On the Productive Value of Biodiversity

Jean-Paul Chavas

Abstract The paper investigates the value of biodiversity as it relates to the productive value of services provided by an ecosystem. The value of biodiversity is linked with a deterioration in the functioning of the ecosystem. The analysis examines how the value of an ecosystem can be "greater than the sum of its parts". First, we propose a general measure of the productive value of biodiversity. The measure involves an experiment that divides the ecosystem into sub-systems that are less diverse, keeping aggregate resources constant. Second, the value of biodiversity is decomposed into four components, reflecting the role of complementarity, scale, convexity, and catalytic effects. This provides new and useful information on the sources, determinants and dynamics of biodiversity value.

Keywords Biodiversity · Productive value · Complementarity · Scale · Convexity · Catalytic effect

JEL Classifications D6 · Q2 · Q5

1 Introduction

Earth ecosystems provide valuable services supporting human life. Since the development of agriculture some 10,000 years ago, earth ecosystems have been modified and managed to satisfy man's nutritional needs. So far, improvements in agricultural productivity have been large enough to allow the earth to feed its growing human population (e.g., Boserup 1965, 1981; Heiser 1990). But the associated alterations in the earth environment have raised

J.-P. Chavas (✉)
University of Wisconsin, Taylor Hall, Madison, WI 53706, USA
e-mail: jchavas@wisc.edu

concerns about the sustainability of current practices. Of particular concern is the decline in biodiversity. Previous research has documented that biodiversity is an important component of ecological systems (e.g., Tilman and Downing 1994; Tilman et al. 1996; Heal 2000; Wood and Lenné 1999). There is empirical evidence that a loss of biodiversity can have adverse effects on the functioning of ecosystems (e.g., Loreau and Hector 2001; Naeem et al. 1994; Tilman and Downing 1994; Tilman et al. 1996). Yet, while the term biodiversity has acquired a positive connotation, its measurement and conceptualization remains difficult (e.g., Mainwaring 2001; Montgomery et al. 1994). A number of indices of biodiversity have been proposed, including the Shannon index and the Simpson index (see Hill 1973; Lande 1996; May 1975; Polasky and Solow 1995; Simpson 1949). These indices have been used extensively in the empirical analysis of biodiversity issues (e.g., Heisey et al. 1997; Meng et al. 1998; Priestley and Bayles 1980; Smale et al. 1998, 2002, 2003; Smale 2006; Wood and Lenné 1999). However, there is a debate on which diversity index is most appropriate (see Keylock 2005; Routledge 1979). At this point, it appears that no particular index is always "better". This is made clear when the value of biodiversity is found to depend on the presence and nature of complementarity among services provided by an ecological system (e.g., Faith et al. 2003; Justus and Sarkar 2002; Loreau and Hector 2001). Weitzman (1992, 1998) has proposed measuring biodiversity through a diversity function based on a measure of dissimilarity. Weitzman (1992) showed that this diversity function is one half of the Shannon index. However, Brock and Xepapadeas (2003) have argued that a more diverse ecosystem can be much more valuable even when the increase in dissimilarity is almost zero.[1] This suggests that the evaluation of biodiversity will typically depend on the nature of the ecosystem being investigated. The globalization of human activities makes such evaluation both important and challenging. The current paper focuses on one of these challenges: how to assess the productive value of biodiversity?

This paper presents a general analysis of the productive value of biodiversity. Our analysis focuses on assessing the productivity effects of biodiversity in an ecological system. For example, a key function of an agroecosystem is to use environmental goods (including ecological capital and environmental services) to produce food. Our approach explores how biodiversity can contribute to enhancing the productivity of an ecosystem. In the process, we gain insights on the factors influencing the productive value of biodiversity.

First, we propose a general measure of the productive value of biodiversity. The measure is designed to answer the question: is the productive value of an ecosystem "greater than the sum of its parts"? This involves an experiment where an ecosystem is split into separate and less diverse sub-systems, holding technology and aggregate resources constant. The experiment provides useful information on how the ecosystem functions and how diversity contributes to the provision of valuable ecological services. The analysis makes use of Luenberger's shortage function (Luenberger 1995) as a multi-input multi-output production function representing the underlying technology. The shortage function has the desirable property of being additive across sub-systems. Our proposed measure of the value of biodiversity has a monetary interpretation that applies when environmental services are non-market goods. And when positive, the value of biodiversity means that the ecosystem is worth more than "the sum of its parts".

Second, we show that the productive value of diversity can be decomposed into four additive parts: one associated with complementarity, one with scale effects, one with convexity effects, and one with catalytic effects. Complementarity means that there is positive synergy

---

[1] Using Bt-corn for motivation, Brock and Xepapadeas (2003) show how biodiversity can stimulate productivity by reducing pest populations.

across sub-systems, where some environmental goods have positive effects on the marginal productivity of others. The role of complementarity has been identified in previous research (e.g., Faith et al. 2003; Justus and Sarkar 2002; Loreau and Hector 2001). Our analysis shows that complementarity is indeed an important component of the value of biodiversity. The scale component establishes linkages between the scale of an ecosystem and its functioning. For example, under increasing returns to scale, we show how an ecosystem can be "too small" to function properly, thus contributing to a positive value of biodiversity. Such a role for scale has been identified in the analysis of ecological fragmentation (e.g., Debinski and Holt 2000; Bissonette and Storch 2002). The convexity component reflects the role and nature of the underlying technology. It shows that diminishing marginal productivity contributes to a positive value of biodiversity. This identifies the role of resource scarcity in the valuation of biodiversity. Finally, the catalytic component measures the possible discontinuous effects of environmental goods around 0. It is relevant when an ecosystem exhibits extreme forms of loss in biodiversity. Overall, our decomposition provides new and useful information on the sources, determinants and dynamics of biodiversity value. We also explore the implications of our analysis for assessing the linkages between diversity and ecosystem resilience.

2 The Productive Value of Environmental Goods

Consider an ecological system as a production sub-system involving m private goods and n environmental goods. Let $z = (z_1, \ldots, z_{m+n}) = (z_a, z_b)$, where $z_a = (z_1, \ldots, z_m) \in \Re^m$ is the quantity of the m private goods, and $z_b = (z_{m+1}, \ldots, z_{m+n}) \in \Re^n$ is the quantity of the n environmental goods. Using the netput notation, quantities are defined to be negative for inputs (i.e., $z_i \leq 0$ when the i-th netput is an input) and positive for outputs (i.e., $z_i \geq 0$ when the i-th netput is an output). The underlying production technology is denoted by the set $Z \subset \Re^{m+n}$, where $z \equiv (z_a, z_b) \in Z$ means that private goods z_a can be feasibly produced in the presence of environmental goods z_b. Throughout, we assume that the set Z is closed, and that it exhibits free disposal with respect to z_a (where free disposal in z_a means that, for any $z \equiv (z_a, z_b) \in Z$, $z_a' \leq z_a$ implies that $(z_a', z_b) \in Z$).[2] However, we do not assume that the set Z is convex, or that it exhibits free disposal with respect to z_b. Thus, our analysis applies under non-convexity and under scenarios where the environmental goods z_b do not exhibit free-disposal. By not assuming free disposal in z_b, we allow environmental netputs to have a negative marginal productivity (i.e., the case the noxious environmental goods). Finally, while we first develop our approach in a static framework, Sect. 6 extents it by incorporating dynamics and uncertainty in the analysis.

Our analysis applies to a very general technology characterizing the productivity of the ecological system. This is illustrated in Fig. 1, where the upper bound of the feasible set Z is given by the line ABCDEFGH. Along the line EFG, both z_a and z_b are considered as outputs. This would apply to a healthy ecosystem that allows for the production of valuable ecological services (e.g., clean water) as well as private goods (e.g., food). Along the line BCDE, the environmental good z_b is an input in the production of z_a. This corresponds to situations where the ecological system is used to produce private goods. An example is an agroecosystem using ecological services to produce food (as discussed in Sect. 5 below). The line GH corresponds to a case of environmental enhancement where the private good z_a is an input into the production of the environmental good z_b (e.g., protecting the ecological system that provides clean water for New York City). Finally, along the line AB, both z_a and z_b are

---

[2] Free disposal in z_a implies that netputs z_a have a non-negative marginal productivity.

Fig. 1 An illustration of the technology
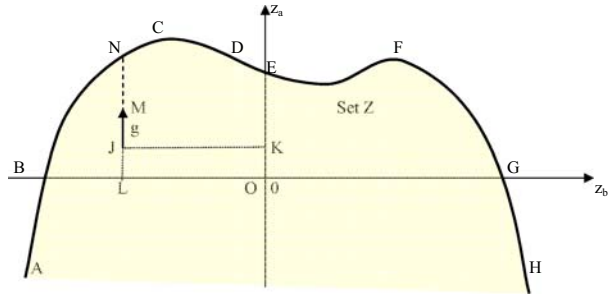
inputs. This would correspond to unproductive ecological systems where the production of private goods becomes impossible (e.g., on Mount Everest). Note that the technology Z in Fig. 1 is not convex (e.g., along the line CDEF). And it does not exhibit free disposal in the environmental good z_b along the lines ABC and DEF (e.g., the case of noxious environmental goods). For example, along the line BC, the environmental good $|z_b|$ is an input that exhibits negative marginal productivity: without free disposal, $|z_b|$ has an adverse impact on the production of the private good z_a.

We are interested in providing a general representation of the frontier technology given by the boundary of Z. This can be done using a multi-input multi-output production function. Below, we rely on the shortage function (proposed by Luenberger 1995) as a convenient representation of the multi-input multi-output production technology.

Define $g \equiv (g_1, \ldots, g_m) \in \Re_+^m$ to be a reference bundle of private goods satisfying $g \geq 0$, and $g \neq 0$. Following Luenberger (1995), for a given g, the shortage function $S(z, g)$ evaluated at point $z \equiv (z_a, z_b)$ is

$$S(z, g) = \min_\alpha \{\alpha : (z_a - \alpha g, z_b) \in Z\}, \text{ if there is a scalar } \alpha \text{ such that } (z_a - \alpha g, z_b) \in Z\},$$
$$= +\infty \quad \text{otherwise.} \tag{1}$$

The shortage function $S(z, g)$ measures the number of units of the reference bundle g reflecting the distance between point $z \equiv (z_a, z_b)$ and the frontier technology. It provides a convenient measure of technical efficiency (reflecting how far point z is from the frontier technology). Under technological change, it also provides a measure of productivity: for a given z, the change in $S(z, g)$ evaluated before and after a technological change reflects the extent of the shift (measured in number of units of the bundle g) in the frontier technology. As we will see below, the shortage function will provide a convenient basis for analyzing the productivity effects of biodiversity.

The shortage function has some useful properties (see Luenberger 1995):

1. $z \in Z$ implies $S(z, g) \leq 0$,
2. Under free disposal in z_a, $Z = \{z : S(z, g) \leq 0\}$,
3. Under free disposal in z_a, $S(z_a, z_b, g)$ is non-decreasing in z_a,
4. $S(z_a + \alpha g, z_b, g) = \alpha + S(z, g)$, for any α.

Property 1 shows that $S(z, g) \leq 0$ is associated with the feasibility of the netputs $z \equiv (z_a, z_b)$. Under free disposal in z_a, property 2 implies that $S(z, g) \leq 0$ provides a complete characterization of the technology. In this case, $S(z, g) = 0$ if and only if z is on the upper bound of the feasible set Z, with $S(z, g) = 0$ providing a multi-input multi-output functional representation of the underlying frontier technology. Under free disposal in z_a, property 3 states that the shortage function $S(z_a, z_b, g)$ is non-decreasing in the private goods z_a. Note

that, in general, $S(z_a, z_b, g)$ can be either increasing or decreasing in the environmental goods z_b. As suggested by property 3, it would be non-decreasing in z_b if the technology exhibited free disposal in z_b. But it would be decreasing in z_b in regions where free disposal in z_b fails to hold. Finally, if $S(z_a, z_b, g)$ is twice differentiable in z, property 4 implies that $\frac{\partial S(z_a, z_b, g)}{\partial z_a} g = 1$ and that $\frac{\partial^2 S(z_a, z_b, g)}{\partial (z_a, z_b) \partial z_a} g = 0$.

The shortage function is illustrated in Fig. 1. Consider evaluating it at point J, where the private good $z_a > 0$ is an output (represented by the distance OK in Fig. 1) and the environmental good $z_b < 0$ is an input (where $|z_b|$ is given by the distance OL). Given the reference bundle g (represented by JM in Fig. 1), the shortage function $S(z, g)$ evaluated at point J is given by $-JN/JM$.

As a further illustration, consider the case where $g = (1, 0, \ldots, 0)$. Then $S(z, g) = \min_\alpha \{\alpha : (z_1 - \alpha, z_2, \ldots, z_{m+n}) \in Z\} = z_1 - G(z_c)$ where $z_c = (z_2, \ldots, z_{m+n})$ and $G(z_c) = \max\{z_1 : (z_1, z_c) \in Z\}$. In this context, $G(z_c)$ is a classical production function for z_1: it gives the largest possible z_1 that can be obtained given other netputs z_c. This shows that the shortage function provides a generalization of the standard production function to a multi-input multi-output context. Note that $S(z, g) = z_1 - G(z_c)$ implies that $\partial S/\partial z_1 = 1$ and $\partial S/\partial z_c = -\partial G/\partial z_c$. It suggests that $(-\partial S/\partial z_c)$ can be interpreted as measuring the marginal product of z_c. This interpretation will prove useful in our discussion below.

Besides providing a representation of the frontier technology in a multi-input multi-output context, the shortage function $S(z, g)$ in (1) has one attractive property: when the reference bundle of private goods g is constant, $S(\cdot)$ can e be meaningfully added across ecological subsystems. As we argue below, this desirable "additivity property" is the main reason why we rely on the shortage function in this paper.[3]

For a given $z \equiv (z_a, z_b)$, the shortage function $S(z, g)$ in (1) provides a convenient basis for analyzing the productive value of the environmental goods z_b. To see that, consider a change in environmental goods from z_b^1 to z_b^2. Then, define

$$P(z_a, z_b^1, z_b^2, g) = S(z_a, z_b^1, g) - S(z_a, z_b^2, g). \qquad (2)$$

Starting from the point $z \equiv (z_a, z_b^1)$, $P(z_a, z_b^1, z_b^2, g)$ in (2) measures the number of additional units of the reference bundle g that can be obtained from changing environmental goods from z_b^1 to z_b^2. To illustrate, consider the case where z_b are inputs (with $z_b < 0$) and (2) is evaluated under a technology exhibiting free disposal in z_b. As suggested by property 3, $S(z_a, z_b, g)$ would be non-decreasing in z_b. Then, with $z_b < 0$, any increase in the environmental inputs from $|z_b^1|$ to $|z_b^2|$ would mean a decrease in z_b, implying that $P(z_a, z_b^1, z_b^2, g) \geq 0$ in (2). In this case, increasing environmental input z_b can make it possible to produce more of the private goods z_a, with $P(z_a, z_b^1, z_b^2, g) \geq 0$ measuring the additional number of units of the private goods g that can be produced.

To note the role of free disposal for the environmental goods z_b, consider the case of an increase in the environmental input from point J in Fig. 1. With $z_b < 0$, increasing

[3] It should be noted that alternative measurements are possible. They include the directional distance function proposed by Chambers et al. (1996): $\bar{D}(z, g) = \max_\beta \{\beta : (z_a + \beta g, z_b) \in Z\}$. Comparing it with (1), the directional distance function $\bar{D}(z, g)$ is simply the negative of the shortage function: $\bar{D}(z, g) = -S(z, g)$. It means that, except for changing the sign, the analysis presented below could be written equivalently using the directional distance function $\bar{D}(z, g)$. Other functional representations of the frontier technology include the Shephard input distance function and the Shephard output distance function (see Shephard 1970). The Shephard distance functions are related to the shortage function and the directional distance function (see Chambers et al. 1996; Färe and Grosskopf 2000). However, they are defined in terms of proportional changes in inputs or outputs. This means that they do not satisfy the "additivity" property of possibility frontiers. As such, they do not provide attractive options for evaluating the productive value of biodiversity.

the environmental input $|z_b|$ means a decrease in z_b from point J, implying an increase in the shortage function. This reflects the fact that free disposal in z_b does not hold in the region BC of Fig. 1, and that the shortage function $S(z_a, z_b, g)$ is now decreasing in z_b in the neighborhood of point J. In this case, any increase in the environmental input $|z_b|$ implies that $P(z_a, z_b^1, z_b^2, g) < 0$ in (2). This illustrates that, without free disposal, increasing environmental input $|z_b|$ can reduce the ability to produce the private goods z_a, with $P(z_a, z_b^1, z_b^2, g) < 0$ measuring the associated reduction in the number of units of g that can be produced.

In the case where the private goods z_a are also market goods with prices $p = (p_1, \ldots, p_m) \in \Re_+^m$, a monetary evaluation of $P(z_a, z_b^1, z_b^2, g)$ in (2) is

$$
\begin{aligned}
V(z_a, z_b^1, z_b^2, p, g) &= P(z_a, z_b^1, z_b^2, g)(p \cdot g) \\
&= [S(z_a, z_b^1, g) - S(z_a, z_b^2, g)](p \cdot g),
\end{aligned} \tag{3}
$$

where "$\cdot$" denotes the inner product, and $(p \cdot g) = \sum_{i=1}^{m} p_i g_i$ is the monetary value of one unit of the bundle g. Starting from the point $z \equiv (z_a, z_b^1)$, $V(z_a, z_b^1, z_b^2, p, g)$ in (3) gives a monetary value of the private goods that can be obtained when environmental goods change from z_b^1 to z_b^2. Then, comparing (2) and (3) gives the following result.

Proposition 1 *When the reference bundle g is chosen to have unit value* (with $p \cdot g = 1$), $P(z_a, z_b^1, z_b^2, g)$ *in* (2) *gives a monetary value of changes in environmental goods from z_b^1 to z_b^2.*

This provides some guidance for choosing the reference bundle g. When g is chosen such that $p \cdot g = 1$, Proposition 1 shows that $P(z_a, z_b^1, z_b^2, g)$ in (2) measures the monetary value of changes in environmental goods. This measure is attractive on several grounds: it allows the analysis of environmental goods as "non-market goods" (i.e., goods with no observable price); it allows for a general technology underlying the productivity implications of an ecological system; it does not require the technology to be convex; and it does not require that the environmental goods satisfy "free disposal".

As shown in Proposition 1, Eqs. 2 and 3 provide *absolute measures* associated with changes in environmental goods. Note that these measures can be easily modified into relative measures. To see that, consider the case where $z_b^1 = 0$ and $z_b^2 = z_b$. Then, Eq. 2 becomes

$$
P(z_a, 0, z_b, g) = S(z_a, 0, g) - S(z_a, z_b, g), \tag{2'}
$$

where $P(z_a, 0, z_b, g)$ measures the *total value* of the environmental goods z_b when $p \cdot g = 1$. In situations where $P(z_a, 0, z_b, g) \neq 0$, a relative measure of changes in environmental goods from z_b^1 to z_b^2 can be written as

$$
\begin{aligned}
R_1(z_a, z_b^1, z_b^2, g) &\equiv P(z_a, z_b^1, z_b^2, g)/P(z_a, 0, z_b^2, g) \\
&= [S(z_a, z_b^1, g) - S(z_a, z_b^2, g)]/[S(z_a, 0, g) - S(z_a, z_b^2, g)].
\end{aligned} \tag{4}
$$

$R_1(z_a, z_b^1, z_b, g)$ in (4) measures the value of the change from z_b^1 to z_b^2 as a proportion of the total value of z_b^2 given in (2'). Finally, note that, in situations where (z_a, z_b^2) is on the upper bound of the feasible set, then $S(z_a, z_b^2, g) = 0$ and Eq. 4 reduces to

$$
R_1(z_a, z_b^1, z_b^2, g) = S(z_a, z_b^1, g)/S(z_a, 0, g), \tag{4'}
$$

showing that a ratio of shortage functions provides a simple relative measure of the value of environmental changes.

3 The Value of Biodiversity

Equations (2) and (3) measure the productive value associated with a change in environmental goods. However, it is often of interest to know more about the source of this value. The concerns about biodiversity provide a good example. Indeed, biodiversity issues typically arise when it is believed that the value of an ecosystem is greater than the value of its parts. This suggests the need to evaluate the value of environmental goods both for their "total value" and for the "sum of their parts". To address this issue, consider an experiment where the ecological system is split into K separate sub-systems, keeping technology and the total amount of resources constant. The experiment is chosen such that each of the K sub-systems exhibits a loss of diversity. In this context, the key question is: is the original system more productive than the K sub-systems? With technology and aggregate resources being held constant, any productivity change is due entirely to changes in the functioning of the ecosystem. With the K sub-systems being less diverse, the experiment provides a relevant basis for assessing the productive value of biodiversity.

To define the K sub-systems, denote by I_b the set of environmental goods in z_b, and consider a partition of the set $I_b = \{I_{b1}, I_{b2}, \ldots, I_{bK}\}$, with $2 \le K \le n$. Let $z_{bk} = \{z_j : i \in I_{bk}\}$ denote the environmental goods in the subset I_{bk}, $k = 1, \ldots, K$, with $z_b = (z_{b1}, \ldots, z_{bK})$. For a given $z \equiv (z_a, z_b) \in Z$, consider K situations where $z^k \equiv (z_a^k, z_b^k) \neq 0$ for $k = 1, \ldots, K$. Using the shortage function (1), we propose the following measure of diversity

$$D(z, g) = \sum_{k=1}^{K} S(z^k, g) - S(z, g), \qquad (5)$$

where $z = \sum_{k=1}^{K} z^k$. Equation (5) compares two situations involving netputs z: one where the netputs z are involved in an integrated ecosystem; and the other situation where there are K separate sub-systems, with z_k being the netputs used in the k-th sub-system. With $z = \sum_{k=1}^{K} z^k$, it follows that, in each situation, the same aggregate amounts of resources are used to produce the same aggregate netputs. In this context, Eq. 5 provides a measure of the number of units of the reference bundle g that can be saved by producing z in an integrated ecosystem compared to producing the same aggregate netputs z from K separate and less diverse sub-systems.[4] Note that Eq. 5 involves adding the shortage functions $S(z^k, g)$ across all K sub-systems. For a given reference bundle g, this reflects the additivity property of the shortage function: $\sum_{k=1}^{K} S(z^k, g)$ simply measures the total number of units of g needed to get to the production possibility frontier for all K sub-systems. In this context, holding technology and aggregate resources constant, $D(z, g) > 0$ in (5) implies that there are productivity gains associated with the provision of ecological services in an integrated ecosystem (as opposed to K fragmented and less-diverse sub-systems). This reflects that $D(z, g) > 0$ corresponds to situations where "the whole is worth more than the sum of the parts". From (5), this would be associated with the subadditivity of the shortage function.

To help further motivate (5), consider the case where $p \cdot g = 1$. Then, use Eq. 2 to define $P_k \equiv S(z_a, 0, g)/K - S(z^k, g)$ as measuring the value of the environmental goods in z^k, $k = 1, \ldots, K$, where $\sum_{k=1}^{K} z_a^k = z_a$. Note that $S(z_a, 0, g)$ is divided by K to reflect the fact that the original ecosystem is being evaluated in the context of K separate systems.

---

[4] Note that $S(z^k, g)$ can be either positive or negative. It would be positive when z^k is not feasible in the k-th sub-system, reflecting that z^k is "too high" by the quantity $[S(z^k, g)g]$.

Then, the value of the "sum of the parts" across the K systems is

$$\sum_{k=1}^{K} P_k = S(z_a, 0, g) - \sum_{k=1}^{K} S(z^k, g),$$
$$= P(z_a, 0, z_b, g) - D(z, g),$$

using (2′) and (5). It follows that $D(z, g) = P(z_a, 0, z_b, g) - \sum_{k=1}^{K} P_k$. This shows that the value of diversity $D(z, g)$ in (5) is indeed the difference between the total value of the environmental goods z_b, $P(z_a, 0, z_b, g)$, and the value of the "sum of its parts", $\sum_{k=1}^{K} P_k$.

Using arguments similar to the ones presented in Proposition 1 yields the following result.

Proposition 2 *When the reference bundle g is chosen to have unit value (with* $p \cdot g = 1$*), then* $D(z, g)$ *in (5) is a monetary measure of the value of diversity.*

The value of diversity $D(Z, g)$ in (5) can be used to measure the value of biodiversity when the $z^k \equiv (z_a^k, z_b^k)$'s are chosen such that the K subsystems exhibit a loss of biodiversity.[5]

As indicated in Proposition 2, when the reference bundle g is chosen such that $p \cdot g = 1$, then $D(z, g)$ in (5) provides a monetary measure of the value of diversity. Then, Eq. 5 provides an absolute measure of the value of diversity. This can be easily modified to obtain a relative measure. In situations where the total value $P(z_a, 0, z_b, g)$ in (2′) is non-zero, a relative measure of diversity can be written as

$$R_D(z, g) \equiv D(z, g)/P(z_a, 0, z_b, g)$$
$$= \left[\sum_{k=1}^{K} S(z^k, g) - S(z, g) \right] / \left[S(z_a, 0, g) - S(z_a, z_b, g) \right]. \tag{6}$$

where $z \equiv (z_a, z_b) = \sum_{k=1}^{K} z^k$. $R_D(z, g)$ in (6) measures the value of diversity as a proportion of the total value of z_b given in (2′). In situations where $z \equiv (z_a, z_b)$ is on the upper bound of the feasible set, then $S(z_a, z_b, g) = 0$ and Eq. 6 reduces to

$$R_D(z, g) = \sum_{k=1}^{K} S(z^k, g)/S(z_a, 0, g), \tag{6′}$$

showing that a ratio of shortage functions provides a simple relative measure of diversity.

4 A Decomposition

Equation (5) defines a general measure of diversity under an experiment generating diversity loss while keeping aggregate resource constant. When applied to biodiversity, such a measure will typically depend on both the ecosystem being analyzed and the extent of diversity loss generated by the experiment. In this section, we examine the factors that may affect the value of biodiversity. This is done by proposing a decomposition of the value of biodiversity D in (5).

Since the value of biodiversity typically depend on the nature of diversity loss, it will be useful to define $z^k \equiv (z_a^k, z_b^k)$ in (5) in a more specific way. We would like to do so with

---

[5] Note that, in general, $D(z, g)$ in (5) can be either positive or negative. Finding $D(z, g) < 0$ would imply that diversify loss would contribute to increased productivity. It is only when $D(z, d)$ is positive that diversity would be found be valuable from a productivity viewpoint.

two objectives in mind: 1/ focus our attention on diversity loss related only to environmental goods; and 2/ preserve some flexibility in characterizing the extent of "diversity loss". We consider choosing

$$z_a^k = z_a/K, \tag{7a}$$

and

$$z_i^k = \beta\, z_i \quad \text{if } i \in I_{bk}, \tag{7b}$$
$$= z_i(1-\beta)/(K-1) \quad \text{if } i \in I_b \backslash I_{bk}, \tag{7c}$$

$k = 1, \ldots, K$, for some $\beta \in (1/K, 1]$. First, note that Eqs. 7a–7c always satisfy $z = \sum_{k=1}^{K} z^k$. This guarantees that the same aggregate netputs are involved in both situations. Second, Eq. 7a divides the market goods z_a equally among the K sub-systems. This imposes "no diversity" in the use of the private goods z_a across the K sub-systems. This means that the associated diversity measure will necessarily reflect changing diversity in environmental services.

Third, Eqs. 7b–7c establish the patterns of distribution in the environmental goods z_b. The parameter β in (7b) represents the proportion of the original environmental netputs $\{z_i : i \in I_{bk}\}$ that are present in the k-th sub-system. And from (7c), $(1 - \beta)/(K - 1)$ represents the proportion of the original netputs $\{z_i : i \in I_b \backslash I_{bk}\}$ present in the k-th sub-system. When $\beta = 1$, this corresponds to the case of complete fragmentation where the k-th sub-system relies exclusively on environmental netputs in the subset I_{bk} (with $z_i^k = z_i$ if $i \in I_{bk}$) with $z_i^k = 0$ for $i \in I_b \backslash I_{bk}$. In such situations, each of the K sub-systems is associated with a complete loss of biodiversity in environmental goods z_b across elements of the partition $I_b = \{I_{b1}, I_{b2}, \ldots, I_{bK}\}$. However, by allowing β to depart from 1, Eqs. 7 can apply under more general conditions. In particular, when $\beta \in (1/K, 1)$, Eqs. 7 allows for partial fragmentation. Then, each of the K sub-system is associated with a partial loss of biodiversity in environmental goods z_b across elements of the partition $I_b = \{I_{b1}, I_{b2}, \ldots, I_{bK}\}$. Thus, the parameter $\beta \in (1/K, 1]$ allows for varying amount of fragmentation in the environmental netputs among the K sub-systems.[6] Alternatively stated, it allows for varying amount of biodiversity loss across the K sub-systems. In general, the degree of fragmentation in each sub-system increases with β. This means that the loss in biodiversity in the K sub-systems also increases with β.

With $z^k \equiv (z_a^k, z_b^k)$ given in (7a)–(7c), Eq. 5 becomes

$$D(z, \beta, g) = \sum_{k=1}^{K} S(z^k, g) - S(z, g), \tag{8}$$

where $\beta \in (1/K, 1]$. Equation (8) provides a measure of the value of biodiversity. It measures the number of units of the reference bundle g that can be saved when the environmental goods z_b are part of an integrated ecosystem (compared to the case where the environmental goods z_b are part of K less diverse sub-systems satisfying (7a)–(7c) and producing the same aggregate netputs z).

---

[6] The parameter β in Eqs. 7 has a very desirable characteristic: it allows for a departure from complete fragmentation (when β decreases from 1). This convenient feature will prove useful in the analysis presented in Sect. 5. However, note that Eqs. 7 restricts β to be the same across sub-systems. In situations where this condition is found to be overly restrictive, it can be relaxed by using Eq. 5 instead as a measure of diversity (Eq. 5 does not restrict the patterns of fragmentation). In general, we expect the patterns of fragmentation to affect the value of diversity. This means that the decomposition analysis presented in Sect. 5 would not be fruitful unless we are willing to impose some structure on the patterns of fragmentation. Equation (7) provides such a structure.

While Eq. 8 provides a basis to evaluate the productive value of biodiversity, it is of interest to identify the factors affecting this value. Our analysis focuses on the case where the patterns of fragmentation across the K sub-systems are given by Eqs. 7. As noted above, this includes the case of complete fragmentation (when $\beta = 1$) as well as situations of partial fragmentation (when $\beta \in (1/K, 1)$).

Our approach relies on the shortage function $S(z, g)$ defined in Eq. 1. It applies whether the shortage function is continuous or not. Below, we consider the case where the shortage function $S(z, g)$ is continuous in z, except possibly at $z = 0$. We characterize the effects of a discontinuity at $z = 0$ as "catalytic effects". These catalytic effects represent situations where the introduction of a small intervention has large consequences on system functioning. To capture such effects, we decompose the shortage function into two parts: $S(z, g) \equiv S_v(z, g) + S_f(z, g)$. $S_v(z, g)$ is the "variable function" assumed to be continuous in z. And $S_f(z, g)$ is a "fixed function", assumed to be a step function satisfying $S_f(0, g) = 0$, with possible discontinuities at $z = 0$. Thus, $S_f(z, g)$ is constant with respect to z as long as the set of non-zero netputs does not change. The jump-discontinuities of $S_f(z, g)$ (an hence $S(z, g)$) at $z = 0$ reflect the possible presence of catalytic effects of z in the ecosystem.

We start from the partition $I_b = \{I_{b1}, \ldots, I_{bK}\}$, where I_{bk} denotes the environmental goods that the k-th ecological sub-system specializes in, $k = 1, \ldots, K$, with $2 \leq K \leq n$. We use the following notation. Let $z_a = \{z_i : i \in I_a\}$, $z_{bk} = \{z_i : i \in I_{bk}\}$, $z_b = (z_{b1}, \ldots, z_{bK})$, $z_{b \backslash bk} = (z_{b1}, \ldots, z_{b,k-1}, z_{b,k+1}, \ldots, z_{bK})$, and $z_{b,i:j} = (z_{bi}, z_{b,i+1}, \ldots, z_{b,j-1}, z_{bj})$ for $i < j$. From Eqs. 7, it follows that $z^k = (z_a/K, \beta z_{bk}, (1 - \beta) z_{b \backslash bk})$. Our main result is stated next. (See the proof in the Appendix).

Proposition 3 *Given* $S(z, g) \equiv S_v(z, g) + S_f(z, g)$, *assume that* $S_v(z, g)$ *is continuously differentiable in* z_b *almost everywhere. Under Eqs. 7, the value of biodiversity* $D(z, \beta, g)$ *in* (8) *evaluated at netputs* $z = (z_a, z_b)$ *can be decomposed as follows*

$$D \equiv D_C + D_R + D_V + D_A, \tag{9}$$

where

$$D_C \equiv \sum_{k=1}^{K-1} \left\{ \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial S_v}{\partial \gamma} (z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma, \right.$$

$$z_{b,k+1:K}(1-\beta)/(K-1), g) d\gamma$$

$$\left. - \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial S_v}{\partial \gamma} (z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma, \beta z_{b,k+1:K}, g) d\gamma \right\}, \tag{10a}$$

$$D_R \equiv K\, S(z/K, g) - S(z, g), \tag{10b}$$

$$D_V \equiv S(z_a/K, \beta z_b, g) + (K-1)S(z_a/K, z_b(1-\beta)/(K-1), g) - K\, S(z/K, g), \tag{10c}$$

and

$$D_A \equiv \sum_{k=1}^{K} S_f(z_a/K, \beta z_{bk}, z_{b \backslash bk}(1-\beta)/(K-1), g)$$

$$- S_f(z_a/K, \beta z_b, g) - (K-1)S_f(z_a/K, z_b(1-\beta)/(K-1), g). \tag{10d}$$

Proposition 3 gives a decomposition of the value of biodiversity $D(z, g)$ in (8) into four additive terms: D_C given in (10a), D_R given in (10b), D_V given in (10c), and D_A given in (10d). As discussed below, each term reflects different aspects of ecosystem functioning contributing to the value of diversity.

The term D_C in (10a) depends on how $z_{b\backslash bk}$ affects the marginal shortage of z_{bk}, $k = 1, \ldots, K$. It reflects the presence of complementarity among environmental netputs in z_b. To see that, consider the case where the shortage function is twice continuously differentiable in z_b. Then, Eq. 10a can be written as

$$D_C \equiv -\sum_{k=1}^{K-1} \int_{z_{b,k+1:K}(1-\beta)/(K-1)}^{\beta z_{b,k+1:K}} \int_{z_{bk}(1-\beta)(K-1)}^{\beta z_{bk}} \frac{\partial^2 S_v}{\partial \gamma_1 \partial \gamma_2}$$
$$\times (z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma_1, \gamma_2, g) d\gamma_1 d\gamma_2. \tag{10a$'$}$$

Equation (10a$'$) makes it clear that the sign of D_C depends on the sign of $\partial^2 S/\partial z_{bk} \partial z_{b\backslash bk}$, $k = 1, \ldots, K$. As discussed above, the marginal shortage can be interpreted as the negative of the marginal product. In this context, define complementarity between z_{bk} and $z_{b\backslash bk}$ as any situation where the shortage function satisfies $\partial^2 S/\partial z_{bk} \partial z_{b\backslash bk} < 0$. Indeed, with $\partial S/\partial z_{bk}$ reflecting the negative of the marginal product of z_{bk}, complementarity (with $\partial^2 S/\partial z_{bk} \partial z_{b\backslash bk} < 0$) means that z_{bk} has positive effects on the marginal product of $z_{b\backslash bk}$, implying positive synergies between z_{bk} and $z_{b\backslash bk}$. Then, it is clear from (10a) that $D_C > 0$ if the shortage function exhibits complementarity between z_{bk} and $z_{b\backslash bk}$, $k = 1, \ldots, K$.

Thus, Proposition 3 establishes that complementarity among environmental netputs (as reflected by the term D_C) is one of the components of the value of biodiversity. This supports the arguments that complementarity is an important contributing factor to the value of biodiversity (e.g., Faith et al. 2003; Justus and Sarkar 2002; Loreau and Hector 2001).

To interpret the term D_R in (10b), we make use of lemma 1 in the Appendix. Given $K \geq 2$, lemma 1 implies that

$$K\, S(z/K, g) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} S(z, g) \text{ under } \begin{Bmatrix} \text{decreasing returns to scale (DRTS)} \\ \text{constant returns to scale (CRTS)} \\ \text{increasing returns to scale (IRTS)} \end{Bmatrix}.$$

It follows that

$$D_R \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \text{ under } \begin{Bmatrix} \text{DRTS} \\ \text{CRTS} \\ \text{IRTS} \end{Bmatrix}. \tag{10b$'$}$$

Equation (10b$'$) implies that D_R vanishes under CRTS, but is positive (negative) under IRTS (DRTS). Thus, the term D_R can be interpreted as capturing scale effects generated as the netput vector z is produced in more specialized ways. Also, Eq. 10b$'$ shows that $D_R \geq 0$ under non-decreasing returns to scale. Intuitively, more fragmented sub-systems involve smaller scales of operation. Under IRTS, such sub-systems (associated with lower biodiversity) would appear less productive (their scale of operation is "too small") as their relative ability to provide ecological services deteriorates. In this case, the scale effect contributes positively to the value of biodiversity ($D_R > 0$). Alternatively, under DRTS, the fragmented sub-systems would appear more productive (as the scale of operation of the integrated sub-system is "too large"), implying a negative scale effect ($D_R < 0$).

Thus, Proposition 3 establishes how the scale of an ecosystem and the nature of returns to scale can affect the value of biodiversity. This supports the arguments that scale effects can play an important role in the evaluation of ecological fragmentation (e.g., Debinski and Holt 2000; Bissonette and Storch 2002).

The term D_V in (10c) reflects the effect of convexity. To show it, we make use of lemma 2 in the Appendix. Lemma 2 states that the shortage function $S(z, g)$ is convex in z when the feasible set Z is convex. It follows that, under the convexity of Z, the shortage function

satisfies $\sum_{j=1}^{K} \theta_j \, S(z^j, g) \geq S(\sum_{j=1}^{K} \theta_j z^j, g)$ for $\theta_j \in [0, 1]$ satisfying $\sum_{j=1}^{K} \theta_j = 1$. Choosing $\theta_j = 1/K$, $z^1 = (z_a/K, \beta z_b, g)$ and $z^j = (z_a/K, z_b(1 - \beta)/(1 - K), g)$ for $j = 2, \ldots, K$, it follows from (10c) that $D_V \geq 0$. Thus, a convex technology is sufficient to imply that $D_V \geq 0$. Intuitively, a convex technology means diminishing marginal productivity, a standard characterization of resource scarcity. This suggests that the term D_V reflects the role of resource scarcity. In this context, Proposition 3 shows that resource scarcity contributes positively to the value of biodiversity. Alternatively, our analysis indicates that $D_V < 0$ can arise only under a non-convex technology. The identification of such effects seems to be new in the literature.

Finally, the term D_A in (10d) reflects catalytic effects around $z = 0$. Indeed, in the absence of discontinuity of the shortage function $S(z, g)$, then $S_f(z, g) = 0$ and thus $D_A = 0$ in Eq. 10d. When S_f is non-zero, note that D_A can be positive, zero, or negative. Since we assume that $S_f(z, g)$ is step function with possible discontinuities only around $z = 0$, $S_f(z, g)$ is a constant as along as the set of non-zero netputs does not change. Then, from Eq. 10d, $\beta \in (1/K, 1)$ implies $D_A = 0$. Alternatively, the catalytic component D_A can be non-zero only when $\beta = 1$. It means that the role of catalytic effects is relevant in the value of biodiversity only when $\beta = 1$, i.e., only under a complete loss of biodiversity. In the case where $\beta = 1$, from Eq. 10d, D_A is positive if and only if $\sum_{k=1}^{K} S_f(z_a/K, z_b^k, 0, g) > S_f(z_a/K, z_b, g) + (K - 1)S_f(z_a/K, 0, g)$. Then, catalytic effects contribute to the value of biodiversity. This corresponds to situations where a complete loss of biodiversity generates a discontinuous decrease in the productivity of the fragmented sub-systems.

Proposition 3 provides useful information on conditions contributing to the value of biodiversity. It generates the following result.

Corollary 1 *Sufficient conditions for a positive productive value of biodiversity are*:

(1)   there is complementarity between z_{bk} and $z_{b \backslash bk}$, $k = 1, \ldots, K$, ($D_C > 0$),
(2)   the technology exhibits non-decreasing returns to scale ($D_R \geq 0$),
(3)   the technology Z is convex (with $D_V \geq 0$), and
(4)   the catalytic effect is non-negative ($D_A \geq 0$).

This shows that the value of biodiversity can arise from complementarity among environmental goods in z_b ($D_C > 0$), from increasing returns to scale ($D_R > 0$), from a convex technology ($D_V \geq 0$), and/or from catalytic effects (when $D_A \geq 0$). This identifies the role of complementarity as an important contributing factor to the value of biodiversity. However, it also shows that complementarity is in general neither necessary nor sufficient to generate a positive value for biodiversity. For example, under decreasing returns to scale (DRTS), Eq. 10b′ implies that $D_R < 0$. This reflects the fact that, under DRTS, the smaller and more fragmented sub-systems require fewer resources to produce the same aggregate outputs. When this scale effect dominates the other components in (9), then $D < 0$, i.e. biodiversity would have a negative value even in the presence of complementarity. Alternatively, B_V can become negative under a non-convex technology. Again if this negative convexity effect dominated the other components in (9), then $D < 0$, and biodiversity would have a negative value even in the presence of complementarity. Finally, we have shown that the catalytic effect D_A is present only under a complete loss of biodiversity in environmental goods. Scenarios where D_A is positive and large can arise when a complete loss of biodiversity is associated with a large decline in the productivity of the fragmented sub-systems. In such cases, the value of biodiversity can be positive even in the absence of complementarity. This illustrates the usefulness of the decomposition provided in Proposition 3.

Apple production
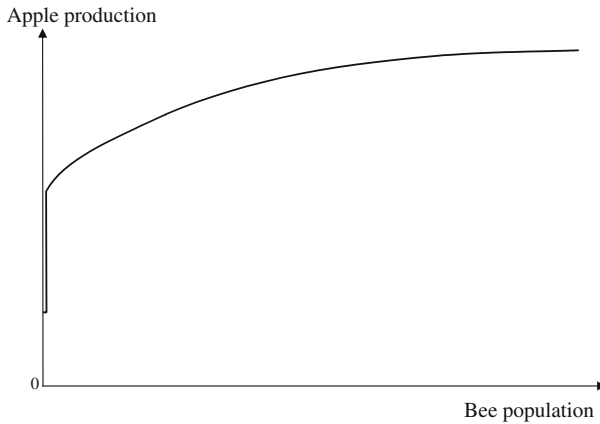
0   Bee population

**Fig. 2** The effect of bee population on apple production

5 An Illustration

This section presents a graphical illustration of our analysis. Our illustration focuses on the case of an agroecosystem where land is allocated between two activities: apple orchard and cereal production. The system faces given agro-climatic conditions and relies on a set of resources (including land, labor, and capital). The ecosystem also involves a bee population. Bees are the world's dominant pollinators. Pollination is one of the best-known examples of a valuable ecological service provided by an insect population. Southwick and Southwick (1992) have estimated the economic value of bee pollination in the US to be in the range from $1.6 billion to $5.2 billion. Other estimates are $8.3 billion (Robinson et al. 1989) and $14.6 billion (Morse and Calderone 2000). Although these estimates are somewhat imprecise, their magnitude does stress the importance of the pollination service provided by bees.[7] This establishes that bee populations play a significant role in increasing agricultural productivity. We use pollination services to illustrate their implications for the value of biodiversity.

While bees do not play a significant role in cereal production, they are vital in apple production. The reason is that apple trees require cross-pollination and bees are the most important carriers of pollen between compatible cultivars. It means that, without bees, apple yields would be much lower. This is illustrated in Fig. 2. Figure 2 shows two characteristics. First, an increase in bee population near an apple orchard tends to improve apple production. Second, bees are social insects living in colonies. Each colony lives in a bee hive and consists of several thousand bees that specialize and cooperate in nest building, food collection, and brood rearing. The bees in a colony can pollinate many apple trees in their neighborhood. Assuming that the region is not too large, this means that a switch between zero and one colony can yield a discrete change in orchard productivity (see Fig. 2). The implications of this discrete change for the valuation of biodiversity are explored below.

For simplicity, assume that the only useful function of bees (from a human viewpoint) is their pollination service (e.g., we neglect the amount of honey they may produce). Also, we assume that the amount of land used by each bee hive is small and negligible. In this

---

[7] For a more general evaluation of the economic value of ecological services provided by insects in the United States, see Losey and Vaughan (2006).
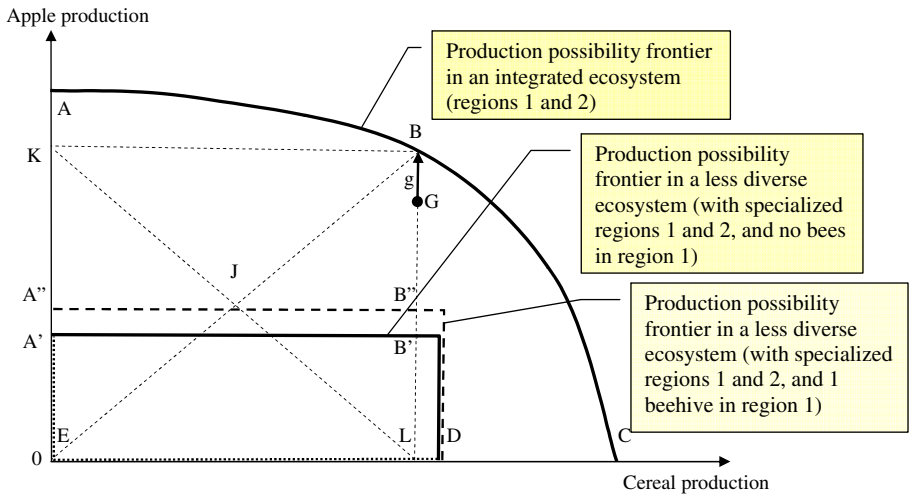
Fig. 3 Decomposition of the productive value of biodiversity

context, the agroecosystem productivity can be represented by the production possibility frontier given by ABC in Fig. 3, where point B is the currently observed outputs.[8]

Next, we consider an experiment where the original ecosystem is split into two separate subsystems. The separation takes place so that each subsystem exhibits a loss of diversity. This is done by dividing the land equally into two "more specialized" regions, where region 1 has no bees, while region 2 has no orchard.[9] The experiment is chosen such that the aggregate amount of resources does not change.[10] The two regions having equal superficies, region 1 now specializes in cereal production and bees, while region 2 specializes in apple production, aggregate land use remaining constant. Under this experiment, there is a loss of biodiversity as the agroecosystem becomes more fragmented: the apple orchards are now found only in region 1, and the bees only in region 2. This is case of complete specialization (corresponding to $\beta = 1$ and $K = 2$ in Eqs. 7).[11]

Next, we use Fig. 3 to illustrate the implications of our biodiversity-reducing experiment for the productivity of the agroecosystem. After dividing the land equally between the two regions, Fig. 3 shows the regional production possibility frontiers under the experiment. The production possibility frontiers are given by the line EA′ for region 1 (with apple production, but no cereal), and by the line ED for region 2 (with cereal production but no apple orchard). To evaluate the effect of the experiment on the agroecosystem, we need to "add" these two

---

[8] For simplicity, we assume that at point B, land is equally allocated between apple orchards (producing apples) and cereal fields (producing cereal).

[9] If the two regions are contiguous, a "buffer zone" is created along the border between the two regions. In the experiment, this buffer zone grows only cereal (i.e., it has no bees and no apple orchards) and is chosen so that the bees in region 2 cannot pollinate the apple orchard in region 1.

[10] In the experiment, we want to stress the importance of keeping aggregate resources constant. Indeed, if the experiment involved changes in aggregate resources, such changes would likely affect the ecosystem "internal functioning". In this case, the productivity implications of the experiment would reflect changes that are unrelated to biodiversity issues.

[11] While the conditions specified in the experiment are very specific, note there are scenarios that may approximate the experiment through a natural occurrence. For example, if we neglect honey production from bees, any natural occurrence leading to the extermination of all beehives in the ecosystem (e.g., from inappropriate use of pesticides) would simulate something close to our experiment.

frontiers. The aggregate production possibility frontier under the experiment is given by the line A′B′D in Fig. 3.[12] The relevant comparison is the difference between the frontier ABC under the original agroecosystem and the frontier A′B′D under the experiment. The original system is one of integration between bees and apple orchards (which benefit from the pollination services provided by the bees). Under the experiment, the agroecosystem has become less diverse and more fragmented. The spatial separation of the bees from the apple orchards means that the pollination services are no longer being performed by the bees. This missing ecological service implies an inward shift in the production possibility frontier from ABC to A′B′D. Importantly, both frontiers correspond to the same aggregate land use. Thus, the shift from ABC to A′B′D is due entirely to a deterioration in the functioning of the agroecosystem.

To measure the distance between the lines ABC and A′B′D, start from the observed point B in Fig. 3. If we choose a reference bundle g equal to 1 lb. of apple (as represented by the distance GB), the distance between ABC and A′B′D can be measured by BB′. Thus, BB′ provides a measure of the productivity loss generated by the experiment. It has the following interpretation: BB′/GB is the number of units of the reference bundle g (i.e., the number of lbs of apples) that can no longer be produced due to the deterioration in the functioning of the agroecosystem. Note that transforming this information into an economic value is straightforward: it suffices to multiply BB′/GB by the unit value of the reference bundle g (i.e., by the price of 1 lb. of apple).

Next, we show that the distance BB′ in Fig. 3 indeed measure D(z, g) in Eq. 5 using the shortage function (1) (with g being 1 lb. of apple). To see that, denote by S_F^X the shortage function evaluated at point X relative to the frontier technology F in Fig. 3. In our bee-orchard example, Eq. 5 or 8 becomes $D = S_{EA'}^K + S_{ED}^L - S_{ABC}^B$, where S_{ABC}^B is the shortage function evaluated at point B relative to the frontier technology ABC of the original ecosystem, $S_{EA'}^K$ is the shortage function evaluated at point K relative to the frontier EA′, and S_{ED}^L is the shortage function evaluated at point L relative to the frontiers ED.[13] Note that $S_{ABC}^B = 0$ (since B is on the frontier ABC), $S_{EA'}^K$ = distance KA′, and $S_{ED}^L = 0$ (since L in the frontier ED). This gives D = distance KA′, which is equal to distance BB′ in Fig. 3. This shows that the value of diversity D in (5) indeed provides a measure of biodiversity.

Figure 3 can also illustrate the decomposition of D presented in Proposition 3 under a scenario of complete fragmentation (where $\beta = 1$ and K=2). As just noted, the value of biodiversity in Fig. 3 is given by $D = S_{EA'}^K + S_{ED}^L - S_{ABC}^B$. The decomposition given in Eqs. 9–10 states that D can be alternatively written as $D = D_C + D_R + D_V + D_A = (S_{EA''}^K + S_{ED}^L - S_{ABC}^B - S_E^E) + (2S_{A'B'D}^J - S_{ABC}^B) + (S_{ABC}^B + S_E^E - 2S_{A'B'D}^J) + (S_{EA'}^K - S_{EA''}^K)$, where $D_C = (S_{EA''}^K + S_{ED}^L - S_{ABC}^B - S_E^E)$, $D_R = (2S_{A'B'D}^J - S_{ABC}^B)$, $D_V = (S_{ABC}^B + S_E^E - 2S_{A'B'D}^J)$, and $D_A = [(S_{EA'}^K + S_{ED}^L - S_{ABC}^B - S_E^E) - (S_{EA''}^K + S_{ED}^L - S_{ABC}^B - S_E^E)] = (S_{EA'}^K - S_{EA''}^K)$.[14] In the case where g is chosen such that p · g = 1, this shows that the value of diversity D as

---

[12] Figure 3 also shows the production possibility frontiers if there is just one beehive in region 1. As suggested by Fig. 2, moving from zero beehive to one beehive in region 1 means a shift in the production possibility from EA′ to EA″ for region 1, and a shift from A′B′D to A″B″D for both regions. The implications of this shift for evaluating the productivity of the agro-ecosystem are explored below.

[13] Note that technology is being held constant in Fig. 3. The frontiers given in Fig. 3 shift across evaluation points because each point involves different resource use (e.g., reflecting changes in apple orchard and cereal fields). Figure 3 controls explicitly (in the vertical and horizontal axes) only for apple production and cereal production. In this context, holding technology constant, changing land allocation does affect production possibilities.

[14] In a way consistent with Fig. 2, the term D_A reflects the discontinuity of the production frontier going from zero to one beehive in region 1. Indeed, starting with no bees in region 1, the frontiers EA′ and A′B′D in Fig. 3 shift to EA″ and A″B″D, respectively, with the introduction of one beehive in region 1.

well as the value of its components (D_C, D_R, D_V, and D_A) simply involves the evaluation of the shortage function at different points.

First, consider the value of complementarity $D_C = (S^K_{EA''} + S^L_{ED} - S^B_{ABC} - S^E_E)$.[15] As shown in (10a$'$), this expected to be positive if bees' pollination services increase the marginal productivity of apple orchard. Noting that $(S^K_{EA''} - S^E_E)$ and $(S^B_{ABC} - S^L_{ED})$ reflect the marginal product of apples, each for a different level of cereal activities (see Fig. 3), this shows that the complementarity term D_C indeed involves the evaluation of how marginal products change under different diversity scenarios. Noting that $S^L_{ED} = S^B_{ABC} = S^E_E = 0$ in Fig. 3 (since each point is on the corresponding frontier), it follows that $D_C = S^K_{EA''} = $ distance $KA' > 0$. Thus, in our bee-orchard example, complementarity does contribute positively to the value of biodiversity.

Second, consider the scale effect $D_R = (2S^J_{A'B'D} - S^B_{ABC})$, where the shortage function S^B_{ABC} is evaluated at point B relative to the production frontier ABC, while $S^J_{A'B'D}$ is evaluated at point J relative to the frontier A$'$B$'$D. Noting that $S^B_{ABC} = 0$ (since point B is on the frontier ABC) and $S^J_{A'B'D} > 0$ (since point J is above the frontier A$'$B$'$D), it follows that $D_R > 0$. As shown in (10b$'$), this corresponds to a situation of IRTS, where the fragmented sub-systems are less productive than the original ecosystem. Thus, in the case illustrated in Fig. 3, the scale effect D_R contributes positively to the value of biodiversity.

Third, consider the convexity effect $D_V = (S^B_{ABC} + S^E_E - 2S^J_{A'B'D})$. This shows that convexity is evaluated along the ray BJE. The evaluation given in Fig. 3 indicates that $S^B_{ABC} = S^E_E = 0$, and that $S^J_{A'B'D} > 0$ (since J is above the frontier A$'$B$'$D), implying that $D_V = -2S^J_{A'B'D} < 0$. In this case, D_V contributes negatively to the value of biodiversity. As discussed in Sect. 4, this identifies the presence of non-convexity in the underlying technology.

Fourth, consider the catalytic effect $D_A = (S^K_{EA'} - S^K_{EA''})$. As shown in Fig. 2, orchard productivity drops significantly when the number of bee hives drops from 1 to 0. This drop is due to three factors: 1/ bees are very effective pollinators of apple trees; 2/ the bee hive (consisting of thousands of bees) is the smallest unit that can support the bees' pollination services; and 3/ as long as the region is not too large, a change between 0 to 1 bee hive can generate a discrete change in orchard productivity. Then, the catalytic effect $D_A = (S^K_{EA'} - S^K_{EA''})$ reflects the jump-effect on the value of biodiversity of switching from 0 beehive to 1 beehive in region 1. Note that $(S^K_{EA'} - S^K_{EA''}) = $ distance $A''A' > 0$ in Fig. 3. It follows that $D_A = $ distance $A''A' > 0$, implying (as expected) that the catalytic effect contributes positively to value of biodiversity. This indicates that a complete destruction of the bee population would contribute to increasing the value of biodiversity (beyond the bee-apple complementarity effects discussed above). This illustrates how a complete loss of biodiversity can affect the value of biodiversity through the catalytic component D_A.

Note that our bee-orchard example does not satisfy all four conditions stated in Corollary 1. Indeed, in the context of Fig. 3, we have shown that $D_C = S^K_{EA''} = $ distance $KA'' > 0$, $D_R = 2S^J_{A'B'D} > 0$, and $D_A = (S^K_{EA'} - S^K_{EA}) = $ distance $A''A' > 0$. Thus, conditions 1, 2 and 4 in Corollary 1 are satisfied. But we have shown that $D_V = -2S^J_{A'B'D} < 0$, implying that condition 3 is not satisfied (due to a non-convex technology). Yet, the value of biodiversity is positive: it is $D = D_C + D_R + D_V + D_A = S^K_{EA''} + 2S^J_{A'B'D} - 2S^J_{A'B'D} + (S^K_{EA'} - S^K_{EA''}) = S^K_{EA'} = $ distance KA'. This gives two interesting results. First, in our bee-orchard example, the scale component D_R and the convexity component D_V cancel each other (as they satisfy $D_R + D_V = 0$). This means that, in this case, the value of biodiversity reduces to $D = D_C + D_A$.

---

[15] Note that $S^K_{EA''}$ is evaluated relative to the frontier EA$''$ (and not EA$'$). The difference between the two frontiers reflects the jump-discontinuity at "0 beehive", which is captured by the catalytic effect D_A discussed below.

Second, our analysis provides useful information about the role of the complementarity and catalytic effects. Our bee-orchard example shows that both effects contribute positively to the value of biodiversity. In addition, our analysis provides a basis for evaluating their relative contributions. Noting that D_C = distance KA'' and that D_A = distance $A''A'$ in Fig. 3, this indicates that the complementarity effect is the main contributor to the value of biodiversity, the catalytic effect being a more minor contributor. This illustrates the usefulness of our decomposition.

6 Extensions

Our analysis so far has neglected the role of dynamics and uncertainty. Below, we explore how introducing such factors in the analysis can generate some useful insights.

6.1 Dynamics

This section introduces ecosystem dynamics and explores the linkages between dynamics and biodiversity. In a way consistent with our analysis, let $z_t = (z_{1t}, z_{2t}) \in Z$ denote the vector of netputs at time t, where z_{1t} is a vector of state variables at time t (e.g., physical and ecological capital), z_{2t} is a vector of control variables at time t, and Z is the feasible set for (z_{1t}, z_{2t}). In general, the dynamics of the ecosystem can be represented by the state equation $z_{1,t+1} = h(z_{1t}, z_{2t})$ describing how the system evolves over time. In short run situations focusing on a given time t, the analysis presented above apply directly, with feasibility being represented by $z_t = (z_{1t}, z_{2t}) \in Z$.

To investigate the role of dynamics, consider the case of an ecosystem facing initial conditions (z_{10}, z_{20}) at time t=0, and where z_{2t} is kept constant: $z_{2t} = z_{20}$ for all $t \geq 0$. Let $h_1(z_{10}, z_{20}) \equiv h(z_{10}, z_{20})$, and $h_t(z_{10}, z_{20}) \equiv h(h_{t-1}(z_{10}, z_{20}), z_{20})$ for $t = 2, 3, \ldots$ Then, for a given (z_{10}, z_{20}), the state variables z_{1t} evolve over time along the forward trajectory $\{z_{1t} : t = 0, 1, 2, \ldots\} \equiv \{z_{10}, h_1(z_{10}, z_{20}), h_2(z_{10}, z_{20}), \ldots\}$. Assuming the existence of a steady state, the long run equilibrium of the ecosystem is given by $z_1^e(z_{10}, z_{20}) \equiv \lim_{t \to \infty}\{h_t(z_{10}, z_{20})\}$. The key questions are: How do the system dynamics affect the functioning of the ecosystem? And how do they affect the value of biodiversity?

At time t=0, the ecosystem is at point $z_0 \equiv (z_{10}, z_{20})$. This corresponds to a short run situation where the system has not had an opportunity to evolve. At the other extreme, when t is large, the ecosystem converges to its long run equilibrium $z_\infty \equiv (z_1^e(z_{10}, z_{20}), z_{20})$. To examine the role of dynamics, it will be useful to contrast the performance of the ecosystem between its short run and its long run situations (represented by z_0 and z_∞, respectively).[16]

First, consider the total value of environmental services $P(z_a, 0, z_b, g)$ in Eq. 2'. The productive value of the environmental goods provided by the ecosystem is given by $P(z_{a0}, 0, z_{b0}, g)$ in the short run, and by $P(z_{a\infty}, 0, z_{b\infty}, g)$ in the long run. Whether $P(z_{a\infty}, 0, z_{b\infty}, g)$ is larger or smaller than $P(z_{a0}, 0, z_{b0}, g)$ is largely an empirical issue (e.g., see Loreau 2000, p. 7). In situations where the ecosystem is not actively managed, its dynamics reflect the natural selection taking place over time. Then, we would have $P(z_{a\infty}, 0, z_{b\infty}, g) < P(z_{a0}, 0, z_{b0}, g)$ when natural selection has a negative effect on ecosystem productivity (e.g., the case of invasive species). Alternatively, we would have $P(z_{a\infty}, 0, z_{b\infty}, g) > P(z_{a0}, 0, z_{b0}, g)$ when natural selection contributes to increasing ecosystem productivity (e.g., Tilman and Downing 1994; Tilman et al. 1996). And in situations where the ecosystem is actively managed, one objective

---

[16] The analysis can be easily extended to examine intermediate run scenarios describing how the system evolves between a short run situation and its long run equilibrium.

of its managers is to increase its long run productivity, i.e. to identify and implement strategies that increase its productive value $P(z_a, 0, z_b, g)$ over time (e.g., the case of agriculture where species are actively selected to increase food production).

Second, consider the value of biodiversity $D(z, g) = \sum_{k=1}^{K} S(z^k, g) - S(z, g)$ given in Eq. 5 or 8. For a given $z_0 \equiv (z_{10}, z_{20})$ and at time t = 0, the k-th subsystem faces netputs $z_0^k \equiv (z_{10}^k, z_{20}^k)$, where $\sum_{k=1}^{K} z_0^k = z_0$, $k = 1, \ldots, K$. It follows that the short run value of biodiversity is $D(z_0, g) = \sum_{k=1}^{K} S(z_0^k, g) - S(z_0, g)$. In the long run, each subsystem evolves over time and eventually converges to its long run equilibrium (assuming that it exists). The long run equilibrium of the k-th sub-system is $z_\infty^k \equiv (z_1^{ek}(z_{10}^k, z_{20}^k), z_{20}^k)$, where $z_1^{ke}(z_{10}, z_{20}) \equiv \lim_{t\to\infty}\{h_t(z_{10}^k, z_{20}^k)\}$. It follows that, in the long run, the value of biodiversity in the ecosystem is $D(z_\infty, g) = \sum_{k=1}^{K} S(z_\infty^k, g) - S(z_\infty, g)$. Again, whether $D(z_0, g)$ is larger or smaller than $D(z_\infty, g)$ is largely an empirical issue. However, our decomposition of the value of biodiversity can provide useful information on this issue. With $D = D_C + D_R + D_V + D_A$ from Eq. 9, we can identify scenarios where the value of biodiversity is likely to be larger in the long run than in the short run: $D(z_\infty, g) > D(z_0, g)$. For example, this could occur when complementarities within the ecosystem take time to develop (e.g., through natural selection), implying that $D_C(z_\infty, g) > D_C(z_0, g)$. When the difference between these two complementarity effects is large enough, this would imply that $D(z_\infty, g) > D(z_0, g)$. Under such a scenario, we would expect to see an increase in the value of biodiversity over time. Alternatively, there are scenarios where the value of biodiversity may be larger in the short run than in the long run: $D(z_0, g) > D(z_\infty, g)$. For example, this could occur when the dynamics of the ecosystem involve a process of (at least partial) collapse. If that process is slow, it may not have a large effect on the provision of environmental services in the short run. However, being below some sustainability threshold, part of the ecosystem may stop functioning (e.g., the case of species extinction). In situations exhibiting a complete loss of biodiversity and where the lost services have positive catalytic effects, such effects are likely to be larger in the short run (when the ecosystem is still functioning) than in the long run, implying that $D_A(z_0, g) > D_A(z_\infty, g)$. Similar arguments may apply to complementarity effects, with $D_C(z_0, g) > D_C(z_\infty, g)$. When the differences between these short run and long run effects are large enough, this would imply that $D(z_0, g) > D(z_\infty, g)$. Under such a scenario, a declining value of biodiversity over time would parallel the declining provision of valuable environmental services.

6.2 Uncertainty and Resilience

What about the role of uncertainty? To introduce uncertainty, consider the case where the underlying technology is represented by the set $Z(e)$, where e is random variable representing uncertain factors (e.g., weather effects) affecting the feasible set, e having a given probability distribution. Then the shortage function in (1) becomes $S(z, e, g)$, and the value of environmental goods z_b in (2') becomes $P(z_a, 0, z_b, e, g)$. This makes it clear that the productive value of the ecosystem is uncertain (as it depends on e).

As discussed in Sect. 3, part of the value of the environmental goods z_b can be associated with the diversity of the ecosystem. More specifically, Eq. 5 or 8 provides a measure of the value of the ecosystem that can be attributed to its diversity. Under uncertainty and using Eqs. 7 (with β measuring the degree of fragmentation), it follows that Eq. 8 becomes $D(z, \beta, e, g)$. This shows that the value of diversity depends on the random variable e. It means that the value of diversity is itself a random variable that can be characterized by its moments. Let E denote the expectation operator, and $M_i(z, \beta)$ denote the i-th central moment of $D(z, \beta, e, g)$,

i $= 1, 2, 3, \ldots$. Then, $M_1(z, \beta) = E[D(z, \beta, e, g)]$ is the mean value of diversity, $M_2(z, \beta) = E\{[D(z, \beta, e, g) - M_1(z, \beta)]^2\}$ is its variance, and $M_3(z, \beta) = E\{[D(z, \beta, e, g) - M_1(z, \beta)]^3\}$ is its third central moment (representing skewness). Of special interest are the effects of the "fragmentation parameter" β on the moments $M_i(z, \beta)$, $i = 1, 2, 3, \ldots$ In situations where diversity has positive value, one may expect that $M_1(z, \beta)$ will be decreasing in β: a higher level of fragmentation is expected to be associated with a lower mean value of diversity. But how would greater fragmentation affect the second and third moment of the value of diversity $D(z, \beta, e, g)$? While answering this question is largely an empirical matter, there has been much interest in studying the linkages between the resilience of an ecosystem and its biodiversity (e.g., Holling 1973; Tilman and Downing 1994; Naeem and Li 1997; Petchey et al. 1999; Allison 2004; Hooper et al. 2005; France and Duffy 2006; Zhang and Zhang 2006). Resilience reflects the ability of a system to resist a shock (and especially an adverse shock).

Our analysis suggests that resilience can be expressed in terms of the following two hypotheses: H1: the variance $M_2(z, \beta)$ increases with β; and H2: the skewness $M_3(z, \beta)$ decreases with β. Hypothesis H1 indicates that a higher fragmentation β increases the variance of the value of diversity. This means that, under H1, the productive value of biodiversity becomes more variable as the ecosystem becomes less diverse (or more fragmented). Alternatively, under H1, diversity would contribute to reducing the variability in ecosystem productivity. And hypothesis H2 indicates that a higher fragmentation β decreases the skewness $M_3(z, \beta)$ (as the distribution of the value of diversity $D(z, \beta, e, g)$ becomes more "skewed to the left"). This means that, under H2, a less diverse (or more fragmented) ecosystem has a higher change to becoming much less productive. Alternatively, under H2, a more diverse ecosystem would exhibit lower odds of facing a large decrease in productive value.

This indicates how our analysis can help refine current understanding of the interactions between uncertainty, diversity and ecosystem productivity. In particular, it can help develop better insights into the linkages between diversity and resilience. For example, in situations where diversity contributes to improved resilience, our decomposition of the value of diversity (presented in Proposition 3) may help identify which factors play a role (e.g., how complementarity benefits vary with the shock e). Finally, in the presence of significant dynamics (as discussed in Sect. 6.1), the approach can also be used to investigate how the effects of shocks on ecosystem productivity evolve in the intermediate term and long term.

7 Concluding Remarks

We have presented an analysis of the productive value of biodiversity in an ecosystem. The analysis applies under general conditions, allowing for non-convexities, lack of free disposal (allowing for noxious environmental goods), and dynamics. In this context, we rely on Luenberger's shortage function (providing a representation of the frontier of what is feasible) to obtain a measure of the productive value of biodiversity. When positive, this value reflects the fact that the productivity of an ecosystem is worth more than the "sum of its parts". We showed that this value can be decomposed into four additive components, reflecting complementarity effects, scale effects, convexity effects, and catalytic effects. Our analysis provides useful information on the sources, determinants and dynamics of the productive value of biodiversity. We have illustrated in an example how the different components (especially complementarity and catalytic effects) contribute to the value of biodiversity. The identification of these components indicates that biodiversity value can be expected to vary across ecosystems. Our analysis provides a framework to support a refined investigation of

the effects of biodiversity on ecosystem productivity. As such, it should help guide future investigations of biodiversity and its implications for ecosystem management.

Appendix

Proof of Proposition 3
From Eq.18, the value of biodiversity is

$$D \equiv \sum_{k=1}^{K} S(z_a/K, \beta z_{bk}, z_{b \backslash bk}(1-\beta)/(K-1), g) - S(z, g) > 0. \tag{A1}$$

Define

$$d_1 \equiv S(z_a/K, \beta z_{b1}, z_{b \backslash b1}(1-\beta)/(K-1), g) + S_v(z_a/K, z_{b1}(1-\beta)/(K-1), \beta z_{b \backslash b1}, g).$$

And letting $z_{b,i:j} = (z_{bi}, z_{b,i+1}, \ldots, z_{b,j-1}, z_{bj})$ for $i < j$, define

$$d_k \equiv S(z_a/K, \beta z_{bk}, z_{b \backslash bk}(1-\beta)/(K-1), g) + S_v(z_a/K, z_{b,1:k}(1-\beta)/(K-1), \beta z_{b,k+1:K}, g)$$
$$- S_v(z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \beta z_{b,k:K}, g),$$

for $k = 2, \ldots, K-1$. Using $d_1, \ldots, d_{K-1}$, and given $S(z, g) = S_v(z, g) + S_f(z, g)$, expression (A1) can be alternatively written as

$$D \equiv \sum_{k=1}^{K-1} d_k + S_f(z_a/K, \beta z_{b,K}, z_{b \backslash bK}(1-\beta)/(K-1), g) - S(z, g). \tag{A2}$$

When $S_v(z, g)$ is continuous in z_b everywhere and continuously differentiable in z_b almost everywhere, note that d_1 can be alternatively written as

$$d_1 = S(z_a/K, \beta z_{b1}, z_{b \backslash b1}(1-\beta)/(K-1), g) + S_v(z_a/K, z_{b1}(1-\beta)/(K-1), \beta z_{b \backslash b1}, g)$$
$$- S_v(z_a/K, \beta z_b, g) + S_v(z_a/K, \beta z_b, g)$$
$$- S_v(z_a/K, z_b(1-\beta)/(K-1), g) + S_v(z_a/K, z_b(1-\beta)/(K-1), g),$$
$$= S_f(z_a/K, \beta z_{b1}, z_{b \backslash b1}(1-\beta)/(K-1), g)$$
$$+ \int_{z_{b1}(1-\beta)/(K-1)}^{\beta z_{b1}} \frac{\partial S_v}{\partial \gamma}(z_a/K, \gamma, z_{b \backslash b1}(1-\beta)/(K-1), g) d\gamma$$
$$- \int_{z_{b1}(1-\beta)/(K-1)}^{\beta z_{b1}} \frac{\partial S_v}{\partial \gamma}(z_a/K, \gamma, \beta z_{b \backslash b1}, g) d\gamma$$
$$+ S_v(z_a/K, \beta z_b, g) + S_v(z_a/K, z_b(1-\beta)/(K-1), g). \tag{A3}$$

Similarly, d_k can be alternatively written as

$$d_k \equiv S(z_a/K, \beta z_{bk}, z_{b \backslash bk}(1-\beta)/(K-1), g) + S_v(z_a/K, z_{b,1:k}(1-\beta)/(K-1), \beta z_{b,k+1:K}, g)$$
$$- S_v(z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \beta z_{b,k:K}, g)$$

$$-S_v(z_a/K, z_b(1-\beta)/(K-1), g) + S_v(z_a/K, z_b(1-\beta)/(K-1), g),$$

$$= S_f(z_a/K, \beta z_{bk}, z_{b\backslash bk}(1-\beta)/(K-1), g)$$

$$+ \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma, z_{b,k+1:K}(1-\beta)/(K-1), g)d\gamma$$

$$- \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma, \beta z_{b,k+1:K}, g)d\gamma$$

$$+ S_v(z_a/K, z_b(1-\beta)/(K-1), g), \tag{A4}$$

k = 2, ..., K − 1. Substituting (A3) and (A4) into (A2) yields

$$D \equiv S_v(z_a/K, \beta z_b, g) + (K-1)S_v(z_a/K, z_b(1-\beta)/(K-1), g)$$

$$+ \sum_{k=1}^{K} S_f(z_a/K, \beta z_{bk}, z_{b\backslash bk}(1-\beta)/(K-1), g)$$

$$+ \sum_{k=1}^{K-1} \left\{ \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma, z_{b,k+1:K} \right.$$

$$\times (1-\beta)/(K-1), g)d\gamma - \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma,$$

$$\left. \beta z_{b,k+1:K}, g)d\gamma \right\} - S(z, g). \tag{A5}$$

Given $S_v(z, g) = S(z, g) - S_f(z, g)$, it follows that (A5) can be written as

$$D \equiv D_C + D_R + D_V + D_A,$$

where

$$D_C \equiv \sum_{k=1}^{K-1} \left\{ \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma, z_{b,k+1:K} \right.$$

$$\times (1-\beta)/(K-1), g)d\gamma$$

$$\left. - \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma, \beta z_{b,k+1:K}, g)d\gamma \right\},$$

$$D_R \equiv K\, S(z/K, g) - S(z, g),$$

$$D_V \equiv S(z_a/K, \beta z_b, g) + (K-1)S(z_a/K, z_b(1-\beta)/(K-1), g) - K\, S(z/K, g),$$

$$D_A = \sum_{k=1}^{K} S_f(z_a/K, \beta z_{bk}, z_{b\backslash bk}(1-\beta)/(K-1), g)$$

$$- S_f(z_a/K, \beta z_b, g) - (K-1)S_f(z_a/K, z_b(1-\beta)/(K-1), g). \qquad \square$$

Lemma 1 *For any* k ∈ (0, 1),

$$S(k\,z, g) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} k\,S(z, g) \; under \begin{Bmatrix} DRTS \\ CRTS \\ IRTS \end{Bmatrix}.$$

Proof By definition, the technology Z exhibits IRTS, CRTS or DRTS if $\alpha Z \subset Z, \alpha Z = Z$, or $\alpha Z \supset Z$, respectively, for all $\alpha > 1$. Let k ∈ (0, 1). Consider the case where there is a γ

satisfying $(k\,z_a - \gamma\,g,\ k\,z_b) \in Z$. Then

$$S(k\,z, g) = \min_\gamma \{\gamma : (k\,z_a - \gamma\,g,\ k\,z_b) \in Z\},$$
$$= k\,\min_\delta \{\delta : (z_a - \delta\,g,\ z_b) \in (1/k)Z\}, \quad \text{where } \delta = \gamma/k,$$

$$\begin{Bmatrix} < \\ = \\ > \end{Bmatrix} k\,S(z, g) \text{ when } (1/k)\,Z \begin{Bmatrix} \supset \\ = \\ \subset \end{Bmatrix} Z, \text{ i.e., under } \begin{Bmatrix} \text{DRTS} \\ \text{CRTS} \\ \text{IRTS} \end{Bmatrix}. \qquad \square$$

Lemma 2 *If the set* Z *is convex, the shortage function* $S(z, g)$ *is convex in* z.

Proof Consider any two netput vectors $z \in R^{n+m}$ and $z' \in R^{n+m}$. First assume that $S(z, g)$ and $S(z', g)$ are finite. It follows that $(z - S(z, g)g) \in Z$ and $(z' - S(z', g)g) \in Z$. Let $z'' = \theta z + (1 - \theta)z'$, for any scalar $\theta \in [0, 1]$. If the set Z is convex, it follows that

$$[z'' - \theta S(z, g)g - (1 - \theta)S(z', g)g] \in Z.$$

The shortage function being defined as a minimum in (1), this yields

$$S(z'', g) = S(\theta z + (1 - \theta)z', g) \leq \theta S(z, g) + (1 - \theta)S(z', g).$$

Second, consider the case where $S(z, g)$ and/or $S(z', g)$ are infinite. Then, the above inequality always holds. Thus, the function $S(z, g)$ is convex in z. $\square$

References

Allison G (2004) The influence of species diversity and stress intensity on community resistance and resilience. Ecol Monogr 74:117–134

Bissonette JA, Storch I (2002) Fragmentation: is the message clear. Conserv Ecol 6:14 (online at http://www.consecol.org/vol16/iss2/art14)

Boserup E (1965) The conditions of agricultural growth. Allen and Unwin, London

Boserup E (1981) Population and technological change: a study of long-term trends. The University of Chicago Press, Chicago, IL

Brock WA, Xepapadeas A (2003) Valuing biodiversity from an economic perspective: a unified economic, ecological and genetic approach. Am Econ Rev 93:1597–1614

Chambers RG, Chung Y, Färe R (1996) Benefit and distance functions. J Econ Theor 70:407–419

Debinski DM, Holt RD (2000) A survey and overview of habitat fragmentation experiments. Conserv Biol 14:342–355

Faith DP, Carter G, Cassis G, Ferrier S, Wilkie L (2003) Complementarity, biodiversity viability analysis and policy-based algorithms for conservation. Environ Sci Policy 6:311–328

Färe R, Grosskopf S (2000) Theory and application of directional distance functions. J Prod Anal 13:93–103

France KE, Duffy JE (2006) Diversity and dispersal interactively affect predictability of ecosystem functions. Nature 441:1139–1143

Heal G (2000) Nature and the marketplace: capturing the value of ecosystem services. Island Press, New York

Heiser CB Jr (1990) Seeds to civilization: the story of food. Harvard University Press, Cambridge, MA

Heisey PW, Smale M, Byerlee D, Souza E (1997) Cereal rusts and the costs of genetic diversity in the Punjab of Pakistan. Am J Agric Econ 79:726–737

Hill MO (1973) Diversity and evenness: a unifying notation and its consequences. Ecology 54:427–432

Holling CS (1973) Resilience and stability of ecological systems. Annu Rev Ecol Systemat 4:1–23

Hooper DU, Chaplin FS, Ewel JJ, Hector A, Inchausti P, Lavorel S, Lawton JH, Lodge DM, Moreau M, Naeem S, Schmid B, Setala H, Symstad AJ, Vandermeer J, Wardle DA (2005) Effects of biodiversity on ecosystem functioning: a consensus of current knowledge. Ecol Monogr 75:3–35

Justus J, Sarkar S (2002) The principle of complementarity in the design of reserve networks to conserve biodiversity: a preliminary history. J Biosci 27:421–435

Keylock CJ (2005) Simpson diversity and the Shannon-Wiener index as special cases of generalized entropy. Oikos 109:203–207

Lande R (1996) Statistics and partitioning of species diversity, and similarity among multiple communities. Oikos 76:5–13

Loreau M (2000) Biodiversity and ecosystem functioning: recent theoretical advances. Oikos 91:3–17

Loreau M, Hector A (2001) Partitioning selection and complementarity in biodiversity experiments. Nature 412:72–76

Losey JE, Vaughan M (2006) The economic value of ecological services provided by insects. BioScience 56:311–323

Luenberger D (1995) Microeconomic theory. McGraw-Hill, Inc., New York

Mainwaring L (2001) Biodiversity, biocomplexity and the economics of genetic dissimilarity. Land Econ 77:79–93

May RM (1975) Patterns of species abundance and diversity. In: Cody ML, Diamond JM (eds) Ecology and evolution of communities. Harvard University Press, pp 81–120

Meng ECH, Smale M, Bellon MR, Grimanelli D (1998) Definition and measurement of crop diversity for economic analysis. In: Smale M (ed) Farmers, gene banks, and crop breeding. Kluwer, Boston, MA, pp 19–31

Montgomery CA, Brown GM, Adams DM (1994) The marginal cost of species preservation: the northern spotted owl. J Environ Econ Manage 26:111–128

Morse RA, Calderone NW (2000) The value of honey bees as pollinators of U.S. crops in 2000. Bee Cult 128:1–15

Naeem S, Li SB (1997) Biodiversity enhances ecosystem reliability. Nature 390:507–509

Naeem S, Thompson LJ, Lawler SP, Lawton JH, Woodfin RM (1994) Declining biodiversity can affect the functioning of ecosystems. Nature 368:734–737

Petchey OL, McPhearson PT, Casey TM, Morin PJ (1999) Environmental warming alters food-web structure and ecosystem function. Nature 402:69–72

Polasky S, Solow AR (1995) On the value of collection of species. J Environ Econ Manage 29:298–303

Priestley RH, Bayles RA (1980) Varietal diversification as a means of reducing the spread of cereal diseases in the United Kingdom. J Natl Inst Agric Bot 15:205–214

Robinson G, Willard S, Nowogrodski R, Morse RA (1989) The value of honey bees as pollinators of US crops. Am Bee J 129:477–487

Routledge RD (1979) Diversity indices:which ones are admissible?J Theor Biol 76:503–515

Shephard RW (1970) Theory of cost and production functions. Princeton University Press, Princeton

Simpson EH (1949) Measurement of species diversity. Nature 163:688

Smale M (ed) (2006) Valuing crop biodiversity:on-farm genetic resources and economic change. CABI Publishing, Cambridge, MA

Smale M, Hartell J, Heisey PW, Senauer B (1998) The contribution of genetic resources and diversity to cereal production in the Punjab of Pakistan. Am J Agric Econ 80:482–493

Smale M, Reynolds MP, Warburton M, Skovmand B, Trethowan R, Singh RP, Ortiz-Monasterio I, Crossa J (2002) Dimensions of diversity in modern spring bread cereal in developing countries from 1965. Crop Sci 42:1766–1779

Smale M, Meng E, Brennan JP, Hu R (2003) Determinants of spatial diversity in modern cereal:examples from Australia and China. Agric Econ 28:13–26

Southwick EE, Southwick L (1992) Estimating the economic value of honey bees as agriucltural pollinators in the United States. Econ Entomol 85:621–633

Tilman D, Downing JA (1994) Biodiversity and stability in grasslands. Nature 367:363–365

Tilman D, Wedin D, Knops J (1996) Productivity and sustainability influenced by biodiversity in grassland ecosystems. Nature 379:718–720

Weitzman ML (1992) On diversity. Q J Econ 107:363–405

Weitzman ML (1998) The Noah's ark problem. Econometrica 66:1279–1298

Wood D, Lenné JM (eds) (1999) Agrobiodiversity:characterization, utilization and management. CABI Publishing, Cambridge, MA

Zhang QG, Zhang DY (2006) Resource availability and biodiversity effects on the productivity, temporal variety and resistance of experimental algal communities. Oikos 114:385–396