of the school year

Interview Tasks	September	May
Transitivity		
Sticks	13	16
Tower	3	16
Inverted T	8	13
Unit iteration		
Tower	0	4
Inverted T	4	6
Conservation		
Number	10	14
Length	7	9
Social knowledge[a]	24	34

[a] Social knowledge values are reported as means. The possible range for social knowledge scores was 0–41 (actual range for September was 14–32; actual range for May was 25–40)

Student-initiated Measurement Activities

Researchers observed very few student-initiated measurement activities. While there were no recorded instances of student-initiated measurement activities during the fall semester (September-December), there were four examples recorded in the spring semester (January–May) indicating that some first graders do express a need to use measurement in the course of their everyday activities in and outside of school. First graders do have ideas about measurement that can be supported with opportunities in the school day to express and act on those ideas. Based on the findings of this study, measurement activities can be incorporated into the first grade curriculum in ways that will engage students' thinking, including their transitive thinking and their ability to iterate units. However, it would be unreasonable to expect all first graders to understand the logic involved in measurement.

Measurement Themes

Analysis of field notes and students' work samples indicated 6 themes of students' understanding of measurement. These are presented with accompanying examples from students' work.

Theme 1

Students approached measurement activities with the empirical procedures of counting, adding, and multiplying numbers on their own with no teacher instruction to do so.

During one teacher-initiated activity, students listened to the book, *The Bunyans*, and were asked to use a 9 foot long paper foot print to find out how many feet it would be from the classroom to the office. Students lined up the foot print and counted the number of times the foot print was repeated along the distance from the classroom to the office. They found it was repeated 32 times. They were then asked how they might use the foot print to figure out how many feet (12 in segments) it was from the classroom to the office. Students laid 9 rulers end-to-end down the middle of the foot print and then placed 9 more rulers extended in front of the foot print intending to continue the process all the way to the office. One girl exclaimed, "All you have to do is add 9 thirty-two times!" They abandoned the rulers and began working with paper and pencils or on marker boards. One girl wrote the numeral 9 thirty-two times on her paper and began to add the 9's together. The problem was too much for her and she abandoned the adding. Another girl changed the 9's to 10's and added thirty-two 10's getting 320. She was satisfied with the answer and didn't show a need to subtract 32 from the total. Another girl drew an equation on her marker board:

32 9s = (- -). A boy created a similar problem:

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 32
 X9
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He said he was doing multiplication. After a few minutes, he said that he had "forgotten" his math, and decided to get a calculator. He returned to the problem with a calculator and added 32 nine times getting 288. Another boy began to add 18's together and said, "It's sixteen 18s!" The students seemed satisfied with the calculated numbers with no need to clarify that the numbers represent feet. The original problem seems to have been lost along the way of adding up all the numbers.

The idea that measurement involved adding and multiplying was very common. After choosing an object (a book) to measure, one girl wrote in her measurement journal, "I figured out that measuring is math."

One student-initiated activity originated from a boy who wanted to know the length of the classroom. His question led to action on the day a local scientist brought tadpoles and frogs to the class and remarked that a frog could jump about one half the length of the room. Students wanted to know how far that would be. One girl counted her tiny footsteps as she walked to the point in the room she estimated would be halfway. But most students worked in teams of two or three and laid yard sticks end-to-end to their estimated halfway point. They then counted the inches along the line of yardsticks. One pair of students attempted to line up yard sticks the entire length of the room. When they ran into the file cabinets at one end, they estimated the length of the file cabinets but did not add it to the number of inches they counted along the yard sticks.

One boy who worked alone calculated that the entire length of the room was five yard sticks, 60 in each. He added five 60's and got 368 in. Then he tried to calculate half of 368 by deriving doubles for each numeral. He said that 4 is a double for 8, 3 is a double for 6, but was stumped on a double for 3. He then thought of the 6 as a 16 and found 8 was a double for 16. That left a 2 with 1 being the double for 2. He concluded that half the length of the room was 184 in.

Theme 2

Students measured objects by outlining them using plastic links, rulers, or tape measures.

It was quite common to observe students involved in measuring an object such as their foot, another student's height, the class tent which was to be set up in the room, or a book by positioning the chosen measurement tools, such as rulers, tape measures, or plastic links, around the object in such a way as to outline the object with the measuring tools. Once the object was outlined, it was common to hear students say the object was so many rulers (links, tape measures) long, not recognizing the confounding of length with width and area.

For example, one teacher-initiated activity asked small groups of students to determine whether a tent located in the hallway would fit in a corner of the classroom. Each group with no knowledge of what other groups were doing approached this problem by measuring the tent using rulers or tape measures to outline it while it was either lying on the floor or propped up against the wall. After outlining the tent, someone would count the number of rulers or tape measures used. Occasionally a student would count the numerals printed on the rulers or tape measures and give a final count with no indication of knowing that the numbers represented inches. In one group, students decided to lie down on the floor positioning their bodies around the tent. The same group also used their shoes to outline the tent.

Theme 3

Students used body parts in measuring.

A common idea among the students was the thought that 1 in could be represented by the segment of the index finger from the joint to the knuckle. The source of this idea was unknown, although it might have originated from an adult in the school. They also used the distance between their two index fingers to show an inch and then used this inconsistent length to measure an object such as a friend's height or the distance of the paper airplanes they flew one day. Some used their feet and others used their hands counting them as a measure of the length their planes flew.

Another common idea was that one's foot was the length of the standard unit for foot (12 in). Some students used their feet without precision to step off the length of the classroom. When questioned by the researchers about whether it made a difference whose foot was used, students did not indicate a need to account for differences in foot length. Class discussions may eventually help students consider the issue of consistency in measurement (Sedzielarz & Robinson, 2007).

Theme 4

Students chose a measuring tool based on whether it was the same length or a bit longer than the object to be measured.

During the interview when students were asked what they would use to measure different objects such as a shoe or pencil, a typical response was to chose the measurement tool that was at least as long as the object to be measured. Initially they would select the ruler, then discard it when it was seen to be shorter than the object.

When measuring body parts, students would begin using rulers that were quickly discarded because they were considered too long for measuring. Students preferred to use plastic links chosen because they were the same length as body parts and because they could be easily counted. One boy remarked, "If a link is too small, you can add links, but you can't add to a ruler."

With the exception of using repetition of the Bunyan foot print to measure the distance to the office, students would typically gather as many yard sticks or tape measures as they could find to lay end-to-end to measure things such as the length of the room. The idea of using one ruler or one yard stick to measure by repeating it along the length of the object (unit iteration) was rarely observed. Even when they ran out of yard sticks, they did not figure out how to complete the measurement with the ones they already had. They would search for more or abandon the task altogether.

Theme 5

Students engaged in many direct comparisons of objects.

Direct comparisons, placing two objects side by side, fall within a broad definition of measurement that includes judging one thing by comparing it to another (Dougherty & Venenciano, 2007). Students were continuously comparing one object to another in terms of height, length, size, and weight. They stood back-to-back to compare who was taller. They compared the individual plants they were growing with others' plants in height and size. They compared the height and weight of individual pumpkins. Direct comparisons were made without measuring with standard units.

During a teacher-initiated activity in which students were asked to find an object in the classroom that was the same length as a 12 in strip they were given, students made direct comparisons by holding the strip of paper next to objects to determine if they were the same length. Only one girl measured her strip with a tape measure, then used the tape measure to measure objects the same length as her strip.

In a similar teacher-initiated activity, each student was given a set of paper strips in different colors. There was a 12 in blue strip marked off in inches, a 6 in gold strip marked off in inches, and 12 one inch strips of six different colors. Students were asked what they noticed about the strips. They made direct comparisons among the strips and concluded that some were longer than others. One boy observed that the 6 in strip was half as long as the 12 in strip. Some sorted them by color or from longest to shortest. Several of them made pictures with the strips of houses and picture frames. Several of them counted 30 units (in) in the entire set of strips. All students demonstrated one-to-one correspondence. However, one girl showed how you could get a different answer to the question of how long a flat stick was by changing the number of smaller units used to place on top of the stick. She demonstrated by spreading out 3 one inch strips along the stick and getting the answer "3"; then by placing closer together 5 one inch strips and getting the answer "5". She concluded that the way you measure determines the answer you get.

Theme 6

Children's literature played a role in student-initiated measurement activities.

There were only four student-initiated measurement activities observed and all involved boys. One mentioned previously was the idea to measure the length of the classroom. The remaining three were influenced by children's books read either by the teacher to the class or by individual boys.

Children's books containing elements of measurement, such as *Inch by Inch*, prompted measurement questions in students that led to student-initiated measurement activities. The book, *Me and the Measure of Things* shows a page that equates 5,280 feet to 1 mile. This page led to a connection made by one boy who remembered his father using a measurement wheel to mark off a mile in their neighborhood. He spent time working on the problem of just how long is a mile.

Another student-initiated activity occurred when a boy was reading a book about fish. He read that a blind catfish is 10 in long and he wanted to know how long that would be. He found out by counting off 10 in on a ruler. The same book said that a whale shark is 40 feet long. In order to find out how long that would be, he lined up 40 rulers along a wall in the hallway. At first, he placed pieces of paper next to each 8 in segment of ruler and started adding up all the 8 in segments. He said he chose the number 8 because it was easier than other numbers. Then he realized that the line of 40 rulers represented forty 12 in segments. He wrote out the numeral 12 forty times and added up the 12's. Then he found he could add up twelve 40's and get the number of inches in the length of the whale shark.

Another book on beetles, which said the length of a beetle was about 6 in, the size of an adult's hand, prompted one boy to find out the length of 6 in. The boy used a ruler to measure the teacher's hand to see for himself if 6 in could be compared to his teacher's hand.

Discussion

Our findings are similar to those of Piaget et al. (1960) and Kamii and Clark (1997) who found that unit iteration develops out of transitivity and that first graders are in the process of developing transitivity and unit iteration, two necessary cognitive abilities involved in the understanding of measurement. Our findings also support the work of Stephan and Clements (2003) who found that children approach measurement problems through counting and of Inhelder et al. (1974) who found that students make measurement judgments based on counting ideas.

In addition, our findings indicate that although first graders typically do not often initiate measurement questions, problems, or activities, there were a few occasions when they did. These occasions all involved boys, three of whom were influenced by children's literature to ask and answer measurement questions. Although the expectation that all first graders demonstrate competency in measurement is unreasonable, it is important that they experience opportunities to think about measurement problems and express measurement questions as they arise within the context of everyday experiences such as children's literature. In fact, the use of children's literature in first grade may be an effective tool for engaging students in discussions and debates about measurement. It may be even more effective than other types of teacher-initiated measurement activities.

The students' approach to measuring objects by outlining the object with the measurement tool may indicate an attempt to cover the whole object (area) or perimeter of the object showing a confusion of the various dimensions of length, width, and area. More likely it may be a result of their unidimensional (linear) thinking (Kamii 1996) in which they are focused on measurement of an object as making a "line" around the object indicating an inability to conceptualize the object in more than one dimension. This finding warrants further research.

Students' choice of a measurement tool based on equal length of tool-to-object and their need to have enough tools (rulers, tape measures, or yard sticks) to extend the length of the object to be measured indicate a lack of development of unit iteration. At the end of the first grade year, in spite of the curricular attention to measurement activities, less than half the class demonstrated the ability to iterate units. Kamii and Clark found that most children construct unit iteration out of transitive reasoning by fourth grade. Therefore, first grade mathematics programs and assessments should not include exercises that require the ability to iterate units. This does not seem to be an appropriate objective for first grade.

Although students made progress during the year in their use of and understanding of measurement terms and tools, most of them did not demonstrate a need to measure objects with precision. Students' frequent direct comparisons of one object to another without measurement using standard units and their use of body parts without precision in measuring the length of objects show not only a lack of understanding of standard units but also a lack of conservation of length. Conservation of number and length are constructed around age 6–8 (Wadsworth 1971). Educators should not assume that all first graders can conserve number and length. Teachers need to reconsider whether activities they plan for children require conservation, transitivity, and unit iteration and the appropriateness of such activities for first grade including test items that require these abilities.

Based on results of this study, it is recommended that the focus of first grade measurement activities be on what it means to measure rather than on how to measure or empirical procedures. First grade teachers should hold appropriate student expectations for measurement. Children's books with elements of measurement may be used to stimulate thinking and subsequent student-initiated measurement activities. There should be a variety of objects to measure plus a variety of measurement tools available, possibly even a measurement center or table area in the classroom. Measurement problems involving indirect comparisons, such as asking children how to figure out whether or not an object located in another part of the school might fit into their classroom, should be used to encourage thinking and considering alternative points of view. Interviewing children about their understanding of measurement and encouraging them to keep measurement books or journals in which they write and draw their measurement ideas that can be read and analyzed will contribute to teachers' better understanding of children's understanding of measurement.

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