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# Permit Markets with Political and Market Distortions

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**Abstract**
This article investigates cap-and-trade markets in the presence of both political and market distortions. We create a model where dominant firms have the ability to rent seek for a share of pollution permits as well as influence the market equilibrium with their choice of permit exchange because of market power. We derive the equilibrium and show the interaction of these two distortions has consequences for the resulting marginal inefficiency—the extent to which a re-allocation of permits between firms can reduce equilibrium abatement costs. We find that if the regulator is not very responsive to rent seeking then marginal inefficiency reduces relative to the case without rent seeking. When the regulator is very responsive to rent seeking, if dominant rent-seeking firms are all permit buyers (sellers) then marginal inefficiency reduces (increases) relative to the case without rent seeking.

**Keywords** Pollution market · Market power · Rent seeking

**JEL Classification** D43 · D72 · Q58

## 1 Introduction

Cap-and-trade markets have been promoted as a cost-effective form of pollution control. The varied scope of these markets can be observed within the European Union and United States, among others.[1] Although permit markets have performed relatively well, the implementation of these schemes has raised a number of concerns. First, concern exists over the potential market power of participating firms that may result in allocative inefficiency (e.g., Hintermann 2017). Participating firms are normally sourced from a small number of concentrated industries (such as the energy sector), which may increase the likelihood of market-power effects. Second, it has been well documented that investments in rent-seeking

[1] While notable successes exist, such as the Regional Greenhouse Gas initiative (RGGI) and the European Emissions Trading Scheme (EU-ETS), many schemes have been less successful (Schmalensee and Stavins 2013) or have not been implemented, such as the Australian carbon pricing scheme.

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efforts have been used to alter the distribution of initial permit allocations, with clear evidence from the US Acid Rain Program (Joskow and Schmalensee 1998) as well as the European Union Emissions Trading Scheme (Zapfel 2007; Rode 2021). Thus, within contemporary cap-and-trade markets, there exists the potential for both political and market distortions. Yet it is a priori unclear how these distortions interact and the consequences for the cost effectiveness of pollution control.

In this article we investigate the interactions between political and market distortions within a cap-and-trade market. To achieve this, we create a model of a cap-and-trade market with large dominant players and a competitive fringe of individually insignificant players. The dominant players have the ability to: (1) rent seek for their initial allocation of permits and the aggregate supply of permits; as well as (2) choose their level of permit exchange. We allow the contestability of the initial allocation of pollution permits and the determination of the aggregate emissions cap to vary from being non-contestable to fully contestable, where we capture contestability using a rent-seeking process. This allows us to consider how alternative initial allocation mechanisms operate when there is a difference in sensitivity towards rent seeking or, indeed, to consider alternative regulatory systems where the degree of rent seeking varies. We focus on the direct comparison between two environments: when permits are either contestable or non-contestable. In order to highlight the relevant channels of influence we start with the simplest case of a single dominant firm and then extend the model to allow for competition in rent seeking and permit exchange. We define the idea of marginal inefficiency to capture the change in inefficiency in the permit market as a result of rent seeking, after which the total abatement required may be different to that in the absence of rent seeking. This is given by the maximum cost saving that can be achieved by re-allocating a single permit between firms, i.e., from a firm with the lowest marginal abatement cost to a firm with the highest marginal abatement cost. We show that the resulting marginal inefficiency in a contestable environment depends on the composition of the permit market: in the presence of active rent seeking, if the dominant firm is a permit buyer then marginal inefficiency improves compared to what is observed in a non-contestable environment. Whereas when the firm is a seller, marginal inefficiency worsens relative to the non-contestable environment. We find that the introduction of competition in rent seeking and permit exchange results in similar findings with the addition that the regulator's responsiveness towards rent seeking has a pivotal impact: if the regulator is very responsive then the impact on marginal inefficiency is similar to the single dominant firm case; but if the regulator is not very responsive to rent seeking then marginal inefficiency improves regardless of the composition of the market.

Our model focuses on a market structure that includes dominant firms and a competitive fringe of small firms. The competitive-fringe framework detailed herein is consistent with existing permit markets that include rent seeking. For example, relating to the EU-ETS Phase I allocation process, Zapfel (2007, p. 29) states that "rent-seeking behaviour played a major role, in particular with regard to larger companies that could afford to allocate sufficient human resources to influence allocation processes. Smaller companies were largely 'price-takers' in the allocation process." Another key aspect of our analysis is that the dominant firms have the ability to rent seek for their own initial allocation . This was observed, for example, in the initial phases of the EU-ETS (specifically Phase I), where dominant players successfully lobbied to obtain additional initial permit allocations that subsequently increased the aggregate supply (National Allocation Plan) (see Zapfel (2007) for specific details of this occurring within individual Member States). As we present in the model, each firm has a non-contestable component of initial allocation as well as a contestable part. One possible interpretation, for example, is that the initial endowment allocation is in draft legislation (which is the fixed baseline for all firms). As the dominant firms are the only entities with any rent-seeking power, it is intuitive to consider that additional permits are created and distributed to those dominant firms, which we capture as the contestable component.

The examination of dominant firms within cap-and-trade markets is well developed. The main branch of literature extends the work of Hahn (1984) that focuses on a dominant firm with a competitive fringe of price-taking firms (e.g., Misiolek and Elder 1989; von der Fehr 1993; Westskog 1996; Sartzetakis 1997; Liski and Montero 2011; Hintermann 2011, 2016, 2017; D'Amato et al. 2017; Dickson and MacKenzie 2018). Hahn (1984) shows the dominant firm will select their permit holdings to decrease (increase) the permit price if it is a permit buyer (seller). Consequently, the existence of market power results in increased costs of pollution control. Although the literature on market power is well established, it does not extend the analysis to incorporate rent-seeking activities. Yet, in a market with a concentrated set of dominant firms, it is plausible that rent seeking would be more prevalent than in a perfectly competitive market. In this article, we advance the literature on market power within cap-and-trade markets by allowing the dominant firm(s) to additionally rent seek for their initial permit allocation as well as the aggregate supply of permits to the market. Our analysis shows that the inefficiency normally associated with market power is significantly altered by the presence of political distortions. That is, whether the regulator is open and responsive to rent seeking has significant consequences for the cost effectiveness of the permit market.

The rent-seeking literature focusing on cap-and-trade markets is also well established (Dijkstra 1998; Lai 2007, 2008; Hanley and MacKenzie 2010; MacKenzie and Ohndorf 2012; MacKenzie 2017; Rode 2021).[2] The majority of this literature focuses on rent-seeking for initial permit endowments, but assumes that firms are price-takers in a competitive permit market. In particular, Hanley and MacKenzie (2010) provide a model where firms first rent seek for permits, where rent seeking influences both the distribution of permits and the aggregate number of permits (in a linear fashion), and then go on to trade permits in a perfectly competitive market where they are assumed to be price takers. Their main conclusion is that the regulator's responsiveness may have ambiguous effects on social welfare. If a regulator becomes more responsive to rent seeking, emissions will increase (thus increasing pollution damages and potentially lowering social welfare) but the price per permit will fall (reducing rent seeking costs and potentially improving social welfare). This approach allows a thorough investigation of how the regulator's responsiveness to rent seeking (i.e., how the number of permits is influenced by lobbying) influences social welfare, but it misses the important feature of how rent seeking can influence and distort trade, and consequently efficiency, quite simply because under the assumption of perfect competition the permit market is always allocatively efficient. While they do include a discussion of a dominant firm, they don't formally investigate the equilibrium to establish the link between rent seeking, market power, and efficiency. Studying this intersection of rent seeking *and* market power within permit markets—which has so far been largely ignored in this literature—is precisely our focus here. The main aim of this article is to provide

[2] Our focus here is on how rent seeking impacts the working and efficiency of the market. Other distinct literature exists on lobbying over environmental policy targets as well as the choice of policy instrument (see, for example, Buchanan and Tullock (1975), Aidt (1998), Aidt (2010)). For a comprehensive review of the literature see Oates et al. (2003). For a comprehensive survey of the rent-seeking literature see Konrad (2009).

the first comprehensive analysis of a cap-and-trade market with dual distortionary effects (rent seeking and market power), and to identify the conditions where efficiency is either improved or reduced by investment in rent-seeking activity.

We therefore provide a model that bridges the gap between the rent-seeking and market-power literatures. We follow the market-power literature by developing a competitive-fringe framework with one dominant firm, but later extend the model to allow for competition between firms with market power. Our model has two stages. In the first stage, the dominant firms invest in rent-seeking effort in order to obtain a share of the initial allocation of permits. Rent-seeking efforts are sunk costs, such as lobbying and persuasive activities, that alter both the distribution of the initial permit allocation and the aggregate number of permits supplied to the market by the regulator. We develop this process as a strategic contest (e.g., Tullock 1980; Hillman and Riley 1989; Konrad 2009; Dickson et al. 2018), where the equilibrium share of permits is determined by a firm's rent-seeking efforts relative to total outlays and the regulator's permit supply response is dependent on the level of aggregate rent seeking. We allow firms' initial permit allocations to be determined by a contestable component and a non-contestable component thus allowing a continuum between these two extremes, which can capture realistic institutional setups of initial allocation processes. While it is clear that firms can rent seek for their permits, there can also be restrictions on the initial allocation of permits. We introduce a parameter $\alpha$ that captures this degree of contestability. This could represent the varying degrees of rent-seeking culture, such as how responsive bureaucrats are to the rent-seeking process. Further, it could represent specific rules from legislation that allow for the earmarking of permits or specific allocation mechanisms like auctioning or grandfathering.[3]

Our major innovation is thus the investigation of permit markets in the presence of both market and political distortions. This may provide insight to policymakers regarding the contestability of allocated permits and the likely consequences for the operation—and cost effectiveness—of the permit market. If rent seeking was shown to be counterproductive towards the marginal inefficiency in the permit market, it may be desirable for the regulator to adjust the initial allocation process so that the permits become less contestable. Possible options would be to allow for more auctioning of permits, or to change laws to either prohibit or limit rent-seeking activity (such as changes to ministerial codes of conduct or lobbying acts (such as the US Lobbying Disclosure Act 1995 and the Honest leadership and Open Government Act 2007)). Equally, if it was found that rent seeking improved the marginal inefficiency of the market, then implementing these above-mentioned actions would actually worsen the cost effectiveness of the market. It may also be the case that regulatory interventions and laws that attempt to influence rent-seeking behaviors are simply neither practical nor enforceable. In such a case, our analysis then provides a comparative analysis between different regulatory systems and rent-seeking cultures that highlights how different intensities of rent seeking can impact on the aggregate cost of pollution reduction.

The article is organized as follows. In Sect. 2 the model is outlined with one dominant firm and we detail the results that link rent-seeking activity and market power to changes in marginal inefficiency. Section 3 introduces competition to both rent seeking and permit exchange and Sect. 4 provides the equilibrium analysis. Section 5 provides the results with

[3] For an analysis of auction and grandfathering aspects related to market power in the permit market see Álvarez and André (2015). Further, the consequences of market power within a multi-unit auction process usually take the form of lower clearing prices due to firms' lower submitted bids (Khezr and MacKenzie 2018a, b).

competition. Section 6 discusses the inclusion of auctioned permits and Sect. 7 concludes. Proofs of the theoretical results are contained in the "Appendix".

## 2 Single Dominant Firm

### 2.1 Preliminaries

Consider a pollution permit market with $N+1$ regulated firms. There is a single dominant firm denoted by $i$ and a competitive fringe of $N > 1$ firms each denoted by index $f$. We initially begin with one dominant firm to highlight the important channels of influence but later extend this to allow for competition. The dominant firm's pollution abatement is $a^i \equiv e - [\omega^i + x^i]$, where $e$ is the level of unconstrained emissions, $\omega^i$ is the initial allocation of permits and $x^i$ represents permits transacted in the market: $x^i > 0$ for purchases and $x^i < 0$ for sales. We assume the cost of abatement is quadratic of the form $C^i(a) = \frac{1}{2}c^i a^2$ with $c^i > 0$, where $C^{i\prime}(a) = c^i a$, $C^{i\prime\prime} = c^i$ and $C^{i\prime\prime\prime} = 0$. We use analogous notation for the fringe firms and impose the same assumptions on their cost functions. Our framework consists of two stages. In Stage 1, the dominant firm invests in rent-seeking effort in order to alter its initial allocation of permits from the regulator. In Stage 2, firms engage in a market for trading pollution permits in which the dominant firm has market power.

In Stage 1 the dominant firm can engage in rent seeking to increase its initial allocation of permits from the regulator. We model this by supposing that the dominant firm chooses a level of (costly) rent-seeking effort $k^i \geq 0$, which results in the regulator allocating the dominant firm additional permits via an increase in the aggregate emissions cap. We follow the rent-seeking literature and assume the cost of rent seeking for the dominant firm is $v^i(k^i)$ and impose $v^i(k^i) = h^i k^i$ with $h^i > 0$. To allow for a wide variety of institutional settings in which the degree of contestability of permits varies, we specify that the initial allocation of permits, to the dominant firm and fringe respectively, is determined by[4]

$$\omega^i(k^i) = \gamma\left[[1-\alpha]\Omega^0 + \alpha\Omega(k^i)\right] \text{ and}$$
$$\omega^f = \frac{1-\gamma}{N}\Omega^0.$$

The function $\Omega(k^i)$ is the regulator's response to rent seeking, where $\Omega(0) = \Omega^0$ is the baseline number of available permits and we assume $\Omega' > 0$, $\Omega'' \leq 0$, and $\Omega' \to 0$ as $k^i \to \infty$. The parameter $\gamma \in (0,1)$ represents the weight of the dominant firm relative to the fringe (who each receive their share of the baseline number of permits).[5] The parameter $\alpha \in [0,1]$

[4] As is natural, the competitive fringe consists of many small firms, where each individual firm has no rent-seeking influence over its initial permit endowment. As such, we implicitly assume that each fringe member can't overcome the collective action problem so fringe members can't coordinate their actions to rent seek. Note that we later analyze the case with two dominant firms (in the presence of a competitive fringe), the results from which are indicative of the consequences of a subset of the fringe members solving the collective action problem to coordinate their actions.

[5] In the model, as stated, the dominant firm does not explicitly 'take' permits from the fringe firms in the rent-seeking process. However, our model could be interpreted as the reduced form of a framework where all firms—both dominant and fringe—have a contestable and non-contestable part to their allocation, but because the fringe firms are small they cannot effectively rent seek to retain their contestable part. As such, all the contestable allocation is directed toward the dominant rent-seeking firm, thus leaving the fringe firms with only the non-contestable component.

captures the responsiveness of the regulator towards rent seeking: if $\alpha = 0$ rent seeking is completely ineffective and the dominant firm simply receives its share of the baseline number of permits $\Omega^0$, whereas if $\alpha > 0$ there is scope for the dominant firm to increase its initial allocation of permits through rent seeking according to the function governing the regulator's response to this rent-seeking activity $\Omega(k^i)$.[6] Note that

$$\omega^{i\prime} = \alpha\gamma\Omega' > 0,$$

and that the total emissions cap is

$$N\omega^f + \omega^i(k^i) = \Omega^0 + \alpha\gamma[\Omega(k^i) - \Omega^0].$$

This framework allows us to consider a wide range of initial allocation mechanisms that differ in the degree of contestability. In Sect. 6 we consider how the degree of contestability in our model can be linked to the proportion of permits that are auctioned.

We now turn to solve for the equilibrium permit exchange and rent-seeking choices.

## 2.2 Stage 2: Equilibrium Permit Exchange

In Stage 2, once the initial permit allocations and the aggregate number of pollution permits in circulation become known, the dominant firm exchanges permits within a competitive-fringe model. Market clearing requires

$$x^i + Nx^f = 0.$$

If $p$ is the price of permits, each firm in the competitive fringe will seek to minimize the cost of abatement to solve

$$\min_{x^f} C^f(e - [\omega^f + x^f]) + px^f.$$

The necessary (and, given our assumptions, sufficient) first-order condition is $c^f[e - [\omega^f + x^f]] = p$ and therefore equilibrium demand from the fringe takes the form

$$\tilde{x}^f(p) = e - \omega^f - \frac{p}{c^f}. \tag{1}$$

Since the market clearing price will satisfy $x^i + N\tilde{x}^f(p) = 0$ it follows that inverse demand is linear and takes the form

$$\tilde{p}(x^i) = c^f[e - \omega^f] + \frac{c^f}{N}x^i, \tag{2}$$

with $\tilde{p}'(x^i) = \frac{c^f}{N} > 0$.

[6] The structure of the regulator's response to rent seeking can be intuitively understood by noting that there may exist external forces in the political environment that attempt to contain aggregate emissions (e.g., environmental lobby groups). Thus the regulator—faced with competing special interests—may increase permit supply when faced with rent seeking from the dominant firm, but only at a decreasing rate. Note that the regulator's response function is sufficiently general to allow for varying degrees of concavity, which can be interpreted as allowing variation in the effectiveness of the external political force (or, more generally, the responsiveness of the regulator to political rent seeking).

The dominant firm will choose a permit exchange in the market to minimize the overall cost of pollution, accounting for the effect on the price of permits. In the second stage (given the choice of $k^i$ from the first stage) the firm thus faces the optimization problem

$$\min_{x^i} C^i(e - [\omega^i(k^i) + x^i]) + x^i\tilde{p}(x^i) + v^i(k^i). \tag{3}$$

The optimal solution, which depends on the level of rent-seeking activity from the first stage, will satisfy the necessary first-order condition and is therefore given by[7]

$$\tilde{x}^i(k^i) = \left\{x^i \,:\, -c^i[e - [\omega^i(k^i) + x^i]] + \tilde{p}(x^i) + x^i\tilde{p}'(x^i) = 0\right\}. \tag{4}$$

Using this optimal solution allows us to determine the following proposition.

**Proposition 1** *Suppose $\alpha > 0$, then for any $k^i > 0$ there exists a unique cost minimizing permit exchange for the dominant firm $\tilde{x}^i(k^i)$. The dominant firm's demand for (supply of) permits is decreasing (increasing) in rent seeking activity, $\tilde{x}^{i\prime} < 0$, but overall permit holdings $\omega^i(k^i) + \tilde{x}^i(k^i)$ are increasing in rent-seeking activity.*

From Proposition 1, it is interesting to note that when a firm engages in more rent seeking in the first stage, which *ceteris paribus* yields more permits, there is either a reduction in demand or increase in supply to the market. However, overall, an increase in rent seeking will result in an increase in the number of permits a firm holds in equilibrium. Thus it is clear that more rent seeking by a firm leads to less pollution abatement by that firm.

### 2.3 Stage 1: Equilibrium Rent-Seeking Choices

In this subsection, we analyze the equilibrium rent-seeking choices of the dominant firm, accounting for the subsequent second-stage permit-market equilibrium. The firm can be seen as choosing rent-seeking effort to minimize the overall cost of emissions that include abatement costs, the net cost of permit purchases, and rent-seeking costs. Firm $i$'s problem is therefore to

$$\min_{k^i \geq 0} C^i(e - [\omega^i(k^i) + \tilde{x}^i(k^i)]) + \tilde{x}^i(k^i)\tilde{p}(\tilde{x}^i(k^i)) + v^i(k^i).$$

For ease of notation, define the cost of emissions as

$$G^i(k^i) = c^i[e - [\omega^i(k^i) + \tilde{x}^i(k^i)]] + \tilde{x}^i(k^i)\tilde{p}(\tilde{x}^i(k^i)).$$

The effect of increasing rent-seeking effort on the cost of emissions is given by

$$G^i_{k^i} = \frac{\partial G^i}{\partial k^i} + \frac{\partial G^i}{\partial x^i}\tilde{x}^{i\prime}.$$

From the first-order optimality condition for permit market transactions (see (4)), $\frac{\partial G^i}{\partial x^i} = 0$, and $\frac{\partial G^i}{\partial k^i} = -c^i\omega^{i\prime}$. As such, the optimal rent-seeking effort is given by

[7] While an explicit solution can be derived, we retain the implicit solution to avoid complicating the presentation with unnecessary algebra.

$$k^{i*} = \left\{k^i : l^i(k^i) \equiv -c^i[e^i - [\omega^i(k^i) + \tilde{x}^i(k^i)]]\alpha\gamma\Omega'(k^i) + h^i = 0\right\}.$$

This allows us to deduce the following result.

**Proposition 2** *For any $\alpha > 0$, the dominant firm has a unique cost-minimizing first-stage rent-seeking effort $k^{i*}$ resulting in permit market trade $\tilde{x}^i(k^{i*})$. $k^{i*} > 0$ if the marginal cost of rent seeking is low enough: $h^i < c^i[e^i - [\omega^i(0) + \tilde{x}^i(0)]]\alpha\gamma\Omega'(0)$.*

## 2.4 The Effect of Permit Contestability on Marginal Inefficiency: The Single-Firm Case

When trade in a permit market results in an allocation of permits in which marginal abatement costs are not equalized, there will be an excess cost of abatement relative to the least-cost solution for the particular level of aggregate abatement under consideration. To measure the inefficiency implications in the permit market of prior rent seeking (which changes the amount of aggregate abatement required) we define a measure of inefficiency, that we call marginal inefficiency. This is given by the maximum cost saving that could be achieved in equilibrium by re-allocating a single permit between firms, i.e., from a firm with the lowest marginal abatement cost to a firm with the highest marginal abatement cost (if there are multiple firms that satisfy this criteria, as there will be with identical fringe firms, one is chosen to give or receive the permit). Since the marginal abatement cost is always equal to the price for fringe firms, this is given by[8]:

$$\begin{aligned} MI &\equiv \max\{c^i a^{i*}, p^*\} - \min\{c^i a^{i*}, p^*\}, \\ &= |c^i a^{i*} - p^*| \\ &= |x^{i*}|\tilde{p}'^*, \end{aligned}$$

since the first-order condition of the dominant firm (4) implies $c^i a^{*i} - p^* = x^{i*}\tilde{p}'^*$. Now, since $\tilde{p}'$ is constant (due to the assumption that abatement costs are quadratic which implies inverse demand is linear) marginal inefficiency in the presence of a single dominant firm is simply measured by $|x^{i*}|$, the firm's absolute magnitude of trade in the permit market, à la Hahn (1984).

In this section we want to compare the effect of contestability of permits, as represented by $\alpha > 0$, with the case where $\alpha = 0$. If permits are not contestable, the dominant firm will not engage in rent seeking. By contrast, when permits are contestable, the firm will engage in rent seeking (so long as the condition on the cost of rent seeking in Proposition 2 is satisfied, which we assume) and by doing so it will increase its initial allocation of permits as $\omega^{i\prime} > 0$. From Proposition 1, our analysis then allows us to conclude that the dominant firm's net trade in pollution permits will fall relative to the case with no contestability and consequently no rent seeking, as $\tilde{x}^{i\prime} < 0$.

Thus, if the dominant firm is a buyer of permits and it engages in rent seeking then it will acquire more permits from the initial allocation and will consequently buy less in the market, thereby reducing marginal inefficiency. By contrast, if the dominant firm is a seller

[8] Here and henceforth, with a slight abuse of notation, we denote equilibrium values of variables by $*$.

of permits then rent-seeking activity will mean increased sales of permits in the market, thereby increasing marginal inefficiency. This is summarized as follows:

**Proposition 3** *If the dominant firm is a net buyer of permits then marginal inefficiency decreases in the presence of active rent seeking, whereas if it is a net seller of permits marginal inefficiency increases as a result of active rent seeking.*

Thus in the presence of rent seeking, we find that the characteristics of the dominant firm—how its (marginal) abatement cost relates to the permit price—play a pivotal role in understanding the effect on marginal inefficiency. In particular, if the dominant firm is a monopsonist (i.e., its marginal abatement cost is relatively high), then marginal inefficiency improves, whereas when the firm is a monopolist (i.e., its marginal abatement cost is relatively low) marginal inefficiency worsens. It is interesting to contrast this finding with the early work of Hahn (1984), in which allocative inefficiency exists for both monopolist and monopsonist and the magnitude of exchange determines the severity of allocative inefficiency. In our approach, however, the presence of rent seeking—and the impact on initial endowments and respective permit exchange—dampens the marginal inefficiency effects of a monopsonist but worsens it for a monopolist. Intuitively, the source of this result is the ability of the firm's rent seeking to alter its initial endowment and its resultant trade in permits.

## 3 Competition in Rent Seeking and Permit Exchange

Up to this point we have assumed that there is a single dominant firm that engages in rent seeking to increase its initial permit allocation, and then engages in trade in pollution permits. We now want to consider the effects of competition between two dominant firms, accounting for both political distortions in rent seeking for permits—which will now account for the distribution of the spoils of rent seeking between firms—and market distortions in trading permits. To model this we maintain a fringe of $N$ firms that have weight $1-\gamma$ and whose initial allocation of permits is determined as before in Sect. 2. Denote two dominant firms by $i, j \in \{1, 2\}$. All cost of abatement functions satisfy the same assumptions as stated previously, and in addition we assume without loss of generality that $c^1 \geq c^2$, so firm 1 has a weakly higher marginal abatement cost than firm 2.

As previously detailed, in Stage 1 the dominant firms invest in rent-seeking effort in order to alter their initial permit allocation from the regulator. In Stage 2, the initial permit allocations become common knowledge and firms subsequently engage in permit exchange in the presence of a competitive fringe of small firms.

### 3.1 Stage 1: Rent Seeking Over Pollution Permits

The two dominant firms each engage in rent seeking, choosing rent-seeking efforts $k^i, k^j$. As previously noted, rent seeking increases the number of permits that are allocated, according to the regulator's response function $\Omega(K)$ where $K = k^i + k^j$ is the total rent-seeking effort. Additionally—due to the existence of competition—relative rent-seeking efforts now also determine the distribution of the contestable permits between these two dominant firms according to the rules of a rent-seeking sharing contest (Tullock 1980; Hillman and

Riley 1989; Konrad 2009; Dickson et al. 2018). As such, a typical dominant firm's initial allocation is given by

$$\omega^i(k^i, k^j) = \gamma\left[[1-\alpha]\frac{\Omega^0}{2} + \alpha\phi(k^i, k^j)\Omega(K)\right],$$

where

$$\phi(k^i, k^j) = \begin{cases} \frac{k^i}{k^i+k^j} & \text{if } K > 0, \\ \frac{1}{2} & \text{if } K = 0. \end{cases}$$

We suppose that $\Omega(0) = \Omega^0$, $\Omega' > 0$,[9] $\Omega'' \leq 0$, and also that the regulator's response function is inelastic: $K\Omega'/\Omega < 1$.[10] The total emissions cap is $\Omega^0 + \alpha\gamma[\Omega(K) - \Omega^0]$.

Note that we can write $k^j = K - k^i$ and therefore so long as $K > 0$,

$$\begin{aligned} \omega^i_{k^i} &= \frac{\alpha\gamma}{K}\left[\Omega - \frac{k^i}{K}[\Omega - K\Omega']\right] > 0 \text{ and} \\ \omega^i_{k^j} &= -\frac{\alpha\gamma}{K^2}k^i[\Omega - K\Omega'] < 0, \end{aligned} \tag{5}$$

where the second inequality follows from our assumption that the regulator's response function is inelastic.

With a single firm, it receives the entirety of the contestable permits; rent seeking can increase this number, but all these are received by the single dominant firm. By contrast, with firms in competition over contestable permits, rent-seeking efforts determine not only the number of permits to be allocated to the dominant firms, but also the distribution of permits between these firms.

### 3.2 Stage 2: Permit Exchange

Once initial permit allocations in the first stage have been determined they become common knowledge, and the firms then engage in the permit market within a competitive fringe model, just as in the single dominant firm case, but where the net demand for permits from the dominant firms is $X \equiv x^i + x^j$. Market clearing works in the same way as stated previously, and therefore the permit price will be given by $\tilde{p}(X) = c^f[e - \omega^f] + \frac{c^f}{N}X$.

9 We consider the case where $\Omega' = 0$ so the emissions cap is fixed in Sect. 5.1. For a discussion of endogenous rents within rent-seeking contests see Dickson et al. (2018).

10 An inelastic response from the regulator is intuitive as one would expect the responsiveness of the regulator to be constrained by external political forces that aim to contain aggregate emissions (e.g., environmental lobby groups). Our focus here is to consider the realistic scenario where rent seeking occurs over the level of aggregate permit supply while simultaneously resulting in competition over the distribution of permit endowments. In contrast, under an elastic regulator's response function, competition over the distribution of permits is reduced because there are positive externalities from a rival's rent seeking that increase the firm's permit endowment over-and-above any distributional concerns (i.e., firm $i$'s permit endowment will increase if the rival invests more in rent seeking). This resembles aspects of a public/club good game, where free riding may exist. This scenario is best captured in our previous framework in which a single dominant firm rent seeks to increase its permit endowment and consequently the aggregate permit supply. In such a case the two dominant firms act as one entity with either an internal agreement of endowment share and effort or there exists intra-group rent seeking that determines the outcome, as presented in MacKenzie and Ohndorf (2012).

Each dominant firm cares about their overall cost of emissions, which includes their abatement cost taking into account the effect of their actions on the permit price and the cost of rent seeking. For firm $i$ this cost takes the form:

$$C^i(e - [\omega^i(k^i, k^j) + x^i]) + x^i\tilde{p}(x^i + x^j) + v^i(k^i) \text{ for } i = 1, 2, i \neq j. \tag{6}$$

## 4 Equilibrium Analysis

We now start the equilibrium analysis of the game using backward induction. In particular, we first derive the Nash equilibrium of the permit market exchange given the choice of rent-seeking efforts from Stage 1, and then turn attention to equilibrium rent-seeking choices.

### 4.1 Stage 2: Permit Market Choices

In a Nash equilibrium of the Stage 2 game, firms can be seen as choosing their permit allocation to minimize the cost detailed in (6). The solution to this, which yields each firm's reaction function, is given by

$$\tilde{x}^i(x^j;\omega^i(k^i, k^j)) = \left\{x^i : -c^i[e - [\omega^i(k^i, k^j) + x^i]] + \tilde{p}(x^i + x^j) + x^i\tilde{p}'(x^i + x^j) = 0\right\}, \tag{7}$$

subject to the second-order condition being satisfied. Let us define $\tilde{l}^i(x^i, x^j;\omega^i(k^i, k^j))$ as the left-hand side of the first-order condition:

$$\tilde{l}^i(x^i, x^j;\omega^i(k^i, k^j)) \equiv -c^i[e - [\omega^i(k^i, k^j) + x^i]] + \tilde{p}(x^i + x^j) + x^i\tilde{p}'(x^i + x^j). \tag{8}$$

Before we engage in a discussion of the nature of the equilibrium, let us make the preliminary observations that

$$\tilde{l}^i_{x^i} = c^i + 2\tilde{p}' > 0, \text{ and}$$
$$\tilde{l}^i_{x^j} = \tilde{p}' > 0.$$

Noting $\tilde{l}^i_{x^i} > 0$ allows us to conclude that the second-order condition is indeed satisfied so the reaction function is identified by (7).

To understand how the dominant firms interact, we utilize the Implicit Function Theorem to deduce the slope of the reaction function:

$$\tilde{x}^i_{x^j} = -\frac{\tilde{l}^i_{x^j}}{\tilde{l}^i_{x^i}} = -\frac{\tilde{p}'}{c^i + 2\tilde{p}'} \in (-1/2, 0), \tag{9}$$

so this is a game of strategic substitutes with downward-sloping reaction functions. Note that when $x^i = 0$, $\tilde{l}^i(0, x^j;\omega^i(k^i, k^j)) = -c^i[e^i - \omega^i(k^i, k_j)] + \tilde{p}(x^j)$. Recalling that $\tilde{l}^i_{x^i} > 0$, if this is greater than zero then $\tilde{x}^i$ will be negative ($i$ will be a seller of permits) for this given $x^j$; while if it is less than zero then $\tilde{x}^i$ will be positive ($i$ will be a buyer of permits). In terms of firm $i$'s reaction function, the point at which $\tilde{l}^i(0, x^j;\omega^i(k^i, k^j)) = 0$ determines where firm $i$'s reaction function crosses the horizontal axis, i.e., the $x^j$ such that $c^i[e^i - \omega^i(k^i, k^j)] = \tilde{p}(x^j)$. Given its negative slope, for any $x^j$ smaller than this, firm $i$ will be a buyer of permits, while for any $x^j$ larger, firm $i$ will be a seller. Figure 1 illustrates one

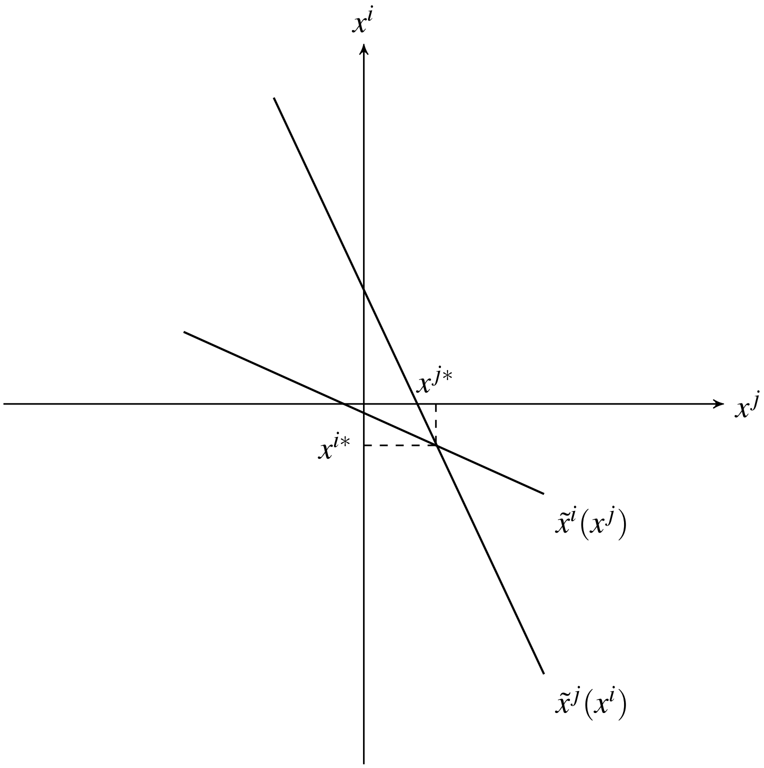


**Fig. 1** Reaction functions where, in equilibrium, firm $i$ is a permit seller and firm $j$ is a permit buyer

possible example where firm $i$ is a net permit seller in the equilibrium whereas firm $j$ is net buyer of permits.

We now derive the unique Nash equilibrium of permit exchange and provide comparative statics of how the equilibrium changes relative to endowments and rent-seeking effort chosen in Stage 1.

**Proposition 4** *Suppose* $\alpha > 0$, *then for any* $k^1, k^2 > 0$ *there exists a unique Nash equilibrium in the permit market that we denote by* $\{x^{1*}(k^1, k^2), x^{2*}(k^1, k^2)\}$ *with the property that* $\frac{dx^{i*}}{d\omega^i} < 0$ *and consequently*

$$x^{i*}_{k^i} < 0, x^{i*}_{k^j} > 0 \text{ for } i = 1, 2, i \neq j.$$

Proposition 4 shows that as a firm's permit endowment increases the purchase of permits decreases (or more permits are sold). As such, as a firm's rent seeking increases this—through a channel of obtaining more permits as $\omega^i_{k^i} > 0$—results in a decrease in permit purchases or increase in permits sold. By contrast, increased rent-seeking activity from a rival results in an increase in permit purchases or reduction in permits sold, as the rival's actions imply a reduction in the allocation of permits to the firm in question.

When a firm engages in more rent seeking in the first stage, which *ceteris paribus* yields more permits, there is an offsetting reduction in permit market transactions in the second stage. However, overall, an increase in rent seeking will result in an increase in the number of permits a firm holds in equilibrium. After some manipulation it can be shown that

$$\underbrace{\omega^i_{k^i}}_{>0} + \underbrace{x^{i*}_{k^i}}_{<0} = \frac{1}{[c^i + 2\tilde{p}'][c^j + 2\tilde{p}']}[4[\tilde{p}']^2\omega^i_{k^i} + c^j\tilde{p}'[2\omega^i_{k^i} + \omega^j_{k^i}]]. \tag{10}$$

From (5) we know that $\omega^i_{k^i} > 0$ and $\omega^j_{k^i} < 0$, but utilizing these expressions (noting that the indices in that for $\omega^i_{k^j}$ must be switched) we can deduce that

$$\begin{aligned}\omega^i_{k^i} + \omega^j_{k^i} &= \frac{\alpha\gamma}{K}\left[\Omega - \frac{k^i}{K}[\Omega - K\Omega'] - \left[1 - \frac{k^i}{K}\right][\Omega - K\Omega']\right],\\ &= \alpha\gamma\Omega' \geq 0,\end{aligned}$$

which allows us to conclude that the expression in (10) is strictly positive.

Note that in the permit-market equilibrium, we could either have both firms on one side of the market, or firms on different sides of the market. From the first-order condition presented in (7) observe that if $c^i a^i > \tilde{p}^*$ in equilibrium then it must be the case that $x^{i*} > 0$ and if $c^i a^i < \tilde{p}^*$ it must be the case that $x^{i*} < 0$. Thus, depending on the relationship between the equilibrium levels of $c^i a^i, c^j a^j$, and $\tilde{p}^*$, we could have both firms on the same side of the market (if both firms' equilibrium marginal abatement costs are larger, or smaller, than the equilibrium permit price), or firms on opposite sides of the market, in which case our assumption that firm 1 always has a larger marginal abatement cost than firm 2 will imply it is firm 1 that will be the buyer of permits while firm 2 will be the seller of permits.

### 4.2 Stage 1: Rent-Seeking Choices

In this subsection, we analyze the equilibrium rent-seeking choices of the dominant firms, accounting for the second-stage permit-market equilibrium. Firms can be seen as choosing their rent-seeking effort to minimize the overall cost of emissions that include abatement costs, the net cost of permit purchases, and rent-seeking costs. Firm $i$'s problem is therefore to

$$\min_{k^i \geq 0} C^i(e - [\omega^i(k^i, k^j) + x^{i*}(k^i, k^j)]) + x^{i*}(k^i, k^j)\tilde{p}(x^{i*}(k^i, k^j) + x^{j*}(k^i, k^j)) + v^i(k^i).$$

For ease of notation, define the cost of emissions as

$$G^i(k^i, k^j) = C^i(e - [\omega^i(k^i, k^j) + x^{i*}(k^i, k^j)]) + x^{i*}(k^i, k^j)\tilde{p}(x^{i*}(k^i, k^j) + x^{j*}(k^i, k^j)).$$

As such, firm $i$'s reaction function will take the form

$$\hat{k}^i(k^j) = \{k^i : l^i(k^i, k^j) \equiv G^i_{k^i}(k^i, k^j) + h^i = 0\}.$$

The effect of increasing rent-seeking effort on the cost of emissions is given by

$$G^i_{k^i} = \frac{\partial G^i}{\partial k^i} + \frac{\partial G^i}{\partial x^{i*}}x^{i*}_{k^i} + \frac{\partial G^i}{\partial x^{j*}}x^{j*}_{k^i}.$$

From the first-order optimality condition in Stage 2 (see (7)), $\frac{\partial G^i}{\partial x^{i*}} = 0$. To proceed we focus attention on settings where the indirect second-stage strategic effect is small enough to be considered negligible, so $x^{j*}_{k^i} \approx 0$. Noting that in this case $\frac{\partial G^i}{\partial k^i} = -c^i[e^i - [\omega^i(k^i, k^j) + x^{i*}(k^i, k^j)]]\omega^i_{k^i}$, the expression for firm $i$'s reaction function is

$$\hat{k}^i(k^j) = \left\{k^i : l^i(k^i, k^j) \equiv -c^i[e^i - [\omega^i(k^i, k^j) + x^{i*}(k^i, k^j)]]\omega^i_{k^i} + h^i = 0\right\}. \quad (11)$$

Rather than work with reaction functions—whose properties are difficult to determine—we instead take an aggregative games approach.[11] This works by first considering individual behavior that is consistent with an equilibrium in which the aggregate rent-seeking effort, $K \equiv k^i + k^j$, takes a particular value (individual consistency), and then identifies an equilibrium by ensuring the aggregate rent-seeking effort is such that the sum of the individual rent-seeking efforts consistent with this aggregate effort is exactly equal to the aggregate effort (aggregate consistency). The approach is detailed in the proof of Proposition 5 within the "Appendix". We work with expressions that consider an individual's share of the aggregate rent-seeking effort, $\sigma^i \equiv k^i/K$, consistent with equilibrium (as opposed to the level) because identifying the equilibrium becomes straightforward as aggregate consistency requires individuals to have shares of the aggregate effort that sum to 1. As such, the properties of the aggregation of share functions are instructive as to the existence and uniqueness of equilibrium.

The following proposition utilizes expression (11) to demonstrate that there is a unique equilibrium in rent-seeking choices.

**Proposition 5** *For any $\alpha > 0$, there is a unique equilibrium with first-stage rent-seeking efforts $k^{1*}, k^{2*}$.*

To proceed with the analysis, let us impose some structure on the nature of firms, which will suppose they are asymmetric in abatement costs (the symmetric case, as we discuss in Proposition 7, is a logical extension). In particular, suppose that the marginal cost of rent seeking ($h^i$) is the same for each firm (as is standard within the rent-seeking literature,[12]) and that firm 1—who has the higher marginal abatement cost for a given level of abatement ($c^1 > c^2$)—always values the outcome of rent seeking more than firm 2 (as we would intuitively expect). This, along with an additional technical assumption, is as follows[13]:

**Assumption 1** The marginal cost of rent seeking is the same for each firm: $v^{1\prime} = v^{2\prime}$. Moreover, (a) for combinations of $\sigma^1, \sigma^2$ and $K$ where the first-order conditions are satisfied the equilibrium marginal abatement cost of firm 1 exceeds that of firm 2:

$$\begin{aligned} &c^1[e - [\omega^1(\sigma^1 K, [1-\sigma^1]K) + x^{1*}(\sigma^1 K, [1-\sigma^1]K)]] \\ &\quad > c^2[e - [\omega^2(\sigma^2 K, [1-\sigma^2]K) + x^{2*}(\sigma^2 K, [1-\sigma^2]K)]]; \end{aligned}$$

and (b) $\Omega'(K)$ is small enough such that

[11] See the Introduction of Dickson (2017) for a concise survey of ideas, and Cornes and Hartley (2007) and Buchholz et al. (2011) for applications related to public goods.

[12] Identical marginal costs of rent seeking can be interpreted as each firm expending a monetary cost to influence the initial endowment of pollution permits (Congleton et al. 2008). Note that firms continue to have heterogeneous values of the outcome of the rent-seeking process.

[13] If Assumption 1 is not satisfied we could end up with a situation where the firm with the higher marginal abatement cost (firm 1) engages in less rent seeking, which appears unrealistic.

$$c^1[e-[\omega^1(K,0)+x^{1*}(K,0)]]\Omega'(K) < c^2[e-[\omega^2(0,K)+x^{2*}(0,K)]]\frac{\Omega(K)}{K}.$$

With this structure on the nature of firms, we can then derive the composition of rent seeking between the dominant firms.

**Proposition 6** *In the unique equilibrium*, $k^{1*} > k^{2*} > 0$.

Proposition 6 is consistent with the rent-seeking literature in that if firm 1 values the outcome of rent seeking more then a higher level of rent-seeking effort is observed in equilibrium. Having established the composition of rent-seeking effort between the dominant firms, we now turn to investigate how the degree of permit contestability influences inefficiency in the permit market.

## 5 The Effect of Permit Contestability on Marginal Inefficiency

We now consider how permit contestability affects marginal inefficiency when there is competition in rent seeking and permit exchange. Again, we want to define marginal inefficiency to capture the maximum cost saving that can be achieved in equilibrium by re-allocating a single permit from a firm with the lowest marginal abatement cost to a firm with the highest marginal abatement cost, which is therefore given by

$$MI^C = \max\{c^1a^{1*}, c^2a^{2*}, p^*\} - \min\{c^1a^{1*}, c^2a^{2*}, p^*\} \tag{12}$$

since, for the fringe firms, marginal abatement cost is always equal to the equilibrium price. Again, if more than one firm satisfies the criteria of having the highest or lowest marginal abatement cost, one is chosen to receive or give a permit. If the dominant firms are symmetric and $c^1a^{1*} = c^2a^{2*} = c^ia^{i*}$, say, then this definition becomes equivalent to that used in the single firm case, and so marginal inefficiency is given by $|x^{i*}|\tilde{p}'^*$. This allows us to conclude the following.

**Proposition 7** *Suppose the dominant firms are symmetric. If the firms are net buyers of permits then marginal inefficiency decreases in the presence of active rent seeking, whereas if they are net sellers of permits marginal inefficiency increases as a result of active rent seeking.*

We now turn to the case where the dominant firms are asymmetric. The nature of the definition of marginal inefficiency in (12) will depend on the composition of the market. Noting that part (a) of Assumption 1 implies $c^1a^{1*} > c^2a^{2*}$, we have three cases to consider: (1) firm 1 is a buyer of permits and firm 2 a seller,[14] in which case $c^1a^{1*} > p^* > c^2a^{2*}$; (2) both firms are buyers of permits, in which case $c^1a^{1*} > c^2a^{2*} > p^*$; and (3) both firms are sellers of permits, in which case $p^* > c^1a^{1*} > c^2a^{2*}$. Consequently, our measure of marginal inefficiency depends on the case under consideration. Using the first-order condition

[14] The reverse cannot be true under Assumption 1. Suppose, by contrast, that $x^{1*} < 0 < x^{2*}$. Then $\tilde{p}^* + x^{1*}\tilde{p}^{*\prime} < \tilde{p}^* + x^{2*}\tilde{p}^{*\prime}$ but then the stage 2 first-order conditions imply $c^1[e^1 - [\omega^{1*} + x^{1*}]] < c^2[e^2 - [\omega^{2*} + x^{2*}]]$, but this is in direct contradiction to Assumption 1.

(7) and noting that in case 1 $c^1a^{1*} - c^2a^{2*} = c^1a^{1*} - p^* + |c^2a^{2*} - p^*|$, marginal inefficiency is given by

$$MI^C = \begin{cases} [x^{1*} + |x^{2*}|]p'^* & \text{in case 1,} \\ x^{1*}p'^* & \text{in case 2,} \\ |x^{2*}|p'^* & \text{in case 3.} \end{cases} \tag{13}$$

As documented in Proposition 6, $k^{1*} > k^{2*}$ and therefore when firms engage in rent seeking we have that

$$\omega^1(k^{1*}, k^{2*}) = \gamma\left[[1-\alpha]\frac{\Omega^0}{2} + \alpha\frac{k^{1*}}{k^{1*} + k^{2*}}\Omega(K)\right] > \gamma\frac{\Omega^0}{2},$$
$$\omega^2(k^{1*}, k^{2*}) = \gamma\left[[1-\alpha]\frac{\Omega^0}{2} + \alpha\frac{k^{2*}}{k^{1*} + k^{2*}}\Omega(K)\right] \gtreqless \gamma\frac{\Omega^0}{2}.$$

Thus, relative to the case of no rent seeking firm 1's initial allocation will always increase. Depending on how responsive the regulator is to rent seeking firm 2's initial allocation following rent seeking may increase or decrease. To see this, note that firm 2's initial allocation is smaller (larger) than without rent seeking when $2\frac{k^{2*}}{k^{1*}+k^{2*}}\frac{\Omega(K)}{\Omega^0} < (>)1$. When the initial allocation is smaller than the no rent-seeking case, this means that the increase in the cap $\frac{\Omega(K)}{\Omega^0} > 1$ is counteracted by a larger reduction in firm 2's distributional component $2\frac{k^{2*}}{k^{1*}+k^{2*}} < 1$. Analogously, firm 2's initial allocation could be larger with rent seeking if the regulator's responsiveness is sufficiently high. In this case the effect from the regulator's increase in the overall cap offsets any losses firm 2's obtains from the distributional contest over permits.

Let us denote $\Delta\omega^i \equiv \omega^i(k^{i*}, k^{j*}) - \gamma\frac{\Omega^0}{2}$ as the change in initial allocation in the case of rent seeking versus no rent seeking for firm $i = 1, 2$. Thus we denote a regulator as 'not very responsive' to rent seeking if $\Delta\omega^1 > 0$ and $\Delta\omega^2 < 0$ and as 'very responsive' if $\Delta\omega^1 > 0$ and $\Delta\omega^2 > 0$.

The effect of rent-seeking on marginal inefficiency depends both on the composition of the market, and on the responsiveness of the regulator, as we document in the following proposition.

**Proposition 8** *Marginal inefficiency is lower in the presence of active rent seeking if both firms are buyers of permits on the market. If both firms are sellers, it is lower if the regulator is not very responsive to rent seeking, but higher if the regulator is very responsive to rent seeking. If firm* 1 *is a buyer and firm* 2 *is a seller then marginal inefficiency decreases so long as the regulator is not very responsive to rent seeking* (*i.e.*, $\Delta\omega^1 > 0 > \Delta\omega^2$), *but it may increase if the regulator is very responsive* (*i.e.*, $\Delta\omega^1, \Delta\omega^2 > 0$).

Proposition 8 shows that when a regulator is very responsive to rent seeking how marginal inefficiency changes under the presence of active rent seeking is similar to what is found in the single dominant firm case. It is clear that, in the presence of active rent seeking, marginal inefficiency is reduced if all dominant firms are buyers but is increased if all dominant firms are sellers. Intuitively, if dominant firms are permit buyers, the existence of rent seeking will result in a reduction in permit demand and, analogously to Hahn (1984), marginal inefficiency is reduced. Equally, if both firms are net sellers, then under the existence of rent seeking, they obtain more permits and increase their sale of permits,

**Table 1** Changes in marginal inefficiency due to rent seeking

| Regulator's response | Market Composition | | |
|---|---|---|---|
| | (1) Buyer and seller | (2) Both buyers | (3) Both sellers |
| Low | Decrease | Decrease | Decrease |
| High | Ambiguous | Decrease | Increase |

thereby increasing marginal inefficiency. An important distinction, however, is when the market composition consists of both a dominant buyer and seller, where these two effects are now competing against each other. This can be summarized in Table 1. Proposition 8 and Table 1 also show that if the regulator has a relatively low responsiveness towards rent seeking then marginal inefficiency is reduced for all possible market compositions.

For policymakers, then, it is important to realize that in environments where firms have market power, the effect of entertaining the possibility of rent seeking on the degree of cost effectiveness in pollution control may rely on the market composition of dominant players. With a very responsive regulator, allowing rent seeking in the presence of dominant net permit sellers may exacerbate inefficiencies, while if the dominant firms have more diverse positions, are net buyers of permits, or the regulator has a low response to rent seeking, allowing for rent seeking can be an efficiency-enhancing policy.

In our analysis we have abstracted from the social cost of rent seeking. To include this, note that social welfare will be comprised of, among other things, the net social benefits (damages) of pollution abatement (emissions), the aggregate costs of emissions compliance, and the social cost of rent seeking. While rent-seeking activity can reduce the aggregate cost of pollution control, a key determinant to understand the consequences for social welfare is the social planner's functional form for social welfare along with any weights the planner has on pollution costs, rent-seeking costs, and net damages associated with emissions. Clearly, more weight on pollution abatement costs (and less on rent seeking and net damages) may result in improvements in social welfare and vice versa.

### 5.1 Special Case: A Fixed Aggregate Emissions Cap

Throughout this article we have allowed the dominant firm(s) to not only affect the distribution of initial permit endowments but also influence the aggregate permit supply. Although allowing firms to influence the aggregate supply of permit is intuitively appealing, it is not necessarily always appropriate when one considers how environmental policy is politically and legislatively determined.

Often the legislative approach of environmental policy may involve bifurcation—either through time or government entity—that separates aspects of the proposed policy into separate components. For example, it may occur that the legislature debates and decides the aggregate level of emissions and then after this stage is complete the regulator—in charge of actual implementation of the policy—may be responsible for the distributional aspects of permit allocation. Thus it may be feasible that firms—at some stage in the policy—can influence the distribution of initial endowments but not their aggregate availability. Even if bifurcation does not occur across government entities, it can exist over the timeline of the policy. For instance the legislature may first debate and conclude on the policy level before attempting to pass the distributional aspects of a bill.

A clear example of this form of bifurication resulting in a fixed aggregate policy level was under the 1990 Title IV of the Clean Air Act Amendments (CAAA) in the 101st US Congress (S.1630) (Joskow and Schmalensee 1998; MacKenzie and Ohndorf 2012).[15] This bill focused on the control of acid deposition (i.e., acid rain). Within this process each installation was initially allocated a fixed amount of permits, where the aggregate target was fixed early within the legislative proceedings. Due to the usual rent seeking over the course of the bill around 30 provisions (detailed under Section 405 of the bill) had resulted in an increase in the aggregate level of emissions. Yet the bill (Section 403 (a)) provides a 'ratchet' provision whereby the aggregate target would reduce to the original target level and all installations would have their permit allocation reduced *pro rata*. Thus rent seeking was entirely distributional.

In our analysis the bifurication effects would mean that the aggregate emissions target $\Omega(K)$ would be fixed at $\Omega^0$. In such a case firm $i$'s initial permit endowment is

$$\omega^i(k^i, k^j) = \gamma\Omega^0\left[\frac{[1-\alpha]}{2} + \alpha\phi(k^i, k^j)\right].$$

Note that if dominant firms are symmetric then marginal inefficiency is *independent* of permit contestability. To see this note that if $c^i = c^j$ then $k^{i*} = k^{j*}$ and consequently $\phi(k^i, k^j) = \frac{1}{2}$. Thus each firm's initial endowment is $\frac{\gamma\Omega^0}{2}$, which is independent of $\alpha$. As such, the equilibrium in the permit market $\{x^{i*}, x^{j*}\}$ doesn't depend on $\alpha$, so inefficiency doesn't depend on $\alpha$. If $\alpha$ increases, which increases the contestability of permits, the equilibrium still awards each firm with 1/2 of $\alpha\Omega^0$, hence each individual $\omega^i$ remains constant. This can be contrasted with the previous findings with symmetric firms, where marginal inefficiency is dependent on the dominant firms; namely, marginal inefficiency decreases if dominant firms are buyers but increases if they are sellers.

When firms are asymmetric and the aggregate emissions cap is fixed, how marginal inefficiency is impacted by rent seeking can be directly observed in the findings of Proposition 8. Namely, when the aggregate emissions cap is fixed, then this is a special case of the regulator being 'not very responsive'. Indeed it is the most extreme case of being 'not responsive', where rent seeking only influences the distribution of permits among firms but not the size of aggregate permit supply. Thus, in this special case of a fixed aggregate emissions cap, Proposition 8 finds that marginal inefficiency reduces for all market compositions.

## 6 The Inclusion of Auctioned Permits

Up to this point we have assumed the initial allocation process has been a free allocation system in which permits are both contestable and non-contestable, the proportion of which was determined by an exogenously given $\alpha$. In this section we consider an intuitive extension of this framework to account for permits being in part auctioned, with the remainder being freely allocated. We consider that auctioned permits are non-contestable (but purchased according to the rules of the auction), so in the context of our model $\alpha$ captures the proportion of permits that are *not* auctioned but are freely allocated (and contestable).

[15] Another example is the UK Phase I National Allocation Plan in the EU-ETS (Rode 2021).

The majority of permit markets take a hybrid approach to initial allocation, consistent with this approach. For example, the Regional Greenhouse Gas Initiative (RGGI) initially allocates 94% of the permits via auction with the remainder freely allocated. In the European Union Emissions Trading Scheme (EU-ETS) the manufacturing industry receives approximately 30% of permits for free whereas in the aviation sector 85% of permits are freely allocated. In the California Cap-and-Trade Program 46% of permits are auctioned, 50% freely allocated, with the remaining 4% being used for the Allowance Price Containment Reserve.[16]

We now demonstrate that the model we have presented is consistent with this approach. First, let us consider the two extreme regimes of full (contestable) free allocation (i.e., $\alpha = 1$); and full auctioning (i.e. $\alpha = 0$). We write the equilibrium total cost of acquiring permits for firm $i$ with free allocation as

$$TC^{i,G}(k^*;\alpha = 1) \equiv C^i(e - [\omega^i(k^{i*}) + \tilde{x}^i(k^{i*})]) + \tilde{x}^i(k^{i*})\tilde{p}(\tilde{x}^i(k^{i*})) + v^i(k^{i*}),$$

while their total cost when all permits are auctioned (noting that they will not engage in rent-seeking) is

$$TC^{i,A}(k^* = 0;\alpha = 0) \equiv C^i(e - [\omega^i(0) + \tilde{x}^i(0)]) + \tilde{x}^i(0)\tilde{p}(\tilde{x}^i(0)) + \gamma\beta^i\Omega^0 p^A,$$

where $p^A$ is the equilibrium auction price, and $\beta^i$ is the share of auctioned permits that firm $i$ wins.

If $TC^{i,G}(k^*;\alpha = 1) > TC^{i,A}(k^* = 0;\alpha = 0)$ firm $i$ prefers that permits are auctioned to freely allocated, whereas if the reverse inequality holds they prefer free allocation to auctioning.[17] Now let us define

$$V^i \equiv TC^{i,G}(k^*;\alpha = 1) - TC^{i,A}(k^* = 0;\alpha = 0),$$

which is the net benefit of auctioning over contested free allocation.

Now consider a situation where firms engage in lobbying over the value of $\alpha$. Suppose that each firm has a strict preference for one scheme or the other: one firm for auctioning ($V^i > 0$) and one for free allocation ($V^i < 0$).[18] This will be determined by the structure of each firm's abatement and rent-seeking costs.[19] Given the values of the two extreme schemes $\{V^i, V^j\}$ it is well known that a unique Nash equilibrium in lobbying efforts will exist (Nti 2004; Dickson et al. 2018), where the equilibrium proportion of free versus auction permits is $\alpha^* \in [0, 1]$.

With this proportion of free (contested) allocation and the implied proportion of auctioned permits, each firm's equilibrium payoff will given by

$$\Pi^G(k^*;\alpha^*) \equiv C^i(e - [\omega^i(k^{i*}) + \tilde{x}^i(k^{i*})]) + \tilde{x}^i(k^{i*})\tilde{p}(\tilde{x}^i(k^{i*})) + \gamma\beta^i[1 - \alpha^*]\Omega^0 p^A + v^i(k^{i*}).$$

Note the main change in the total cost function is a lump-sum term $\gamma\beta^i[1 - \alpha^*]\Omega^0 p^A$ that reflects the payment for auctioned permits. Note that this the lump-sum payment does not

[16] For an overview of pollution auction design see MacKenzie (2022).

[17] Note that a firm has an incentive to invest in rent-seeking effort in the free allocation game, as the payoff is higher than with no rent seeking.

[18] We also suppose, for the sake of this discussion, that firms' costs are monotonic in $\alpha$ so each firm prefers full auctioning or full free allocation to a mixture.

[19] If firms have aligned preferences, then no lobbying would take place.

impact the equilibrium rent-seeking effort $k^{i*}$ nor the equilibrium demand/supply of permits $\tilde{x}^i(k^{i*})$ and consequently marginal inefficiency is not impacted.

This framework has many possible fruitful research directions. For example, we suppose firms have a strict preference over free allocation versus auctioned permits as their costs are monotonic in $\alpha$; an interesting direction would be to consider other cases where firms have a preference for a mixed allocation system. Furthermore, we abstract from how the auction price $p^A$ is determined. One possible alternative would be to link it to the secondary market price $\tilde{p}(\tilde{x}^i(k^i))$. By following the recent literature on pollution auctions, future research directions could focus on the dynamic determination of the auction price and the optimal bidding behavior of firms in a market that exhibits market power and rent seeking (Khezr and MacKenzie 2018a, b, 2021).

## 7 Concluding Remarks

The purpose of this article is to investigate how the interaction of political and market distortions affect the efficiency of cap-and-trade markets. We develop a two-stage permit market model where dominant firms, first, have the ability to invest in rent-seeking effort to obtain an initial allocation of permits (as well as influencing the aggregate supply of permits) and then, second, choose their level of permit exchange. We derive the unique equilibrium of the game. We analyze the case of one dominant firm and then introduce competition in rent seeking and permit exchange.

In the permit market, for a given initial allocation of permits, a unique equilibrium exists where equilibrium permit holdings are decreasing (purchasing less; selling more) in the level of the initial allocation. Further, a dominant firm's equilibrium permit holdings increase in the firm's level of rent seeking. We show that if a dominant firm invests in rent seeking then (1) there is an increase in initial permit endowment towards that firm but (2) there is also an offsetting decrease (increase) in permit demand (supply). We show the increase in initial endowment offsets any change in demand/supply: rent seeking will therefore result in less individual pollution abatement. In the rent-seeking stage, we derive the unique equilibrium accounting for the second-stage permit market exchange.

Our main focus is whether permit contestability—how vulnerable the initial allocation of permits and the aggregate supply of permits is to rent seeking—affects the marginal inefficiency of the market (i.e., the maximum cost saving that can be achieved by re-allocating one permit between firms). In a market with pre-existing market-power issues, we compare how marginal inefficiency is altered when permits become contestable. We show how the cost effectiveness of the market is altered depends on the permit exchange activity of the dominant firms as well as the regulator's responsiveness towards rent seeking. In particular, when the regulator is very responsive, we show that if dominant firms are permit buyers in equilibrium then marginal inefficiency actually improves, whereas if all dominant firms are sellers then marginal inefficiency worsens. If the regulator is not very responsive to rent seeking we show that marginal inefficiency improves irrespective of market composition.

## Appendix

***Proof of Proposition 1*** Defining

$$\tilde{l}^i(x^i, \omega^i(k^i)) \equiv -c^i[e - [\omega^i(k^i) + x^i]] + \tilde{p}(x^i) + x^i\tilde{p}'(x^i), \tag{14}$$

we can deduce that[20]

$$\tilde{l}^i_{x^i} = c^i + 2\tilde{p}' > 0 \text{ and}$$
$$\tilde{l}^i_{\omega^i} = c^i.$$

From this last expression we can deduce that $\frac{\partial \tilde{l}^i}{\partial k^i} \equiv \tilde{l}^i_{\omega^i}\omega^{i\prime} = c^i\alpha\gamma\Omega' > 0$. Since $\tilde{l}^i_{x^i} > 0$ the objective function is strictly convex and will therefore obtain a unique minimum. By the Implicit Function Theorem,

$$\tilde{x}^{i\prime} = -\frac{\frac{\partial \tilde{l}^i}{\partial k^i}}{\tilde{l}^i_{x^i}} < 0,$$

hence as the dominant firm increases rent seeking it reduces its permit purchases if it is a buyer ($x^i > 0$), or increases its sales if it is a seller ($x^i < 0$). Note that we can write

$$\frac{d}{dk^i}\{\omega^i(k^i) + \tilde{x}^i(k^i)\} = \omega^{i\prime}\left[1 + \frac{d\tilde{x}^i}{d\omega^i}\right].$$

By the Implicit Function Theorem

$$\frac{d\tilde{x}^i}{d\omega^i} = -\frac{\tilde{l}^i_{\omega^i}}{\tilde{l}^i_{x^i}} = -\frac{c^i}{c^i + 2\tilde{p}'} \in (-1, 0)$$

so the expression in the square parenthesis is positive, allowing us to conclude that overall permit holdings $\omega^i(k^i) + \tilde{x}^i(k^i)$ are increasing in rent-seeking activity. □

***Proof of Proposition 2*** It is clear that

$$l^{i\prime} = c^i[\omega^{i\prime} + \tilde{x}^{i\prime}]\alpha\gamma\Omega' - C^{i\prime}\alpha\gamma\Omega'' > 0$$

because $\Omega'' \leq 0$, and we demonstrated in Proposition 1 that $\omega^{i\prime} + \tilde{x}^{i\prime} > 0$. As such, the first-order condition is both necessary and sufficient in identifying the unique cost-minimizing level of rent-seeking effort. If the condition in the statement of the proposition is not true then $l^i(0) > 0$ and since $l^i$ is increasing in $k^i$ there is no $k^i > 0$ where $l^i(k^i) = 0$ and consequently $k^{i*} = 0$. Conversely, when the condition is satisfied $l^i(0) < 0$ and so, noting that $\lim_{k^i \to \infty} l^i(k^i) > 0$ (as $\lim_{k^i \to \infty} \Omega'(k^i) = 0$), we can deduce from the Intermediate Value Theorem that there will be a strictly positive $k^i$ whereby $l^i(k^i) = 0$. □

***Proof of Proposition 4*** Since reaction functions have a slope whose absolute value is less than 1 they will intersect once and only once, which identifies the unique Nash equilibrium that we denote by $\{x^{1*}(k^1, k^2), x^{2*}(k^1, k^2)\}$: for any $k^i, k^j$ there is a unique Nash equilibrium in the second-stage subgame. Applying the Implicit Function Theorem to the first-order

[20] By convention, for a function of many variables we use subscripts to denote the derivative with respect to the variable highlighted; for functions of single variables we use ′s to indicate derivatives.

condition (7) allows us to understand the effect of a change in the permit endowment on the reaction function:

$$\tilde{x}^i_{\omega^i} = -\frac{\tilde{I}^i_{\omega^i}}{\tilde{I}^i_{x^i}} = -\frac{c^i}{c^i + 2\tilde{p}'} < 0. \tag{15}$$

So if firm $i$ is a buyer of permits it buys fewer with a larger endowment, and if it is a seller a larger endowment means it will sell more. Considering how equilibrium permit transactions change, we note that this can be decomposed into the direct and strategic effects:

$$\begin{aligned}\frac{dx^{i*}}{d\omega^i} &= \tilde{x}^i_{\omega^i} + \tilde{x}^i_{x^j}\tilde{x}^j_{x^i}\tilde{x}^i_{\omega^i}\\ &= \tilde{x}^i_{\omega^i}[1 + \tilde{x}^i_{x^j}\tilde{x}^j_{x^i}].\end{aligned}$$

Since this is a game of strategic substitutes (as we showed in (9)), $\tilde{x}^i_{x^j}\tilde{x}^j_{x^i} > 0$, which allows us to conclude that $\frac{dx^{i*}}{d\omega^i} < 0$.

Turning now to understand the effect of a change in $k^i$ on the permit-market equilibrium, we know that

$$x^{i*}_{k^i} = \tilde{x}^i_{\omega^i}\omega^i_{k^i} + \tilde{x}^i_{x^j}\tilde{x}^j_{\omega^j}\omega^j_{k^i}.$$

Utilizing the expressions for $\tilde{x}^i_{x^j}$ in (9) and $\tilde{x}^i_{\omega^i}$ in (15) above allows us to deduce that

$$x^{i*}_{k^i} = -\frac{1}{c^i + 2\tilde{p}'}\left[c^i\omega^i_{k^i} - \frac{c^j\tilde{p}'}{c^j + 2\tilde{p}'}\omega^j_{k^i}\right] < 0 \tag{16}$$

using the inequalities in (5).

Similarly, for firm $j$,

$$x^{j*}_{k^i} = \tilde{x}^j_{\omega^j}\omega^j_{k^i} + \tilde{x}^j_{x^i}\tilde{x}^i_{\omega^i}\omega^i_{k^i}.$$

This is given by

$$x^{j*}_{k^i} = \frac{1}{c^j + 2\tilde{p}'}\left[\frac{c^i\tilde{p}'}{c^i + 2\tilde{p}'}\omega^i_{k^i} - c^j\omega^j_{k^i}\right] > 0, \tag{17}$$

where the inequality is again implied using (5). □

***Proof of Proposition 5*** Recall from (11) the expression for firm $i$'s reaction function, which is derived by assuming that the indirect second-stage strategic effect $x^{j*}_{k^i} \approx 0$ so is negligible.[21] First, we replace $k^j$ with $K - k^i$ in this expression, where $K \equiv k^i + k^j$ is the aggregate rent-seeking effort. This would yield firm $i$'s rent-seeking effort consistent with an equilibrium in which the aggregate rent-seeking effort is $K$. But rather than working with levels, we want to work with shares of the aggregate effort, and so we take an additional step

[21] This is necessary for tractability of the problem. While firms undertake backward induction to understand the Nash equilibrium that emerges from any combination of rent-seeking efforts, when a firm considers marginally changing its own effort it accounts for the direct effect of this on its own permit market activity, but perceives the indirect effect on the permit market activity of the other firm is negligible.

by replacing $k^i$ with $\sigma^i K$ where $\sigma^i \equiv k^i/K$ represents firm $i$'s share of the aggregate rent-seeking effort. Firm $i$'s *share function* consequently takes the form:

$$
\begin{aligned}
s^i(K) = \big\{\sigma^i \,:\, l^i(\sigma^i K, [1-\sigma^i]K) \equiv -c^i[e^i - [\omega^i(\sigma^i K, [1-\sigma^i]K) \\
+x^{i*}(\sigma^i K, [1-\sigma^i]K)]]\omega^i_{k^i} + h^i = 0\big\}.
\end{aligned}
\tag{18}
$$

We want to develop an understanding of the properties of these share functions. To begin, we show the following.

**Lemma 1** $\frac{dl^i}{d\sigma^i} > 0$ *and* $\frac{dl^i}{dK} > 0$.

***Proof of Lemma 1*** Note that

$$
\begin{aligned}
\frac{dl^i}{d\sigma^i} &= \left[\frac{d\omega^i}{d\sigma^i} + \frac{dx^{i*}}{d\sigma^i}\right] c^i\omega^i_{k^i} - c^i[e - [\omega^i + x^{i*}]]\frac{d\omega^i_{k^i}}{d\sigma^i} \text{ and} \\
\frac{dl^i}{dK} &= \left[\frac{d\omega^i}{dK} + \frac{dx^{i*}}{dK}\right] c^i\omega^i_{k^i} - c^i[e - [\omega^i + x^{i*}]]\frac{d\omega^i_{k^i}}{dK}.
\end{aligned}
\tag{19}
$$

From (5), we can deduce that

$$
\begin{aligned}
\omega^i_{k^i} &= \frac{\alpha\gamma}{K}[\Omega - \sigma^i[\Omega - K\Omega']], \\
\omega^i_{k^j} &= -\frac{\alpha\gamma}{K}\sigma^i[\Omega - K\Omega'], \\
\omega^i_{k^j} - \omega^i_{k^i} = \omega^j_{k^i} - \omega^j_{k^j} &= -\frac{\alpha\gamma\Omega}{K}, \\
\frac{d\omega^i_{k^i}}{d\sigma^i} &= -\frac{\alpha\gamma}{K}[\Omega - K\Omega'] < 0, \text{ and} \\
\frac{d\omega^i_{k^i}}{dK} &= \alpha\gamma\left[[1-\sigma^i]\frac{K\Omega' - \Omega}{K^2} + \sigma^i\Omega''\right] < 0.
\end{aligned}
\tag{20}
$$

The two inequalities stated last imply that the second term in each of the expressions in (19) is positive, and therefore we can focus attention on the first terms, and in particular the terms in the square brackets since $\omega^i_{k^i} > 0$.

Now, since equilibrium permit market actions are written as $x^{i*}(\sigma^i K, [1-\sigma^i]K)$,

$$
\begin{aligned}
\frac{dx^{i*}}{d\sigma^i} &= K[x^{i*}_{k^i} - x^{i*}_{k^j}] \text{ and} \\
\frac{dx^{i*}}{dK} &= \sigma^i x^{i*}_{k^i} + [1-\sigma^i]x^{i*}_{k^j},
\end{aligned}
$$

recall from (16) and (17) (with the indices reversed) in the proof of Proposition 1 that

$$
\begin{aligned}
x^{i*}_{k^i} &= -\frac{1}{c^i + 2\tilde{p}'}\left[c^i\omega^i_{k^i} - \frac{c^j\tilde{p}'}{c^j + 2\tilde{p}'}\omega^j_{k^i}\right] \text{ and} \\
x^{i*}_{k^j} &= \frac{1}{c^i + 2\tilde{p}'}\left[\frac{c^j\tilde{p}'}{c^j + 2\tilde{p}'}\omega^j_{k^j} - c^i\omega^i_{k^j}\right].
\end{aligned}
$$

This allows us to deduce that

$$\begin{aligned}\frac{dx^{i*}}{d\sigma^i} &= \frac{K}{c^i + 2\tilde{p}'}\left[c^i[\omega^i_{k^j} - \omega^i_{k^i}] + \frac{c^j\tilde{p}'}{c^j + 2\tilde{p}'}[\omega^j_{k^i} - \omega^j_{k^j}]\right] \\ &= -\frac{\alpha\gamma\Omega}{c^i + 2\tilde{p}'}\left[c^i + \frac{c^j\tilde{p}'}{c^j + 2\tilde{p}'}\right]\end{aligned}$$

using the third line in (20). As such, since $\frac{d\omega^i}{d\sigma^i} = \alpha\gamma\Omega$, it follows after some manipulation that

$$\frac{d\omega^i}{d\sigma^i} + \frac{dx^{i*}}{d\sigma^i} = \frac{\alpha\gamma\Omega}{[c^i + 2\tilde{p}'][c^j + 2\tilde{p}']}[4[\tilde{p}']^2 + c^j\tilde{p}'] > 0,$$

allowing us to conclude that $\frac{dl^i}{d\sigma^i} > 0$.

Similarly,

$$\frac{dx^{i*}}{dK} = \frac{1}{c^i + 2\tilde{p}'}\left[-c^i[\sigma^i\omega^i_{k^i} + [1-\sigma^i]\omega^i_{k^j}] + \frac{c^j\tilde{p}'}{c^j + 2\tilde{p}'}[[1-\sigma^i]\omega^j_{k^j} + \sigma^i\omega^j_{k^i}]\right].$$

Now, using expressions in (20), we can show after some manipulation that

$$\begin{aligned}\sigma^i\omega^i_{k^i} + [1-\sigma^i]\omega^i_{k^j} &= \sigma^i\alpha\gamma\Omega' \text{ and} \\ [1-\sigma^i]\omega^j_{k^j} + \sigma^i\omega^j_{k^i} &= [1-\sigma^i]\alpha\gamma\Omega'\end{aligned}$$

so

$$\frac{dx^{i*}}{dK} = \frac{\alpha\gamma\Omega'}{c^i + 2\tilde{p}'}\left[-\sigma^i c^i + [1-\sigma^i]\frac{c^j\tilde{p}'}{c^j + 2\tilde{p}'}\right].$$

Since $\frac{d\omega^i}{dK} = \sigma^i\alpha\gamma\Omega'$, we can then deduce that

$$\begin{aligned}\frac{d\omega^i}{dK} + \frac{dx^{i*}}{dK} &= \sigma^i\alpha\gamma\Omega'\left[1 - \frac{c^i}{c^i + 2\tilde{p}'}\right] + [1-\sigma^i]\frac{\alpha\gamma\Omega' c^j\tilde{p}'}{[c^i + 2\tilde{p}'][c^j + 2\tilde{p}']} \\ &= \frac{2\sigma^i\alpha\gamma\Omega'\tilde{p}'}{c^i + 2\tilde{p}'} + [1-\sigma^i]\frac{\alpha\gamma\Omega' c^j\tilde{p}'}{[c^i + 2\tilde{p}'][c^j + 2\tilde{p}']} > 0\end{aligned}$$

allowing us to conclude that $\frac{dl^i}{dK} > 0$. □

Next we turn to consider where the share function is defined. Abusing notation, we write the left-hand-side of the first-order condition that defines share functions in (18) as $l^i(\sigma^i, K)$. For a given $K > 0$, we look for the value of $\sigma^i$ where $l^i$ (which we showed in Lemma 1 is an increasing function of both $\sigma^i$ and $K$) is equal to zero. Share functions are bound to lie between 0 and 1: they cannot be below zero as rent-seeking efforts must be non-negative; and they cannot exceed 1 by definition (individual effort cannot exceed aggregate effort).

If $K$ is such that $l^i(1, K) < 0$ then the fact that $l^i$ is increasing in $\sigma^i$ means $l^i(\sigma^i, K) < 0$ for all $\sigma \in [0, 1]$, so share functions are not defined (the suggestion in this case is that the share function exceeds one, which as noted is ruled out by definition). Let us define

$$\underline{K}^i \equiv \{K : l^i(1, K) = 0\}. \tag{21}$$

Then the fact that $l^i$ is increasing in $K$ means that $l^i(1, K) < 0$ (and therefore $l^i(\sigma^i, K) < 0$ for all $\sigma \in [0, 1]$) for all $K < \underline{K}^i$ so the share function is undefined. For $K = \underline{K}^i$, $s^i(\underline{K}^i) = 1$ since $l^i(1, \underline{K}^i) = 0$.

If $K$ is such that $l^i(0, K) > 0$ then the fact that $l^i$ is increasing in $\sigma^i$ means $l^i(\sigma^i, K) > 0$ for all $\sigma \in [0, 1]$ (the suggestion in this case is that the share function is negative, but this would require negative rent-seeking effort which is not allowed and in fact the firm would choose $k^i = 0$). Let us define

$$\bar{K}^i \equiv \{K : l^i(0, K) = 0\}. \tag{22}$$

Then the fact that $l^i$ is increasing in $K$ means that $l^i(0, K) > 0$ (and therefore $l^i(\sigma^i, K) > 0$ for all $\sigma \in [0, 1]$) for all $K > \bar{K}^i$. For $K = \bar{K}$, $l^i(0, \bar{K}) = 0$ and therefore $s^i(\bar{K}^i) = 0$, and we define $s^i(K) = 0$ for all $K > \bar{K}^i$.

Note that $\underline{K}^i < \bar{K}^i$, which can be proved by straightforward contradiction using the facts presented in Lemma 1: suppose $\underline{K}^i \geq \bar{K}^i$, then we would have

$$\begin{aligned} l^i(1, \underline{K}^i) &= l^i(0, \bar{K}^i) \\ &\leq l^i(0, \underline{K}^i) \\ &< l^i(1, \underline{K}^i) \end{aligned}$$

a contradiction, where the first line comes from the definitions of $\underline{K}^i$ and $\bar{K}^i$, the first inequality comes from the fact that $l^i$ is increasing in $K$ and the second inequality comes from the fact that $l^i$ is (strictly) increasing in $\sigma^i$.

When $K \in (\underline{K}^i, \bar{K}^i)$, $l^i(0, K) < 0$ and $l^i(1, K) > 0$ and so the fact that $l^i$ is strictly increasing in $\sigma^i$ implies there is a unique $\sigma^i \in (0, 1)$ where $l^i(\sigma^i, K) = 0$ by the Intermediate Value Theorem, so $s^i(K)$ is single-valued. By the Implicit Function Theorem

$$s^i_K = -\frac{\frac{dl^i}{dK}}{\frac{dl^i}{d\sigma^i}},$$

and so where they are defined and positive, share functions are strictly decreasing in $K$.

We now seek to identify a Nash equilibrium. This requires the sum of individual rent-seeking efforts to be equal to the aggregate rent-seeking effort (aggregate consistency), or, dividing both sides of this equation by the aggregate rent-seeking effort, for the sum of share functions to be equal to 1. We define this aggregate share function as $S(K) \equiv s^1(K) + s^2(K)$, which is defined for $K > \max\{\underline{K}^1, \underline{K}^2\}$ where both firms' share functions are defined.

When $K = \max\{\underline{K}^1, \underline{K}^2\}$ the aggregate share function $S(K)$ takes a value no smaller than 1, and when $K = \max\{\bar{K}^1, \bar{K}^2\}$ it takes the value zero. Since it inherits the property of individual share functions, which are strictly decreasing in $K$, it follows from the Intermediate Value Theorem that there is a unique $K^* \in [\max\{\underline{K}^1, \underline{K}^2\}, \max\{\bar{K}^1, \bar{K}^2\})$ where $S(K^*) = 1$, and therefore a unique Nash equilibrium in which the shares of the two players are $s^{1*} = s^1(K^*)$ and $s^{2*} = s^2(K^*)$ and their rent-seeking efforts are $k^{1*} = K^* s^{1*}$ and $k^{2*} = K^* s^{2*}$. If $K^* < \min\{\bar{K}^1, \bar{K}^2\}$ both firms will be active in the rent-seeking equilibrium.

***Proof of Proposition 6*** We again abuse notation by writing the left-hand-side of the first-order condition that defines share functions in (18) as $l^i(\sigma^i, K)$.

First, we show that for any $K$ where they are both defined, $s^1(K) > s^2(K)$, which we prove by contradiction. When $h^1 = h^2$ and part (a) of Assumption 1 is satisfied, $l^1(\sigma, K) < l^2(\sigma, K)$. It then follows that $0 \equiv l^1(s^1(K), K) < l^2(s^1(K), K)$, but then if $s^1(K) \leq s^2(K)$, the fact that $l^2$ is increasing in $\sigma^2$ (Lemma 1) implies $l^2(s^1(K), K) \leq l^2(s^2(K), K) \equiv 0$, giving rise to a contradiction.

Note that these same assumptions also imply that $\bar{K}^1 > \bar{K}^2$. Again this can be proved by contradiction. Referring back to the definition of $\bar{K}^i$ in (22) $0 \equiv l^1(0, \bar{K}^1) < l^2(0, \bar{K}^1)$ as the assumptions imply $l^1(\sigma, K) < l^2(\sigma, K)$. But then the fact that $l^2$ is increasing in $K$ (Lemma 1) implies that if $\bar{K}^1 \leq \bar{K}^2$ then $l^2(0, \bar{K}^1) \leq l^2(0, \bar{K}^2) \equiv 0$, a contradiction. We can similarly prove $\underline{K}^1 > \underline{K}^2$ (omitted).

Next we demonstrate that $\underline{K}^1 < \bar{K}^2$, again by contradiction. Recall the definitions of $\underline{K}^i$ in (21) and $\bar{K}^i$ in (22), and note that $h^1 = h^2$ and part (b) of Assumption 1 implies $l^1(1, K) > l^2(0, K)$. Then we have $0 \equiv l^1(1, \underline{K}^1) > l^2(0, \underline{K}^1)$. Then since $l^2$ is increasing in $K$ (Lemma 1), if $\bar{K}^2 \geq \underline{K}^1$ we also have $l^2(0, \underline{K}^1) \geq l^2(0, \bar{K}^2) \equiv 0$, leading to a contradiction.

These deductions allow us to conclude that the aggregate share function $S(K)$ is defined only for $K \geq \underline{K}^1$. Since $\bar{K}^2 > \underline{K}^1$, $S(\underline{K}^1) > 1$ and $S(\bar{K}^2) < 1$. As such, $K^* \in (\underline{K}^1, \bar{K}^2)$ at which $s^1(K^*) > s^2(K^*) > 0$, implying $k^{1*} = K^* s^1(K^*) > K^* s^2(K^*) = k^{2*} > 0$. Thus, both firms are active in the rent-seeking game, and firm 1's rent-seeking effort exceeds that of firm 2. □

***Proof of Proposition 7*** $c^i = c^j$ is ruled out by Assumption 1, but following through the logic of the identifying equilibrium using the share function approach implies that $k^{i*} = k^{j*}$ and consequently $\omega^i(k^{i*}, k^{j*}) = \gamma \left[ [1-\alpha]\frac{\Omega}{2} + \alpha \frac{\Omega(K)}{2} \right]$ for $i = 1, 2$. We want to compare the case of no contestability with the case where permits are contestable through rent seeking prior to permit exchange. With $\alpha = 0$, it follows that $\omega^i = \gamma \frac{\Omega^0}{2}$. With $\alpha > 0$, we need to consider the contest equilibrium. It is clear that

$$\omega^1(k^{1*}, k^{2*}) = \omega^2(k^{1*}, k^{2*}) = \gamma \left[ [1-\alpha]\frac{\Omega^0}{2} + \alpha \frac{\Omega(K)}{2} \right] > \gamma \frac{\Omega^0}{2}$$

Given our earlier results on how $x^{i*}$ changes with $\omega^i$ in Proposition 4, we can therefore directly conclude, abusing notation slightly, that:

$$x^{1*}\Big|_{\alpha>0} < x^{1*}\Big|_{\alpha=0}$$
$$x^{2*}\Big|_{\alpha>0} < x^{2*}\Big|_{\alpha=0}.$$

Thus, if the dominant firms are permit buyers with $x^{i*} > 0$ then when $\alpha > 0$ their permit purchases decrease which results in a decrease in marginal inefficiency. By contrast, if the dominant firms are permit sellers ($x^{i*} < 0$) then when $\alpha > 0$ the number of permits sold increases and therefore marginal inefficiency increases. □

***Proof of Proposition 8*** We want to compare the case when there is rent seeking ($\alpha > 0$) with the case of no rent seeking ($\alpha = 0$). We first make the preliminary observation that $\Delta\omega^1 = -\Delta\omega^2 + \alpha\gamma[\Omega(K) - \Omega^0]$, where $\Delta\omega^i \equiv \omega^i(k^{i*}, k^{j*}) - \gamma\frac{\Omega^0}{2}$ is the change in initial allocation in the case of rent seeking versus no rent seeking. To see this note that we can write

$$\begin{aligned}\Delta\omega^1 &= \gamma\left[-\alpha\frac{\Omega^0}{2} + \alpha\frac{k^{1*}}{k^{1*}+k^{2*}}\Omega(K)\right]\\ &= -\gamma\left[-\alpha\frac{\Omega^0}{2} + \alpha\frac{k^{2*}}{k^{1*}+k^{2*}}\Omega(K)\right] + \alpha\gamma[\Omega(K)-\Omega^0]\\ &= -\Delta\omega^2 + \alpha\gamma[\Omega(K)-\Omega^0]\end{aligned}$$

by adding and subtracting $\alpha\Omega^0$ and $\alpha\frac{k^{2*}}{k^{1*}+k^{2*}}\Omega(K)$.

Moreover, recall from Proposition 4 that

$$\frac{dx^{i*}}{d\omega^i} = \tilde{x}^i_{\omega^i}[1 + \tilde{x}^i_{x^j}\tilde{x}^j_{x^i}] < 0$$

where $\tilde{x}^i_{\omega^i} = -\frac{c^i}{c^i+2\tilde{p}'}$.

As noted, there are three cases of market composition to consider in which the measure of marginal inefficiency takes a different form, as detailed in (13). These are (1) where firm 1 is a buyer and firm 2 a seller of permits; (2) where both firms are permit buyers; and (3) where both firms sell permits.

Moreover, there are two cases of regulator responsiveness to consider. We begin by dealing with the case where the regulator is not very responsive to rent seeking so $\Delta\omega^1 > 0$ and $\Delta\omega^2 < 0$. Accordingly, in this case we will have:

$$\begin{aligned} x^{1*}\Big|_{\alpha>0} &< x^{1*}\Big|_{\alpha=0}\\ x^{2*}\Big|_{\alpha>0} &> x^{2*}\Big|_{\alpha=0}. \end{aligned}$$

Using (13), in case 1 $MI^C = [x^{1*} + |x^{2*}|]p'^*$. Since $x^{1*}$ reduces and $x^{2*}$ increases (becomes less negative), this measure declines. In case 2 $MI^C = x^{1*}p'^*$, which reduces since $x^{1*}$ reduces; and in case 3 $MI^C = |x^{2*}|p'^*$ which also reduces as $x^{2*}$ increases, becoming less negative. Consequently, if the regulator is not very responsive to rent seeking, marginal inefficiency improves in the presence of active rent seeking.

We now turn to the case where the regulator is very responsive to rent seeking and so $\Delta\omega^1 > 0$ and $\Delta\omega^2 > 0$. In this case we will have

$$\begin{aligned} x^{1*}\Big|_{\alpha>0} &< x^{1*}\Big|_{\alpha=0}\\ x^{2*}\Big|_{\alpha>0} &< x^{2*}\Big|_{\alpha=0}. \end{aligned}$$

In case 2 where both firms are buyers, $MI^C = x^{1*}p'^*$ and since $x^{1*}$ declines, marginal inefficiency declines.

In case 3 where both firms are sellers $MI^C = |x^{2*}|p'^*$. With a very responsive regulator $x^{2*}$ decreases, i.e., gets more negative, and therefore marginal inefficiency increases.

In case 1 firm 1 is a buyer and firm 2 is a seller ($x^{1*} > 0$ and $x^{2*} < 0$) and we are interested in how $x^{1*} + |x^{2*}|$ changes. Since both $x^{1*}$ and $x^{2*}$ reduce, the effect is ambiguous. Indeed, assuming small changes, we can see that

$$\begin{aligned} \mathrm{d}x^{1*} + |\mathrm{d}x^{2*}| &= \frac{dx^{1*}}{d\omega^1}\mathrm{d}\omega^1 + \left|\frac{dx^{2*}}{d\omega^2}\right|\mathrm{d}\omega^2 \\ &= \left[\frac{dx^{1*}}{d\omega^1} - \left|\frac{dx^{2*}}{d\omega^2}\right|\right]\mathrm{d}\omega^1 + \alpha\gamma[\Omega(K) - \Omega^0]\left|\frac{dx^{2*}}{d\omega^2}\right|. \end{aligned}$$

Now, while $\left|\frac{dx^{2*}}{d\omega^2}\right| > 0$,

$$\begin{aligned} \frac{dx^{1*}}{d\omega^1} - \left|\frac{dx^{2*}}{d\omega^2}\right| &= [\tilde{x}^1_{\omega^1} - |\tilde{x}^2_{\omega^2}|][1 + \tilde{x}^i_{x^j}\tilde{x}^j_{x^i}] \\ &= -\left[\frac{c^1}{c^1 + 2\tilde{p}'} + \frac{c^2}{c^2 + 2\tilde{p}'}\right][1 + \tilde{x}^i_{x^j}\tilde{x}^j_{x^i}] \\ &< 0 \end{aligned}$$

so the overall effect is ambiguous. □

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