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Environmental Taxation and Vertical Cournot Oligopolies: How Eco-industries Matter

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Abstract This article specifies what an optimal pollution tax should be when dealing with a vertical Cournot oligopoly. Polluting firms sell final goods to consumers and outsource their abatement activities to an environment industry. It is assumed that both markets are imperfectly competitive. Thus, the tax is a single instrument used to regulate three sorts of distortions, one negative externality and two restrictions in production. Consequently, the optimal tax rate is the result of a trade-off that depends on the firms' market power along the vertical structure. A detailed analysis of Cournot-Nash equilibria in both markets is also performed. In this context, the efficiency of abatement activities plays a key-role. It gives a new understanding to the necessary conditions for the emergence of an eco-industrial sector.

Keywords Eco-industry · End-of-pipe pollution abatement · Environmental taxation · Vertical Cournot oligopolies

JEL Classifications D43 · H23 · Q58

1 Introduction

Restrictions in production due to imperfect competition among polluting firms can be seen as positive for the environment. They reduce gross emissions and consequently lessen the optimal level of taxation. However, there is one sector where restrictions in production have a direct negative impact on the environment. It concerns the eco-industry, the sector supplying

[1] "Eco-industries may be described as including firms producing goods and services capable of measuring, preventing, limiting or correcting environmental damage such as the pollution of water, air, soil, as well as waste and noise-related problems. They include clean-technologies where pollution and raw-material used is being minimized" (OECD 1999).

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polluting firms in abatement goods and services.[1] Less of these goods means that polluters' abatement incentives are reduced and net emissions are higher than their optimal level.

Examples of imperfect competition both at an upstream and a downstream level are common. For instance, take the case of European chemicals firms which purchase end-of-pipe pollution abatement from one of the three major firms on the water and waste-water management sector—namely Veolia, Onyx and Saur. Both downstream and upstream firms can be suspected of holding a market power on their respective markets. There are other eco-industries that are characterized by oligopolistic competition. Technologies of air pollution control are only supplied by a few firms in Europe and North-America. It is also the case in most countries with plastic and paper waste. In the United States, merger activities within the waste management sector have led to the emergence of an oligopoly based on four main firms—Waste Management, Allied Waste Industries, Republic Services and Onyx North America.

The confrontation of market power at an upstream and a downstream level and its impact on optimal environmental taxation when the pollution tax is the only available instrument makes the specificity of this work. It also puts the eco-industry in context. The conditions of the emergence of this sector under imperfect competition are precised by detailing how the demand faced by eco-industries is the result of a cost-minimization decision by the polluting firms, and how it is the consequence of the presence of a pollution tax.

When competition is perfect, an optimal environmental tax should be chosen equal to marginal social damage (Pigou 1920). However, when the structure of the polluting good market is monopolistic, production is reduced. If the regulator considers both pollution and production in its decision, then it is optimal to choose an environmental tax that falls short of marginal damage, so as to avoid too much restriction in production (Buchanan 1969; Barnett 1980). The gap between marginal environmental damage and pollution tax depends on the price elasticity of demand. This basic approach has been extended to the analysis of Cournot oligopolies—including the case of asymmetric competition—and the main intuitions remain (Levin 1985; Simpson 1995).

In most of the environmental economics literature, pollution abatement is assumed to be set only by polluters. Still, tighter environmental policies have contributed to the emergence of an abatement market.[2] If the importance of the eco-industrial sector has been recognized by numerous reports from national and international institutions (OECD 1996; Berg et al. 1998; Ecotech Research and Consulting Limited 2002; Numeri and R.D.I. 2004; Kennett and Steenblik 2005), and by a few empirical studies (Barton 1997; Baumol 1995), it would be hard to find any trace of the existence of such an industry in the theoretical approach of environmental policy. David and Sinclair-Desgagné (2005) are the first to address the consequences of the market power of the eco-industry on the efficiency of environmental policies. The sector is assumed to be imperfectly competitive. Firms compete in quantity and therefore supply abatement technologies to polluting firms at a higher price than their marginal production costs. So, the cost of reducing pollution for downstream firms is higher than if it had been done internally, which pushes the regulator to set up a tax higher than marginal damage. If not, polluters would choose an abatement level that is too low relative to the first best.

Confronting the impacts of underproduction along the vertical structure of polluting activities seems to be a natural extension. A simultaneous and independent attempt to deal with this has been proposed by Nimubona and Sinclair-Desgagné (2005). Focusing on Lerner indexes, they reconsider Pigouvian taxes in the context of imperfect competition and discuss in which

[2] The goods and services provided by eco-industries represents approximately 2.2% of GDP in the EU-25 area. The total direct and indirect employment due to eco-industries represent approximately 3.4 million full-time job equivalents (European Commission, DG Environment 2006).

cases the tax must be set higher or lower than marginal damage. However, in their model, the environmental demand is treated as exogenous. By detailing how the demand faced by eco-industries comes from the cost-minimization decision of the polluting industry, we underline the role played by technological characteristics on this vertical chain. The demand faced by the eco-industry is directly influenced by the relative efficiency of the depollution function used by polluting firms. Greaker (2006) explains that more stringent environmental policies would lead to a more active abatement sector and more competitiveness for downstream industries. We refine this argument by showing that the existence of environmental policies is only a necessary condition for the emergence of an eco-industry.

We also give sufficient conditions for the existence and uniqueness of Cournot-Nash equilibria in both markets. A lot of literature has already presented more or less restrictive conditions (Kolstad and Mathiesen 1987; Gaudet and Salant 1991; Long and Soubeyran 2000). Their results could have been applied without loss of generality for the downstream market. However, the context induced by the vertical chain needed specific attention. In fact, the demand function in environmental goods is directly determined by downstream firms' clean-up functions. Consequently, conditions characterizing equilibria in the literature are not easily tractable in our context. That is why our work is also devoted to presenting and discussing sufficient conditions to ensure a market equilibrium on the eco-industrial sector. It has already been proven that a unique equilibrium exists in the context of the taxation of a polluting Cournot oligopsonistic oligopoly (Okuguchi 2004). However, the author introduces imperfect competition in all factor markets and for our purposes, competition in the environmental sector is paramount.

Using the above contributions, this article presents the environmental tax chosen by a regulator dealing with pollution and imperfect competition. Once the tax is chosen, we model the consequences in two oligopolistic markets, with firms competing *à la* Cournot.[3] Downstream, firms compete for the supply of a final good, purchased by consumers. Upstream, eco-industry firms compete to sell environmental goods and services to polluting firms. Among the key elements of the model, both price elasticities of demand play a significant role. They indicate to what extent a polluting firm adjusts its level of production and pollution to a tax rate and therefore influence the optimal tax. The number of firms in both markets also matters. As they modify the degree of competition, they influence the regulator's decision. All these variables can be empirically estimated.

We have structured the paper as follows: the following section presents the model and solves the last two stages, notably showing the existence and uniqueness of Cournot equilibria in both markets. Section 3 then derives the optimal pollution tax. It gives some details on which effect dominates so as to decide whether the tax should be set above or below marginal damage. At last, Section 4 concludes and suggests different ways to further develop this work.

2 Vertical Structure and Eco-industry

Given the sphere we have chosen to work in, the following three-stage game is going to be solved, by backward induction:

[3] This modeling introduces firms as Stackelberg followers because they consider the tax rate chosen by the regulator to maximize social welfare as a given (Petrakis and Xepapadeas 2003). We are aware that it is a restrictive assumption, as in some countries, the biggest firms can try to lobby toward a shift in environmental regulation that goes in their interests. This leaves room for work on the role of eco-industry lobbies in the political economy of environmental policy.

1. The regulator chooses an optimal tax to control pollution;
2. eco-industry firms, anticipating the demand, compete in quantity. This gives a price for environmental goods;
3. polluting firms, given the tax and the price of environmental inputs, choose their optimal level of production and pollution.

The last two stages are now presented.

2.1 The Polluting Downstream Firms

There are n symmetric downstream firms, indexed by i, in this vertically related structure which produce a given commodity x at the same cost $c_d(x_i)$. This cost function is assumed to be increasing and convex, i.e. $\forall x_i \in \mathbb{R}_{++}$, $c'_d(x_i) > 0$ and $c''_d(x_i) > 0$. Inactivity is allowed i.e. $c_d(0) = 0$ but marginally $c'_d(0_+) \geq 0$.[4] Production cannot expand to infinity since the marginal cost goes to infinity as x_i becomes large, i.e. $c'_d(+\infty) = +\infty$.

This activity aims to satisfy some demand characterized, as usual in a Cournot setting, by the inverse demand $f(X)$ which is common knowledge and well behaved. $f(X)$ is positive and strictly decreasing in its domain. The basic point of the paper is that this activity generates some pollution which is summarized by an emission function $\varepsilon(x_i)$. This function, identical for all firms is assumed to be increasing and convex with the level of production i.e. $\forall x_i \in \mathbb{R}_{++}$, $\varepsilon'(x_i) > 0$ and $\varepsilon''(x_i) > 0$. Inactivity generates no pollution i.e. $\varepsilon(0) = 0$ but marginally $\epsilon'(0_+) \geq 0$. In other words, pollution appears more and more as the production expands. We therefore exclude any kind of threshold related to the existence of non-convexity in the emission function.[5] Pollution is taxed at a rate t, giving firms an incentive to reduce this undesirable by-product by starting a clean-up activity which requires some specific inputs a_i sold by an upstream eco-industry at a price p. The efficiency of this activity is given by a function $w(a_i)$ which measures the amount of pollution cleaned by the purchase of a_i environmental goods and whose characteristics are crucial in the decision to enter or not into this clean-up process or in other words, in the decision of whether or not to buy the required inputs produced by the eco-industry. So, we assume that the technology is characterized by a decreasing marginal productivity, i.e. $\forall a_i \in]0, +\infty[$, $w'(a_i) > 0$ and $w''(a_i) < 0$. More environmental goods consumed decrease the net amount of pollution, but at a decreasing rate.[6]

From this point of view any downstream firm maximizes the following profit function over two variables x_i and a_i, the individual level of production and the amount of purchased environmental goods, respectively.

$$\Pi_i = f(X)x_i - c_d(x_i) - pa_i - t\left(\epsilon(x_i) - w(a_i)\right) \tag{1}$$

If all downstream firms simultaneously maximize their profits by taking p and t as given, we get an equilibrium in the downstream sector. Moreover, due to the assumption of end-of-pipe

[4] We note, for a given function f, $\lim_{z'\to x, z'>x} f(z) = f(x_+)$ and $\lim_{z'\to+\infty} f(z) = f(+\infty)$. We also precise that all functions are assumed continuous and twice differentiable.

[5] It does not encompass all possible cases of pollution. For instance, ϵ can be first concave and then convex. However, this technical assumption is necessary to provide general conditions on the existence and unicity of a solution.

[6] Abatement activities are assumed additively separable to the production process. The model here closely follows David and Sinclair-Desgagné (2005). End-of-pipe abatement activities include solid waste management and recycling, waste water treatment, air pollution control or noise and vibration control. Even though more preventative approaches are more and more common, end-of-pipe activities still represented two-thirds of the eco-industry's turnover in 2004 (European Commission, DG Environment 2006).

pollution abatement, we are even able to separate the production decision from the decision to purchase environmental goods. The last decision solves:

$$\frac{\partial \Pi_i}{\partial a_i} = -p + tw'(a_i) = 0 \ \ \forall i = 1, \ldots, n \tag{2}$$

Under our assumptions about the clean-up process $w(a_i)$, we even know by the inverse function theorem that the individual demands of environmental goods are such that $\forall i = 1, \ldots, n, \ a_i(p, t) = \left(w'\right)^{-1}\left(\frac{p}{t}\right)$, which depends as expected on the characteristics of the clean-up function.

It now remains to make sure that the whole downstream sector is at equilibrium and to characterize the total equilibrium level of production for later surplus computation. Equilibrium production levels are given by those of a standard Cournot-Nash equilibrium in which the inverse demand is given by $f(X)$ and the production costs are the same and obtained by $c(x_i) = c_d(x_i) + t\varepsilon(x_i)$. Here, technical conditions are needed. First, the slope of the demand function should be lower than $n + 1$, i.e. $e_{f'}(X) := -\frac{f''(X)X}{f'(X)} \leq n + 1$, which means that the demand function should be concave or at least not too convex. Moreover, we suppose that the aggregate marginal revenue of the polluting industry is higher than the aggregate marginal cost function as the overall production level goes to zero, i.e. $nf(0_+) + f'(0_+)0_+ > n(c_d'(0_+) + t\varepsilon'(0_+))$ and that $f(+\infty) > c_d'(0_+) + t\varepsilon'(0_+)$, which means that the demand function always remains higher than the marginal cost function as the individual production level tends to zero. Both conditions impose a limit on the per unit tax rate, i.e. $t < \bar{t} = \min\{\frac{nf(0_+)+f'(0_+)0_+-nc_d'(0_+)}{n\varepsilon'(0_+)}, \frac{f(+\infty)-c_d'(0_+)}{\varepsilon'(0_+)}\}$.[7] It is now a matter of fact to check that all the conditions of Lemma 2 given in Appendix 1 are satisfied. Applying this Lemma, it is therefore proved that:

Proposition 1 *Under our assumptions on the inverse demand $f(X)$, the cost function $c_d(x_i)$ and the emission function $\varepsilon(x_i)$, there exists a unique n-firm Cournot equilibrium in the market of the polluting good and this equilibrium is interior and symmetric.*

2.2 The Upstream Eco-industry

Since the players of the upstream eco-industry are able, as usual in a subgame perfect equilibrium, to anticipate the behaviors of downstream firms, the expected demand for environmental goods is given by:

$$A = \sum_{i=1}^{n} a_i(p, t) = n\left(w'\right)^{-1}\left(\frac{p}{t}\right)$$

where A denotes the expected total amount of environmental goods traded. We assume that polluting firms are price-takers in the eco-industry market. As already underlined in Ishikawa and Spencer (1999), it is open to the criticism that downstream firms recognize their market power as sellers of the final-good, but take price as given as buyers of the environmental input. Relaxing this assumption while keeping a Cournot competition framework increases sharply the difficulty of the analysis. Moreover, it is justified when one considers that the polluting firms are seeking generic abatement goods and services, i.e. goods and services

[7] It is generally assumed that $c_d'(0_+) = \varepsilon'(0_+) = 0$, which simply implies that $f(X)$ must be positive for all $X \in]0, +\infty[$. However, relaxing these assumptions imposes a condition on the per unit tax rate, which is: $t < \bar{t}$. If $t \geq \bar{t}$, there exists no equilibrium in the market and firms will stop producing. See Fig. 1 for an intuitive explanation in the similar case of the eco-industry.

which are not tailored to their specific needs and can be sold to many firms in other industries. Indeed, eco-industry firms are present in more than one segment. For instance, Veolia and Onyx—through different subsidiaries—are present in the water management market as well as in the waste management market. Therefore, it is generally difficult for polluting firms to pretend to have a market power in the input market, as eco-industry firms could otherwise reallocate their activities.[8] In any case, if polluting firms can behave strategically in the upstream market, eco-industry firms lose their rents, leading to the case of perfect competition.

From this point of view, the inverse demand function is $p = tw'\left(\frac{1}{n}\sum_{j=1}^{m} a_j\right)$ where a_j denotes the production of the jth upstream firm. Upstream firms support a production cost $c_u(a_j)$. It is assumed to be the same for each firm, and it satisfies usual restrictions on cost functions. It is increasing and convex, i.e. $\forall a_j \in \mathbb{R}_{++}$, $c_u'(a_j) > 0$ and $c_u''(a_j) > 0$. Inactivity is allowed, i.e. $c_u(0) = 0$, but marginally $c_u'(0_+) \geq 0$ and production cannot expand to infinity since $c_u'(+\infty) = +\infty$. Since the price expectation $tw'\left(\frac{A}{n}\right)$ is commonly shared, each upstream Cournot player simply decides to produce the amount a_j^* of environmental goods which maximizes its profit:

$$\Pi_j = tw'\left(\frac{1}{n}\sum_{j=1}^{m} a_j\right) a_j - c_u(a_j)$$

Everybody acts in the same way, so equilibrium production levels typically form a Nash equilibrium. As in the case of Proposition 1, we make use of Lemma 2 to prove that[9]:

Proposition 2 *Under the double condition that (i) $e_{w''}(A) \leq (m+1)$ and (ii) $t > \underline{t} = max\{\frac{c_u'(0_+)}{w'(+\infty)}, \frac{m}{m-e_{w'}(0_+)}\frac{c_u'(0_+)}{w'(0_+)}\}$, there exists a unique m-firm Cournot equilibrium in the eco-industry market and this equilibrium is symmetric.*

Both conditions have important implications on the analysis of the eco-industry. The first one stipulates that the marginal depollution function should be decreasing and concave (w'' and $w''' < 0$) or at least not too convex (w''' slightly positive). Let us denote $w(a) = \int_0^a \omega(s)d(s)$ the quantity of pollution cleaned up by a units of environmental goods. $\omega(s)$ is the marginal quantity of cleanup following the use of the last unit of environmental goods or services. Consequently, $w''(a) = \omega'(a) < 0$ and $w'''(a) = \omega''(a) < 0$ signifies that $\omega(a)$ decreases more and more quickly. The last unit of environmental good still has a positive impact—$\omega(a) > 0$—but is less and less important and at an increasing rate. The second one allows us to discuss some of the factors explaining the existence of an eco-industry.

2.3 Conditions for the Emergence of an Eco-industry

The second condition in Proposition 2 suggests that the pollution tax should be chosen above a certain threshold. We present an interpretation for this restriction. The first order condition of profit maximization for each eco-industry firm is a continuous function strictly decreasing

[8] When interactions are repeated, for instance in the case of air pollution management in oil refineries, contracts are made between upstream and downstream firms. This is the vertical structure chosen by Hamilton and Requate (2004) in the context of both Cournot and Bertrand competition and strategic environmental trade policies. They show that allowing a contract fixing an input price and a lump-sum payment among firms means that there is no incentive for the regulator to deviate from a tax equals to marginal damage.

[9] We pose that: $e_{w''}(A) = -\frac{w'''(\frac{A}{n})\frac{A}{n}}{w''(\frac{A}{n})}$ and $e_{w'}(A) = -\frac{w''(\frac{A}{n})\frac{A}{n}}{w'(\frac{A}{n})}$.

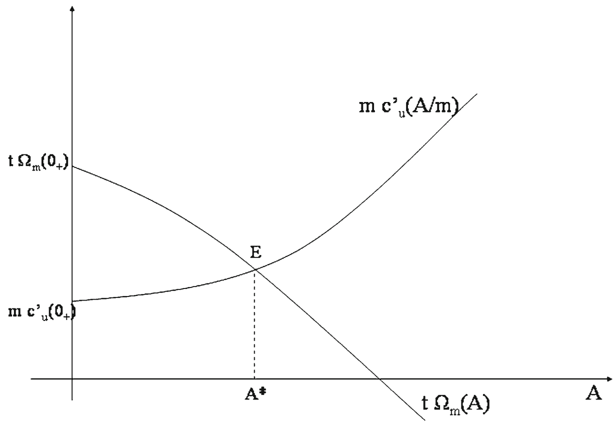

Fig. 1 Aggregate marginal revenue and cost functions

in a_j and tends to $-\infty$ when a_j increases. In order to find a potential unique solution, we need to make sure that this function is positive for low values of a_j. $\forall A \in]0, +\infty[$, we need $tw'(\frac{A}{n}) > c'_u(0_+)$. As the marginal depollution function is decreasing in A by assumption, the pollution tax must be such that $t > \frac{c'_u(0_+)}{w'(+\infty)}$.

Furthermore, let us define $\Omega_m(A) = w''(\frac{A}{n})\frac{A}{n} + mw'(\frac{A}{n}) = w'(\frac{A}{n})(m - e_{w'}(A))$ for $A \geq 0$. $t\Omega_m(A)$ is the aggregate marginal revenue of the eco-industry and $mc'_u(\frac{A}{m})$ the aggregate marginal cost function. Both curves are presented in Fig. 1. According to our assumptions, both functions are C^2 and the aggregate marginal revenue of the eco-industry must be strictly decreasing whereas its marginal cost is strictly increasing and tends to infinity when $A \to +\infty$. Thus, there is a unique equilibrium in the market if for really low values of environmental goods produced, the marginal revenue of the eco-industry is above its marginal cost. In other words, we want that $t\Omega_m(0_+) > mc'_u(0_+)$, which implies $t > \frac{m}{m-e_{w'}(0_+)}\frac{c'_u(0_+)}{w'(0_+)} = g(m)$.

Proposition 3 *(i) The existence of environmental policies is only a necessary condition for the emergence of an eco-industry; (ii) For a given level of pollution tax, an eco-industry is more likely to arise if the marginal cost of producing the first units of abatement goods is low, the marginal efficiency of abatement activities is high and the initial number of firms is high.*

This analysis explains the trade-off faced by polluters between doing nothing and buying environmental goods and services. If the regulator chooses a tax that does not satisfy the condition of Proposition 2, polluters reduce their gross emissions—the pollution tax has a negative impact on X—but no environmental goods or services can be sold and therefore no abatement activities take place. In our context, an eco-industry can only exist if the tax is sufficiently high. Among the factors explaining the emergence of an eco-industry, technological characteristics such as the shape of the clean-up function or the marginal cost of producing abatement goods and services are fundamental parameters. The more efficient the eco-industry is—both in terms of production costs or clean-up efficiency—the lower the right hand side (RHS) of condition 2 in Proposition 2 is. The number of firms that are going to enter the market is also a decisive criteria. $g(m)$ is decreasing in m, so the higher the potential number of eco-industry firms, the more likely the market has an equilibrium.

3 The Regulator's Decision

The regulator wishes to maximize social welfare. This welfare can be specified as follows:

$$W = \int_0^{X^*(t)} f(u)du - nc_d\left(\frac{X^*(t)}{n}\right) - mc_u\left(\frac{A^*(t)}{m}\right)$$
$$-n\nu\left(\varepsilon\left(\frac{X^*(t)}{n}\right) - w\left(\frac{A^*(t)}{n}\right)\right) \tag{3}$$

The first part of this function considers consumers' surplus and DFs' turnover. Then, we take into account DFs' and UFs' cost functions, the supply of environmental goods being only a transfer between firms. The last part of the surplus measures the damage induced by pollution, where ν is the constant marginal environmental damage of each unit of pollution.[10]

3.1 An Optimal Pollution Tax

The optimal pollution tax is found when the following condition is satisfied[11]:

$$0 = f(X)\frac{dX}{dt} - c_d'\left(\frac{X}{n}\right)\frac{dX}{dt} - c_u'\left(\frac{A}{m}\right)\frac{dA}{dt} - \nu\left[\epsilon'\left(\frac{X}{n}\right)\frac{dX}{dt} - w'\left(\frac{A}{n}\right)\frac{dA}{dt}\right] \tag{4}$$

Note that an interior equilibrium, where both sectors supply strictly positive quantities, will only be found if $\underline{t} < t^* < \bar{t}$. Before presenting an expression of the optimal tax, we discuss the impact of a change in the tax on global production levels.

Lemma 1 *An increase in the tax rate always induces a reduction in the output produced in the downstream market and higher levels of environmental goods purchased.*

Proof By totally differentiating the FOCs of DFs' and UFs' programs, we present the expected variations of X and A according to t. The following expressions, and their signs, are explained in Appendix 2:

$$\frac{dX}{dt} = \frac{n\epsilon'\left(\frac{X}{n}\right)}{f''(X)X + (n+1)f'(X) - c_d''\left(\frac{X}{n}\right) - t\epsilon''\left(\frac{X}{n}\right)} < 0 \tag{5}$$

$$\frac{dA}{dt} = \frac{-m\left(w''\left(\frac{A}{n}\right)\frac{A}{nm} + w'\left(\frac{A}{n}\right)\right)}{tw'''\left(\frac{A}{n}\right)\frac{A}{n^2} + \frac{t}{n}(m+1)w''\left(\frac{A}{n}\right) - c_u''\left(\frac{A}{m}\right)} > 0 \tag{6}$$

□

A more stringent pollution tax increases DFs' production costs. Each downstream firm has an interest in reducing production. As the number of firms is assumed to be constant, overall production is also reduced. Conversely, an increase in the tax shifts the demand upward in abatement activities. It leads to an increase in production from UFs.

Using FOCs of profit maximization at an upstream and a downstream level, we substitute $c_d'(x)$ and $c_u'(a)$ in Eq. 4 by their values at equilibrium. Calculations leading to the implicit expression of the optimal tax t can be found in Appendix 3.

[10] Two comments can be made about the taxes collected by the regulator: first we do not take into account any opportunity cost. Second, we assume that taxes are reallocated as lump-sum transfers.

[11] We are now dropping the superscript * and the fact that X and A depend on t.

$$t=\nu\frac{\left[\epsilon'\left(\frac{X}{n}\right)\frac{dX}{dt}-w'\left(\frac{A}{n}\right)\frac{dA}{dt}\right]}{\left[\epsilon'\left(\frac{X}{n}\right)\frac{dX}{dt}-w'\left(\frac{A}{n}\right)\left(1-\frac{e_{w'}(A)}{m}\right)\frac{dA}{dt}\right]}+\frac{\frac{f(X)}{ne_{X/f(X)}}\frac{dX}{dt}}{\left[\epsilon'\left(\frac{X}{n}\right)\frac{dX}{dt}-w'\left(\frac{A}{n}\right)\left(1-\frac{e_{w'}(A)}{m}\right)\frac{dA}{dt}\right]} \tag{7}$$

where $e_{X/f(X)}=\left(\frac{df(X)}{dX}\frac{X}{f(X)}\right)^{-1}$. The first term on the RHS of Eq. 7 is necessarily higher than ν. As the numerator of the second part of the RHS of Eq. 7 is positive and the denominator necessarily negative, we are sure that this term will be negative. Therefore, this equation underlines the trade-off that faces a benevolent regulator.

3.2 A Comparison of Tax and Marginal Damage

In order to better understand the main variables influencing the regulator's decision, Eq. 4 can be rewritten as follows:

$$(t-\nu)\left(\epsilon'\left(\frac{X}{n}\right)\frac{dX}{dt}-w'\left(\frac{A}{n}\right)\frac{dA}{dt}\right)=f'(X)\frac{X}{n}\frac{dX}{dt}+tw'\left(\frac{A}{n}\right)\frac{e_{w'}(A)}{m}\frac{dA}{dt}$$

As the terms into brackets on the LHS are always negative, we can notice that:

$$t\gtreqless\nu\Leftrightarrow f'(X)\frac{X}{n}\frac{dX}{dt}+tw'\left(\frac{A}{n}\right)\frac{e_{w'}(A)}{m}\frac{dA}{dt}\lesseqgtr 0$$

Introducing price elasticities of demand, this trade-off can be rewritten as follows:

$$t\gtreqless\nu\Leftrightarrow\left|\frac{e_f(X)}{n}f(X)\frac{dX}{dt}\right|\lesseqgtr\left|\frac{e_p(A)}{m}p(A)\frac{dA}{dt}\right| \tag{8}$$

The two opposite incentives are now isolated on each side of the inequation. This enables us to emphasize the key elements affecting the trade-off and to do comparative statics. First, the higher the number of downstream (resp. upstream) firms, the more likely the tax will be set above (resp. below) marginal damage. When the number of firms increases in a market, the level of competition increases as well, reducing the mark-up between price and marginal cost. Consequently, the regulator has less reasons to distort the tax from marginal damage. Second, the higher the elasticity in the downstream (resp. upstream) market, the more likely the tax will be set below (resp. above) marginal damage. Compared to the current literature, we underline that the price elasticity in the environment market is influenced by both the environmental tax and the efficiency of abatement activities. The technical characteristics of the abatement process are then at the core of the analysis of the eco-industry's market power. Third, the trade-off is affected by the impact on both market values of a change in the tax rate. Here, the market value is given by the price times the overall variation in production. The following proposition summarizes the regulator's position.

Proposition 4 *(i) In a context of imperfect competition along the vertical structure of polluting activities, an optimal environmental tax is always the result of a trade-off between two antagonistic effects: the inefficient level of production in the final good market tends to induce a lower tax than the Pigouvian one; however, imperfect competition in the upstream market urges the regulator to increase the tax above the marginal damage of emissions; (ii) the overall effect depends on the number of firms in each market, on price elasticities and on the relative value of both markets.*

It can be noted that this model encompasses as special cases the previous literature on Cournot oligopolies and environmental taxation. For instance, let us suppose that polluters have no market power. This is the case when the elasticity of demand in the downstream market is infinite or when the number of firms increases in that market ($n \rightarrow +\infty$). Then, the optimal pollution tax is always greater or equal to marginal damage. As already explained in David and Sinclair-Desgagné (2005, 148), "if the tax t was to be set equal to the marginal damage ν, then the polluter would settle for an abatement level that is too small relative to the first-best". Conversely, we can study the case where the market power of eco-firms disappears. This could happen for two reasons. First, the elasticity of marginal depollution could tend to 0. Then, each polluting firm becomes indifferent to buying the environmental goods or paying the resultant tax. In other words, UFs must take the price of A as given, they cannot manipulate it. Second, the loss of market power occurs when the number of eco-firms increases. In both cases, the optimal tax is necessarily lower or equal to marginal damage. At last, the model is consistent with Pigou's approach: when both sectors do not have market power, the optimal tax is set equal to marginal social damage.

4 Conclusion

This work has precised how imperfect competition in the polluting and the eco-industry sectors are important when the regulator chooses its optimal environmental policy. First, the role of technological characteristics on the existence of an eco-industry is underlined. Once the eco-industry exists, the environmental policy remains a single instrument to regulate three sorts of distortions. An already low level of production in the downstream market is an incentive to lessen the tax. Conversely, there is an incentive for the regulator to increase the tax in order to compensate the low level of environmental goods supplied by an imperfectly competitive upstream industry. A few key elements should decide which effect dominates. Among them, price elasticities in both markets are fundamental parameters. It is notably shown that the elasticity in the upstream market is determined by technological characteristics such as the relative efficiency of clean-up activities. Other key parameters are the number of firms in each market and the overall variations in the value of production in both markets when the tax is modified. All these variables could be estimated empirically, which leaves room for an econometric extension of this work.

Further work is needed in the attempt to consider the eco-industrial sector in environmental economics. Economic analysis should underline the interaction between environmental regulation and the eco-industry. We have seen that environmental policies should take into account of the current structure of the eco-industry so as to avoid market inefficiencies. Therefore, more work is needed to better understand all the consequences of the eco-industry's structure. As already explained, the vertical relationship presented in this document cannot be seen as exclusive. For instance, allowing vertical contracts as in Hamilton and Requate (2004) could modify the payments among downstream and upstream firms. Then, a deviation in the environmental policy would not affect welfare. Second, one could try to check to what extent our results would be changed in an international context. Eco-industry markets have not reached the same stages of development around the world. The impact of these differences on environmental policies remains to be studied. Conversely, the influence of (environmental) regulation on the structure of the eco-industry should not be underestimated. It changes the number and size of incumbent firms, which modifies the degree of competition on the market and has direct impacts on the environment. Moreover, environmental regulations should consider eco-industries' innovation strategies. Each environmental policy instrument gives a specific incentive to innovate, which could lead to different innovation strategies from different eco-industries.

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Appendix

Appendix 1: A Useful Result on Cournot Equilibria

Lemma 2 *Let us consider a Cournot game with n players. Let* $P : R_{++} \to R$ *be a* C^2 *inverse demand function which verifies* $\forall Q \in]0, +\infty[$, $P(Q) > \lim_{q\to 0} c'(q) \geq 0$, $P'(Q) < 0$, $e_{P'}(Q) := -\frac{P''(Q)Q}{P'(Q)} \leq (n+1)$ *and* $\lim_{Q\to 0}(nP(Q) + P'(Q)Q) > \lim_{Q\to 0} c'(Q/n)$ *with* $c(q)$ *a cost function with the property that* $\forall q \in]0, +\infty[$, $c'(q) > 0$, $c''(q) > 0$, $\lim_{q\to 0} c'(q) \geq 0$ *and* $\lim_{q\to +\infty} c'(q) = +\infty$. *Under these restrictions, there exists a unique symmetric Cournot equilibrium in which each firm maximizes its profit given by* $\pi(q_i, q_{-i}) = P(\sum_{i=1}^{n} q_i)q_i - c(q_i)$

Let us construct $H : R_+ \times R_+ \to R$ given by $H(q, Q) = P(Q) + P'(Q)q - c'(q)$ and let us observe that:

- $\forall (q, Q) \in]0, +\infty[^2$, $\partial_q H(q, Q) = P'(Q) - c''(q) < 0$
- $\forall Q \in]0, +\infty[$, $\lim_{q\to 0} H(q, Q) = P(Q) - \lim_{q\to 0} c'(q) > 0$
- $\forall Q \in]0, +\infty[$, $\lim_{q\to +\infty} H(q, Q) = P(Q) + \lim_{q\to +\infty} P'(Q)q - \lim_{q\to +\infty} c'(q) = -\infty$

We can therefore conclude that $\forall Q \in]0, +\infty[$, $\exists q = \phi(Q)$ a unique q with the property that $H(\phi(Q), Q) = 0$. Now remark that $H(q, Q) = 0$ is the FOC of each of the n Cournot players. We deduce by the previous uniqueness result that $\forall i$, $q_i = \phi(Q) = \frac{Q}{n}$. This has two consequences:

- The second order condition is satisfied since this one is given by

$$\begin{aligned}
\partial_{q_i} H\left(q_i, \sum_{i=1}^{n} q_i\right)\Bigg|_{q_i=Q/n} &= \partial_q H(q, Q)\big|_{q_i=Q/n} + \partial_Q H(q, Q)\big|_{q_i=Q/n} \\
&= \left(P'(Q) - c''(\frac{Q}{n})\right) + \left(P'(Q) + P''(Q)\frac{Q}{n}\right) \\
&= \frac{P'(Q)}{n}(2n - e_{P'}(Q)) - c''\left(\frac{Q}{n}\right) < 0
\end{aligned}$$

- A unique Cournot equilibrium exists if $\Gamma(Q) = \sum_{i=1}^{n} H_i(\frac{Q}{n}, Q) = nP(Q) + P'(Q)Q - nc'(\frac{Q}{n}) = 0$ admits a unique solution.

Let us now check this last point. First, we define $\Lambda(Q) = nP(Q) + P'(Q)Q$, for $Q \geq 0$, as the aggregate marginal revenue of the industry and $\Theta(Q) = nc'(\frac{Q}{n})$ its aggregate marginal cost. There exists an equilibrium if $\Gamma(Q^*) = 0 \Leftrightarrow \Lambda(Q^*) = \Theta(Q^*)$. We first observe that $\forall Q \in]0, +\infty[$:

$$\frac{d\Lambda(Q)}{dQ} = P'(Q)(1 + n - e_{P'}(Q)) < 0$$

$$\frac{d\Theta(Q)}{dQ} = -c''\left(\frac{Q}{n}\right) > 0$$

So, the aggregate marginal revenue of the industry is strictly decreasing and the aggregate marginal cost is strictly increasing. Now remark that:

- $\lim_{Q\to+\infty} \Lambda(Q) = -\infty$ since $\lim_{Q\to+\infty} P'(Q)Q = -\infty$ and $\lim_{Q\to+\infty} P(Q)$ must be finite otherwise $P'(Q) < 0$ makes no sense.
- $\lim_{Q\to+\infty} \Theta(Q) = +\infty$ since $\lim_{Q\to+\infty} c'(\frac{Q}{n}) = +\infty$
- $\lim_{Q\to 0} \Lambda(Q) = \lim_{Q\to 0} = nP(Q) + P'(Q)Q > 0$
- $\lim_{Q\to 0} \Theta(Q) = \lim_{Q\to 0} c'(\frac{Q}{n}) \geq 0$

Therefore, there exists a unique equilibrium value $Q^* > 0$, for which each firm produces $q^* = \frac{Q^*}{n}$.

Appendix 2: Variations of A and X According to t

The first step consists in making the total differentiation of the second DFs' FOCs, using optimal values for x_i. We have:

$$f'(X)\,dX + \frac{f'(X)}{n}dX + \frac{f''(X)\,X}{n}dX - \frac{c_d''\left(\frac{X}{n}\right)}{n}dX - \epsilon''\left(\frac{X}{n}\right)dt - t\frac{\epsilon''\left(\frac{X}{n}\right)}{n}dX = 0 \quad (9)$$

Rearranging this expression, we get:

$$\frac{dX}{dt} = \frac{n\epsilon'\left(\frac{X}{n}\right)}{f''(X)\,X + (n+1)\,f'(X) - c_d''\left(\frac{X}{n}\right) - t\epsilon''\left(\frac{X}{n}\right)} \quad (10)$$

The numerator is always positive, and in order to find a unique Nash equilibrium, the denominator has to be negative.

Let us now differentiate UFs' FOCs. We get:

$$\begin{aligned} 0 = {} & w''\left(\frac{A}{n}\right)\frac{A}{nm}dt + tw'''\left(\frac{A}{n}\right)\frac{A}{n^2m}dA + tw''\left(\frac{A}{n}\right)\frac{1}{nm}dA \\ & + w'\left(\frac{A}{n}\right)dt + t\frac{w''\left(\frac{A}{n}\right)}{n}dA - \frac{c_u''\left(\frac{A}{m}\right)}{m}dA \end{aligned} \quad (11)$$

Rearranging this expression, we find:

$$\frac{dA}{dt} = \frac{-m\left(w''\left(\frac{A}{n}\right)\frac{A}{nm} + w'\left(\frac{A}{n}\right)\right)}{tw'''\left(\frac{A}{n}\right)\frac{A}{n^2} + \frac{t}{n}(m+1)\,w''\left(\frac{A}{n}\right) - c_u''\left(\frac{A}{m}\right)} \quad (12)$$

A positive Cournot equilibrium needs that $w''(\frac{A}{n})\frac{A}{nm} + w'(\frac{A}{n})$ must be positive. So, the numerator, at the equilibrium, is necessarily negative. As far as the denominator is concerned, we also know that a unique equilibrium will be found if it is negative. Consequently, an increase in t always induces an increase in the size of the market for eco-firms.

Appendix 3: An Expression of the Optimal Tax

We know, from stage 2 and stage 3 of our game, that:

$$c_d'\left(\frac{X}{n}\right) = f(X) + f'(X)\frac{X}{n} - t\epsilon'\left(\frac{X}{n}\right) \quad (13)$$

$$c_u'\left(\frac{A}{m}\right) = tw'\left(\frac{A}{n}\right) + tw''\left(\frac{A}{n}\right)\frac{A}{nm} \tag{14}$$

So, we can rewrite Eq. 4 as follows:

$$\begin{aligned} 0 = &-f'(X)\frac{X}{n}\frac{dX}{dt} + t\epsilon'\left(\frac{X}{n}\right)\frac{dX}{dt} - \left(tw'\left(\frac{A}{n}\right) + tw''\left(\frac{A}{n}\right)\frac{A}{nm}\right)\frac{dA}{dt} \\ &-\nu\left[\epsilon'\left(\frac{X}{n}\right)\frac{dX}{dt} - w'\left(\frac{A}{n}\right)\frac{dA}{dt}\right] \end{aligned} \tag{15}$$

We also know that:

$$\frac{w''\left(\frac{A}{n}\right)}{w'\left(\frac{A}{n}\right)}\frac{A}{nm} = \frac{e_{w'}(A)}{m} \tag{16}$$

Rearranging the expression and introducing price elasticities of demand gives Eq. 7.

References

Barnett AH (1980) The Pigouvian tax rule under monopoly. Am Econ Rev 70(5):1037–1041

Barton JR (1997) The north–south dimension of the environment and cleaner technology industries. Discussion paper, Institute for New Technologies, United Nations University, Maastricht

Baumol WJ (1995) Environmental industries with substancial start-up costs as contributors to trade competitiveness. Annu Rev Energy Environ 20:71–81

Berg DR, Ferrier G, Paugh J (1998) The U.S. environmental industry. U.S. Department of Commerce Buchanan, Office of Technology Policy

Buchanan JM (1969) External deseconomies, corrective taxes, and market structures. Am Econ Rev 59:174–177

David M, Sinclair-Desgagné B (2005) Environmental regulation and the eco-industry. J Regul Econ 28(2):141–155

Ecotech Research and Consulting Limited (2002) Analysis of the EU eco-industries, their employment and export potentials. Technical report, European Commissions, DG Environment

European Commission, DG Environment (2006) Eco-industry, its size, employment perspectives and barriers to growth in an enlarged EU. Ernst and Young Environment and Sustainability Services

Gaudet G, Salant SW (1991) Uniqueness of Cournot equilibrium: new results from old methods. Rev Econ Stud 58(2):399–404

Greaker M (2006) Spillovers in the development of new pollution abatement technology: a new look at the Porter-hypothesis. J Environ Econ Manage 56:411–420

Hamilton SF, Requate T (2004) Vertical structure and strategic environmental trade policy. J Environ Econ Manage 47(2):260–269

Ishikawa J, Spencer B (1999) Rent-shifting export subsidies with an imported intermediate product. J Int Econ 48:199–232

Kennett M, Steenblik R (2005) Environmental goods and services: a synthesis of country studies. OECD Trade and Environment Working Papers 2005/3

Kolstad CD, Mathiesen L (1987) Necessary and sufficient conditions for uniqueness of a Cournot equilibrium. Rev Econ Stud 54(4):681–690

Levin D (1985) Taxation within Cournot oligopoly. J Public Econ 27:281–290

Long NV, Soubeyran A (2000) Existence and uniqueness of Cournot equilibrium: a contraction mapping approach. Econ Lett 67:345–348

Nimubona A-D, Sinclair-Desgagné B (2005) The Pigouvian tax rule in the presence of an eco-industry. FEEM. Nota de lavoro (57-2005)

Numeri I. and R.D.I. (2004) Panorama des eco-entreprises. Technical report, Ministère de l'économie et des finances

OECD (1996) The global environmental goods and services industry. Technical report

OECD (1999) The Environmental oods and services industry: manual for data collection and analysis. OCDE Editions, Paris

Okuguchi K (2004) Optimal pollution tax in Cournot oligopsonistic oligopoly. Working paper, Departments of Economics and Information, Gifu Shotoku Galuen University

Petrakis E, Xepapadeas A (2003) Location decisions of a polluting firm and the time consistency of environmental policy. Resour Energy Econ 25:197–214
Pigou AC (1920) The economics of welfare. Macmillan, London
Simpson R (1995) Optimal pollution tax in a Cournot duopoly. Environ Resour Econ 6:359–369