

Emission Permit Trading Between Imperfectly Competitive Product Markets

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Abstract The present paper analyzes the efficiency of emission permit trading between two imperfectly competitive product markets. Even if firms are price takers in permit markets, the integration of permit markets can decrease welfare because of imperfect competition in product markets. If there is asymmetric information between the regulator and firms, the integration of the permit markets could have a positive effect related to the flexibility of an integrated market; this flexibility can justify integrating the permit markets.

Keywords Emission permit market · Demand uncertainty · Imperfect competition · Market integration

JEL Classification D82 · D43 · L13

1 Introduction

With the implementation of the European Union Emission Trading Scheme (EUETS) and the Kyoto protocol, emission permit markets are used at an unprecedented scale to regulate externalities, and it is likely that they will play a key role in any future international agreement concerning greenhouse gas emissions. The EUETS covers several sectors in all European countries, and among the output markets covered, some are concentrated and considered imperfectly competitive; this is the case of several geographically isolated electricity markets (Wolfram 1999; Newbery 2002; DG COMP 2007). Theoretically, if markets are perfectly competitive, a unique global permit market that covers all polluting activities would be efficient to allocate an aggregate emissions level. If markets are not perfect, however, and if firms enjoy market power, several permit markets may be more efficient than an integrated one.

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This paper analyzes the efficiency of an integrated permit market between two imperfectly competitive output markets. Firms are assumed to be price takers in the permit market but engage in Cournot competition in output markets. We examine two outputs that are non-substitutable with a fixed total quantity of emissions to determine whether this constraint is better allocated with two isolated permit markets or with an integrated one. After having established that the integration of permit markets could reduce welfare, asymmetric information about output demands between a benevolent regulator and firms is introduced to analyze the choice between the two options. Although a perfectly informed regulator can always outperform an integrated permit market with two isolated ones, this is not always the case when asymmetric information is introduced. However, let us first review the connected literature.

The inefficiency of permit trading that arises in the present paper is indirect and is related to imperfect competition in output markets but not in the permit market. The inefficiency of permit trading due to imperfect competition in the permit market itself, sometimes combined with imperfect competition in the output market, has been previously analyzed (Hahn 1984; Misiolek and Elder 1989). In such a case, the initial allocation of permits to a firm influences the outcome, and Eshel (2005) has analyzed how these initial allocations should be set to maximize the efficiency of the permit market. In the present paper, strategic manipulations of permit markets are not considered, and the discussed allocation is not the initial allocation to a particular firm in a permit market but rather the allocation of an emission cap between two output markets that could be done either with an integrated permit market or with two isolated ones.

Indeed, because of the assumption of perfectly competitive permit markets, the equivalence between tax and quantity regulation holds as long as asymmetric information is not considered (Weitzman 1974), and the inefficiency of an integrated permit market could be paralleled with the inefficiency of a Pigouvian tax to regulate a monopoly. In such a case, the optimal tax is lower than marginal environmental damage, and the difference could be interpreted as a subsidy to indirectly—and imperfectly—correct the exercising of market power in the output market (Barnett 1980).¹ Because the subsidy is specific to output demand and the firm's costs, discriminatory taxes should be used to regulate heterogeneous firms (Innes et al. 1991; Requate 1993). For similar reasons, an integrated perfectly competitive permit market that equalizes the permit price across firms might be less efficient than command and control or isolated permit markets when firms are strategic and heterogeneous.

The possible inefficiency of a perfectly competitive permit market when output markets are imperfectly competitive has been previously established by Malueg (1990), Hung and Sartzetakis (1998), and Sartzetakis (2004). These three articles are the closest to the present one. Malueg (1990) examined a Cournot oligopoly of heterogeneous producers; he did not explicitly model the permit market; instead, assuming that permit trading lowers the marginal net production costs of all producers, he showed that permit trading, even if it increases output, could reduce welfare by reallocating production from an efficient producer to an inefficient one. Sartzetakis (2004) considered a duopoly; he explicitly modeled firms' choice of emissions and established under which conditions permit trading enhances or reduces welfare. Hung and Sartzetakis (1998) studied permit trading between a monopoly output market and a competitive one; they established that with a permit market, the monopoly emits less than the optimal quantity. Similar to the research by these authors, the present paper analyzes the consequences of allowing permit trading between imperfectly competitive firms, finding that

¹ Barnett (1980) analyzes the case of a monopoly and established that the subsidy is equal to the price cost margin times the rate of change of output with respect to the regulated input. In oligopoly the optimal tax could actually be larger than marginal damage if firms are heterogeneous (Simpson 1995), or at a long-run equilibrium with free entry (Katsoulacos and Xepapadeas 1995).

even though the market configuration is different, the inefficiency of permit trading is fundamentally similar. We establish here how this inefficiency could be corrected by a subsidy of emission permits for firms of one output market, and above all, we deepen the analysis of the policy implications by introducing asymmetric information between a benevolent regulator and firms.

In the three articles mentioned, the policy implications were not fully derived. Given the possible inefficiency of permit trading outlined in these articles, one can deduce that a regulator should set initial allocations and keep permit markets isolated to maximize welfare. However, this solution requires the regulator to be perfectly informed while, as mentioned by [Hung and Sartzetakis \(1998, p.45\)](#), “the implementation of a tradable permit market has no information requirement”. To understand the benefits of permit trading fully, information asymmetry should be introduced. We consider here that firms have better information than the regulator about output demands. The regulator has to decide *ex ante* whether to integrate permit markets, and if he keeps permit markets isolated, he is likely to misallocate the emissions cap because of uncertainty about the outputs demands. The comparison of the two options in a quadratic setting allows us to isolate the effect of information asymmetry. It is shown that when information asymmetry is introduced, the integration of permit trading could enhance welfare. Furthermore, in the case of an integrated market, welfare increases as the risks of the output demands become more negatively correlated. These results highlight the fundamental role of markets as the information ‘processors’ described by [Hayek \(1945\)](#). According to him, “We need decentralization because only thus can we ensure that the knowledge of the particular circumstances of time and place will be promptly used” ([Hayek 1945, p. 534](#)). However, asymmetric information does not always favor the integration of permit markets, and a particular case where it does not is exhibited.

Finally, these results are close to those obtained on decentralization in a context of asymmetric information and strategic behavior. Concerning the design of the EUETS, [Malueg and Yates \(2009\)](#) analyzed the decentralization process of the allocation of allowances between trading and non-trading sectors within each Member State. In their framework, firms are not strategic, but states are. Decentralization not only allows states to use their private information about abatement costs but also allows them to allocate emissions rights strategically; the trade-off between these two effects determines whether centralization is preferred to decentralization.

Let us begin to introduce the model with general specifications (Sect. 2) and analyze the effect of market power within this framework (Sect. 3). Then, asymmetric information is introduced in a simpler version with Leontief technologies in order to analyze the choice of the regulator (Sect. 4).

2 The Model

2.1 Set up

Let us consider two polluting product markets indexed $i = 1, 2$, with inverse demand functions $P_i(Q_i)$, $i = 1, 2$, where Q_i is the aggregate quantity of good i produced. The gross surplus from consumption of good i is $S_i(Q_i)$ with $S'_i(Q_i) = P_i$. For each $i = 1, 2$ the function P_i is non-negative, decreasing, and twice differentiable; it satisfies $P'_i + Q_i P''_i < 0$ when positive. The last assumption signifies that the price function is not too convex. It implies that quantities are strategic substitutes and ensures existence and uniqueness of Cournot equilibrium when firms have convex cost ([Novshek 1985; Amir 2005](#)).

On market $i = 1, 2$, there are n_i firms that produce the good. Emissions are modeled as an input for the production of the good; the individual cost of production is $C_i(q_i, e_i)$ where q_i and e_i are quantities of output produced and emissions used by an individual firm. The following assumptions are made on the cost function:²

$\forall q_i \geq 0, e_i \geq 0$:

- it is worth producing: $C_i(0, e_i) = 0$ and $P_i(0) > \partial C_i(0, e_i)/\partial q_i$;
- costs are increasing and convex with respect to output quantity: $\partial C_i/\partial q_i > 0, \partial^2 C_i/\partial q_i^2 > 0$;
- costs are decreasing and convex with respect to emissions: $\partial C_i/\partial e_i < 0$ and $\partial^2 C_i/\partial e_i^2 \geq 0$;
- and $\partial^2 C_i/\partial q_i^2 \cdot \partial^2 C_i/\partial e_i^2 - (\partial^2 C_i/\partial q_i \partial e_i)^2 \geq 0$.

The last assumption ensures that the gross cost of a firm is convex (cf [Appendix A](#)) by limiting the effect of emissions on marginal cost.³

On each market $i = 1, 2$ an emission permits market is implemented, and the local price of emission is denoted as σ_i , hence a firm on market i that produces q_i with e_i emissions has a profit:

$$\pi_i = P_i(Q_i)q_i - C_i(q_i, e_i) - \sigma_i e_i. \tag{1}$$

The quantity of emissions in market $i = 1, 2$ is E_i and the aggregate quantity of emissions available for both sector is \bar{e} . The quantity \bar{e} is assumed fixed and the issue is the allocation of this cap. Environmental damage is not explicitly introduced because it is assumed to only depend on the aggregate quantity of emissions and not its allocation.

On each output market, all equilibria considered are symmetric, so quantities of output and emissions are equally distributed among firms:⁴ individual quantities are Q_i/n_i and E_i/n_i . Welfare is the sum of surpluses net of production costs on both polluting sectors:

$$W(Q_1, Q_2, E_1, E_2) = \sum_i [S_i(Q_i) - n_i C_i(Q_i/n_i, E_i/n_i)]. \tag{2}$$

Initial allocation to a market $i = 1, 2$ is denoted \hat{E}_i : it is the quantity of permits available to a market if there is no trade of permits across markets.

If firms are price takers in both output and permit markets, for any initial distribution of permits among firms, the integration of permit markets always enhances welfare ([Montgomery 1972](#)). If the regulator does not perfectly know each market characteristic such as costs, demand, and emission rates an integrated permit market minimize the cost of reaching a given emission cap.

² I do not introduce upper bound on emissions to ensure that at equilibria considered the emissions constraints are always binding.

³ These assumptions are satisfied for the two common specifications:

1. $C(q, e) = c(e/q)q$ with $c' < 0, c'' > 0$,
2. $C(q, e) = q + c(q - e)$ with $c' > 0, c'' > 0$ for $q > e$.

The first specification represents the choice of a technology: a firm can lower its emission rate (e/q) by increasing its marginal cost. And the second assumes separability between production and abatement ($q - e$), so there is a technology to produce unitary abatement or emissions permits, such as clean development mechanisms or carbon sequestration.

⁴ I do not consider that the regulator can discriminate among firms in a sector by allocating different quantities of permits. Even if firms are symmetric this could increase welfare as established by [Amir and Nannerup \(2005\)](#).

3 Imperfect Competition

In output markets, firms engage in Cournot competition by strategically choosing output quantities, whereas in the permit markets, they are price takers. Let us first describe the equilibrium with two isolated permit markets before considering the optimal allocation of emissions among markets. We then show that an integrated permit market allocates emissions differently and can therefore decrease welfare.

Assumptions regarding price and cost functions ensure the existence of a unique symmetric Cournot equilibrium for any permit price σ_i (see [Appendix A](#)) on each market. At this equilibrium, all firms produce the same quantity of output and consume the same quantity of emissions. The total quantity produced can therefore be written as a function of total emissions $Q_i^C(E_i)$; it is the unique solution of:

$$P_i + P'_i \frac{Q_i}{n_i} = \frac{\partial C_i}{\partial q} (Q_i/n_i, E_i/n_i), \quad i = 1, 2. \tag{3}$$

And for a local price $\sigma_i, i = 1, 2$, the demand for permits of firms of market i is $E_i^C(\sigma_i)$, which solves the following:

$$\sigma_i = -\frac{\partial C_i}{\partial e_i} \left(Q_i^C \left(E_i^C(\sigma_i) \right) / n_i, E_i^C(\sigma_i) / n_i \right), \quad i = 1, 2. \tag{4}$$

As firms are price takers in the permit market; the initial distribution of permits between firms does not influence the market outcome. Therefore, it is equivalent to consider that the regulator gives an allocation \hat{E}_i/n_i to each firm or an aggregate amount of \hat{E}_i to all firms on market i , which is allocated between firms by the emission permit market. At equilibrium the permit price is such that $\hat{E}_i = E_i^C(\sigma_i)$.

The optimal allocation of emissions denoted (E_1^*, E_2^*) solves the following:

$$\max_{E_1, E_2} W(Q_1^C(E_1), Q_2^C(E_2), E_1, E_2) \text{ subject to } E_1 + E_2 \leq \bar{e}.$$

On each market $i = 1, 2$ an additional emission increases the local net surplus by:

$$\left(P_i - \frac{\partial C_i}{\partial q_i} \right) \frac{\partial Q_i^C}{\partial E_i} - \frac{\partial C_i}{\partial e_i}. \tag{5}$$

The second term is the direct increase of surplus related to the decrease of production costs, and the first term is an indirect effect related to market power. This indirect effect is composed of two factors: the price-marginal cost difference and the sensitivity of production to emissions. In addition to decreasing costs, an additional permit increases production and because of market power, this has a strictly positive effect. [Barnett \(1980\)](#) has analyzed this indirect effect when studying the optimal taxation of a pollutant monopoly, for a symmetric Cournot oligopoly, using Eq. (3), the price-marginal cost margin is $-P'_i Q_i/n_i$ which is decreasing with respect to the number of firms. If interior, the optimal allocation of emissions satisfies the first order condition,

$$\left(P_1 - \frac{\partial C_1}{\partial q_1} \right) \frac{\partial Q_1^C}{\partial E_1} - \frac{\partial C_1}{\partial e_1} = \left(P_2 - \frac{\partial C_2}{\partial q_2} \right) \frac{\partial Q_2^C}{\partial E_2} - \frac{\partial C_2}{\partial e_2}, \tag{6}$$

which signifies that marginal net surpluses are equalized between sectors. Let us denote the difference of indirect effects, from 6 by using 3:

$$s^* = -P'_1 \frac{Q_1^C(E_1^*)}{n_1} \frac{\partial Q_1^C}{\partial E_1} + P'_2 \frac{Q_2^C(E_2^*)}{n_2} \frac{\partial Q_2^C}{\partial E_2}. \tag{7}$$

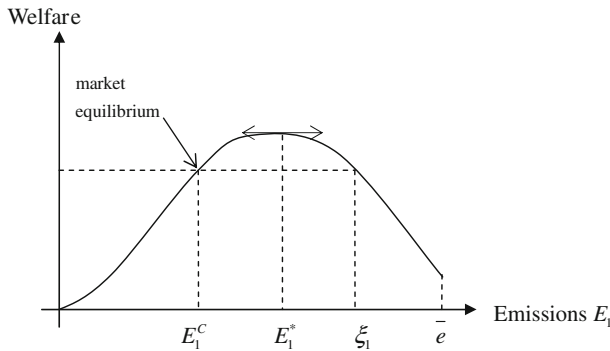


Fig. 1 Welfare with respect to market 1 emissions E_1 . E_1^C is the integrated market equilibrium, ξ_1 is defined by $W(\xi_1) = W(E_1^C)$

Proposition 1 *If $s^* \neq 0$, welfare is strictly lower with an (uncorrected) integrated permit market than it is with two isolated markets with initial allocations $\hat{E}_i = E_i^*$, $i = 1, 2$. A perfectly informed regulator perform better than an integrated emission permit market.*

Proof With an integrated permit market, local permit prices are equalized: $\sigma_1 = \sigma_2$, and the equilibrium price σ clears the permit market: $E_1^C(\sigma) + E_2^C(\sigma) = \bar{e}$. At this equilibrium, the marginal values of emissions for each firm are equalized across output markets:

$$-\frac{\partial C_1}{\partial e_1}(Q_1^C(E_1)/n_1, E_1/n_1) = \sigma = -\frac{\partial C_2}{\partial e_2}(Q_2^C(E_2)/n_2, E_2/n_2).$$

If $s^* \neq 0$, the market allocation does not satisfy (6) and welfare is lower than with the allocation (E_1^*, E_2^*) . □

Even if firms are price takers in permit markets, the integration of these markets does not increase welfare in general. The benefit of integrating permit markets depends on initial allocations \hat{E}_i . This is illustrated in Fig. 1 where welfare, assumed quasi-concave, is represented with respect to E_1 , while $E_2 = \bar{e} - E_1$. If the initial allocations are close to the optimal ones, i.e. $\hat{E}_1 \in [E_1^C, \xi_1]$ in Fig. 1, the integration of permit markets decreases welfare, but if the initial allocations depart sufficiently from the optimal ones, i.e. $\hat{E}_1 < E_1^C$ or $\hat{E}_1 > \xi_1$ in Fig. 1, integration could be beneficial. The issue of permit markets integration thus boils down to the analysis of the choice of initial allocations. These initial allocations can be sub-optimal if the regulator lacks information about the output markets' conditions when setting allocations. Before analyzing this situation, let us first analyze how the regulator can correct the integrated permit market.

Instead of allocating emissions to each sector and keeping markets isolated, the regulator can use a price instrument to correct the integrated market and implement the optimal allocation.

Corollary 1 *The optimal allocation can be established with an integrated permit market if permits in market 1 are subsidized; the subsidy is then s^* .*

Proof With a subsidy s on permits in market 1, the price of permits faced by firms in market 1 is $\sigma - s$ and by firms in market 2 it is σ , at equilibrium the price σ clears the market: $E_1^C(\sigma - s) + E_2^C(\sigma) = \bar{e}$ and:

$$\frac{\partial C_1}{\partial e_1} = \frac{\partial C_2}{\partial e_2} + s.$$

For any s there is a unique allocation that satisfies this equation (uniqueness is mainly due to the strict convexity of cost with respect to e_i for $i = 1, 2$). For a subsidy s^* , the allocation (E_1^*, E_2^*) satisfies this equation (cf Eq. 6), so s^* decentralizes the optimum. \square

The subsidy s^* reflects the difference in market power between the sectors. This subsidy is an indirect way to correct market power, which is less efficient than a direct subsidy on production. More generally, if the regulator cannot directly correct market power by subsidizing production, he can do it indirectly by subsidizing inputs and ‘distorting’ the input markets. The greater the number of subsidized inputs, the more efficiently market power is corrected. Here, emissions are the subsidized input, and a local subsidy would be $-P'_i Q_i / n_i \partial Q_i^C / \partial E_i$ in each market $i = 1, 2$. The regulator can either set such a subsidy on each market or only a subsidy s^* on market 1 because the only relevant variable is the difference between the two subsidies. If s is the difference between market 1’s permit price and the one of market 2, there is a unique allocation that satisfies $\partial C_1 / \partial e_1 = \partial C_2 / \partial e_2 + s$, no matter how s is implemented. Consequently, there are infinite corrective policies that could be implemented, as the only relevant feature of such a policy is that the price of emissions faced by firms in market 1 and market 2 are different and the difference is equal to s^* . This indeterminacy is due to the use of a partial equilibrium approach, with a slightly more general framework this would not hold.

Which market is underemitting with an uncorrected integrated permit market depends on the extent of market power and the relationship between production and emissions. For instance, [Hung and Sartzetakis \(1998\)](#) considered the case of a monopoly in a market, $n_1 = 1$, and competitive firms in the other, $n_2 = +\infty$. In that case $s^* > 0$ and the monopoly’s permits should be subsidized to indirectly correct its tendency to underproduce.

4 Imperfect Information

As stated by proposition 1, integrating emission permit markets can decrease welfare because of market imperfections in the output markets. A perfectly informed regulator is able to perform better than an integrated market by initially allocating the constraint and keeping the permit markets isolated. However, if the regulator lacks information about the market conditions, he is unable to set optimal allocations and permit markets integration might improve efficiency despite the presence of imperfect competition. To obtain explicit formulas, we develop a quadratic framework with additive uncertainty to analyze the appeal of the permit markets’ integration.

The approach used is similar to the one developed by [Weitzman \(1974\)](#) to compare price and quantity regulatory instruments. It is assumed that when firms trade, they have better information than the regulator when he designs the permit market. Previous notations are extended to account for imperfect information. The regulator ignores some characteristics θ_1 and θ_2 when designing the permit market that firms know when producing and trading permits. In market $i = 1, 2$, the gross consumer surplus is $S_i(Q_i, \theta_i)$, where θ_i is a random parameter with $\mathbb{E}\theta_i = 0$; here, uncertainty is assumed to be additive,

$$S_i(Q_i, \theta_i) = (a_i + \theta_i)Q_i - \frac{b_i}{2}Q_i^2 \quad (8)$$

and the costs are

$$C_i(q_i, e_i) = \begin{cases} 0 & \text{if } q_i \leq e_i \\ 0.5c_i(q_i - e_i)^2 & \text{otherwise} \end{cases} \quad \text{for } i = 1, 2; \tag{9}$$

An explanation for this cost function is that the production cost, net of abatement costs, is null and the emission rate of the production process is 1, but a firm can reduce its emissions by $q_i - e_i$ at a quadratic cost. There is an abatement technology that is separable from the production process, such as carbon sequestration. This specification has been used elsewhere (e.g., [Montero 2002](#); [Sartzetakis 2004](#)), it is highly tractable and it enables the disentangling of the effects at stake. However, the following analysis could be reproduced with a more general quadratic cost function.⁵

In each output market $i = 1, 2$, production and demand for permits are contingent on θ_i ; the total production $Q_i^C(E_i, \theta_i)$ solves (3) with the price function $P_i(Q_i, \theta_i) = a_i + \theta_i - b_i Q_i$. If $a_i + \theta_i$ is sufficiently high,⁶ the production is

$$Q_i^C(E_i, \theta_i) = \frac{n_i(a_i + \theta_i) + c_i E_i}{(n_i + 1)b_i + c_i}. \tag{10}$$

The sensitivity of production to emissions is independent of the random market condition and is denoted as α_i :

$$\alpha_i = \frac{\partial Q_i^C}{\partial E_i} = \frac{c_i}{(n_i + 1)b_i + c_i}. \tag{11}$$

And the demand for permits $E_i^C(\sigma_i, \theta_i)$ solves (4), i.e., $c_i(Q_i^C - E_i^C)/n_i = \sigma_i$ so

$$E_i^C(\sigma_i, \theta_i) = \frac{1}{\beta_i} [\alpha_i(a_i + \theta_i) - \sigma_i]. \tag{12}$$

where

$$\beta_i = \frac{(n_i + 1)b_i}{n_i} \alpha_i. \tag{13}$$

Welfare in a particular state (θ_1, θ_2) can be written as a function of emissions:

$$W(E_1, E_2, \theta_1, \theta_2) = \sum_{i=1,2} S_i(Q_i^C(E_i, \theta_i), \theta_i) - n_i C_i(Q_i^C(E_i, \theta_i)/n_i, E_i/n_i). \tag{14}$$

Parameters θ_1, θ_2 can be interpreted in several ways. First, with additive uncertainty and quadratic specifications, they can encompass uncertainties about marginal consumer surpluses or marginal costs. More precisely, the formulation is equivalent to a formulation that would include an additive uncertainty in marginal production cost, θ_i would be the difference between demand and cost uncertainties. Second, the relevant feature of the model is that when firms decide how much to produce, they have more information than the regulator when he designs the policy. Consequently, the parameter θ_i represents either asymmetric information between the regulator and firms or uncertainties about future market conditions due to the

⁵ With quadratic costs of the form $C_i(q, e) = c_i q_i^2 - \gamma_i q_i e_i + \delta_i e_i^2$, with $c_i \delta_i > \gamma_i^2$, α_i defined by (11) should be replaced by $\alpha'_i = \gamma_i / (c_i + b_i(n_i + 1))$ and β_i defined by (13) should be replaced by $\beta'_i = [b_i(n_i + 1) \frac{\delta_i}{\gamma_i} + (\delta_i c_i / \gamma_i - \gamma_i)] / n_i$. The expression (19b) is unchanged but the expression (19a) becomes $\alpha'_i + \alpha_i^2 b_i / \gamma_i$. The calculations done on welfare are unchanged, the expressions (28) and (31) of I still hold; and the propositions 2 to 4 and the lemma 1 are still valid.

⁶ If the quantity of emissions is negative and lower than $n_i(a_i + \theta_i)/c_i \leq -E_i$ it is not worth producing. And if the quantity of emissions is higher than the unregulated production $n_i(a_i + \theta_i)/(n_i + 1)$, firms produce this quantity (the unconstrained Cournot production).

time lag between the design of the permit markets and market interactions. Both are relevant and are simultaneously at stake for the EU ETS: it is reasonable to assume that firms have superior knowledge of market conditions to that held by the regulator and that future trends of sectors covered are uncertain when the regulator designs emission permit markets.

4.1 Regulatory Options

The regulator does not know the value of random parameters θ_i when deciding whether to implement an integrated market for emission permits. If he keeps markets isolated, he has to determine the allocation to each, and if he implements an integrated permit market, he can set a subsidy ex ante. As it is unlikely that such a subsidy will be implemented, the option of an integrated market without subsidy is also considered.

The three regulatory options are as follows:

1. The regulator allocates emissions \hat{E}_i to firms of market $i = 1, 2$ and emission permit markets are isolated so that for all θ_i production is $Q_i^C(\hat{E}_i, \theta_i)$ and the expected welfare is

$$\hat{W} = \mathbb{E} \left[W(\hat{E}_1, \hat{E}_2, \theta_1, \theta_2) \right] = \max_{E_1, E_2} \mathbb{E} [W(E_1, E_2, \theta_1, \theta_2)]. \tag{15}$$

2. Emission permit markets are integrated, and a subsidy s on market 1 emissions is fixed ex ante. Firms choose production and emissions once θ_1, θ_2 are revealed, the permit price $\sigma(\theta_1, \theta_2, s)$ clears the market for emissions:

$$E_1^C(\sigma - s, \theta_1) + E_2^C(\sigma, \theta_2) = \bar{e} \tag{16}$$

For any s , expected welfare is

$$W_I(s) = \mathbb{E} \left[W(E_1^C, E_2^C, \theta_1, \theta_2) \right], \tag{17}$$

and the regulator sets s to maximize $W_I(s)$.

3. Emission permit markets are integrated and no subsidy is set so expected welfare is $W_I(0)$.

Assumptions are required on the parameters to ensure that in all regulatory options considered and in all states, the emission constraint is binding and both goods are produced. Those conditions (listed in [Appendix B](#)) consist of mainly that a_i ($i = 1, 2$) are sufficiently high for the emission constraint to be binding and are similar enough so that both goods are produced. The support of θ_i ($i = 1, 2$) should be restricted to ensure that these conditions are satisfied in all demand states.

4.2 Comparison of Regulatory Options

To begin this section, let us first consider what a perfectly informed regulator would do. With expression (10) of output quantities, the local net surplus can be expressed in a quadratic reduced form as a function of emissions:

$$S_i(Q_i^C, \theta_i) - C_i(Q_i^C, E_i) = (A_i(a_i + \theta_i) - 0.5B_i E_i) E_i + K_i, \tag{18}$$

where

$$A_i = \alpha_i + \alpha_i^2 \frac{b_i}{c_i}, \tag{19a}$$

$$B_i = \beta_i - \alpha_i^2 \frac{b_i}{n_i}, \tag{19b}$$

$$K_i = \left[a_i - 0.5(b_i + c_i) Q_i^C(0, \theta_i) \right] Q_i^C(0, \theta_i). \tag{19c}$$

Coefficients A_i and B_i encompass the social value of emissions due to both the reduction of production costs and the correction of market power; the marginal social value of a permit in market $i = 1, 2$ is:

$$\begin{aligned} A_i(a_i + \theta_i) - B_i E_i &= \left[-\frac{\partial P_i}{\partial Q_i} \frac{Q_i^C}{n_i} \right] \frac{\partial Q_i^C}{\partial E_i} - \frac{\partial C_i}{\partial e_i} \\ &= \alpha_i b_i \frac{Q_i^C}{n_i} + [\alpha_i(a_i + \theta_i) - \beta_i E_i]. \end{aligned} \tag{20}$$

The effect of market power is reflected in the first term of the right-hand side of the equation; this term represents the difference between firms perceived marginal benefits from a permit and the social marginal benefit and explains the difference between A_i, B_i and α_i, β_i . The ex post optimal allocation that satisfies (6) in any states (θ_1, θ_2) is, from (20),

$$E_i^*(\theta_1, \theta_2) = \frac{A_i(a_i + \theta_i) - A_j(a_j + \theta_j) + B_j \bar{e}}{B_1 + B_2}, \quad i, j = 1, 2, i \neq j. \tag{21}$$

This is the allocation that would be set by a perfectly informed regulator, the expected welfare obtained in this case is

$$W^* = \mathbb{E} [W(E_1^*, E_2^*, \theta_1, \theta_2)]. \tag{22}$$

Let us now turn to the problem of the imperfectly informed regulator. In the first regulatory option, the regulator allocates emissions \hat{E}_i to firms of market $i = 1, 2$. The regulator maximizes the expected welfare, so he sets an allocation that satisfies

$$\mathbb{E} \left[-\frac{\partial P_1}{\partial Q_1} \frac{Q_1^C}{n_1} \frac{\partial Q_1^C}{\partial E_1} - \frac{\partial C_1}{\partial e_1} \right] = \mathbb{E} \left[-\frac{\partial P_2}{\partial Q_2} \frac{Q_2^C}{n_2} \frac{\partial Q_2^C}{\partial E_2} - \frac{\partial C_2}{\partial e_2} \right]$$

so from expression (20),

$$\hat{E}_i = \frac{A_i a_i - A_j a_j + B_j \bar{e}}{B_i + B_j}, \quad i, j = 1, 2, j \neq i. \tag{23}$$

With this option, the allocation of emissions is independent of realized market conditions. There is a welfare loss associated with the lack of flexibility of this option, which is the difference between \hat{W} and W^* , from (18):

$$W^* - \hat{W} = \frac{B_1 + B_2}{2} \mathbb{E} \left(\hat{E}_1 - E_1^* \right)^2 = \frac{\mathbb{E} \left[(A_1 \theta_1 - A_2 \theta_2)^2 \right]}{2(B_1 + B_2)}, \tag{24}$$

the right-hand-side term represents the value of information for the regulator with the option of two isolated permit markets. As we shall see, part of this value could be recovered with an integrated permit market.

With the second and third options, an integrated permit market is implemented. With these options, the allocation is conditional on random market conditions: when production and emissions are chosen parameters $\theta_i, i = 1, 2$ are known⁷ and the allocation of emissions is determined via the permit price. If a subsidy s is set on emissions in market 1, from the permit market-clearing Eq. (16) and from (12), the equilibrium quantity of market 1 emissions is

$$E_1^C(\theta_1, \theta_2, s) = \frac{1}{\beta_1 + \beta_2} [\alpha_1 (a_1 + \theta_1) + s - \alpha_2 (a_2 + \theta_2) + \beta_2 \bar{e}]$$

$$= \mathbb{E} [E_1^C] + \frac{\alpha_1 \theta_1 - \alpha_2 \theta_2}{\beta_1 + \beta_2}. \tag{25}$$

This allocation is flexible and varies with output demands; the subsidy only modifies the expected allocation and not its variable component. The comparison of expected welfare between this option and the optimal case is

$$W^* - W_I(s) = 0.5(B_1 + B_2)\mathbb{E} \left(E_1^C - E_1^* \right)^2. \tag{26}$$

With the quadratic framework, it is possible to isolate the influence of the variable part of the allocation (25) and with the expressions of welfare (24) and (26) and emissions (21), (23) and (25):

$$W_I(s) - \hat{W} = 0.5 (B_1 + B_2) \mathbb{E} \left[\left(\hat{E}_1 - E_1^* \right)^2 - \left(E_1^C - E_1^* \right)^2 \right]$$

$$= I - 0.5 (B_1 + B_2) \left(\mathbb{E} [E_1^C] - \hat{E}_1 \right)^2, \tag{27}$$

where

$$I = \frac{B_1 + B_2}{2} \mathbb{E} \left[\left(\frac{A_1 \theta_1 - A_2 \theta_2}{B_1 + B_2} \right)^2 - \left(\frac{\alpha_1 \theta_1 - \alpha_2 \theta_2}{\beta_1 + \beta_2} - \frac{A_1 \theta_1 - A_2 \theta_2}{B_1 + B_2} \right)^2 \right]. \tag{28}$$

I in (27) and (28) represents the value of the *adaptation* of the allocation to θ_1 and θ_2 with an integrated permit market (options 2 and 3). It is the difference between the gains from an optimal adaptation and the loss due to the suboptimality of the market one. It encompasses the influence of asymmetric information on the comparison of an integrated market and two separate ones; it is the part of the value of information that could be recovered with an integrated market.

Proposition 2 *With asymmetric information:*

- (i) *with a corrective subsidy, expected welfare is greater with permit markets integration (option 2) than it is with two permit markets (option 1) if and only if $I \geq 0$;*
- (ii) *without any corrective subsidy, expected welfare is greater with permit markets integration (option 3) than it is with two permit markets (option 1) if and only if*

$$(B_1 + B_2) \left(\mathbb{E} [E_1^C] - \hat{E}_1 \right)^2 \leq 2I. \tag{29}$$

Proof (i) With the second option, the regulator sets s to maximizes $W_I(s)$. The subsidy influences the expected allocation ($\mathbb{E}E_1^C, \mathbb{E}E_2^C$) and not its variable components.

⁷ On each market $i = 1, 2$, firms know θ_i but not necessarily $\theta_j, j \neq i$. However, this information can be inferred from the permit price.

With the comparison (27), $W_I(s)$ is maximized for s such that $\mathbb{E}E_1^C = \hat{E}_1$ and for this subsidy $W_I(s) = \hat{W} + I$.

- (ii) From expression (27) of the difference of expected welfare. □

This proposition stresses the two effects at stake when evaluating market integration. On the one hand there is a welfare loss related to market power; on the other hand, there is a possible welfare gain related to uncertainty and the flexibility of an integrated market. The first effect can be canceled by a subsidy that ensures that the average market allocation is equal to the average optimal one, and the second effect could justify market integration. In a context of asymmetric information, the integration of permit markets can enhance welfare because it ensures that the allocation of the emissions cap is flexible and depends on information that firms have. However, the value of this flexibility, represented by I , is not necessarily positive and the rest of this section is devoted to the analysis of its sign and its monotocity properties. To proceed it is useful to introduce the ratio of the optimal variation to the market one:

$$u_i = \frac{A_i}{B_1 + B_2} \frac{\beta_1 + \beta_2}{\alpha_i}, i = 1, 2. \tag{30}$$

These ratios are larger than one ($u_i > 1$ as $A_i > \alpha_i$ and $B_i < \beta_i$) because the optimal allocation is more sensitive than the market allocation to a change in output demand. This difference of sensitivity is related to the price-marginal cost markup. If an output demand rises, both this markup and the effect of emissions on cost increase; the optimal allocation is influenced by both these changes (cf. 5) whereas firms' demand for permits, and consequently the market allocation, is solely influenced by the latter. Replacing these ratios into the expression (28) of I gives

$$I = \frac{B_1 + B_2}{2(\beta_1 + \beta_2)^2} \mathbb{E} \{ (\alpha_1 \theta_1 - \alpha_2 \theta_2) [\alpha_1 (2u_1 - 1) \theta_1 - \alpha_2 (2u_2 - 1) \theta_2] \}. \tag{31}$$

This expression illustrates that even if they are not optimal, the changes of the allocation of permits could enhance welfare. The sign of the braced term points out whether in a particular demands state (θ_1, θ_2) the variation of the market allocation enhances welfare. For instance, if the demand for good 1 is higher than expected while the other is lower than expected ($\theta_1 \geq 0$ and $\theta_2 \leq 0$), there are more permits allocated to the production of good 1 and there is an efficiency gain from this change (the braced term is positive). This observation gives the intuition of the results of the following proposition.

Proposition 3 *I decreases with respect to the correlation of demand states, and if this correlation is negative I is positive and increases with respect to the variance of each state.*

Proof By expanding the expression (31) of I :

$$I \frac{(\beta_1 + \beta_2)^2}{B_1 + B_2} = \sum_{i=1,2} \alpha_i^2 \left(u_i - \frac{1}{2} \right) \text{var} \theta_i - \alpha_1 \alpha_2 (u_1 + u_2 + 1) \text{cov}(\theta_1, \theta_2).$$

The correlation of θ_1 and θ_2 is $\text{cov}(\theta_1, \theta_2) / (\text{var} \theta_1 \text{var} \theta_2)^{0.5}$. Therefore, I is decreasing with respect to the correlation, and if this correlation is negative, I increases with respect to the variance of $\theta_i, i = 1, 2$. □

When output demands are negatively correlated and possibly independently distributed, I is positive. In such a case, a change in output demands induces a reallocation of permits that

is welfare improving even if not optimal. Furthermore, more negatively correlated demand states lead to a larger value of I . When output demands are negatively correlated, a large demand in one market ($\theta_i > 0$) is likely to be associated with a low demand in the other market ($\theta_j < 0, j \neq i$), in such a case there is much to gain from reallocating permits to the flourishing sector. I also increases with respect to the variance of either demand. An increase of the variance could be interpreted as an increase in the regulator’s uncertainty, and the more he is uncertain, the more he will benefit from an integrated market for allocating permits.

The issue is more complicated if demand states are positively correlated, a situation which is likely to occur if uncertainties are related to macroeconomic conditions that influence output demands in similar ways. If both output demands are shown to be high ($\theta_1 > 0; \theta_2 > 0$), ideally, it is the sign of the weighted difference $A_1\theta_1 - A_2\theta_2$ that determines whether output 1 needs more permits. However, the permit market allocates permits according to the difference $\alpha_1\theta_1 - \alpha_2\theta_2$, which could have the opposite sign. In such a case, the adaptation of the allocation with an integrated market reduces welfare. This divergence is indeed rooted in the fact that firms’ demand for permits is only determined by the effect of permits on costs, whereas the optimal allocation is related not only to this effect but also to the market-power corrective effect. Lemma 1 exhibits a range of situations where demands are positively correlated and I is negative.

Lemma 1 *If $n_1 + c_1/b_1 < n_2 + c_2/b_2$, then $u_1 > u_2$, and I is negative if $\theta_1 = x\theta_2$ with*

$$\frac{2u_2 - 1}{2u_1 - 1} \frac{\alpha_2}{\alpha_1} < x < \frac{\alpha_2}{\alpha_1}$$

Proof The condition $n_1 + c_1/b_1 < n_2 + c_2/b_2$ is equivalent to $((n_1 + 1)b_1 + c_1)/b_1 < ((n_2 + 1)b_2 + c_2)/b_2$ and from expression (11) it implies that $\alpha_1 b_1/c_1 > \alpha_2 b_2/c_2$. Then using the expressions (19a) and (30) $u_1 > u_2$. By replacing $\theta_1 = x\theta_2$ into the expression (31) of I we obtain that the sign of I is the sign of:

$$(\alpha_1 x - \alpha_2) (\alpha_1 (2u_1 - 1)x - (2u_2 - 1)\alpha_2).$$

Next, as $u_1 > u_2$ and $u_i > 1, i = 1, 2$ there is x that satisfies both inequalities in the proposition and for such x , I is negative because $\alpha_1 x - \alpha_2 < 0$ and $\alpha_1 (2u_1 - 1)x > (2u_2 - 1)\alpha_2$. □

If demand states are perfectly correlated, I can be negative if the ratio of demand states is neither too high nor too low. In such cases, there is no gain from the integration of permit markets; the flexibility of the allocation associated with an integrated market does not justify integration because the permit market makes the allocation vary inefficiently. If $n_1 + c_1/b_1 < n_2 + c_2/b_2$, when both demands increase ($\theta_1 = x\theta_2 > 0$) the permit market increases the allocation to market 2 because of cost reduction (c_2 is high compared to c_1) whereas it is socially beneficial to reduce this allocation because the correction of market power in market 1 is more important than it is in market 2 (n_1 is low compared to n_2).

With a perfect correlation, the sign of I is not determined by the amplitude of variations but by the ratio of these variations, denoted x in Lemma 1. For large or small ratios of demand variations, I is still positive, even with perfect positive correlation. In such cases, the variations of the demand of one output dominate and this is as if only one output demand were variable; I is then positive and increasing with respect to the variance of demand variations. Lemma 1 provides a step to a general result on parameters values.

Proposition 4 *I is positive for any distribution of demand states if and only if $n_1 + c_1/b_1 = n_2 + c_2/b_2$.*

Proof (\Rightarrow) Proof by contrapositive: if $n_1 + c_1/b_1 \neq n_2 + c_2/b_2$ then by lemma 1 there is a demand states distribution such that $I < 0$.

(\Leftarrow) If $n_1 + c_1/b_1 = n_2 + c_2/b_2$ then $u_1 = u_2$ (cf. proof of lemma 1) and with expression (31) of I and the fact that $u_i > 1$ (for $A_i > \alpha_i$ and $B_i < \beta_i$) $I > 0$. □

This proposition provides a condition on parameters that ensures that there is always a gain from integrating permit market whatever the distribution of demand states. The condition is restrictive, but as previously discussed even if this condition is not satisfied there are many situations with restrictions on demand states distribution where I is positive. Furthermore, as articulated in the following corollary, the condition is satisfied in the particular case where production is equal to emissions and there is no possibility of abatement but to reduce production. In this case production costs are null (resp. infinite) if production is lower (resp. larger) than emissions, that is $c_1 = c_2 = +\infty$ in (9), and the sensitivity α_i of production to emissions is 1 and by (19a) $A_i = 1$, so $u_1 = u_2$ and the variations of the allocation with a permit market enhance welfare.

Corollary 2 *With asymmetric information and production equal to emissions:*

(i) *with an optimal ex ante subsidy, markets integration always increases expected welfare and the welfare gain is :*

$$\frac{\text{var}(\theta_1 - \theta_2)}{(\beta_1 + \beta_2)^2} \left[\left(\frac{1}{2} + \frac{1}{n_1} \right) b_1 + \left(\frac{1}{2} + \frac{1}{n_2} \right) b_2 \right].$$

(ii) *without any subsidy an integrated market improves expected welfare if and only if:*

$$\text{var}(\theta_1 - \theta_2) > \frac{[b_1(a_1 - a_2 + b_2\bar{e})/n_1 - b_2(a_2 - a_1 + b_1\bar{e})/n_2]^2}{[(1 + 2n_1)b_1 + (1 + 2n_2)b_2](b_1 + b_2)};$$

Calculations are in Appendix C. In that particular case, with emissions equal to production, the total production of both outputs is fixed by the emission cap; therefore, compared to the optimal allocation, firms in one market produce too much and firms in the other do not produce enough. Market power misallocates the constraint rather than decreasing production. Both the optimal allocation and the integrated market allocation are determined by the relative marginal consumer surplus and are linear functions of the difference $\theta_1 - \theta_2$; thus, whatever the realized demand state the adaptation of the market allocation has a positive welfare effect. I is always positive and proportional to the variance of the difference of marginal consumers surplus.

The misallocation of the emission cap is related to concentration in each market and some comparative statics can be done for the particular case of Corollary 2. Consider a given state (θ_1, θ_2) , firms in one sector overproduce while firms in the other underproduce. An increase of the number of overproducing (resp. underproducing) firms reduces (resp. enhances) welfare in that particular state. The effect of such a change on expected welfare is ambiguous, however, because firms that overproduce in a state could be underproducing in another state. Some more insights can be gained, however, by decomposing the effect of such a change into its effect on I and its effect on welfare in the average state $(\theta_1 = \theta_2 = 0)$. It appears that an increase of either type of firms increases I . Therefore, an increase of the number of firms that underproduce in the average state will unambiguously enhance expected welfare,

while an increase of the number of firms that overproduce in the average state will have an ambiguous effect. Remark finally that if an ex-ante subsidy is put in place to correct for the average misallocation, an increase in the number of firms in any market enhances welfare.

5 Conclusion

This paper has analyzed how imperfect competition in two output markets influences the efficiency of an integrated market of emission permits. If output markets are imperfectly competitive, the integration of permit markets can decrease welfare. The inefficiency of an integrated permit market arises from the divergence between price and marginal cost and the sensitivity of production to emissions. This inefficiency can be corrected by a subsidy on the emissions of one sector.

A benevolent, welfare maximising, perfectly informed regulator can perform better with two isolated permit markets than he can with an uncorrected integrated market. However, it is more realistic to consider that there is asymmetric information between the regulator and firms regarding market conditions. This informational cap can justify market integration. By introducing, in a quadratic specification of the model, uncertainty about output demands the issue of permit markets integration under asymmetric information has been analyzed. In addition to the misallocation previously stressed, an integrated permit market allows firms to adapt the allocation to their information. There is a value associated to the flexibility of an integrated market and the decision of whether to integrate permit markets should be based on the comparison of this value with the average misallocation due to market power. The value of flexibility is decreasing with output demand correlation and in many cases it is positive.

This analysis showed that the integration of two imperfect systems can be counterproductive and should be accomplished carefully. The scope of an emission market can thus be limited to avoid imperfections ‘contagion’. The analysis was based on the observation of the European emission permit market but is helpful in considering the development of an international regulation of emissions. Whether an integrated international emission permit market should be preferred to several isolated submarkets is a major issue, and the present paper has provided some elements to be discussed.

Appendix A: Existence and Uniqueness of Cournot Equilibrium

Let $i \in \{1, 2\}$, I establish that for any permit price σ_i there exists a unique equilibrium which is symmetric on market i . To do so I consider the gross cost function $\Gamma(q_i, \sigma_i) = \min_{e_i} \{C_i(q_i, e_i) + \sigma_i e_i\}$.

This function is convex with respect to q_i :

$$\frac{\partial \Gamma}{\partial q_i} = \frac{\partial C_i}{\partial q_i}(q_i, e_i) \text{ and } \frac{\partial^2 \Gamma}{\partial q_i^2} = \frac{\partial^2 C_i}{\partial q_i^2} - \left(\frac{\partial^2 C_i}{\partial q_i \partial e_i} \right)^2 \left(\frac{\partial^2 C_i}{\partial e^2} \right)^{-1} \geq 0$$

Thanks to the assumption on the price function $\forall Q_i, P_i' + P_i'' Q_i < 0$ if $P_i > 0$ there is a unique Cournot equilibrium for all σ_i (Novshek 1985).

Appendix B: Conditions with Quadratic Specifications

Without Uncertainty

In an isolated market with a quantity of emissions E_i , the emissions constraint is binding if and only if $n_i a_i < (n_i + 1)b_i E_i$, and for production to be positive it must be the case that $E_i > -n_i a_i / c_i$.

In all cases considered, the emissions constraint is binding if it is lower than the aggregated unconstrained production: $\bar{e} < n_1 a_1 / (n_1 + 1)b_1 + n_2 a_2 / (n_2 + 1)b_2$. A sufficient condition is:

$$\bar{e} < 0.5(a_1/b_1 + a_2/b_2). \tag{32}$$

With an integrated and uncorrected permit market, to ensure that both sectors produce a positive quantity, the equilibrium quantity E_i^C given by (25) for $(\theta_1 = \theta_2 = 0)$ should be higher than $-n_i a_i / c_i$ i.e.:

$$\alpha_i a_i - \alpha_j a_j + \beta_j \bar{e} > -n_i a_i (\beta_1 + \beta_2) / c_i \tag{33}$$

dividing both sides by β_j , and using the relations $1/\beta_j = n_j/c_j + \alpha_j/\beta_j$ this inequality is equivalent to

$$\frac{n_j}{(n_j + 1)b_j} (a_j - a_i) \leq \bar{e} + a_i \left(\frac{n_1}{c_1} + \frac{n_2}{c_2} \right),$$

a sufficient condition is

$$-b_2 \bar{e} \leq a_1 - a_2 \leq b_1 \bar{e}. \tag{34}$$

Note that if $a_1 > a_2$ the left inequality is always satisfied and the right one is $a_1 - b_1 \bar{e} < a_2$.

With two isolated markets, \hat{E}_i should be larger than $-n_i a_i / c_i$ i.e.: $A_i a_i - A_j a_j + B_j \bar{e} > -n_i a_i (B_1 + B_2) / c_i$, with the expression of A_i and B_i (19a) and (19b), and given that (33) is satisfied, the above inequality is satisfied if:

$$\alpha_i^2 \frac{b_i}{c_i} a_i - \alpha_j^2 \frac{b_j}{c_j} a_j + \alpha_j^2 \frac{b_j}{n_j} \bar{e} > -\frac{n_i a_i}{c_i} \left(\alpha_i^2 \frac{b_i}{n_i} + \alpha_j^2 \frac{b_j}{n_j} \right),$$

and this is so if

$$\bar{e} > n_1 a_1 / c_1 - n_2 a_2 / c_2. \tag{35}$$

A condition satisfied if c_1, c_2 are sufficiently large.

With Uncertainty

With uncertainty, the distributions of demand states should be restricted to ensure that in all regulatory options emission constraints are binding and production positive.

- With the third option of an uncorrected integrated market, conditions (32) and (34) should be satisfied in all demand states:

$$\begin{aligned} \forall(\theta_1, \theta_2) : (a_1 + \theta_1)/b_1 + (a_2 + \theta_2)/b_2 &> 2\bar{e}, \\ -b_2 \bar{e} < a_1 + \theta_1 - a_2 - \theta_2 &< b_1 \bar{e}. \end{aligned}$$

- With the first option of two isolated markets, production is positive and the emission constraint is binding in all demand states if:

$$\forall \theta_i, (a_i + \theta_i)/2b_i > \hat{E}_i > -n_i(a_i + \theta_i)/c_i.$$

- And finally in the second option, the condition is:

$$\forall \theta_i, \frac{n_i}{n_i + 1} \frac{a_i + \theta_i}{2b_i} > \hat{E}_i + \frac{\alpha_i \theta_i - \alpha_j \theta_j}{\beta_1 + \beta_2} > -n_i \left(\frac{a_i + \theta_i}{c_i} \right).$$

Appendix C: Proof of Corollary 2

If emissions are equal to production $Q_i^C(E_i, \theta_i) = E_i$ and $\alpha_i = 1$. This situation corresponds to $c_1 = c_2 = +\infty$. With a subsidy s the equilibrium production can be found by replacing $\alpha_i = 1$ into the expression (25):

$$\begin{aligned} Q_1^C = E_1^C &= \frac{1}{\beta_1 + \beta_2} \left[(a_1 - a_2) + \frac{n_2 + 1}{n_2} b_2 \bar{e} + s + (\theta_1 - \theta_2) \right] \\ &= \bar{E}_1^C + \frac{\theta_1 - \theta_2}{\beta_1 + \beta_2} \end{aligned} \tag{36}$$

For a market $i = 1, 2$ local welfare is simply $(a_i - 0.5b_i E_i)E_i$ i.e. $A_i = 1$ and $B_i = b_i$. Therefore

$$I = \frac{\text{var}(\theta_1 - \theta_2)}{(\beta_1 + \beta_2)^2} \left[\beta_1 + \beta_2 - \frac{1}{2}(b_1 + b_2) \right],$$

and the difference between Cournot production and \hat{E}_i is:

$$\begin{aligned} \bar{E}_1^C - \hat{E}_1 &= \frac{1}{\beta_1 + \beta_2} \left[(a_1 - a_2) + \frac{n_2 + 1}{n_2} b_2 \bar{e} \right] - \frac{1}{b_1 + b_2} [(a_1 - a_2) + b_2 \bar{e}] \\ &= \frac{1}{(\beta_1 + \beta_2)(b_1 + b_2)} \left[\frac{b_1}{n_1}(a_2 - a_1 + b_2 \bar{e}) + \frac{b_2}{n_2}(a_1 - a_2 + b_1 \bar{e}) \right] \end{aligned}$$

Therefore, the difference between command and control and an integrated permits market without subsidy is:

$$\begin{aligned} W_I(0) - \hat{W} &= \frac{\text{var}(\theta_1 - \theta_2)}{(\beta_1 + \beta_2)^2} \left((\beta_1 + \beta_2) - \frac{1}{2}(b_1 + b_2) \right) \\ &\quad - \frac{1}{2(\beta_1 + \beta_2)^2 (b_1 + b_2)} \left[\frac{b_1}{n_1}(a_2 - a_1 + b_2 \bar{e}) + \frac{b_2}{n_2}(a_1 - a_2 + b_1 \bar{e}) \right]^2 \end{aligned}$$

And an integrated market increases welfare if and only if:

$$\text{var}(\theta_1 - \theta_2) \geq \frac{[b_1(a_2 - a_1 + b_2 \bar{e})/n_1 + b_2(a_1 - a_2 + b_1 \bar{e})/n_2]^2}{((1 + 2/n_1)b_1 + (1 + 2/n_2)b_2)(b_1 + b_2)}.$$

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