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The Role of Information Provision as a Policy Instrument to Supplement Environmental Taxes

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Abstract This paper examines, within a dynamic framework, the role of information provision as a policy instrument to supplement environmental taxation. Several products are responsible for health as well as environmental damages. Many consumers do not possess the required information to optimally substitute away from these products. However, as the stock of information regarding the negative effects of these products builds up, an increasing fraction of consumers behaves optimally. The government uses two policy instruments, environmental taxation and information provision. We show that as the accumulated stock of information increases, the optimal tax rate declines over time. Information provision can shift market demand towards environmentally friendly goods over time, and thus reduce the required level of the tax rate. Our results provide strong evidence in support of information campaigns as a policy instrument to supplement traditional environmental policies.

Keywords Information provision · Environmental taxation

JEL Classification Q53 · Q58 · D62 · D82

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1 Introduction

Balancing human needs with the health of consumers and the natural environment may be the most pressing global concern of the twenty-first century. Consumers are becoming more and more concerned about products containing substances that are toxic, carcinogenic or in general harmful to them in their everyday use and at the same time dangerous to the environment. Many products, including food, electric and electronic products, tools and toys have been proven to generate health problems to their users as well as environmental damages. Individuals have incentives to reduce the consumption of such products by choosing, if available, less harmful substitutes. However, consumers are bewildered by—and often very skeptical about—the many health and environmental claims made by manufacturers and retailers for their products. Although consumer associations and environmental groups could play a role in bridging the information gap, their effectiveness, in most cases, is limited. Thus, there is a clear need for government intervention to resolve the information asymmetry, which over the years has taken different forms, including taxation and provision of information.

The present paper examines the role that information provided by the government to consumers could play in supplementing environmental taxation, and specifically the question of choosing the optimal mix of taxation and information provision. We use a dynamic framework in order to be able to take into account the lengthy process through which information affects consumers' habits and attitudes.

In particular we examine the case of a differentiated product offered in two types, produced by two firms competing in prices. During its lifetime this product generates environmental externalities (external damages) and at the same time imposes damages on each individual user (individual damages). The magnitude of both types of damages depends on the product type. We normalize by assuming that one type of the product does not generate damages (clean good), while the other type of the product (dirty good), generates both types of damages. We assume that consumers take into account individual damages if they have correct information. However, consumers' knowledge (perception) of individual damages is imperfect. For simplicity, we assume that there are two groups of consumers, those that have perfect knowledge of the individual damages and those that do not. Informed consumers always hold true beliefs and, for given prices, substitute away from the dirty and towards the clean good.

Within this framework the government imposes at each time period a tax $\tau(t)$ and provides a *flow* of information $a(t)$. We assume that consumers' behavior at each time period depends on the accumulated stock of information $A(t)$, rather than on currently provided information. More specifically, we assume that the stock of accumulated information $A(t)$ influences the composition of the two groups of consumers. The higher the stock of information is, the higher is the fraction of consumers that become informed. However, the stock of information depreciates over time. Following the mainstream of the advertising literature we assume that consumers' response to the stock of information is S-shaped. At the initial stages of the information provision campaign, consumers are more responsive to the information they receive, while as the campaign develops consumers' responsiveness slows down.

The main policy result of our analysis is that the optimal tax rate declines over time as the accumulated stock of information increases. Therefore, if the government invests in shifting consumption habits and attitudes through improving information, there is no need to regulate as strictly as before. Taxation alone does not have long-lasting effects; the same tax level has to be imposed in each time period in order to be effective. Furthermore, apart from bureaucratic costs, taxation results in efficiency losses which are increasing at the tax level. On the contrary, information provision accumulates over time and does have long-lasting effects.

Therefore, our analysis indicates that there are strong arguments for using information provision to support environmental taxation.

We derive the optimal paths and the steady states for the two policy instruments. We show that the optimal level of the stock of information is higher and the tax rate is lower, the smaller is the rate at which information depreciates over time and the lower is the cost of information provision. The optimal level of the policy instruments depends also on behavioral parameters and the level of individual and external damages. We also show that, under certain conditions, which include high cost of information provision and high depreciation rate of advertisement, if we start with sufficiently low stock of information, the system could be trapped to zero information provision. That is, it is optimal for the government to rely only on taxation and make no effort to inform consumers. In such cases, the system might exit from the trap only if cost or behavioral parameters change.

In this paper we examine information provision as a public policy instrument adopting the view that emphasizes the information disseminating role of advertisement.[1] There is however, another strand of the literature which asserts that advertisement alters consumers' tastes, resulting in higher demand or lower demand elasticity for the advertised product.[2] Bagwell (2007) offers an excellent review of the different views of advertising in the economic literature. Recently, Glaeser and Ujhelyi (2010) examine the effects of misleading advertisement by the producers regarding the health consequences of their products and evaluate different policy responses including taxation and advertising by the government. They find that, in an oligopolistic environment, misleading advertisement may increase welfare by offsetting the market imperfection. On the policy side, if the government can apply a tax or a ban on misleading advertising, any additional policy—including government advertising—cannot improve welfare. These policy recommendations are based on the assumption that the information asymmetry arises from firms' misleading advertising and thus, taxing or banning advertising is optimal. In the present paper we assume that firms do not mislead consumers and asymmetry is due to consumers' lack of information. In addition, we assume that the products in question generate environmental externalities. In such a setting, information provision that supplements environmental taxation is welfare improving.

The environmental economics literature has examined eco-labeling, certifying products' environmental attributes, as a response to fraudulent "green" advertising by firms.[3] This literature though, does not consider the negative effects on consumer's health. There are only a few papers addressing the issue of government advertising as an environmental policy instrument. Petrakis et al. (2005) show, within a static framework, that information provision could dominate, in some cases, environmental taxation in terms of welfare and that a combination of these two policies is welfare improving. They also examine the way in which each group of consumers is affected by information provision. The present paper differs considerably since it employs a richer structure of the way in which information provision affects consumers' behavior and it also uses a dynamic framework. This allows us to generalize, strengthen and extent the results in Petrakis et al. (2005). An earlier study by Kennedy et al. (1994) also examines environmental information provision. They consider goods that generate only environmental damages (there are no individual damages) and consumers cannot with certainty relate these damages to their utility. Information is provided to consumers at a cost by

[1] For example, Nelson (1974); Kotowitz and Mathewson (1979); Kihlstrom and Riordan (1984) and Stigler (1961) assume that consumers are not fully informed and they receive complete, costless and instantly validated information through advertisement.

[2] See for example Galbraith (1958) and Dixit and Norman (1978).

[3] See for example the recent paper by Hamilton and Zilberman (2006).

private firms. The informed consumers know the true marginal external damage and take into account the effect that their own consumption has on their utility. However, the framework of their analysis differs substantially from the current paper, which focuses on the role of information provided by the government.

The rest of the paper is organized as follows. Section 2 presents examples of goods that pose threat to consumers' health and damages to environment and examines the various policy responses. Section 3 describes the model, with Sect. 3.1 presenting the policy options available to the government. Section 4 derives the optimal policy mix. Section 5 presents analytical results in the case of linear demands, constant marginal production costs and constant marginal external damages. Section 6 illustrates our results using simulations of the linear model. Section 7 concludes the paper.

2 Information Asymmetry and Policy Responses

Polyvinyl chloride (PVC), one of the largest-selling plastics in the world, is a prominent example of products posing both environmental and health risks. Both of these attributes of PVC, especially its health consequences, have been discussed extensively over the past decade. PVC is widely used in building, packaging, consumer goods (including office supplies and toys), electronics industries and even in agriculture. During all phases of PVC production, as well as during its use and disposal, poisonous chemicals (dioxins) linked to cancer and birth defects are released. Therefore, PVC generates environmental damages—such as groundwater contamination and air pollution—at the same time it poses health risks—including angiosarcoma of the liver, lung cancer, brain cancer, lymphomas, leukemia, and liver cirrhosis—to its users as well as to certain groups of people such as workers in the PVC industry and residents in the nearby areas.

Similar problems are encountered with lead used in paints, asbestos used in buildings and many other elements used in the production of goods. Furthermore, many household goods, including electric and electronic devices, contain toxic substances harmful to their users at the same time that their production and disposal generates environmental damages. In the food sector one could think of fruits and vegetables grown with the use of pesticides, which generate environmental externalities during production and also health problems to consumers from residues of pesticides.

In response to these problems, governments have implemented a variety of policies. In some cases governments have used direct policies banning the use of particular substances in products.[4] For example, in response to PVC's toxic threats, many governments around the world have passed policies to ban PVC from use in certain products (with priority given to toys and food packaging) and switch to safer, healthier alternatives. Some governments have also used environmental taxes to provide economic incentives for reducing the demand for such products. One such example is the Danish government's tax on PVCs.

The above examples of policy responses indicate a general transition of environmental policy from the smokestacks and effluent pipes towards the process of production and finally to the consumption patterns, enriching at the same time, the policy instruments options with market-based approaches. However, due to the large number of products that generate health and environmental damages and the complexity of their effects, it is difficult to address the problems only with direct policies and/or economic instruments. For example, in the process

[4] Such an example is the EU Directive on the Restriction on Use of Hazardous Substances (RoHS) which can be accessed at http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:32002L0095:EN:HTML.

of switching to PVC-free products, the provision of information to consumers regarding the health risks of PVC has been proven extremely important. Information provision is still very important in countries that have not yet banned these products. Moreover, the importance of information provision has been established by many studies in the case of public antismoking campaigns.[5]

The role of information provision in complementing traditional environmental policies has been recognized by Tietenberg and Wheeler (2001). They offer a number of examples of products and processes that generate damages to individual consumers and to the environment and they also review the empirical literature. They conclude that information provision can be an effective policy instrument. Since consumers have incomplete and inaccurate information regarding the health and environmental effects of particular products, the government could intervene and provide reliable information to consumers.

3 The Model

Consider a product that generates individual and external environmental damages. The product is offered in two horizontally differentiated types, and the magnitude of both individual and external damages differs between them. For simplicity, we normalize emission units so that the *clean* type of the product does not generate any damages, while the *dirty* type generates positive individual and external damages. The external damages are given by $D(Q_{dt})$, with $\frac{\partial D}{\partial Q} > 0$ and $\frac{\partial^2 D}{\partial Q^2} \geq 0$, where Q_{dt} is the total quantity of the dirty type of product produced in period t. As regards the individual damages, we assume that an informed consumer takes into account these damages in making her consumption choices. The informed consumer derives higher utility from the consumption of the clean relative to the dirty good, with the parameter θ being a measure of the utility differential per unit of product consumed.

At each time period t, the utility of the representative informed consumer over the two product types is[6]

$$U(q_c, q_d; \theta, \gamma)\ , \tag{1}$$

where $q_j,\ j = c, d$ are the quantities consumed of the clean and the dirty good respectively and γ measures the degree of substitutability between the two types of the product.[7] Thus, we allow for two dimensions of product heterogeneity, vertical product differentiation as reflected by individual damages differentials[8] and horizontal product differentiation as reflected by individual product type features. We assume that the utility function in (1)

[5] For example, Choo and Clark (2006) find that information plays an important role in encouraging particular groups of smokers to quit smoking. Their study is based on data from an antismoking campaign in the US and Canada in the early 1990s. Farrelly et al. (2005) using data from an antismoking campaign in the US in early 2000, find that the campaign accounted for a significant portion of the decline in youth smoking in the period after the campaign. Pierce et al. (1990) report similar results for an antismoking campaign in Sydney, Australia in 1983, and in Melbourne in 1984.

[6] We assume that the utility function is linearly separable in the two types of the product and the other goods. That is, utility is given by $U(q_c, q_d; \theta, \gamma) + I$, where I is the utility derived from the consumption of other goods. This assumption implies that there are no income effects and allows us to perform partial equilibrium analysis (see Sect. 5 for a particular specification of this utility function).

[7] For instance, in the linear model we employ in Sect. 5, we assume that the two products are substitutes and thus, γ is strictly positive. If $\gamma = 0$, each firm has monopolistic market power, while if $\gamma = 1$, the products are perfect substitutes.

[8] We assume that the only measure of quality in a vertical sense that is different between the two types of the product is θ.

exhibits the standard properties that yield negatively sloped and strictly convex indifference curves.

We further assume that initially only a fraction of the consumers are informed about the dirty good's health effects. For simplicity, there are two groups of consumers, those with perfect knowledge of the negative health effects associated with the dirty good and those that have no knowledge at all. The informed consumers, which form μ fraction of the population, make their choices based on the correct value of the parameter θ, while the uninformed consumers behave as if θ was equal to 0. Thus, the uninformed consumers are unable to distinguish the two product types in terms of individual damages and they differentiate between them based on γ alone. The total consumers' population is normalized to unity.

Maximization of (1) subject to the budget constraint yields the per period t demand function for each type of the product,

$$q_j(p_j, p_k; \gamma, \theta), \tag{2}$$

where $j, k = c, d$, and $j \neq k$. The total demand for the clean and the dirty type is $Q_d(p_j, p_k; \gamma, \theta, \mu) = \mu q_{di} + (1 - \mu) q_{dn}$ and $Q_c(p_j, p_k; \gamma, \theta, \mu) = \mu q_{ci} + (1 - \mu) q_{cn}$ respectively, where q_{di} and q_{dn} (q_{ci} and q_{cn}) are the quantities of the dirty (clean) good consumed by the informed and the uninformed consumer, respectively. That is, $q_{ji} \equiv q_j(p_j, p_k; \gamma, \theta)$ and $q_{jn} \equiv q_j(p_j, p_k; \gamma, 0)$, where $j = c, d$.

At each time period t, the product is offered by two firms, each offering only one product type. The firms compete in prices in the market. We assume for simplicity that they are endowed with identical and constant over time, production cost functions $C(Q)$, with $C_Q > 0$ and $C_{QQ} \geq 0$.[9]

In the absence of any regulatory intervention, at any time period t, firm j's profit maximization problem is

$$\max_{p_j} \pi_j \left(p_j, p_k; \mathbf{z}\right) = p_j Q_j(p_j, p_k; \gamma, \theta, \mu) - C(Q_j(p_j, p_k; \gamma, \theta, \mu)), \tag{3}$$

where $\mathbf{z}$ is a vector of demand and cost parameters that includes γ, θ and μ, and $j = c, d,\ j \neq k$. Assuming that each duopolist's profit function is strictly concave on its own price and that there is an interior solution to the maximization problem for all parameter values, the first order condition of (3) yields duopolist j's reaction function, $p_j = R_j(p_k; \mathbf{z}),\ j \neq k$. Assuming further that $\frac{\partial^2 \pi_j}{\partial p_j \partial p_k} > 0$, the slope of the reaction function is positive, $\frac{\partial R_j}{\partial p_k} > 0$, thus making duopolists' prices strategic complements. Assuming finally that $\frac{\partial R_j}{\partial p_k} < 1$, the two firms' reaction functions yield the Nash equilibrium prices, $p_j(\mathbf{z}),\ j = c, d$. Clearly in this case, equilibrium prices, quantities of both product types, firms' profits, external damages and social welfare remain unchanged over time.

3.1 Policy Options

In the absence of any regulatory intervention, we have two distortions related to the characteristics of the dirty good. Firstly, an information asymmetry, since only a fraction of the consumers has the required information to take into account individual damages. Secondly, a negative externality on the environment that cannot be eliminated even when all consumers are informed. Assuming that the government is unable to intervene separately in order to

[9] One could assume instead that the two firms differ in terms of fixed costs, with the dirty good being cheaper than the clean good in terms of fixed production costs. This however would not qualitatively alter our results.

correct the additional distortion arising from imperfect market competition, this distortion must also be taken into account by a welfare maximizing regulator. In what follows, we examine the case in which the regulator uses a combination of a tax on the dirty good and information provision.

We model information provision as follows. The regulator provides a level of information $a(t)$ at each time period t. The cost of providing information to consumers, $K(a)$, with $K(0) = 0$, is assumed to be increasing, $K_a > 0$, at an increasing rate, $K_{aa} > 0$.[10] The provision of information increases the fraction of consumers that behave as informed consumers. However, it is not the level of currently provided information that affects the fraction of informed consumers but rather the stock of information accumulated at time t. We denote the stock of information at time t by $A(t)$, which summarizes current and past information provision efforts. It is reasonable to assume that information provided in the past is less effective than currently provided information. That is, while information provision directly affects uninformed consumers, some of the currently informed consumers tend to forget and behave as uninformed.[11] We model the latter by treating information provision as a capital good,

$$\dot{A} = a - \delta A \, , \tag{4}$$

assuming a constant rate of depreciation $0 < \delta < 1$.[12]

When the stock of information accumulated at time t is $A(t)$, then a fraction $\phi(A(t))$ of the uninformed consumers become informed. The following properties for the informed consumers generating function (ICGF) $\phi(A)$ are assumed:

$$\phi(A) : \mathbb{R}_+ \to [0, 1], \ \phi(0) = 0, \ \phi\left(\bar{A}\right) = 1, \ \bar{A} \leq \infty, \tag{5}$$

$$\phi_A(A) \geq 0 \text{ for all } A \geq 0, \quad \lim_{A \to \bar{A}} \phi_A(A) = 0, \tag{6}$$

$$\exists \underline{A} : \phi_{AA} \geq 0 \text{ for } A \in [0, \underline{A}] \text{ and } \phi_{AA} < 0 \text{ for } A \in (\underline{A}, \bar{A}). \tag{7}$$

These assumptions imply that an increase in the stock of accumulated information will never turn informed consumers to uninformed. Moreover, that zero information stock could not generate informed consumers, while there may exist a finite level of accumulated information stock at which all consumers become informed. Further, the ICGF does not exhibit diminishing returns for all A. In fact, the ICGF shares common characteristics with the sales response function to advertising which is commonly used in the advertisement literature, to which we resort in order to characterize its shape. The view that advertisement exhibits some degree of economies of scale is widely acceptable by both theoreticians and practitioners and an S-shaped response to advertisement function has been used extensively in the literature

[10] Grossman and Shapiro (1984) use an advertisement cost function with the same properties in a model of product differentiation. To support the $K_{\alpha\alpha}$ assumption, they argue that ".. it becomes increasingly expensive to reach higher fractions of the population, either because preferred media become saturated, or because the target population is heterogeneous along a second dimension, namely, the tendency to view ads" (p. 66).

[11] We assume that this decay in the number of informed consumers does not apply to the initial fraction of consumers that behave as informed. These consumers have acquired their information through different channels and their behavior is not affected by the government's information provision policy. Glaeser and Ujhelyi (2010) make the same assumption.

[12] The classic paper by Nerlove and Arrow (1962) introduced the following model of the dynamic effects of advertising: $\dot{A} = a - \delta A$, where A is the level of "goodwill" at time t, which affects consumers demand, a is the level of advertising (in monetary terms) at time t and δ is the depreciation rate of "goodwill". This model has been used extensively in the advertisement literature.

(see for example Feinberg 2001).[13] The S-shaped response function implies increasing marginal returns to advertising for low advertising levels followed, after an inflection point, by decreasing marginal returns. Despite the continuing debate on the shape of the advertising response function,[14] we adopt the view that consumers' response to the current stock of accumulated information is S-shaped with $\underline{A}$ denoting the point of inflection.

Taking into account the impact of information provision, the fraction of the informed consumers $m(t)$ at each point in time is

$$m(t) = \mu + (1-\mu)\,\phi(A(t))\ ,$$

where μ is the initial fraction of informed consumers, $0 \leq \mu < 1$.

Within this policy framework, that is, a tax on the dirty good, $\tau(t)$, and information provision $a(t)$ (which contributes to the formation of the information stock $A(t)$), the two firms' profit maximization problems at each time period t are[15]

$$\max_{p_c} \pi_c(p_c, p_d; \tau, \phi(A), \mathbf{z}) = p_c Q_c(\bullet) - C(Q_c(\bullet))\ ,$$
$$\max_{p_d} \pi_d(p_c, p_d; \tau, \phi(A), \mathbf{z}) = (p_d - \tau) Q_d(\bullet) - C(Q_d(\bullet)).$$

Making a similar set of assumptions on the firms' profit functions as above, we can obtain the duopolists' reaction functions and then the equilibrium prices at each time period t,

$$p_j(\tau, \phi(A), \mathbf{z}),\ \ j = c, d. \tag{8}$$

The firms' strategic variables are now functions of the two policy instruments, $\tau(t)$ and $a(t)$, in addition to cost and demand parameters presented by the $\mathbf{z}$ vector.

4 Optimal Policy Mix

Substituting $p_c(\tau, \phi(A), \mathbf{z})$ and $p_d(\tau, \phi(A), \mathbf{z})$ from (8) into the demand functions given in (2), yields $q_{ji}(\tau, \phi(A), \mathbf{z})$ and $q_{jn}(\tau, \phi(A), \mathbf{z})$, from which we obtain $Q_j(\tau, \phi(A), \mathbf{z}),\ j = c, d$. Further, substituting these expressions into the representative consumer's utility function (1) yields the (gross) utility of the informed $V_i(\tau, \phi(A), \mathbf{z})$, and the uninformed $V_n(\tau, \phi(A), \mathbf{z})$ consumer, both evaluated at the true value of θ. This means that, in deriving the optimal policy instrument levels, the regulator considers the true cost of the dirty type of product and thus, it uses the true value of the uninformed consumer's utility, even though the consumer does not take into account individual damages when making her choices.

At each time period t, social welfare is the sum of the consumer and producer surplus minus the external damages,

$$v(\tau, \phi(A), \mathbf{z}) = mV_i + (1-m)V_n - C(Q_c) - C(Q_d) - D(Q_d), \tag{9}$$

Thus, the regulator's problem is to choose the optimal time paths for the tax $\tau(t)$ and information provision $a(t)$, that is, to solve

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13 It should be noted however, that there are some empirical studies showing little or no evidence of substantial returns to scale in advertisement (see for example Arnodt and Simon 1983 and Seldon et al. 2000).

14 See for example Cannon et al. (2002) and Dube et al. (2005).

15 For notational simplicity we drop the time variable t.

$$\max_{\{a(t),\tau(t)\}} \int_0^\infty e^{-\rho t} \left[v\left(\phi(A), \tau, \mathbf{z}\right) - K(a) \right] dt \tag{10}$$

subject to:

$$\dot{A} = a - \delta A, \quad A(0) = A_0 \geq 0,$$
$$(a(t), \tau(t)) \geq 0, \forall t \geq 0,$$

where ρ is the discount rate and $K(a)$ the cost of advertisement. This is a formal optimal control problem with current value Hamiltonian function

$$\mathcal{H} = v\left(\phi(A), \tau, \mathbf{z}\right) - K(a) + \lambda\left(a - \delta A\right), \tag{11}$$

where λ is the costate variable reflecting the shadow price of the stock of accumulated information. The necessary conditions for the choice of the optimal policy instruments a and τ yield

$$\frac{\partial \mathcal{H}}{\partial a} \leq 0,\ a^0 > 0 \Rightarrow K_a\left(a^0\right) = \lambda, \text{ and } a^0 = a(\lambda), \tag{12}$$

$$a^0 = 0 \Rightarrow K_a\left(a^0\right) < \lambda \tag{13}$$

$$\frac{\partial \mathcal{H}}{\partial \tau} \leq 0,\ \tau^0 > 0 \Rightarrow \frac{\partial v\left(\phi(A), \tau^0, \mathbf{z}\right)}{\partial \tau} = v_\tau = 0, \text{ and } \tau^0 = \tau(A) \tag{14}$$

$$\tau^0 = 0 \Rightarrow \frac{\partial v\left(\phi(A), \tau^0, \mathbf{z}\right)}{\partial \tau} < 0 \tag{15}$$

The paths for λ and A evaluated at the optimal choices $\left(a^0, \tau^0\right)$ should satisfy

$$\dot{\lambda} = \rho\lambda - \frac{\partial \mathcal{H}}{\partial A}, \dot{A} = \frac{\partial \mathcal{H}}{\partial \lambda}, \tag{16}$$

We assume, without loss of generality, that the cost of advertisement is quadratic, or $K(a) = \frac{1}{2}\omega a^2$ so that $K_{aa}(a) = \omega$. Restricting attention to interior solutions and using (12) and the fact that $\omega\dot{a} = \dot{\lambda}$ to eliminate λ and $\dot{\lambda}$ from (16), the dynamic state-control system associated with problem (10) can be written in the control-state space (a, A) as[16]

$$\dot{a} = (\rho + \delta)\, a - \frac{1}{\omega} D_A v, \tag{17}$$

$$\dot{A} = a - \delta A,\ A(0) = A_0 \geq 0, \tag{18}$$

where $D_A v \equiv \frac{dv(\phi(A),\tau(A),\mathbf{z})}{dA} = v_\phi \phi_A + v_\tau \tau_A$ denotes the rate of change of social welfare with respect to the stock of information provision along a path where the controls a and τ are chosen optimally for every A according to (12) and (14) and $v_\phi = \frac{\partial v(\phi(A),\tau(A),\mathbf{z})}{\partial \phi}$. Since we are choosing the controls optimally, $v_\tau = 0$ according to (14) and $D_A v = v_\phi \phi_A$.

A steady state in the stock A and the flow a of information provision is defined as (A^*, a^*) : $\dot{A} = 0,\ \dot{a} = 0$. As it will be proven later, multiple steady states exist for our problem. To study the properties of these steady states for solutions $A^* \in [0, \bar{A}]$, we make some additional assumptions regarding the structure of the social welfare functional $v\left(\phi(A), \tau(A), \mathbf{z}\right)$ which defines, for given parameters $\mathbf{z}$, a map from the vector space containing ϕ and τ to the real numbers when a and τ are chosen optimally.

[16] The dynamic system associated with problem (10) can be equivalently analyzed in the state-costate space (λ, A). Results carry over from one space to the other since control and state are related by the optimality condition $a^0 = a(\lambda)$.

Assumption A1 $v_\phi \geq 0,\ v_{\tau\tau} < 0,\ v_{\tau\phi} < 0$, with all derivatives bounded, and $\phi_A(0) = 0$.

Assumption A1 implies that an increase in the fraction of the informed consumers does not reduce social welfare. Furthermore, the rate of change in social welfare due to an increase in the tax level is decreasing in both τ and ϕ. The last part of the assumption means that an increase in the stock of the information provision will not increase the fraction of informed consumers when this stock is negligible.

Assumption A2 For $\sigma(A) = v_{\phi\phi}(\phi_A)^2 + v_\phi \phi_{AA}$, (i) $\sigma(0) > 0$, (ii) $\lim_{A \to \bar{A}} \sigma(A) < 0$, and (iii) there is a unique $\hat{A} \in [0, \bar{A}] : \sigma\left(\hat{A}\right) = 0$.[17]

The structure of the steady states is described in the following proposition.

Proposition 1 *Under assumptions A1 and A2 and an S-shaped ICGF, there exist at most two positive steady states, $0 < A_1^* < A_2^* < \infty$. When only one positive steady state exists then it is a saddle point. When two positive steady states exist then the larger is a saddle point while the smaller is unstable. The origin is a steady state, which is a saddle point if two positive steady states exist and unstable if only one positive steady state exists. For a sufficiently large depreciation rate δ there is no positive steady state and the origin is the only nonnegative steady state.*

Proof A steady state occurs at the intersection of the isocline $\psi^a \equiv \psi^a(A)|_{\dot{a}=0} = \frac{v_\phi \phi_A}{(\rho+\delta)\omega}$, defined through $0 = (\rho + \delta)a - \frac{1}{\omega} D_A v$, with the isocline $\psi^A \equiv \psi^A(A)\big|_{\dot{A}=0} = \delta A$ defined through $0 = a - \delta A$. Thus, the steady state equation is

$$\frac{v_\phi \phi_A}{(\rho + \delta)\omega} = \delta A \text{ or } \psi^a = \psi^A.$$

The slope of ψ^a is

$$\frac{d\psi^a}{dA} = \psi_A^a = \frac{\sigma(A)}{(\rho + \delta)\omega},\quad \sigma(A) = v_{\phi\phi}(\phi_A)^2 + v_\phi \phi_{AA}.$$

Assumptions A1 and A2, along with the assumptions about the S-shaped ICGF and continuity, imply that, ψ^a starts at the origin, has an increasing part, reaches a maximum for $A = \hat{A} > 0$, and then declines converging to zero as A becomes large. We do not make any assumptions regarding the sign of $v_{\phi\phi}$. If it is positive, then $\sigma(A)$ is definitely positive up to the inflection point of the ϕ function, i.e. for $A \in [0, \underline{A}]$, and then as A increases further and ϕ_{AA} becomes negative, $\sigma(A)$ decreases and eventually becomes negative. If $v_{\phi\phi}$ is negative, then $\sigma(A)$ is positive for a range of values of A for which $v_\phi \phi_{AA} > \left|v_{\phi\phi}(\phi_A)^2\right|$ and then as A increases further, $\sigma(A)$ becomes negative. In the linear demand case examined in Sect. 6, we show that $v_{\phi\phi} > 0$. The isocline ψ^A is a ray from the origin with positive slope δ. If the slope of ψ^a at the origin is less than δ or $\psi_A^a(0) < \delta$, then ψ^A intersects ψ^a either one or 3 times in the positive quadrant as shown in Fig. 1.[18] For really high levels of δ, they only intersect at the origin. Otherwise they intersect 3 times, one at the origin and two at positive values

[17] This assumption is required for the existence of a steady state. It also simplifies the analysis by excluding the possibility of more than three steady states. The satisfaction of parts (i) and (ii) of this assumption requires that $v_\phi(\phi(0), \tau(0), \mathbf{z}) > 0,\ \phi_{AA}(0) > 0,\ v_\phi\left(\phi\left(\bar{A}\right), \tau\left(\bar{A}\right), \mathbf{z}\right) > 0$. Assumption A2(iii) can be dispensed in a more general model. It is verified that all our assumptions are satisfied in our linear demand example, see Eqs. (25), and the numerical simulations in Sect. 6.

[18] We do not consider hairline cases where the two isoclines are tangent.

of A, points U and Q in Fig. 1. If $\psi_A^a(0) > \delta$ then ψ^A intersects ψ^a 2 times in the positive quadrant, one at the origin and another one at positive values of A, which corresponds to point E in Fig. 1. To study the stability property of a steady state we consider the linearization of the state-control system defined by (17) and (18) at this steady state. The Jacobian matrix of the state-control system is

$$J = \begin{pmatrix} \rho + \delta & -\frac{1}{\omega}\sigma(A^*) \\ 1 & -\delta \end{pmatrix} = J\left(a^*, A^*\right). \tag{19}$$

Since $\mathrm{tr}(J) = \rho > 0$ and the eigenvalues of (19) are $\beta_{1,2} = \frac{1}{2}\rho \pm \sqrt{\rho^2 - 4\det(J)}$, where $\det(J) = -\delta(\rho+\delta) + \frac{\sigma(A^*)}{\omega}$, a steady state will be either unstable or it will have the local saddle point property. If $\det(J) < 0$, then the eigenvalues are real numbers and the steady state is a local saddle point. For $\det(J) < 0$ we need

$$\frac{d\psi^A}{dA} = \delta > \frac{d\psi^a}{dA} = \frac{\sigma(A^*)}{(\rho+\delta)\,\omega},$$

that is, the slope of the ψ^A curve should exceed the slope of ψ^a at the steady state. Thus, a saddle point steady state occurs at the declining part of the ψ^a curve, or more generally when ψ^A intersects ψ^a from below. If $\det(J) > 0$ then the steady state is unstable. The unstable steady state occurs at the increasing part of the ψ^a curve where the slope of ψ^a is larger than δ, or more generally when ψ^A intersects ψ^a from above. If $\rho^2 < 4\det(J(A^*))$ at the unstable steady state, then the eigenvalues at A^* are complex with positive real parts and the trajectories curl away from the unstable steady state as shown in Fig. 2, where the phase diagram suggests that A_1^* is a local unstable focus. Finally, for sufficiently high δ, isoclines ψ^A and ψ^a do not intersect at the positive quadrant and the only nonnegative steady state is the origin. Since in this case ψ^A intersects ψ^a from below, the origin is a saddle point. □

It can be easily shown following the steps of the proof that if $\phi_A(0) > 0$, which means that an increase in the stock of information provision will increase the fraction of informed consumers even when this stock is negligible, then there is the possibility of three positive steady states.[19] In terms of Fig. 1 the assumption $\phi_A(0) > 0$ means that the ψ^a isocline shifts upwards, like the dashed line, so that it has a positive intercept. In this case there could either be three steady states, the middle one being unstable and the other two being local saddle points, or a unique saddle point steady state. In the latter case, the unique steady state will be high for low δ and low for high δ.

We turn now back to the case $\phi_A(0) = 0$. Steady states are shown in Fig. 1 along the solid isocline $\psi^a(A)|_{\dot{a}=0}$. For low δ there exists a unique positive steady state at A_M^* with the saddle point property, while the origin is an unstable focus. Convergence to A_M^* takes place along the stable manifold MM. For any initial value of the information stock A, there exists an initial value of the flow of information provision a such that there is convergence to the optimal steady state on the one-dimensional manifold MEM. For example, if the initial stock of information is A_0, the converging path is along ME. As δ increases the steady state information stock is reduced.[20]

There exists a range of δ such that three steady states could exist, the origin and $\left(A_1^*, A_2^*\right) > 0$. The origin and steady state A_2^* are local saddle points, while A_1^* is unstable. It should be

[19] We use the assumption $\phi_A(0) = 0$ in the main proof in order to have compatibility with our numerical results presented in the Sect. 6, where the ICFG function satisfies this assumption.

[20] This is because the slope of the isocline ψ^A increases, while the isocline ψ^a shifts downward since $\psi^a = \frac{v_\phi \phi_A}{(\rho+\delta)\omega}$ and neither v_ϕ nor ϕ_A are functions of δ.

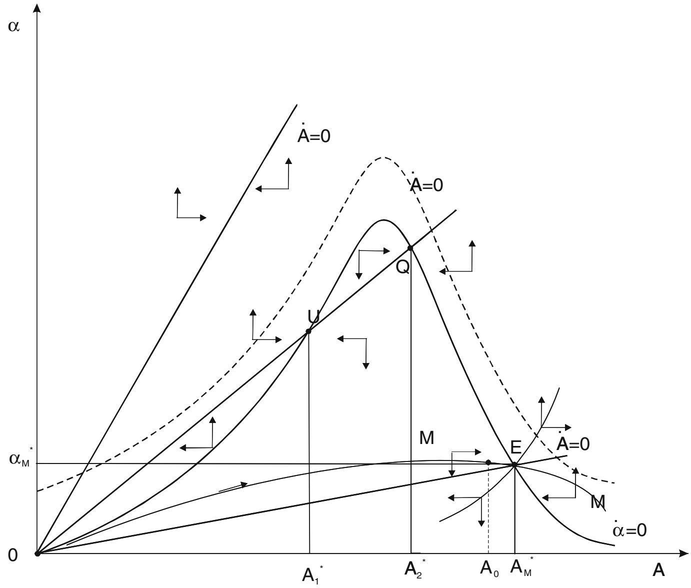

Fig. 1 Multiple steady states

noted that in order to have three steady states, it is necessary that, apart from a sufficiently high δ, the ψ^a curve be convex-concave to the left of its maximum. If ψ^a is concave until its maximum then only one positive steady state exists. For even higher δ there is no positive steady state. The origin is the only nonnegative steady state which is a local saddle point. Thus, when δ increases, the slope of the ψ^A curve increases, while the ψ^a curve shifts downward.[21] Therefore, as might have been expected, the higher the depreciation of the stock of knowledge, the lower, ceteris paribus, the steady state stock of information provision. The rate of change of marginal advertising costs ω has a similar effect on the steady state stock of information provision. The higher ω is, the further ψ^a shifts downward and the more the steady state A is reduced.

Although a full analysis of the dynamics of the state-control system (17)–(18) is beyond the purpose of this paper, there are some interesting insights related to the ranking between multiple steady states. The parameters δ and ω act as bifurcation parameters in the sense that as these parameters are varied, the qualitative behavior of the state-control system changes and moves from a unique to multiple steady states. The existence of multiple steady states indexed by i, with $i = 1, 2, 3$ in our problem, implies that each steady state can be thought as a local maximum of the value function $W(A)$ for the problem. The value function can be defined using the Hamilton–Jacobi equation as

$$\rho W(A) = \max_a \mathcal{H}(A, \lambda(A), a), \tag{20}$$

where $\lambda(A)$ is the optimal stable manifold in the state-costate space, while $\alpha(A)$ will be the corresponding optimal manifold in the contol-state space (e.g. HQ in Fig. 2). Consider any locally optimal trajectory $\{\lambda_i(A), a_i(A)\}$ and associate with it a candidate value function $w_i(A) = \frac{1}{\rho}\mathcal{H}(A, \lambda_i(A), a_i(A))$. The globally optimal steady state i which is reachable from the state A will be the one that corresponds to the $\max_i w_i(A)$ for all i, or

[21] To make the diagram simpler we do not show shifts in the $\psi^a(A)$ curve.

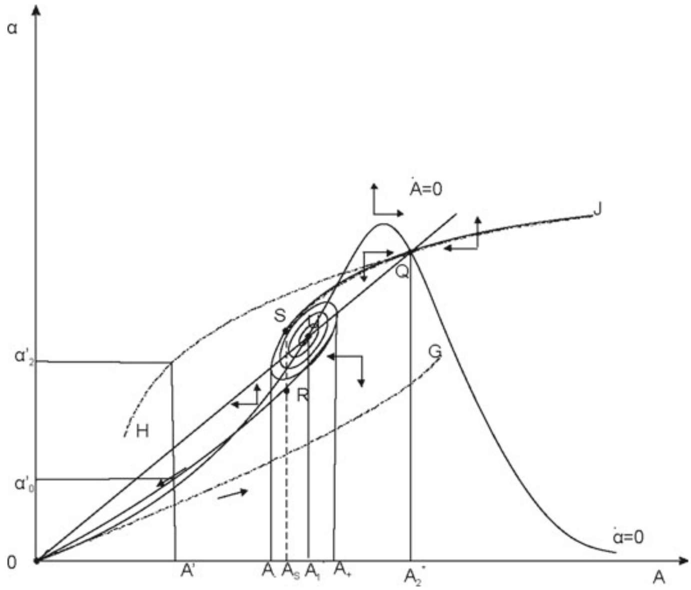

Fig. 2 The zero information provision trap

$i(A) = \arg\max w_i(A)$. In our problem it is interesting to examine whether, in the case of the three steady states of Fig. 2, the globally optimal steady state is attained at A_2^* or at the origin, and how this ranking depends on the initial state A.[22] We proceed by excluding first the possibility of a limit cycle around the unstable steady state A_1^*. In the state-control space, the divergence of the vector field $F = \frac{1}{\omega}\left(\frac{\partial \mathcal{H}}{\partial a}, \rho a - \frac{\partial \mathcal{H}}{\partial A}\right)$ is

$$\operatorname{div} F = \frac{1}{\omega}\left(\frac{\partial}{\partial A}\left(\frac{\partial \mathcal{H}}{\partial a}\right) + \frac{\partial}{\partial a}\left(\rho a - \frac{\partial \mathcal{H}}{\partial A}\right)\right) = \frac{\rho}{\omega} > 0\,,$$

therefore by the Poincaré–Bendixon theorem a limit cycle does not exist around the unstable steady state A_1^*.

Then we compare the candidate steady states by using the "Candidate value function comparison" (Brock and Starrett 2003, Proposition 5),[23] which for our case can be stated as follows.

Index by $i = 0$ and $i = 2$ the candidate value functions, so $w_0(A)$, $w_2(A)$ are the two candidate value functions corresponding to the two local steady states, the origin $A_0^* = 0$ and A_2^*. Then if

$$[\lambda_0(A) - \lambda_2(A)]\frac{dA_0}{dt} \geq 0 \Rightarrow w_0(A) \leq w_2(A)\,. \tag{21}$$

This comparison allows us to study the existence of the so-called Skiba point (Skiba 1978). Assume that for a given parametrization of problem (10) the structure of the steady states is such that one branch of the unstable focus around A_1^* becomes the stable manifold for A_2^*, while a second branch becomes the stable manifold for the origin as shown in Fig. 2. Then it is clear that for initial states to the right of A_+ the manifold USQ that converges to

[22] The analysis is exactly the same for the case where the "low A" steady state is positive.

[23] For a similar result see Wagener (2003, Lemma 4).

A_2^* is optimal, while for initial states to the left of A_- the manifold $UR0$ that converges to the origin is optimal. It follows then that there exists one switch point in $A_s \in [A_-, A_+]$ such that for initial states to the right of A_s it is optimal to converge to A_2^* along the stable manifold SQ, while for initial states to the left of A_s it is optimal to converge to the origin along the stable manifold $R0$. The initial state A_s is a Skiba point.[24] Thus, for a range of parameter values for δ and ω, initial conditions matter and for a certain range of values of the initial stock of advertisement the system can be trapped into a zero (or low if $\phi_A(0) > 0$) steady state stock of information provision. In this case it is optimal to provide very little or no information at all and to rely heavily on taxation for the control of personal and external damages.

Alternatively, there could be steady state structures with stable manifolds pairs like $(HQJ, UR0)$ and $(G0, USQ)$ converging respectively to the origin, or A_2^*, where only one branch emanates from the unstable steady state and becomes a stable manifold. Given an initial state, condition (21) can be used to determine the globally optimal steady state. For example, if the structure corresponds to the pair $(HQJ, UR0)$ and the initial state is $A' < A_-$ (Fig. 2) we have $\lambda_0(A') < \lambda_2(A')$ and $\frac{dA_0}{dt} < 0$[25] then by (21) $w_0(A) \leq w_2(A)$ and the globally optimal path is the one leading to A_2^*. Under an alternative structure the globally optimal path could be the one leading to zero information provision.

The possibility of a trap with zero information provision is policy relevant, since in most cases information provision policies are absent, and thus a realistic initial value for A is zero. In this case if information depreciation δ and the marginal cost of providing information ω are sufficiently low, then the optimal policy will be to follow a path like $0E$ in Fig. 1 and converge to a steady state with positive information provision. If however δ and ω are sufficiently high, it might be optimal not to start any information provision policy and to have the system remain at the initial state $A = 0$. If some positive stock for A exists at the initial state because of previous policies,[26] but the system has two positive steady states as in Fig. 2, then if the initial stock is below A_s it is optimal to let the existing stock fully depreciate. It might be possible that a decrease in δ and/or ω might change that structure of the steady states of the system so that the initial state is now to the right of the Skiba point. In this case the system would be out of the trap so that it could reach the steady state A_2^*.

Having determined the optimal paths for information provision, the corresponding optimal path for the tax $\tau^*(t)$ can be obtained by solving the optimality condition (14) for $\tau(t)$, with A replaced by the optimal path $A^*(t)$. Therefore,

$$\tau^*(t) : \frac{\partial v(\phi(A^*(t)), \tau^*(t), \mathbf{z})}{\partial \tau} = 0. \tag{22}$$

The trade-off between the stock of information provision and taxation at each point in time can be obtained by using the implicit function theorem in (22) to obtain

$$\frac{d\tau^*(t)}{dA^*(t)} = -\left.\frac{v_{\tau\phi}\phi_A}{v_{\tau\tau}}\right|_{A=A^*(t)},$$

[24] The existence of an unstable steady state between two saddle points does not however imply the existence of a Skiba point. As has shown by Wagener (2003) if the dynamic state-control system has a local cusp bifurcation for $\rho = 0$ then a Skiba point exists for small but positive ρ. Identification of the Skiba point relies heavily on numerical analysis (e.g. Wagener 2003; Mäler et al. 2003).

[25] Taking the vertical from A' below A_- up to the points where it intersects $UR0$ and HQ, we have $a_0'(A') < a_2'(A')$ but $\lambda = \lambda(a)$ from the maximization of the Hamiltonian and $d\lambda/da > 0$, therefore $\lambda_0(A') < \lambda_2(A')$. Furthermore $dA_0/dt < 0$ since the path $R0$ is below $\dot{A} = 0$ where $\dot{A} < 0$.

[26] Examples are the cases of antismoking campaigns and information regarding damages related to PVCs, discussed in the introduction.

which is negative, given $\phi_A > 0$ and assumption A1 (that is, $\upsilon_{\tau\tau} < 0$ and $\upsilon_{\tau\phi} < 0$). As the information stock increases along the optimal path leading to the steady state, the optimal tax rate decreases.

5 Analytical Results using Linear Demand and Cost Functions

In this section we resort to specific functional forms for the utility and cost functions in order to obtain tractable results for the model developed above. In order to incorporate both horizontal and vertical differentiation characteristics, the utility of the representative informed consumer is[27]

$$U(q_c, q_d, \theta, \gamma) = \alpha q_c + (\alpha - \theta) q_d - \frac{1}{2}\left(q_c^2 + q_d^2 + 2\gamma q_c q_d\right) + I\ ,$$

where I is the numeraire good produced by a competitive sector. Thus, utility is quadratic in the consumption of the clean and dirty type of the product and linear in the consumption of other goods I. The parameter $\gamma \in [0, 1]$ measures the degree of substitutability between the two types of the product. We assume that the two types of the product are less than perfect substitutes, that is, $\gamma < \gamma_k < 1$, where γ_k is the critical value of the degree of substitutability guaranteeing that both informed and uninformed consumers purchase, in all cases under consideration, positive quantities of both types of the product.

The consumer's utility maximization problem yields the following demand function for each type of product

$$q_c = \frac{\alpha(1-\gamma) + (1+\gamma)\theta - p_c + \gamma p_d}{1-\gamma^2};\ q_d = \frac{\alpha(1-\gamma) - (1+\gamma)\theta - p_d + \gamma p_c}{1-\gamma^2}.$$

As in the general case, the regulator, at each time period, imposes a tax on the dirty good, $\tau(t)$, and provides information $a(t)$. The stock of information accumulated at time t is $A(t)$, and thus a fraction $\phi(A(t))$ of the uninformed consumers becomes informed. For the ICGF we assume a simple algebraic sigmoid function of the following form,

$$\phi(A) = \frac{A^2}{1+A^2}\ , \tag{23}$$

which is illustrated in Fig. 3. This functional form satisfies all the properties we have assumed in the general model, Eqs. (5)–(7) and has an inflection point at $\underline{A} = \frac{1}{\sqrt{3}}$.[28]

The fraction of the informed consumers $m(t)$ at each point of time is $m(t) = \mu + (1-\mu)\phi(A(t))$. The total demand for the clean and the dirty good is $Q_d = m q_{di} + (1-m) q_{dn}$ and $Q_c = m q_{ci} + (1-m) q_{cn}$ respectively, where q_{di} and q_{dn} (q_{ci} and q_{cn}) are the quantities of the dirty (clean) good consumed by the informed and the uninformed consumer, respectively. That is, $q_{ji} \equiv q_j(\theta)$ and $q_{jn} \equiv q_j(\theta = 0)$, where $j = c, d$.

On the production side, we assume that the two firms produce with the same constant marginal cost c. Each firm's profit maximization problem, at each time period t, is

$$\max_{p_c} \pi_c = (p_c - c)\, Q_c\ ,$$
$$\max_{p_d} \pi_d = (p_d - \tau - c)\, Q_d.$$

27 A similar type of utility function has been introduced by Dixit (1979) and used in many works such as Singh and Vives (1984). For a comprehensive and complete presentation, see Martin (2002).

28 In particular, $\phi_A(A) = \frac{2A}{(1+A^2)^2} > 0,\ \forall A > 0,\ \phi_A(0) = 0$ and $\phi_{AA}(A) = \frac{2-6A^2}{(1+A^2)^3}$.

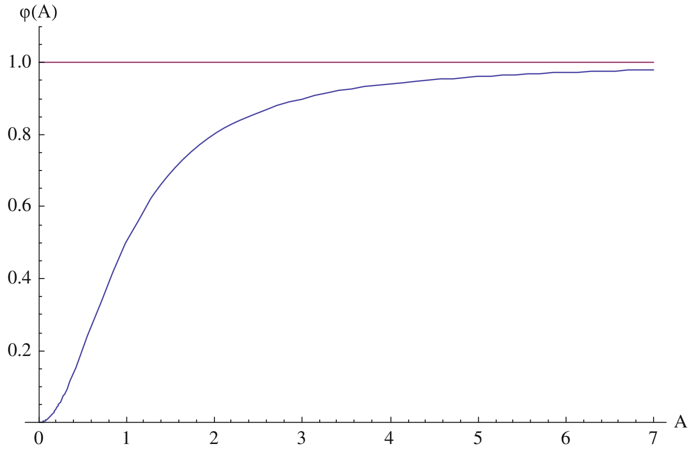

Fig. 3 The ICGF sigmoid function

The reaction functions resulting from the duopolists' profit maximization problems are solved for the prices, at each time period t, as functions of the two policy instruments

$$p_c = \frac{B + m(2-\gamma)(1+\gamma)\theta + \gamma\tau}{4-\gamma^2}, \; p_d = \frac{B - m(2-\gamma)(1+\gamma)\theta + 2\tau}{4-\gamma^2}, \tag{24}$$

where $B = (2+\gamma)\left[(1-\gamma)\,\alpha + c\right],\; j,k = c,d$ and $j \neq k$.

Using similar steps as in the previous section, we obtain, at each time period t, the total demand for the clean and the dirty type of the product and, moreover, the informed and uninformed representative consumer's gross utilities, both evaluated at the true values of θ. To further simplify the analysis we assume that the dirty product's external damages are linear in output, $D(Q_d) = dQ_d$. Given these specifications, the social welfare function given in (9) satisfies all the properties we have assumed in the general model. In particular,

$$\begin{aligned} v_\phi &= \frac{(1-\mu)\theta\left\{(1-\gamma)\Omega_1 + \theta\phi(A)\left[(5-\gamma^2) + \Omega_2/A^2\right]\right\}}{(1-\gamma)\left(4-3\gamma^2\right)}, \\ v_{\phi\phi} &= \frac{(1+\gamma)\,(7-6\gamma)\,(1-\mu)^2\,\theta^2}{(1-\gamma)\left(4-3\gamma^2\right)} > 0, \\ v_{\tau\tau} &= -\frac{\left(4-3\gamma^2\right)}{(1-\gamma^2)\left(4-\gamma^2\right)^2} < 0, \text{ and } v_{\tau\phi} = -\frac{(1-\mu)\,\theta}{(1-\gamma)\,(2+\gamma)^2} < 0, \end{aligned} \tag{25}$$

where $\Omega_1 = (1-\gamma)(a-c)+\gamma d > 0$ and $\Omega_2 = 2(1-\gamma)\left[(1+\gamma)-3\mu\right]+\gamma\,(1-\mu)-1$. Ω_2 is positive for all values of γ that ensure that both types of consumers purchase positive quantities of both types of the product. Since $\Omega_1 > 0$ and $\Omega_2 > 0$, we have that $v_\phi > 0$.

6 Numerical Simulations

In order to illustrate the optimal time paths of the tax $\tau(t)$ and information provision $a(t)$ that solve the maximization problem stated in Sect. 4, we resort to numerical simulations. We use the following values for the model's parameters: $a = 80,\ \theta = 15,\ \gamma = 0.4,\ \mu = 0.3,$

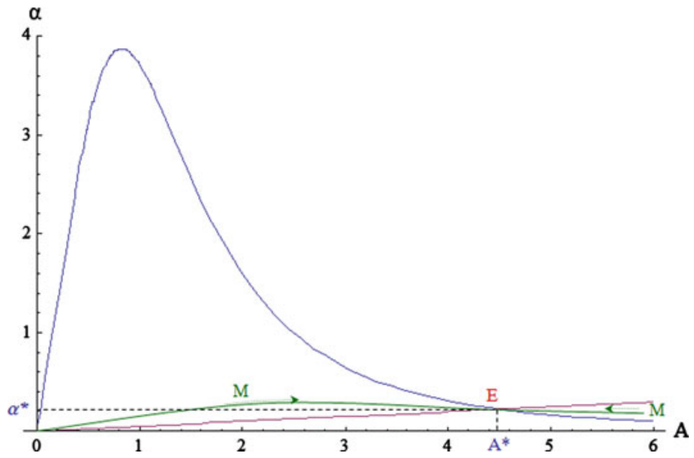

Fig. 4 Steady states in the linear example

$c = 10$, $\rho = 0.03$, $\delta = 0.05$, $\omega = 500$ and $d = 12$.[29] From the solution of the dynamic state-control system, Eqs. (17) and (18), for A and a, we define the isoclines $\psi^a(A)|_{\dot{a}=0}$ and $\psi^A(A)|_{\dot{A}=0}$ which are presented in Fig. 4. for the values of the parameters we use, and given that $\phi_A(0) = 0$ for the ICGF assumed in (23), the two isoclines intersect at the origin and at one more point in the positive quadrant. The coordinates (A, a) of the second intersection, which corresponds to point E in Fig. 4, are (4.49296, 0.224648). Since only one positive steady state exists, it is expected by Proposition 1, that the origin is unstable while the positive steady state is the only saddle point. The simulations confirm this result since the eigenvalues corresponding to point E are real $(0.13829, -0.10829)$ while those corresponding to the origin are complex $(0.015 + 0.714168i, \ 0.015 - 0.714168i)$. Therefore, in our example, the optimal positive steady state with the local saddle point property corresponds to a stock of advertisement $A^* = 4.49296$ and a flow of advertisement $a^* = 0.224648$. At the optimal steady state a fraction $\phi(A^*) = 0.952801$ of uninformed consumers becomes informed and the total fraction of informed consumers is $m = \mu + (1 - \mu)\phi(A^*) = 0.96696$. As in the general case, convergence to the optimal steady state occurs on the manifold MEM. Starting in the neighborhood of E from any level of the stock of information away from the optimal, the government can determine the path of the flow of information $a^*(t)$ leading to the optimal steady state (A^*, a^*).

Figure 5 depicts the tax τ as a function of the stock of information A, derived from the optimality condition (14). The curve depicting the optimal tax response has a horizontally inverted S-shape, that is, there is a fast decrease in the tax rate for low values of A and a slower decrease for higher levels of A. This is exactly what we expected from the general model, where we found that there exists a trade-off between the stock of information provision and taxation, and in particular, $\frac{d\tau^*(t)}{dA^*(t)} = -\left.\frac{v_{\tau\phi}\phi_A}{v_{\tau\tau}}\right|_{A=A^*(t)} < 0$. Given that $\phi_A(A) > 0$ and the values of $v_{\tau\tau}$ and $v_{\tau\phi}$ presented in (25), it is clear that the slope of the curve is negative. Furthermore, since $\frac{v_{\tau\phi}}{v_{\tau\tau}}$ does not depend on A, the slope of the curve will follow the slope of the ICGF, $\phi_A(A)$.

[29] The values of the parameters were chosen so that all constraints of the model are satisfied. Note that the policy conclusions are based on the analytical results presented in the previous sections and do not depend on the specific values of the parameters chosen for the simulations.

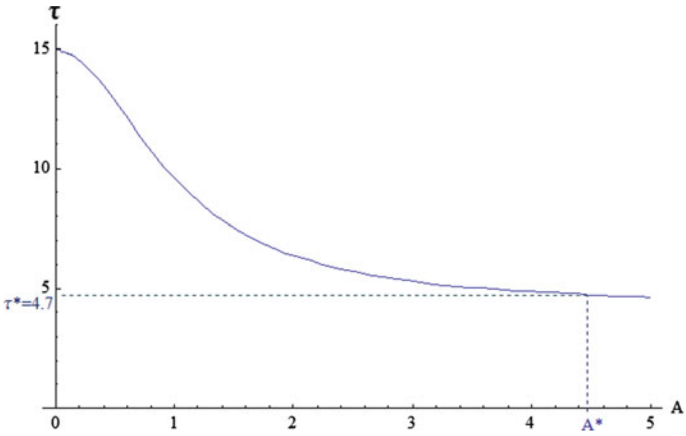

Fig. 5 Tax response to changes in the stock of information

When the stock of advertisement is zero, the tax rate is $\tau(0) = 14.9236$. Notice that taxation is the only policy instrument available to deal with the three distortions present in our setting: the negative environmental externality, the information asymmetry, and the market imperfection. Under perfect competition the tax rate with zero advertisement would be definitely higher, exceeding marginal environmental damage in order to correct the information asymmetry. As the stock of information builds up, the required tax rate decreases, approaching its lowest value as the stock of advertisement approaches its optimal steady state value A^*, $\tau(A^*) = 4.737$. It should be noted that the optimal steady state value of the tax, $\tau(A^*)$, is very close to the optimal tax in the absence of information asymmetry, that is when $\mu = 1$. In such a case, the optimal tax level that takes into account personal damages and the externality is $\tau = 4.23273$. Again, this tax level takes into account the fact that due to market imperfections firms produce lower than the perfectly competitive output, and thus it is below external and personal damages.

The numerical results of the linear demand model confirm the results of the general model. There exists one, under the assumptions of our model, optimal steady state with the saddle point property. As we approach the optimal steady state, increasing the stock of information, the optimal tax level decreases.

Some points regarding the behavior of the informed and uninformed consumers as government's policies are converging to the optimal steady state are worth making. As the stock of information builds up, the market demand for the clean (dirty) product type increases (decreases) and as a result its price increases (decreases). Responding to the change in prices, the uninformed as well as the informed consumers decrease their consumption of the clean product type and increase the consumption of the dirty product type at the individual level. Figures 6 and 7 illustrate the change in the informed and uninformed consumer's demand for the clean and the dirty product type as the stock of information increases. The change in the aggregate demand for the two types of product is also shown in the graphs. In Fig. 6 (7), $Q_c(A)$ ($Q_d(A)$) increases (decreases) as the stock of information moves towards A^*, converging to $q_{ci}(A)$ ($q_{di}(A)$) since the fraction of informed consumers approaches unity. The individual consumer's shift towards the dirty product type depends on the degree of substitutability between the two goods.

The market demand for the clean (dirty) product type increases (decreases) despite the shift towards the dirty good at the individual level, because the fraction of informed consumers

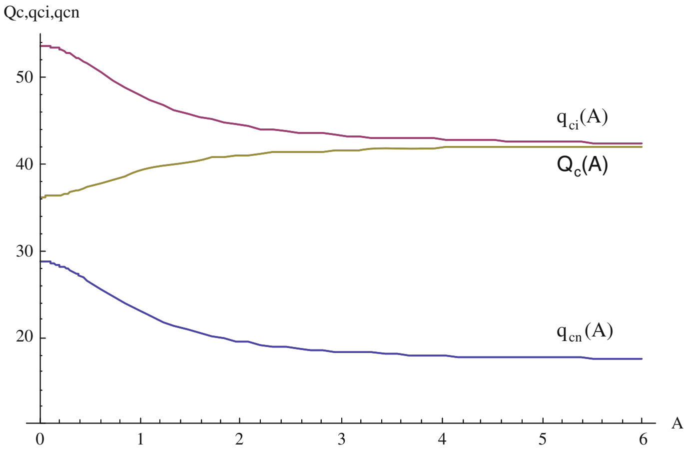

Fig. 6 $Q_c(A)$, $q_{ci}(A)$ and $q_{cn}(A)$

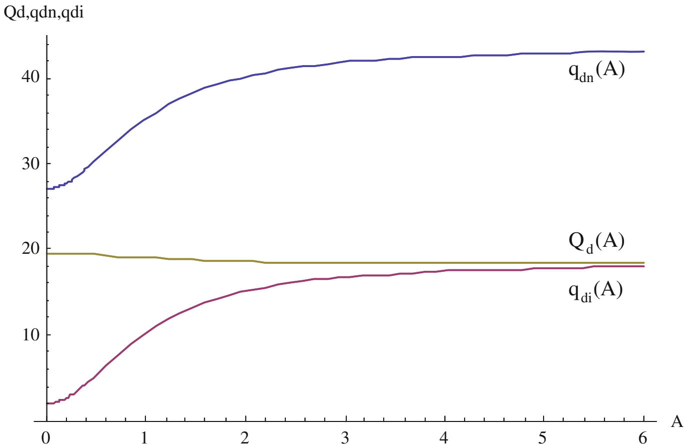

Fig. 7 $Q_d(A)$, $q_{di}(A)$ and $q_{dn}(A)$

increases and their consumption of the clean product type is much higher than the demand of the uninformed consumers as Fig. 6 shows. Given that environmental damages are decreasing as aggregate consumption of the dirty good decreases, the optimal mix of policies leads to the decrease in environmental damages.

Both types of consumers are substituting towards the dirty good as the government's policy develops, in order to maximize their own utility. Social welfare is increasing, as shown in Fig. 8, since the utility of both types of consumers increases and environmental damages are decreasing.

In closing, a note on how the parameters of the model affect the steady state is in order. First, as the fraction μ of informed consumers before the policy intervention increases, the ψ^a isocline shifts downwards and the stock of advertisement at the steady state A^* decreases. The higher is the fraction of informed consumers in the absence of policy, the less aggressive is the optimal information campaign. The first part of Fig. 9 below illustrates the above result. Ceteris paribus, as the fraction of informed consumers initially increases to

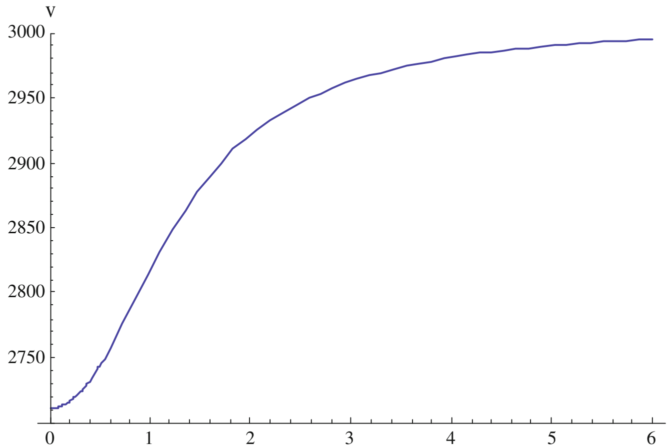

Fig. 8 Optimal social welfare $v(A)$

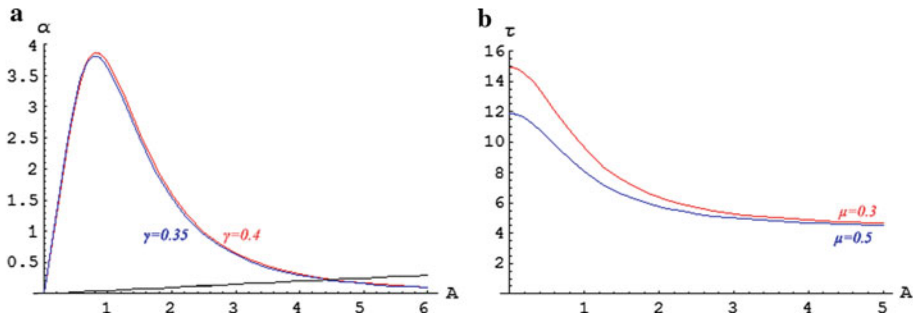

Fig. 9 **a** Effect of μ on ψ^a. **b** Effect of μ on $\tau(A)$

$\mu = 0.5$, the ψ^a isocline shifts downwards and the stock of advertisement at the steady state decreases from $A^*_{|\mu=0.3} = 4.49296$ to $A^*_{|\mu=0.5} = 4.118$. Figure 9b shows how the relationship between A and τ changes when the fraction of initially informed consumers increases to $\mu = 0.5$, yielding a marginal decrease in the steady state tax from $\tau(A^*)_{|\mu=0.3} = 4.737$ to $\tau(A^*)_{|\mu=0.5} = 4.65802$.

Second, the higher are the personal damages, θ, the more the ψ^a isocline shifts upwards and the stock of advertisement at the steady state A^* increases. A decrease in personal damages from 15 to 12, keeping all other parameters constant, yields the decline of the ψ^a isocline shown in Fig. 10a (resulting in a lower stock of advertisement at the steady state $A^*_{|\theta=12} = 4.066$), while it also shifts the function of $\tau(A)$ downwards, which implies a sharp decrease in the steady state tax to $\tau(A^*)_{|\theta=12} = 0.444$.

Finally, the closer substitutes the two types of the product are, the ψ^a isocline shifts upwards and the stock of advertisement at the steady state A^* increases. Figure 11 illustrates the effect of a decrease in γ from 0.4 to 0.35 to the stock of advertisement at the steady state, which decreases slightly to $A^*_{|\gamma=0.35} = 4.44149$ and to the tax level at the steady state, which decreases substantially to $\tau(A^*)_{|\gamma=0.35} = 0.0882$.

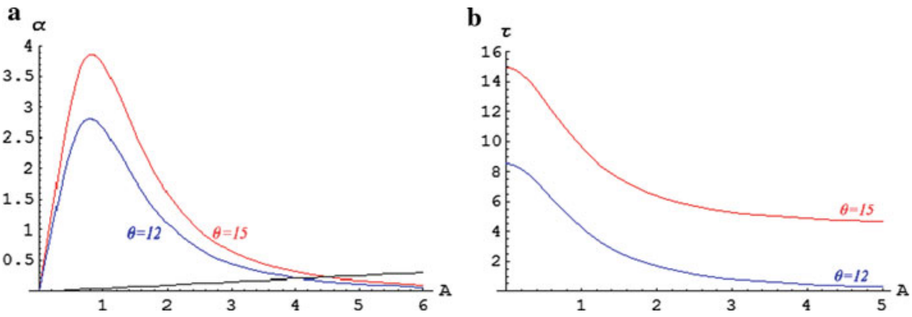

Fig. 10 **a** Effect of θ on ψ^a. **b** Effect of θ on $\tau(A)$

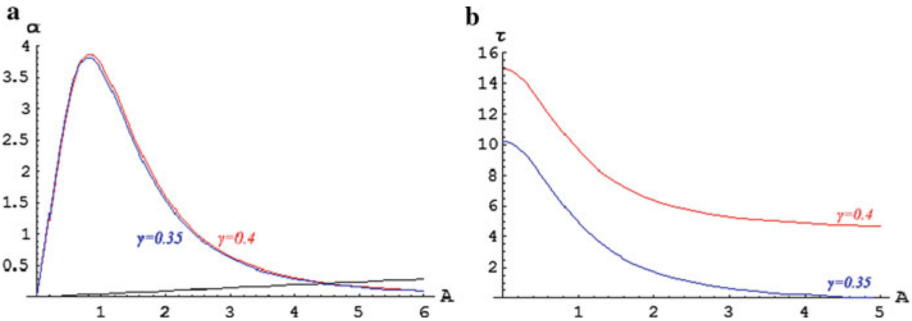

Fig. 11 **a** Effect of γ on ψ^a. **b** Effect of γ on $\tau(A)$

7 Conclusions

Under certain circumstances information about health and environmental risks related to certain products can motivate consumers to shift their consumption towards more healthy and environmentally safe products. However, these risks are rarely common knowledge, and private firms even when they possess this information are unlikely to share it with the public voluntarily. Thus, a government maximizing social welfare has an incentive to provide reliable information in order to complement existing policies. The present paper examines the case of products that are responsible for both environmental and health damages. Given that consumers have incomplete information about health and environmental risks, we examine the role of information provision in supporting environmental taxation. We find that the combination of the two policy instruments enhances efficiency since information provision results in lowering aggregate consumption of the good generating health and environmental damages and thus it reduces the need for environmental taxation. Over time the optimal tax rate declines resulting in lower costs, while the benefits are increasing. The optimal steady state value of the tax depends also on the existence of market imperfections.

Although in the present paper we do not introduce environmental damages in individuals' utility, a natural extension would be to examine cases in which consumers are willing to internalize part of the external damages they generate. In such cases, the government could provide the appropriate information in order to convince the public to, at least partially, internalize the environmental cost. Another very interesting extension would be to examine private firms's incentive to provide information timely, given that there might be uncertainty about health and environmental damages.

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