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Fiscal and Environmental Sustainability: Is Public Debt Environmentally Friendly?

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Abstract

This article assesses the dilemma that most governments face when seeking to ensure the sustainability of their public finances through economic growth while simultaneously protecting the environment. We propose a growth model in which the government finances abatement-spending through taxation or public debt and which follows a fiscal rule that targets the long-run debt-to-GDP ratio. We show that there is a threshold for the debt ratio below which debt and environmental sustainability are secured. In steady state, the debt ratio exerts a nonlinear effect on environmental quality in the form of an inverted U-shaped curve, and the environmental tax is good for the environment when public debt is not. A fiscal rule authorizing a small but strictly positive debt ratio could help the government to implement adaptation policies for environmental protection while supporting long-run economic growth.

Keywords Growth · Environment · Public debt · Environmental tax

JEL Classification O44 · E62 · H63 · Q58

1 Introduction

In the 1990s, one of the most influential research agendas opened by Professor Xepapadeas was the use of economic policies to ensure environmental protection and reduce polluting emissions. His pioneering work concerned the design of policies with incentive systems that discouraged firms' emissions, including emission charges and emission limits on firms' productive input (see, e.g., Xepapadeas 1992a, b, 1995). With this in mind, Professor Xepapadeas wrote the significant book *Advanced Principles in Environmental Policy* (1998) to reassess the literature on pollution-management policies and to expose the use of mathematical tools in environmental economic.

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Professor Xepapadeas's contributions also lie in the area of macroeconomic modeling. For example, he was the first to incorporate an abatement technology sector into existing growth models to investigate the long-run implications of productivity-enhancing knowledge spillovers in emissions reduction (Xepapadeas 1997). His contributions resulted in *Economic growth and the environment* (Xepapadeas 2005), a chapter in the first edition of the *Handbook of Environmental Economics* that provides a pedagogical overview of how to incorporate the environment and nature into general equilibrium models, as well as the key policy messages that can be derived from them. This article is one of the most cited in his bibliography and remains a reference for any student or researcher interested in macroeconomic modeling of the environment.

Professor Xepapadeas is still very involved in this research area. Together with his coauthor George Economides, he developed the first monetary growth model integrating climate change to determine whether monetary policy could help address global warming (Economides and Xepapadeas 2018). He also used the latest advances in dynamical systems analysis to study the complex relationship between economic growth and the environment in a simple endogenous growth setup (Menuet et al. 2023).

This article is part of that research agenda. Despite extensive methodological and policy messages, a missing variable in the above-mentioned environmental growth models is probably public debt. This lack of interest in the role of public debt is particularly striking given the impressive amounts of debt accumulated by most countries around the world since the mid-1970s. Our objective is then to assess—from a theoretical perspective—the dilemma facing today's governments, namely ensuring the sustainability of their public finance through economic growth while maintaining environmental protection.

One of the traditional arguments from economists is that governments can overcome the conflict between environmental protection and economic growth by means of public emission-reducing (i.e., abatement) expenditure (see, e.g., Van Ewijk and Van Wijnbergen 1995; Boly et al. 2022). However, if such expenditure is financed by the issuance of public debt, the growth-environment trade-off may reappear because of the crowding-out effect of the debt burden on public expenditure. This is one of the reasons why debt-for-nature swaps have been so popular since the 1980s.[1]

Taking a policy perspective, this paper asks whether a government can ensure both debt and environmental sustainability through its fiscal policy instruments and whether the use of debt-financing for abatement expenditures can be beneficial for environmental protection in the long run.

To address these questions, we develop a second-best theory based on an endogenous growth model with an overlapping generations (OLG) formulation à la Diamond (1965). In our setup, environmental quality is modeled as a stock, following Bovenberg and Smulders (1995), Fullerton and Kim (2008), and Menuet et al. (2023). The environmental quality is a renewable resource that regenerates itself and depletes due to polluting emissions. These emissions come from production activities that require the use of a polluting input, i.e. firms' energy consumptions. The government can mitigate the effect of these emissions by abatement expenditures, which are financed by taxes on firm's energy consumptions and

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[1] The goal of these swaps was indeed to provide developing countries with external debt relief to reduce their debt burden and mitigate the crowding-out effect, in exchange for investment in environmental conservation (see Hansen 1989, for a review). However, Cassimon et al. (2011) have cast doubt on the possibility of scaling-up debt-for-nature swaps.

public debt. On the fiscal side, the government follows a fiscal rule which mandates that the debt-to-GDP ratio must equal a fixed target in the long run.[2]

The motivation for this endogenous growth framework is twofold. First, we aim at studying the impact of fiscal policies on the long-run economic growth rate. In endogenous growth frameworks, capital can be defined broadly to include human capital, which prevents diminishing returns. In our model, the aggregate-level production function is an AK-type, characterized by constant returns to scale, which results in endogenous growth. In this context, where economic growth is endogenously determined and affected by government policies (see Shaw 1992), we can determine whether economic growth and environmental protection are compatible, and study the impact of fiscal/environmental policy on economic growth in the long run. Second, we aim at integrating a growing long-run public debt level into our framework, given that developed economies have consistently accumulated debts, particularly since the 1970s. In endogenous growth theories, the government can run permanent public deficits, so that the level of public debt increases in the long run (see, e.g. Minea and Villieu 2012). In exogenous growth models, on the other hand, public deficits are only transitory, leading to a stable long-run level of public debt (as in Schmitt-Grohé and Uribe 1997; Fodha and Seegmuller 2014; Fodha et al. 2018). Therefore, exogenous growth models feature long-run level effects of public debt on environmental issues, whereas our emphasis is on the growth effects.[3]

When studying a second-best model, the choice of policy goals and instruments is a key issue. Regarding policy objectives, the government has two long-term goals: (i) *debt sustainability*, defining by a steady state with positive economic growth rate and debt-to-GDP ratio; and (ii) *environmental sustainability,* defining by a steady state with positive environmental quality. Regarding policy instruments, the government designs the fiscal rule, i.e. it sets the value of the debt-to-GDP target. We consider the debt target as the fiscal instrument to ensure debt sustainability, as fiscal rules are generally implemented to limit fiscal discretion and promote fiscal discipline (see, e.g., Debrun et al. 2008). On the environmental side, the government has two means to enhance natural capital in the long run: directly, by financing public abatement spending that mitigates the effect of polluting emissions; and indirectly, by taxing firms' energy consumptions through a "polluter pays" principle. An environmental tax-increase reduces the use of polluting inputs and, consequently, lowers the flow of emissions.

Following the Tinbergen rule, the number of policy objectives should match the number of instruments. In our framework, the two sustainability objectives are intertwined via the intertemporal government budget constraint. This budget constraint requires a fiscal variable for adjustment. We consider abatement spending as this adjustment variable, as abatement activities often involve large public investment programs. This choice is consistent with the empirical literature findings, which highlight that fiscal consolidations are more likely to result in reductions in public investment programs rather than tax

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[2] According to the IMF (2018), as of 2015, at least 70 countries worldwide had a fiscal framework with an explicit cap on public debt.

[3] The choice of an endogenous growth framework is also motivated by technical considerations. Studying the effects of growth in endogenous growth models is often simpler, as these models generally involve dynamic systems of smaller dimensions than those of exogenous growth models.

increases (see, e.g., Roubini and Sachs 1989; De Haan et al. 1996; Balassone and Franco 2000).[4] As a result, in our model, the government is tasked with two sustainability goals and has at its disposal two instruments—the debt target and the environmental tax rate.

Our results. We show that the economy converges to a unique long-run sustainable steady state, in which the level of environmental quality is constant and the level of public debt is growing along a balanced-growth path. We reveal that, in the long run, the debt-to-GDP ratio has two conflicting effects on abatement-spending. First, public deficits generate a permanent flow of new resources to finance abatement activities. Second, public debt generates a permanent flow of wasteful spending (the debt burden), which crowds out abatement-spending. For low debt targets, the first effect prevails, resulting in a positive relationship between public debt and abatement-spending (in ratios per unit of output). For high debt targets, the relationship is reversed, as high debt implies a large debt burden and thus the crowding-out effect prevails.

We show that there is a threshold for the debt-to-GDP target, below which the two goals of the government (i.e., debt and environmental sustainabilities) are secured. The intuition is as follows. On the one hand, long-term economic growth is negatively correlated with debt due to a substitution effect in the households' portfolio. Indeed, since government bonds and physical capital are two assets comprising perfect substitutes for households, an increase in debt reduces investment in capital, thereby negatively affecting economic growth. On the other hand, the quality of the environment in the long run fundamentally depends on the level of abatement-spending. Therefore, when the economic growth rate is positive *(debt sustainability*), the crowding-out effect of the debt burden is limited and abatement-spending is sufficient to offset the withdrawal of resources; hence, *environmental sustainability*.

Regarding comparative statics, we highlight a nonlinear relationship between the debt target and the environmental quality in the form of an inverted U-shaped curve.[5] The intuition concerns the link between debt and the abatement-spending. For small debt targets, an increase in debt leads to more abatement-spending and thus to better environmental quality in the long run. However, for high debt targets, the debt burden is so large that the crowding-out effect prevails. Hence, an increase in debt leads to a reduction in abatement-spending and lower environmental quality.

Regarding the effect of taxation, an increase in environmental tax reduces the use of polluting inputs in the production process, with two conflicting effects on the environmental quality. First, the decrease in polluting input reduces the marginal return of physical capital and economic growth, consequently reducing the ratio of public spending to GDP. Second, a reduction of polluting input results in lower polluting emissions. We show that when public debt is detrimental for environmental quality, i.e. the debt target is high, the second effect exceeds the first one, and environmental-tax hikes improve the long-run environmental quality.

Methodologically, this paper provides two messages.

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[4] Alesina and Perotti (1997) explain this evidence by "political realities" which suggest that cutting back investment spending is easier than raising taxes.

[5] Empirically, Carratù et al. (2019) also found non-linearities shaping the interaction between public debt and environmental quality, measured by air pollution. They investigate whether involvement in European Union treaties and the implementation of associated fiscal rules have shaped the relationship between debt and environmental performance. In our model, we establish a non-linear relationship between debt and environmental quality by examining an exogenous change in the debt target when the government is subject to a fiscal rule. In contrast, Carratù et al. (2019) study how exogenous changes in fiscal stance (i.e. the implementation of a fiscal rule) affect environmental quality.

First, the two policy objectives, namely debt and environmental sustainability, are interconnected via the government's budget constraint. Indeed, when the debt target is set at a low level, public debt is sustainable and the permanent public deficits in the budget constraint, used to finance abatement spending, can improve environmental quality over the long run and secure environmental sustainability. Conversely, if the debt target is set at a high level, the economy is plunged into a chronic recession (the steady-state economic growth rate is negative), leading to debt and environmental unsustainability.

Second, in our second-best theory, the two available instruments, namely the debt target and the environmental tax, can work in opposition to achieve the government's objectives. When the economy is subject to a fiscal rule targeting the debt-to-GDP ratio, the government can choose between the environmental tax and the debt ratio as tools, depending on its level of indebtedness. The government should opt for (environmental) tax increases if the debt target is set at a high level, whereas it should resort to debt increases if the target is sufficiently low.

The paper is organized as follows. The following subsection presents the findings of the literature review. Section 2 presents the model, Sect. 3 defines the steady state, and Sect. 4 details the policy implications revealed by comparative statics. Section 5 presents some concluding remarks.

1.1 Related Literature

Our model contributes to the sparse literature on the interaction between environmental and macroeconomic policies and scrutinizes the relationship between sovereign debt and the environment.

Researches closely related to ours consider public debt in environmental OLG models with exogenous growth. Fodha and Seegmuller (2014) and Fodha et al. (2018) examine debt-financed abatement expenditure. In these studies, the level of public debt is constant in the long run and the production processes emit pollution as a by-product. Their findings show that, while there are two steady states, only one is well-determinate. On the policy side, the debt-to-GDP ratio is harmful for the environment in the long run in the well-determined steady state, while a polluting tax can be beneficial. In a different configuration, where pollution is a by-product of capital, Heijdra et al. (2006) consider public spending as lump sum transfers to households, and an exogenous path of public debt. In a multi-sector setup based on numerical simulations, Rausch (2013) shows that the impact of combining climate and debt consolidation policies is limited, as raising environmental tax reduces the debt burden but impedes economic growth.

Our model extends and challenges these results in different ways.

First, since we aim to assess the impact of public debt on both long-run economic growth and the environment, we resort to an endogenous growth model. In our setup, the levels of output and public debt grow in the long run, while there is no long-run endogenous growth, and public deficits are only transitory in the aforementioned papers.

Second, we ensure the uniqueness of the steady state in a one-sector model, as the environmental-modeling methods differ. The depletion of natural resources through human activity is not a by-product of output or capital but a necessary input of production (as in Bovenberg and de Mooij 1997, for example). Hence, the taxation of this input can thus be considered to the price of a permit to pollute or to harvest the resource.

Third, in our setup, public debt can be beneficial for the environment in the long run, provided the debt-to-GDP ceiling in the fiscal rule is not too high. The debt ratio—assumed by Fodha and Seegmuller (2014) and Fodha et al. (2018) to be constant over time—is always detrimental for environmental quality in steady state in the existing models. Our setup is then the first, to the best of our knowledge, to show that debt-financing abatement-spending can be environmentally friendly in the long run.

Our analysis also follows the contribution of Boly et al. (2022). The authors propose an endogenous growth model in the spirit of Tahvonen and Kuuluvainen (1991) and Bovenberg and Smulders (1995), where abatement-spending can be financed by debt issuing, public debt grows perpetually, and the government follows a fiscal rule in which the debt-to-GDP ratio is constant in the long run, as in ours. They show that increasing the debt ratio reduces the quality of the environment in the long run.

Our main methodological contribution is to add an OLG structure to this endogenous growth framework. Our results qualify those of Boly et al. (2022), since in our model, the debt ratio exerts a nonlinear relationship on long-run environmental quality in the form of an inverted U-shaped curve. This feature comes precisely from the OLG structure, which does not require a transversality condition for aggregate asset stocks due to a dynamic inefficiency (see Diamond 1965). However, in Boly et al. (2022), as agents are infinitely lived, individual optimization behaviors require the presence of a transversality condition. The growth rate of public debt must be lower than the real interest rate in the long run for this transversality condition to hold. In short, this condition implies that the crowding-out effect outweighs any positive effect of fiscal deficits; hence, the debt ratio is harmful to public spending. This result is well-established in the literature addressing public debt in endogenous growth models without an environmental module (Minea and Villieu 2012; Menuet et al. 2018).

Finally, our work relates to several endogenous growth models investigating the impact of environmental taxation on economic growth and environmental quality. A large part of the literature shows that environmental taxes can promote growth via two mechanisms. The first is based on productive externalities: environmental taxation improves the quality of the environment, which positively affects total factor productivity, thereby promoting economic growth (see, e.g. Van Ewijk and Van Wijnbergen 1995; Bovenberg and Smulders 1995; Bovenberg and de Mooij 1997). The second relies on endogenous leisure-labor choices (see, e.g., Hettich 1998; Chen et al. 2003). More recent models qualify such results. For example, for Itaya (2008) or Menuet et al. (2023), environmental tax can reduce economic growth and environmental quality in the presence of multiplicity and indeterminacy. Ono (2003) shows that there is a critical tax level, below which economic growth and environmental quality rise with the level of taxation.

In our model, we have neither productive externality nor an endogenous work-leisure choice; so, environmental tax is harmful to growth, as tax increases reduce the return of capital. In addition, in contrast to the above-mentioned papers that disregard public debt, we show that environmental tax can be good for improving long-run environmental quality through a debt-based channel.

2 The Model

We consider an overlapping generations economy, with discrete time indexed by $t = 0, 1, 2, \dots, \infty$. There are three types of agents: households, a representative firm and a government.

2.1 Households

A household's lifespan comprises two periods. In the first period, the household is considered young, and in the second it is old. For simplicity, we assume that the population level remains constant over time, and we normalize this to unity. The preferences of the household born at period t are represented by a log-linear utility function (U_t), defined over consumption when young (C_t) and old (D_{t+1}), and environmental quality when young (E_t) and old (E_{t+1}), namely

$$U_t = \log(C_t) + \varphi \log(E_t) + \rho\left[\log(D_{t+1}) + \varphi \log(E_{t+1})\right], \tag{1}$$

where $\rho \in (0, 1)$ is the discounted factor, and $\varphi \in [0, 1]$ captures environmental preferences.

During the first period of life, a household born at period t supplies inelastically one unit of labor, paid at the competitive real wage w_t. It uses its net income to consume (C_t) and save (S_t) through the two available assets, investing in physical capital (K_{t+1}) or buying government bonds (B_{t+1}), whose returns in the second period are $r_{k,t+1}$ and $r_{b,t+1}$, respectively. During the second period of life, household no longer works, and retires and consumes its remunerate savings. Hence, the household born at t maximizes (1), subject to the following budget constraints:

$$S_t + C_t = w_t, \tag{2}$$

with $S_t = K_{t+1} + B_{t+1}$, and

$$D_{t+1} = r_{k,t+1}K_{t+1} + r_{b,t+1}B_{t+1}. \tag{3}$$

The first-order conditions of the households' program leads to

$$\frac{1}{C_t} = \rho\left(\frac{r_{k,t+1}}{D_{t+1}}\right), \tag{4}$$

$$\frac{1}{C_t} = \rho\left(\frac{r_{b,t+1}}{D_{t+1}}\right). \tag{5}$$

Quite intuitively, the marginal utility of consumption ($1/C_t$) equals the present value of saving in each asset (expressed in terms of marginal utility of consumption $1/D_{t+1}$). From (4) and (5), we deduce the so-called no-arbitrage condition between investing in government bonds versus investing in capital (the two assets are perfect substitutes), namely

$$r_{k,t+1} = r_{b,t+1} =: r_{t+1}. \tag{6}$$

We then derive the savings function

$$\frac{\rho}{1+\rho} w_t = S_t = K_{t+1} + B_{t+1}. \tag{7}$$

2.2 Environment

Environmental quality (E_t) is defined by the stock of natural capital that surround individuals and which contribute directly or indirectly to providing for their needs. Following Tahvonen and Kuuluvainen (1991) and Bovenberg and Smulders (1995), the law of motion of the environmental quality is

$$E_{t+1} - E_t = f(E_t) - P_t. \tag{8}$$

In Eq. (8), P_t is the reduction in environmental quality from the net flow of emissions (called "pollution"). $f(\cdot)$ is an environmental regeneration function that reflects the capacity of the environment to absorb pollution. We consider an usual form $f(E_t) = \varepsilon(\bar{E} - E_t)$, where $\varepsilon \in (0,1)$ is a scale parameter, and the critical level $\bar{E} > 0$ is the "*virgin state*". Without emission ($P_t = 0$), environmental quality reaches its highest possible (finite) level ($\bar{E}$), which is the maximum stock of natural resources that can be kept intact by natural regeneration (for identical assumptions, see e.g. Boly et al. 2022).

However, this virgin state cannot be sustained because economic activity incurs polluting emissions, i.e. the production process uses energy from fossil fuels (the input Z_t) that results in emissions. Nevertheless, such an adverse effect of production can be (at least partially) neutralized by abatement spending. As usual, we assume that this abatement activity is provided by the public sector through government expenditure (G_t). Consequently, the net flow of emission is

$$P_t = \frac{Z_t}{G_t}. \tag{9}$$

In this specification that follows Fullerton and Kim (2008), the input Z_t that provides energy services for the production depends both on emissions and the abatement spending: $Z_t = P_t G_t$; thus, the same Z_t can be achieved with less emissions if the economy has access to more abatement.[6]

2.3 Firm

The representative firm produces the consumption good (Y_t) using three inputs: private man-made physical capital (K_t), human capital (H_t), and the polluting input (Z_t), according to the following Cobb-Douglas production function

$$Y_t = \tilde{A} K_t^{\alpha} Z_t^{\beta} H_t^{1-\alpha-\beta}, \tag{10}$$

where $\tilde{A} > 0$ is a scale parameter, and $\alpha \in (0,1)$ and $\beta \in (0,1)$ are the elasticities of output to private capital and the polluting input, respectively. Following Romer (1986), human capital is produced both by labor (L_t) and by the economy-wide stock of physical capital ($\bar{K}_t$), namely $H_t = L_t \bar{K}_t$.

In a perfect-competition decentralized economy, the representative firm chooses private factors (K_t, L_t, and Z_t) to maximize its profit

$$\Pi_t = Y_t - r_{k,t} K_t - w_t L_t - \tau_p Z_t, \tag{11}$$

where τ_p is a (constant) tax rate on firm's energy consumption (i.e., an environmental tax). This tax can be assimilated to the price of a permit to pollute.

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[6] The Fullerton and Kim (2008)'s specification is $P_t = (Z_t/G_t)^{\mu} \Leftrightarrow Z_t = P_t^{1/\mu} G_t$, where μ is the elasticity of emissions to the energy input, i.e. a pollution-conversion parameter: a lower μ makes emissions more effective, or—equivalently—makes abatement relatively less effective. In our model, to reduce the number of parameters, we consider $\mu = 1$. However, our results do not qualitatively change in the case of $\mu \neq 1$.

The first-order conditions ensure that the price of factors is given by their marginal returns in production

$$r_{k,t} = \alpha \frac{Y_t}{K_t}, \tag{12}$$

$$w_t = (1 - \alpha - \beta) \frac{Y_t}{L_t}, \tag{13}$$

$$\tau_p = \beta \frac{Y_t}{Z_t}. \tag{14}$$

Moreover, since each young household supplies inelastically one unit of labor, we have $L_t = 1$ at equilibrium.

2.4 Government

The government provides public abatement expenditure (G_t), levies taxes ($\tau_p Z_t$), and borrows from households. The deficit is financed by issuing debt (B_{t+1}); hence, the following budget constraint

$$B_{t+1} = r_{b,t} B_t + G_t - \tau_p Z_t. \tag{15}$$

Introducing debt-financed abatement expenditures in an OLG configuration means that the burden of financing environmental protection is shared across generations. The future generation, in order to enjoy a high-quality environment, must repay the burden of the debt that is issued by the government during the previous generation.

Technically, there are two free variables in Eq. (15): public debt (B_t), and abatement-spending (G_t). Hence, to close the model, we must introduce a hypothesis on the public debt path. Therefore, following Minea and Villieu (2013), we specify the fiscal rule governing the changes of the debt-to-output ratio (B_t/Y_t) as follows:

$$\frac{B_{t+1}}{Y_{t+1}} - \frac{B_t}{Y_t} = -\phi \left(\frac{B_t}{Y_t} - \theta \right), \tag{16}$$

where $\theta \geq 0$ is the long-run target of the debt-to-output ratio, and $\phi \in (0, 1)$ the speed of adjustment. This specification reflects stylized facts, since many fiscal rules implemented since the 1980s require an exogenous target of debt-to-output ratio.[7]

In the few OLG environmental models introducing public debt (see, e.g. Fodha and Seegmuller 2014; Fodha et al. 2018), the level of debt (B_t) is constant in the long run and the debt-to-output ratio (B_t/Y_t) is assumed to be constant over time. In our model, we relax this assumption since the debt *ratio* is only constant in the long run (equal to the target θ). During the transition in the short(medium)-run, in contrast, the debt ratio (B_t/Y_t) evolves over time according to the rule (16). In addition, thanks to our endogenous growth

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[7] According to IMF (2018), as of 2015, at least 70 countries worldwide had a fiscal framework with an explicit cap on public debt, with debt ceilings frequently ranging between 60% and 70% of the GDP. For example, the European Union imposed a debt ceiling of 60% of its GDP, while the Central African Economic and Monetary Community and the West African Economic and Monetary Union both imposed caps of 70%.

formulation, the *level* of public debt (B_t) can grow perpetually along a balanced-growth path. This feature is consistent with the data because governments have been accumulating assets over time in both developed and developing countries, especially since the 1970s.

At this stage, from (15) and (16), the so-called Tinbergen rule for fiscal policy applies, because the government has two objectives (debt and environmental sustainability) and two policy instruments: the debt ratio (θ) in the fiscal rule, and the environmental tax (τ_p). Indeed, we assume that the path of abatement spending (G_t/Y_t) is endogenously determined to adjust the government's budget constraint (15).[8]

3 Equilibrium

At equilibrium ($\bar{K}_t = K_t$), from (10) and (14), the aggregate production function is

$$Y_t = AK_t, \tag{17}$$

where $A = \tilde{A}^{\frac{1}{1-\beta}} \tau_p^{-\frac{\beta}{1-\beta}} \beta^{\frac{\beta}{1-\beta}}$. Thanks to constant-returns at the social level, endogenous growth can emerge despite decreasing returns of private capital from the individual firm's perspective. Using (6) and (12), the real interest rate is $r = \alpha A$.

From (7) and (13), the aggregate saving function is

$$\frac{\rho}{1+\rho}(1-\alpha-\beta)Y_t = S_t = K_{t+1} + B_{t+1}, \tag{18}$$

leading to a constant saving-to-output ratio[9]

$$s = \frac{S_t}{Y_t} = \frac{\rho}{1+\rho}(1-\alpha-\beta). \tag{19}$$

To obtain long-run stationary ratios, we deflate public debt and abatement spending by output, namely $b_t = B_t/Y_t$ and $g_t = G_t/Y_t$. Using (6) and (14), the government's budget constraint (15) becomes

$$b_{t+1}\gamma_{t+1} = rb_t + g_t - \beta \Leftrightarrow g_t = b_{t+1}\gamma_{t+1} - rb_t + \beta, \tag{20}$$

where is $\gamma_{t+1} = Y_{t+1}/Y_t$ the growth factor of output. Introducing (20) into (9), we derive the following net flow of emission

$$P_t = \frac{Z_t}{G_t} = \frac{Z_t}{Y_t}\left(\frac{1}{g_t}\right) = \frac{\beta}{\tau_p}\left(\frac{1}{b_{t+1}\gamma_{t+1} - rb_t + \beta}\right), \tag{21}$$

and the law of motion of the environmental quality (8) becomes

$$E_{t+1} - E_t = \varepsilon(\bar{E} - E_t) - \frac{\beta}{\tau_p}\left(\frac{1}{b_{t+1}\gamma_{t+1} - rb_t + \beta}\right). \tag{22}$$

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[8] Alternatively, we could consider a constant abatement-spending-to-output ratio and an adjustment of the government's budget constraint via the environmental-tax rate. This case would not alter our main result, namely the long-run non-linear relationship between the debt ratio and environmental quality.

[9] To ensure a positive long-run economic growth rate, we need to assume $As > 1$. To this end, we consider throughout the paper $A > (1+\rho)/[\rho(1-\alpha-\beta)]$.

The economy is then described by a three-equation dynamic system formed by the saving function (18),[10] the law of motion of the environment (22), and the fiscal rule (16)

$$\begin{cases} \gamma_{t+1}(1+Ab_{t+1}) = As, \\ E_{t+1} - E_t = \varepsilon(\bar{E} - E_t) - \frac{\beta}{\tau_p}\left(\frac{1}{b_{t+1}\gamma_{t+1} - rb_t + \beta}\right), \\ b_{t+1} - b_t = -\phi(b_t - \theta). \end{cases}$$

When the first line is substituted with the second, the reduced form of the model is finally given as a two-dimensional system with two predetermined variables (E_t and b_t), as stated in the following definition.

Definition 1 Given $(E_0, b_0) \in (\mathbb{R}^+)^2$, an intertemporal equilibrium is a sequence $\{E_t, b_t\}_0^{+\infty}$ that satisfies the following system

$$\begin{cases} E_{t+1} = E_t(1-\varepsilon) + \bar{E}\varepsilon - \frac{\beta}{\tau_p}\left(\frac{1}{\frac{Asb_{t+1}}{1+Ab_{t+1}} - rb_t + \beta}\right), & \text{(a)} \\ b_{t+1} = b_t(1-\phi) + \phi\theta. & \text{(b)} \end{cases} \tag{23}$$

4 Steady State

A steady state is an intertemporal equilibrium in which the *level* of environmental quality (E_t) and the debt-to-output *ratio* (b_t) are invariant over time. In steady state, the economy is thus characterized by a balanced-growth path (BGP), where the level of capital, output, debt, and abatement spending grow at a constant (endogenous) rate γ.

Proposition 1 *A steady state* $(b^*, E^*) \in (\mathbb{R}^+)^2$ *satisfies*

$$b^* = \theta, \tag{24}$$

$$E^* = \bar{E} - \frac{\beta}{\varepsilon\tau_p}\left(\frac{1}{\frac{As\theta}{1+A\theta} - r\theta + \beta}\right), \tag{25}$$

and the steady-state growth factor follows

$$\gamma^* = \frac{As}{1+A\theta}. \tag{26}$$

Proof: Setting $b_{t+1} = b_t = b$ and $E_{t+1} = E_t = E$ in system (23).

From (24), according to the fiscal rule, the debt-to-output ratio is equal to its target (θ). From (25), the steady-state value of environmental quality can be written as

$$\varepsilon(\bar{E} - E^*) = \left(\frac{Z}{Y}\right)^* \frac{1}{g^*}. \tag{27}$$

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[10] From (18), we compute $s = \frac{K_{t+1}}{Y_t} + \frac{B_{t+1}}{Y_t} = \gamma_{t+1}\frac{K_{t+1}}{Y_{t+1}} + \gamma_{t+1}\frac{B_{t+1}}{Y_{t+1}} = \gamma_{t+1}\frac{1}{A} + \gamma_{t+1}b_{t+1}$.

Fundamentally, the steady state arises when the depreciation of the environment (the right-hand side of 27) just offsets its (natural) regeneration (the left-hand side). On the depreciation side, the flow of emissions coming from the production process ($(Z/Y)^*$) is mitigated by the abatement spending ($1/g^*$).

4.1 Debt and Environmental Sustainabilities

In our model, *debt sustainability* is defined by a steady state with a positive economic growth rate (i.e. $\gamma^* - 1 > 0$) and a positive debt ratio ($\theta > 0$). *Environmental sustainability* is defined by a steady state with a positive level of environmental quality ($E^* > 0$).

Proposition 2 *Public debt is sustainable if and only if* $0 < \theta < \bar{\theta} := (As - 1)/A$.

Proof. Using (26), the economic growth rate is $\gamma^* - 1 = (As - 1 - A\theta)/(1 + A\theta)$.[11]

In our model, the long-run economic growth rate ($\gamma^* - 1$) negatively depends on the debt-to-GDP ratio (θ). In endogenous growth models with public indebtedness (see, e.g. Minea and Villieu 2012; Menuet et al. 2018), the channel in which debt has an adverse effect on growth comes from a well-known crowding-out mechanism (in the long run, the debt burden cancels out the potential for growth-enhancing effects of debt, such as the financing of productive expenditure). On the other hand, in our setup, the channel is different and comes from a substitution effect in the household's portfolio. The no-arbitrage condition and our AK framework imply that the returns on investment in physical capital or public debt are the same and constant ($r_{k,t} = r_{b,t} = r$). Therefore, as government debt and physical capital are perfect substitutes in the household's portfolio, an increase in the stock of government bonds reduces investment in physical capital. Indeed, in the steady state, the saving ratio writes $s = (K_{t+1} + B_{t+1})/Y_t = \gamma^*(1/A) + b^*\gamma^*$; hence, a negative relationship between b^* and γ^* as s is fixed. Consequently, the target of the debt ratio (θ) must be lower than a threshold ($\bar{\theta}$) to ensure a positive long-term growth rate, as stated in Proposition 2.

Let us now focus on environmental sustainability. From (25), we observe that public debt affects environmental quality via the abatement-spending ratio (g) only, which is written in the steady state, using (20)

$$g^* = \beta + b^*\gamma^* - rb^* = \beta + \frac{As\theta}{1 + A\theta} - r\theta. \tag{28}$$

Without public debt (i.e., $\theta = 0 \Rightarrow b^* = 0$, as in Barro 1990), the spending ratio just corresponds to the tax resources ($g^* = \beta$) according to a so-called balanced-budget rule. In the presence of public debt ($\theta > 0 \Rightarrow b^* > 0$), the spending ratio is affected in two ways. On the one hand, fiscal deficits produce a permanent flow of new resources for financing abatement activities ($b^*\gamma^* = As\theta/(1 + A\theta)$). On the other hand, public debt generates a permanent flow of new useless expenditure (the debt burden $rb^* = r\theta$), which exerts a crowding-out effect on the abatement spending. From (28), we compute

$$\frac{\partial g^*}{\partial \theta} = b^*\frac{\partial \gamma^*}{\partial \theta} - (r - \gamma^*) = -\frac{sA^2\theta}{(1 + A\theta)^2} - \left(r - \frac{sA}{1 + A\theta}\right). \tag{29}$$

Two points deserve particular attention.

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[11] We ensure $\bar{\theta} > 0$ since $As > 1$.

First, if $\gamma^* < r$, public debt is always detrimental to abatement spending (because $\partial\gamma^*/\partial\theta \leq 0$). As $\gamma^* < r \Leftrightarrow \theta > \tilde{\theta} := \frac{1}{A}[\frac{sA}{r} - 1]$, this case occurs for high values of the debt target. Effectively, economic growth negatively depends on θ; so for $\theta > \tilde{\theta}$, the growth factor falls below the interest rate.

Second, if $\gamma^* > r$, in contrast, there is a way for public debt to be beneficial for abatement spending in the long run. However, the condition $\gamma^* > r$ is necessary but not sufficient for debt and abatement spending to be positively-linked, as $\partial\gamma^*/\partial\theta \leq 0$.

From (29), we derive[12]

$$\frac{\partial g^*}{\partial\theta} \geq 0 \Leftrightarrow \theta \leq \hat{\theta} := \frac{1}{A}\left[\sqrt{\frac{sA}{r}} - 1\right] < \tilde{\theta}.$$

The relationship between the debt ratio and abatement spending is then described by an inverted U-shaped curve, as depicted in Fig. 1. If $\theta < \hat{\theta}$, the effect linked to the positive impact of deficits on public spending prevails over the crowding-out effect, and an increase in debt leads to an expansion in abatement spending. If $\theta > \hat{\theta}$, the relationship is reversed, as high debts lead to large debt burdens and thus to a strong crowding-out effect.

The positive relationship between public debt and abatement spending is a result of a dynamic inefficiency phenomenon. As we have discussed, when $\gamma^* < r$, which means $\theta > \tilde{\theta}$, the economy is in the declining part of the curve (see Fig. 1). This situation is the one in existing endogenous growth models (see, e.g. Minea and Villieu 2012, 2013; Menuet et al. 2018) that do not incorporate an OLG structure, where optimizing individual behavior requires a transversality condition for aggregate asset stocks to hold. This condition precisely states that the growth factor of public debt is less than the real interest rate (i.e., $\gamma^* < r$), eliminating dynamic inefficiency. In this scenario, public debt consistently hinders abatement spending in the long run.

On the other hand, in our model with an OLG structure, there is no transversality condition for aggregate asset stocks due to a dynamic inefficiency (see Diamond 1965), so that the growth factor of public debt can be higher than the real interest rate ($\gamma^* > r$). This configuration that occurs for small levels of debt target ($\theta < \tilde{\theta}$) is necessary for establishing a positive link between the debt ratio and the abatement spending in the long run, as illustrated in Fig. 1. Hence, dynamic inefficiency (i.e., $\gamma^* > r$) can be viewed as the price to pay to make debt issuance beneficial for long-run abatement-spending.

As the debt target θ affects the steady-state environmental quality E^* via the abatement spending only (see the bracketed-term in Eq. 25), the following proposition shows that if debt is sustainable ($\gamma^* - 1 > 0$), then $E^* > 0$.

Proposition 3 *Provided that* $\bar{E}\varepsilon\tau_p > 1$ *debt sustainability leads to environmental sustainability.*

Proof: See "Appendix A".

Without public debt ($b = 0$), the ratio of abatement spending just corresponds to tax resources (β) and the law of motion of the environment is

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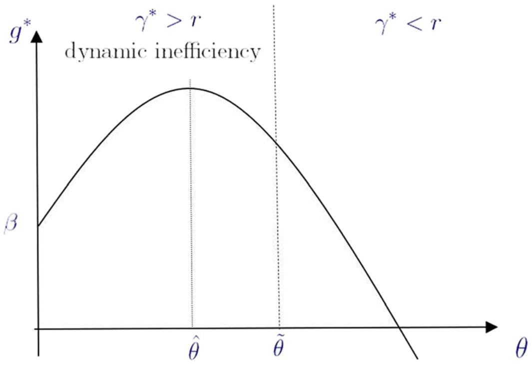
[12] We have $\hat{\theta} > 0$ as $r < 1 < As$.

Fig. 1 Relationship between debt ratio (θ) and spending ratio (g^*) in the steady state



$$E_{t+1} - E_t = \varepsilon(\bar{E} - E_t) - \frac{1}{\tau_p},$$

leading to the following steady-state value

$$E^* = \bar{E} - \frac{1}{\varepsilon\tau_p}.$$

The condition $\bar{E}\varepsilon\tau_p > 1$ means that the environment must be sustainable ($E^* > 0$) in the absence of public debt.[13] Under this assumption, Proposition 3 proves that if public debt is sustainable then long-run environmental quality will be positive. Intuitively, when the economic growth rate is positive, i.e. $\theta < \bar{\theta}$ (*debt sustainability*), the crowding-out effect of the debt burden will be limited and the abatement spending will be sufficient to offset the withdrawal of resources linked to the use of polluting input; hence, *environmental sustainability.*

Interestingly, it is sufficient that $\theta < \bar{\theta}$ to achieve the two government's objectives, namely debt and environmental sustainability. This result suggests that a fiscal rule authorizing a (small but positive) debt in the long run could make environmental and fiscal sustainability compatible, at least in the sense that the economy would develop in the long-term (i.e., a positive growth rate) and resources would not be depleted (positive environmental quality). In this context, the institutional fiscal framework adopted in most countries in the last few years to target the debt-to-GDP ratio—such as the famous 60% ceiling in the Eurozone and West African countries—would not inevitably be detrimental to the environment.

4.2 Uniqueness and Stability

The rest of this article focuses on the sustainable configuration ($\theta < \bar{\theta}$). The dynamics analysis is based on a linearization in the neighborhood of the unique steady state. In this case, system (23) behaves according to $(E_{t+1}, b_{t+1}) = \mathbf{J}(E_t - E^*, b_t - b^*)$, where $\mathbf{J}$ is the Jacobian matrix. The reduced form includes two predetermined variables (E_t and

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[13] If we relax this assumption, the long-run debt ratio (θ) would be comprised between two bounds (say, $\underline{\theta} < \theta < \bar{\theta}$) and our results would not be qualitatively affected. To keep the model as simple as possible, we assume that $\bar{E}\varepsilon\tau_p > 1$.

b_t). Hence, for the steady state to be well-determined, the Jacobian matrix must contain two eigenvalues whose absolute value is less than one.

Proposition 4 *The steady state is unique and well-determined.*

Proof: See "Appendix A".

The uniqueness and local stability of the steady state are intuitive. In system (23), the law of motion of the debt ratio b_t does not depend on E_t and thus converges autonomously to its target ($b^* = \theta$) as $\phi \in (0, 1)$, hence the vertical line in the diagram phase (see Fig. 2). Furthermore, the stationary locus of environmental quality is depicted by an inverted U-shaped curve, as the debt ratio exerts a non-linear effect on abatement spending, as we have discussed. There is only one crossing point in Fig. 2, defining the unique steady state.

5 Policy Implications

In this section, we perform several comparative statics to study how the government via its two fiscal instruments (the debt target θ, and the environmental-tax rate τ_p) can affect economic growth and the environment in the long run.

5.1 Economic Growth

The following proposition establishes the comparative statics results.

Proposition 5 *Long-run economic growth rate negatively depends on θ and τ_p.*

Proof: Using (19), the steady-state growth factor is

$$\gamma^* = \frac{\rho A(\tau_p)(1 - \alpha - \beta)}{(1 + \rho)(1 + A(\tau_p)\theta)},$$

where $A(\tau_p) = \tilde{A}^{\frac{1}{1-\beta}} \tau_p^{-\frac{\beta}{1-\beta}} \beta^{\frac{\beta}{1-\beta}}$. It is clear that $\partial\gamma^*/\partial\theta \leq 0$. As $A'(\tau_p) \leq 0$, it follows that $\partial\gamma^*/\partial\tau_p \leq 0$. □

The impact of the debt ratio is driven by the substitution effect in the household's portfolio choice. As we have seen, an increase in θ reduces household's investment in physical capital.

The impact of the environmental tax rate is conveyed by A. An increase in the environmental tax incentivizes the firm to reduce the use of polluting inputs in the production process (as $Z/Y = \beta/\tau_p$), which reduces output and the marginal return on physical capital ($Y/K = A(\tau_p)$, with $A'(\tau_p) \leq 0$). This, in turn, decreases physical capital accumulation and economic growth.

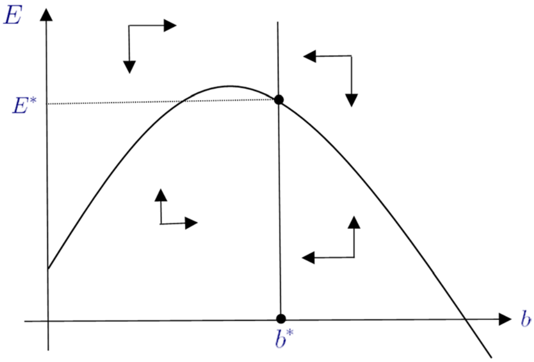

Fig. 2 Diagram phase—in the case where $E^* > 0$

5.2 Environment

The impact of the debt ratio is characterized by the following proposition.

Proposition 6 *The relationship between* θ *and* E^* *is described by an inverted U-shaped curve.*

Proof: See "Appendix B".

The debt-environment relationship is non-linear and the quality of the environment is maximized at the positive debt ratio $\hat{\theta}$, as depicted in Fig. 3. The intuition follows the link between public debt and abatement-spending. The increasing part of the curve comes from the environmental-enhancing effect of debt: for low debt ratios ($\theta < \hat{\theta}$), an increase in debt leads to more abatement-spending and thus better environmental quality in the long run. However, for high debt ratios ($\theta > \hat{\theta}$), the associated debt burden is also high, thus the crowding-out effect prevails. Hence, an increase in debt leads to a reduction in abatement-spending and lower environmental quality.

Three points deserve particular attention.

First, as discussed in the link between public debt and abatement spending, the dynamic inefficiency that arises for small debt targets ($\theta < \hat{\theta}$) is needed for public debt to be environmentally friendly in the long term.

Second, this is the first theoretical model, to the best of our knowledge, in which public debt is shown to be beneficial to the environment in the long run. Existing environmental growth models integrating debt-financing for abatement-spending indicate that public debt is always harmful for the environment.[14] Consequently, our model suggests that a rule authorizing a small but strictly positive level of debt could help the government to implement adaptation strategies by financing abatement-spending, ultimately improving the quality of the environment.

Third, on the policy side, the design of the fiscal rule is useful for determining the trade-off between economic growth and environmental protection. If the long-run target of the debt ratio is small, an increase in this ratio will reduce economic growth but improve environmental quality. There is the traditional trade-off between the goal of economic development and that of environmental protection. However, for high targets, an increase

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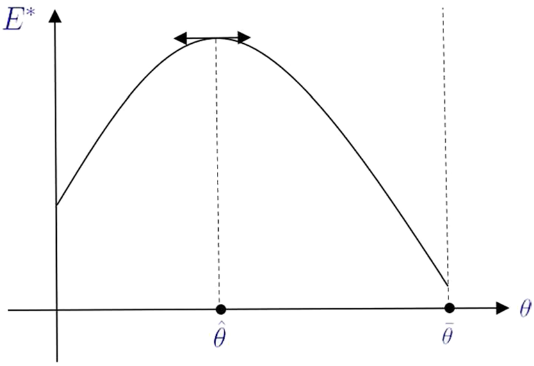
[14] It can be beneficial in an unstable steady state as in Fodha and Seegmuller (2014), but this is of little interest to policy discussions.

Fig. 3 Relationship between θ and E^*



in the debt ratio is detrimental for both economic development and environment, and the trade-off disappears.

Finally, the next proposition establishes the condition in which the environmental tax is environmentally friendly.

Proposition 7 *If* $\partial E^*/\partial\theta \leq 0$ $(i.e., \theta \geq \hat{\theta})$, *then* $\partial E^*/\partial\tau_p \geq 0$.

Proof: See "Appendix B".

The intuition of Proposition 7 is the following. Recall that the steady-state environmental quality writes

$$E^* = \bar{E} - \frac{\beta}{\varepsilon\tau_p}\left(\frac{1}{g^*(\tau_p)}\right), \tag{30}$$

where the abatement spending is

$$g^*(\tau_p) = \theta\gamma^*(\tau_p) - r(\tau_p)\theta + \beta,$$

with $r(\tau_p) = \alpha A(\tau_p)$ the real interest rate. An increase in the environmental tax has two conflicting effects on the abatement spending. Indeed, an increase in τ_p decreases the marginal return of capital, reducing the economic growth $((\gamma^*)'(\tau_p) \leq 0$, see Proposition 5), and the debt burden $(r'(\tau_p) \leq 0)$. Reduced economic growth exerts a downward force on abatement spending, whereas the decreased debt burden exerts an upward force. Proposition 7 shows that when $\theta \geq \hat{\theta}$, the second force prevails (i.e., $(g^*)'(\tau_p) \geq 0$), while the first one prevails for $\theta < \hat{\theta}$ (i.e., $(g^*)'(\tau_p) < 0$). Indeed, if the debt ratio is high, the economic growth is so small that the impact of reducing the debt burden outweighs the effect of reduced economic growth.

Interestingly, the condition $\theta \geq \hat{\theta}$ just corresponds to the negative impact of debt on abatement spending (see Fig. 1). Hence, while increasing public debt is detrimental to abatement spending, increasing environmental tax is not, and vice versa.

Finally, regarding Eq. (30), since an increase in τ_p decreases the use of polluting inputs and the flow of emissions $(Z/Y = \beta/\tau_p)$, an environmental-tax hike is unambiguously favorable for environmental quality when abatement spending improves, i.e. $\theta > \hat{\theta}$. In this way, an environmental tax is good for the environment when public debt is not.

This result has interesting policy implications. From the perspective of environmental protection, there is a conflict between the two instruments. A policy based on tax increases (on polluting inputs) is useful for improving environmental quality in the long run if public debt is bad. From a normative point of view, if debt-to-GDP levels are too high ($\theta > \hat{\theta}$), the government should adopt tax-financing for abatement expenditure, but it should implement debt-financing if debt ratios are sufficiently low ($\theta < \hat{\theta}$). Thus, environmental tax and public debt are two tools available to governments, depending on the level of indebtedness in the economy.

5.3 Adjustment Profiles

Let us now address the question of the impact on steady-state and on the transition path of a permanent increase in the debt target (θ).[15]

Figure 4 depicts the evolution of our key variables—the debt-to-ouput ratio (b_t), the economic growth rate ($\gamma^* - 1$), the public-spending-to-output ratio (g_t), and the environmental quality (E^*)—following two scenarios. Starting to a balanced-budget rule (i.e. $\theta = 0$), we consider a slight increase (from $\theta = 0$ to $\theta = 0.1$, see Fig. 4a), and a sharp increase (from $\theta = 0$ to $\theta = 0.2$, see Fig. 4b) in the debt target. In the two scenarios, the debt ratio and the economic growth rate monotonically converge to their new steady-state values, and they are negatively linked during the transition path. When public debt rises, households will substitute their saving from physical capital to government bonds, resulting in a decrease in the accumulation of physical capital and a slowdown in economic growth. The only change between the two scenarios lies in the paths of public spending and environmental quality.

Two point devote particular attention.

In the long run, following a slight increase in the debt target (from $\theta = 0$ to $\theta = 0.1$), abatement spending and environmental quality are improved. Environmental quality improves from 8.6 without debt ($\theta = 0$) to 10.2 with a 10% debt-to-GDP ratio after 100 time periods (as shown in Fig. 4a). With a larger increase in the debt target (from $\theta = 0$ to $\theta = 0.2$), in contrast, abatement spending and environmental quality are reduced. Environmental quality decreases from still 8.6 to 6.8 for $\theta = 0.2$ (see Fig. 4b). This feature illustrates Proposition 6, showing that the relationship between the debt target and both abatement spending and environmental quality follows an inverted U-shaped curve, with a threshold at $\theta = \hat{\theta}$. Based on our calibration, we compute $\hat{\theta} \approx 0.125$. Consequently, for $\theta \leq 0.125$ (as in in Fig. 4a), an increase in θ enhances abatement spending and environmental quality, while the relationships reverse for $\theta > 0.125$ (as in Fig. 4b).

In the short run, the transition paths of abatement spending and environmental quality go hand-in-hand and describe inverted U-shaped curves. When the debt target increases, abatement spending initially rises due to the new deficits, enhancing the stock of natural capital. However, it subsequently declines after reaching a certain threshold because of the

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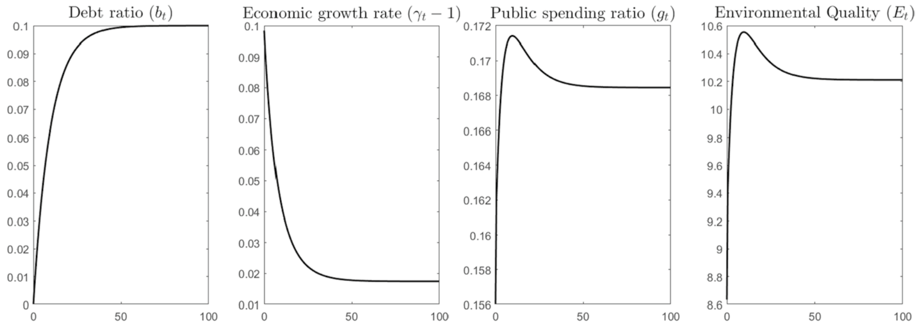
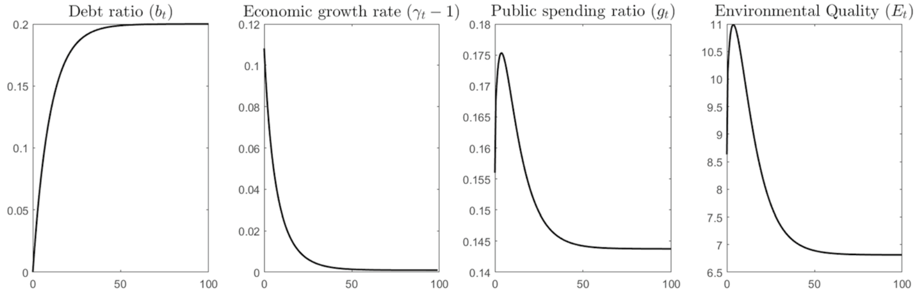
[15] Simulations are performed for $\rho = 0.99$, $\alpha = 0.2$, $\beta = 0.1$, $\tau_p = 0.03$, $\phi = 0.05$, $\bar{E} = 30$, $\epsilon = 0.2$, $\tilde{A} = 0.95$. We numerically ensure that the behavior of our key variables remains unchanged when we slightly modify parameter values.



(a) From $\theta = 0$ to $\theta = 0.1$.



(b) From $\theta = 0$ to $\theta = 0.2$.

Fig. 4 Adjustment profiles following an increase in the debt target

crowding-out effect caused by the debt burden prevails, leading to a reduction in the stock of natural capital.[16]

6 Concluding Remarks

In this article, we have developed a second-best theory based on an OLG endogenous growth model that includes the dynamic of public debt, the environment, and economic growth. We have shown that public debt may be beneficial for the environment but harmful for growth in the long run. Indeed, if the fiscal rule that the government follows does not allow for too much values of the debt-to-GDP ratio, debt issuance could be used to finance abatement expenditure to limit the depletion of natural resources.

From a methodological perspective, the environmentally friendly nature of public debt is due to the positive effect of debt on public (abatement) spending. This effect is due to the OLG structure, in which the economic growth rate can be higher than the interest rate along the balanced-growth path. Indeed, as agents have a finite lifetime, there is no transversality condition in the household's optimization program, contrary to the models with an infinite lifetime representative household. The transversality condition usually ensures that the economy is dynamically efficient and that Ponzi games are not feasible, as the

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[16] Examining the similar adjustment paths of our key variables in response to a permanent environmental tax shock (as, e.g., an increase in τ_p) would not provide meaningful insights. Indeed, the dynamics of the debt ratio b_t (see Eq. 23b) are independent of τ_p, so all variables would instantaneously jump to their new steady-state levels after an environmental-tax shock.

growth rate of public debt is smaller than the interest rate. In our setup, as in Diamond (1965), the stock of debt can increase at a rate higher than the interest rate in a steady state, emphasizing a dynamic inefficiency characterized by an overaccumulation of assets (see O'Connell and Zeldes 1988; King and Ferguson 1993). Such an inefficiency is thus the price to pay for the positive effect of public debt on the environment.

From a policy perspective, our model contributes to recent debates on the role of fiscal rules in addressing environmental challenges (see, e.g. Pereira and Pereira 2017; Carratù et al. 2019; Darvas and Wolff 2021). We have introduced a fiscal rule with a debt-to-GDP ceiling—as implemented in most countries, including with the "Stability and Growth Pact" in European countries—and showed that additional public debt created by deficit-financed green public investment can improve environmental quality, provided the debt ceiling is not too high. However, in this case, there is trade-off between economic growth and environmental preservation, as the debt ratio reduces the long-run growth rate. The implementation of a fiscal rule with a fixed debt ceiling could thus be beneficial for long-run environmental conservation objectives.

Against this background, several extensions of our model could be considered. First, we could assume that the government's budget constraint is adjusted by the environmental tax rate rather than by abatement spending. In this case, the channel through which fiscal policy influences long-run environmental quality would be modified. Whereas the impact of debt on the environment is channelled through public abatement activity, which mitigates the effect of pollution regardless of the flow of emissions, the impact of environmental taxes is channelled through the firm's behaviour, via a polluter-pays principle, regardless of abatement spending.

A second possible extension would be to introduce a *green golden rule* for public finance, as recently suggested by Darvas and Wolff (2021). Under such a rule, a running public deficit could be allowed only to finance green investment. In an endogenous growth setup without an environmental module, Minea and Villieu (2009) have shown that a golden rule allowing a government to run public-investment-oriented fiscal deficit could improve social welfare compared to a strict balanced-budget rule (i.e., zero debt). In the same spirit, we could analyze the impact of a green golden rule on social welfare, comparing this to the fixed rule that we use in this paper.

Discussions of the economic policies designed to address global warming often overlook the policy mix. An interesting extension would be to introduce money and a central bank into this model. Thus, we could study a monetization rule that allowed green-investment expenditure to be financed by money issuance. The recent contributions of Professor Xepapadeas propose various interesting setups, from this perspective (see, e.g Economides and Xepapadeas 2018).

Appendix A. Steady State

Proof of Proposition 3. The long-run environmental quality is given by

$$E^*(\theta) = \bar{E} - \frac{\beta}{\varepsilon \tau_p g^*(\theta)} \tag{A.1}$$

where

$$g^*(\theta) = \beta + \frac{As\theta}{1 + A\theta} - r\theta, \text{ and } \gamma^*(\theta) = \frac{As}{1 + A\theta}.$$

First: $\gamma^*(\theta) - 1 > 0 \Leftrightarrow \theta < \bar{\theta} := (As - 1)/A$. $\bar{\theta}$ is positive as we assume $As > 1$.

Second: $E^*(\theta) > 0 \Leftrightarrow g^*(\theta) - \frac{\beta}{\bar{E}\varepsilon\tau_p} =: h(\theta) > 0$, where

$$h(\theta) := \beta + \frac{As\theta}{1 + A\theta} - r\theta - \frac{\beta}{\bar{E}\varepsilon\tau_p}.$$

Let us suppose that $\bar{E}\varepsilon\tau_p > 1$. Then h is a continuous mapping on $\mathbb{R}^+$, with the following properties: $h(0) = \beta - \beta/\bar{E}\varepsilon\tau_p > 0$, $h(+\infty) = -\infty$, and

$$h'(\theta) = \frac{As}{(1 + A\theta)^2} - r \geq 0 \Leftrightarrow \ \theta \leq \hat{\theta} =: \frac{1}{A}\left[\sqrt{\frac{As}{r}} - 1\right].$$

The threshold $\hat{\theta}$—which is positive as $As > 1 > r$—is the level of the long-run debt target that maximizes the abatement-spending ratio. Hence, h describes an inverted U-shaped curve with a maximum at $\theta = \hat{\theta}$, as described in Fig. 1.

Additionally, we have $h(\bar{\theta}) = \beta + (s - 1) - r(s - 1) - \frac{A\beta}{\bar{E}\varepsilon\tau_p} = \beta + \frac{(As-1)(1-r)}{A} - \frac{\beta}{\bar{E}\varepsilon\tau_p} > \beta - \frac{\beta}{\bar{E}\varepsilon\tau_p} > 0$. Hence, if $\beta > \beta/\bar{E}\varepsilon\tau_p$, then $h(\theta) > 0$, for any $\theta \in (0, \bar{\theta})$.

Consequently, if $\theta < \bar{\theta}$ (i.e., $\gamma^* - 1 > 0$, *debt sustainability*), the *environmental sustainability* (i.e., $E^*(\theta) > 0$) is ensured. □

Proof of Proposition 4. We first prove the uniqueness of the steady state then we focus on the local stability.

Uniqueness. From Eq. (23.b), it is clear that the law of motion of b_t does not depend on E_t; such that b_t monotonically converges to its target $b^* = \theta$ since $\phi \in (0, 1)$. This explains the vertical line in the diagram phase (see Fig. 5). Let $b_{t+1} = b_t = b$ and $E_{t+1} = E_t = E$ in system (23). From Eq. (23.a), the stationary locus of the environmental quality is depicted by the following link between E and b

$$E = E(b) = \bar{E} - \frac{\beta}{\varepsilon\tau_p}\left(\frac{1}{\frac{Asb}{1+Ab} - rb + \beta}\right).$$

Let us suppose that $\theta < \bar{\theta} \Leftrightarrow b < \bar{b}$, namely $E(b) > 0$, as stated in the proof of Proposition 3. It is clear that $E(b)$ is a continuous mapping on $[0, \bar{b}]$.

First, we have

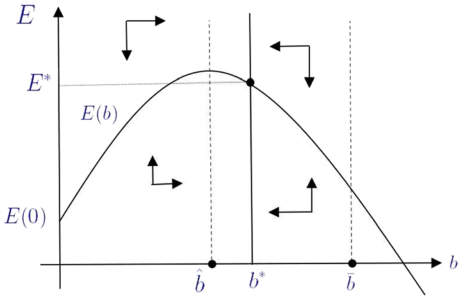
$$E(0) = \bar{E} - \frac{1}{\varepsilon\tau_p} > 0, \text{ and } E(\bar{b}) = \bar{E} - \frac{\beta}{\varepsilon\tau_p[(As - 1)(1 - r)/A + \beta]} > 0,$$

as we assume $\bar{E}\varepsilon\tau_p > 1$, and $As > 1 > r$.

Second, we compute

$$E'(b) \geq 0 \Leftrightarrow b \leq \hat{b} := \frac{1}{A}\left[\sqrt{\frac{As}{r}} - 1\right].$$

Fig. 5 Diagram phase



Hence, the stationary locus of the environmental quality depicts an inverted U-shaped curve on the phase portrait (b, E), with a maximum at $b = \hat{b}$, as depicted in Fig. 5. Consequently, for $b^* \in [0, \bar{b}]$, there is a unique crossing point between b^* and $E(b)$ that defines the unique steady state of the model (b^*, E^*).

Local stability. From system (23), the Jacobian matrix evaluated at the steady state (b^*, E^*) is

$$\mathbf{J} = \begin{bmatrix} 1-\varepsilon & K \\ 0 & 1-\phi \end{bmatrix},$$

where K is a (finite) scalar. Then, the determinant and trace are $\det(\mathbf{J}) = (1-\varepsilon)(1-\phi)$, and $\operatorname{tr}(\mathbf{J}) = 2 - \phi - \varepsilon$. For the steady state to be well determined, we must ensure that $\det(\mathbf{J}) < 1$ and $\operatorname{tr}(\mathbf{J}) < 2$. As $\varepsilon < 1$ and $\phi < 1$, it follows the steady state is well determined. □

Appendix B. Comparative Statics

Proof of Proposition 6. At steady state, the environmental quality is given Eq. (A.1), namely

$$E^*(\theta) = \bar{E} - \frac{\beta}{\varepsilon\tau_p\left[h(\theta) + \frac{\beta}{\bar{E}\varepsilon\tau_p}\right]}.$$

As proved in "Appendix A", $h(\theta)$ describes an inverted U-shaped curve on $\theta \in [0, \bar{\theta}]$. Hence, $E(\theta)$ also describes an inverted U-shaped curve on $\theta \in [0, \bar{\theta}]$, with a threshold at $\theta = \hat{\theta}$. □

Proof of Proposition 7. From Eq. (25), we define $E^* = E^*(\theta, \tau_p)$, where

$$E^*(\theta, \tau_p) = \bar{E} - \frac{\beta}{\varepsilon\tau_p g^*(\theta, \tau_p)}, \tag{B.1}$$

with, using $r = \alpha A(\tau_p)$,

$$g^*(\theta, \tau_p) = \beta + \frac{A(\tau_p)s\theta}{1 + A(\tau_p)\theta} - \alpha A(\tau_p)\theta.$$

We compute

$$\frac{\partial g^*(\theta, \tau_p)}{\partial \tau_p} = A'(\tau_p)\theta \left[\frac{s}{(1 + A(\tau_p)\theta)^2} - \alpha \right].$$

The bracketed-term is negative if and only if

$$\theta \geq \frac{1}{A}\left[\sqrt{\frac{s}{\alpha}} - 1\right] = \frac{1}{A}\left[\sqrt{\frac{As}{r}} - 1\right] = \hat{\theta}.$$

Consequently, as $A'(\tau_p) \leq 0$, it follows that $\partial g^*(\theta, \tau_p)/\partial \tau_p \geq 0 \Leftrightarrow \theta \geq \hat{\theta}$. As shown in "Appendix A", this condition ($\theta \geq \hat{\theta}$) is precisely equivalent to $\partial g^*(\theta, \tau_p)/\partial \theta \leq 0$. In other words, while increasing public debt is detrimental to abatement spending, increasing taxes is not, and vice versa. Finally, from (B.1), if $\theta \geq \hat{\theta}$, we derive $\partial E^*(\theta, \tau_p)/\partial \tau_p \geq 0$.

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