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ORIGINAL PAPER

Measuring welfare losses from interruption and pricing as responses to water shortages: an application to the case of Seville

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Abstract Supply cuts and pricing policies can be used to ration water. The appropriateness of a given policy depends on the losses in social welfare which it generates. We find some drawbacks with the only method in the previous literature which deals with the issue of measuring welfare losses under supply cuts. We propose an alternative method. We compare the welfare losses under supply cuts and a pricing policy during the drought period of 1992–1996 in Seville, Spain, using both methods, and find that the results vary widely from one method to the other.

Keywords Consumer surplus · Household behavior · Rationing · Supply interruptions · Water demand

JEL classification D11 · D12 · D45 · D60 · Q25

1 Introduction

Water has traditionally been considered as one of the most important natural resources in terms of contribution to the development of civilizations (Marshall 1879; Gibbons 1986), and access to water resources is one of the main factors which conditions the location and growth of towns and cities. The importance of water lies in the fact that it satisfies a broad group of needs, both in its role as a necessary good upon which public health and life itself depend, as well its role as a basic input in most agricultural and industrial production processes. This double role of consumption good and factor of production makes water an essential factor in economic activity and obliges the

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authorities to invest in infrastructure which guarantees its availability, often at a high cost.[1]

The fact that water is a limited resource has given rise to numerous conflicts over its allocation between alternative uses (Lee (1999)). Moreover, the relative scarcity of water resources is becoming more pronounced over time due to a variety of factors. Demographic pressure and the process of economic development lead to a continual increase in the consumption demand for water, while at the same time the climatological conditions which limit the availability of water in many geographical zones appear to be worsening as a consequence of the process of climatic change which the planet is undergoing (Frederick and Major 1997; Giupponi and Shechter 2003). These concerns are addressed by Frederick (2002): "The climate affects all aspects of the hydrologic cycle, and a change in the climate is likely to affect water supplies and demands as well as ecosystems that depend on timely supplies of water. Even in the absence of climate change, the adequacy of supplies to meet demands of growing and increasingly affluent populations while sustaining a healthy environment is a matter of concern around the globe".

Despite often large investments in infrastructure, these demographic and climatic factors have the effect that periods of water shortage can be expected to reappear in the future. This in turn will give rise to the need for policies which limit consumption. In particular, one of the most serious causes of water shortage in many regions is drought, and when this cyclical phenomenon reoccurs the entities responsible for water supply often impose water cuts in order to match the available supply with demand (Lund and Reed 1995). Generally, water shortages give rise to the need for rationing, and while the authorities frequently resort to supply cuts, there are various other ways in which this can be achieved (Winpenny 1994).

The effects of policies aimed at limiting water consumption has been the subject of several studies, including Moncur (1987), Woo and Lo (1993) Woo (1994) or Renwick and Archibald (1998), However, none of these, with the exception of the application by Woo (1994), has attempted to quantify the welfare losses associated with various alternative rationing systems in household consumption. In this paper, we identify certain shortcomings in the methodology presented by Woo (1994) and develop an alternative model which allows us to quantify the welfare losses associated with two rationing regimes: supply interruptions and price rises. In an empirical application, we use the model to study the welfare effects caused by the drought which affected the city of Seville (Spain) in the first half of 1990s.

We proceed as follows. Section 2 outlines the model used to analyze the welfare effects associated with different rationing schemes. In section 3 we describe the effects of the drought in Seville in the early 1990s and the main initiatives implemented by the supplier in response. Section 4 contains an empirical application where the households' welfare losses associated with the⁻rationing initiatives implemented in Seville at the time are estimated. Section 5 describes the data set used, and the results are analyzed in sect 6. Section 7 concludes.

[1] In Spain, for example, the contrast between regions in terms of the natural availability of water has led to a policy of diverting water between basins. The National Water Plan approved in 2001, which aims to improve the water supply in regions in the south of Spain on the Mediterranean coast, has an estimated cost of around 3.78 billion euro. This Plan was abandoned by the new government in 2004, which supports conservation policies and desalinization plants in Southeastern Spain as an alternative.

2 Measuring welfare losses from water rationing policies

Woo (1994) has provided the only attempt that has been made in the literature to quantify changes in consumer welfare arising from alternative rationing systems. The tool used in that study is the compensating variation and the supply cut is modeled as a variable that does not affect either consumer income or prices. Our results show, however, that under these assumptions it is not possible to determine the value of the compensating variation (see Appendix).

As in Woo (1994), our objective is to compare the effects on consumer welfare of two rationing systems, namely price rises and supply cuts. Both rationing schemes can be equally effective in reducing the quantity demanded to match it to that available. However, although the quantity of water consumed may be the same under both regimes, they can generate different welfare effects. Under a price rise, consumers are free to consume whenever they wish. A supply cut, on the other hand, reduces the availability of the resource to consumers. This generates a distortion in demand behavior because consumers cannot freely choose the timing of their consumption, which in turn implies that consumer utility is affected.

As a first step toward comparing the welfare losses under these rationing regimes, we design a model which captures the influence of temporary unavailability of the resource on consumer demand. These temporary interruptions have two main observed effects on consumer behavior. First, consumers react to an interruption by acquiring only a proportion of the water that they would have consumed had there been no cut.[2] Second, the longer the duration of the interruption, the lower the proportion of water acquired. It should be noted that consumers can install devices such as storage tanks to soften the impact of interruptions. These expenditures will in turn generally depend on household income. We therefore expect that the reduction in consumption due to the supply interruptions may also depend on household income.

To capture the aforementioned characteristics, we model a marshallian water demand function comprising two components as follows:

$$Q = Q\left[p, p_{-i}, y, s; f\left(c, y\right)\right] = q\left(p, p_{-i}, y, s\right).f\left(c, y\right). \quad (1)$$

The first component, $q(\cdot)$, represents a standard marshallian demand function which depends on the price of the rationed good (p), the vector of prices of other goods ($\mathbf{p}_{-i}$), income (y), and other variables which affect the utility of the consumer ($\mathbf{s}$).

The second component, $f(\cdot)$, is a function which captures how the duration of the interruption (c), and the household income (y) determine the proportion of the desired good that the consumer acquires when a supply cut takes place.

It is assumed in this model that the amount of consumption loss due to the supply cut depends on the planned consumption and therefore on the variables which determine this planned consumption. However, it is also assumed that the consumption lost as a proportion of planned consumption depends only on the duration of the cut and household income. In this sense, the effect of the interruption is similar to that of a proportional rationing scheme (see, for example, Tirole 1988), under which consumers only acquire a certain proportion of their desired quantity of the good.

Figure 1 shows the water demand function consistent with the assumptions maintained in the model under two different scenarios. When there are no interruptions,

[2] Consumers are able to maintain their overall water consumption at pre-interruption levels if they so wish (by consuming at times when water is available). Changes in consumption due to the interruption are therefore a consequence of changes in consumer behavior induced by the cut.

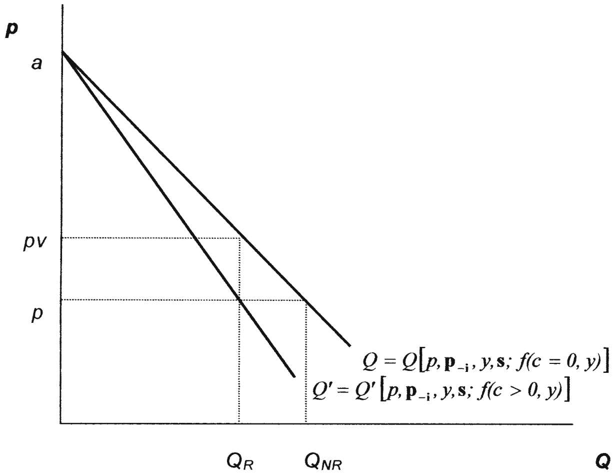

Fig. 1 Welfare losses : prices versus interruptions

the water demand curve is represented by Q, whereas Q' represents the demand curve when supply interruptions are in place. Thus, the introduction of a water cut induces a reaction on the part of consumers such that the water demand curve moves from Q to Q', reducing the quantity demanded, given the price, from $Q_{NR}$to Q_R.[3]

Based on this modeling of the marshallian demand function, we propose using the consumer surplus as an approximation to the welfare losses associated with the distinct rationing systems. In terms of Fig 1, the introduction of a water cut has the effect that consumer surplus is reduced by

$$\nabla W^C = \int_p^a Q\left[p, p_{-i}, y, s; f\left(c = 0, y\right)\right] \mathrm{d}p - \int_p^a Q\prime\left[p, p_{-i}, y, s; f\left(c > 0, y\right)\right] \mathrm{d}p. \quad (2)$$

We compare the effects on welfare of the interruption with those which would arise from a policy of rationing via prices. To do so, we need to determine how much the price should be raised in order to obtain a similar effect to that of the interruption. That is, from the demand function we need to determine the virtual price (pv) which would give rise to a reduction in consumption equivalent to that which would occur under a given interruption. Raising the price to pv therefore represents an alternative, but equally effective, method of rationing, and the corresponding loss in consumer surplus would be:

$$\nabla E^p = \int_p^{\mathrm{pv}} Q\left[p, p_{-i}, y, s; f\left(c = 0, y\right)\right] \mathrm{d}p. \quad (3)$$

However, part of this variation is accounted for by a transfer of rent to the supplier. As water provision is typically regulated by the public sector, these rents may return to

[3] It is implicitly assumed in the model that the intercept of the demand function does not change when a supply cut takes place. This may be questionable in the sense that the intercept of the demand function refers to the most valuable unit of the good and it could therefore be argued that this value may change when an interruption of supply occurs. However, as households are able to store a certain amount of water when the supply cut is announced, this storage capacity will assure the consumption of the most valuable unit and as a result it can be argued that its value does not change.

Table 1 Reserves during the drought periods

	Maximum capacity of the reservoirs (Hm^3)	Reserves during periods with interruptions (%)
1974–1976 Drought	187	5–6
1981–1983 Drought	222	12–34
1992–1996 Drought	390	5–15

EMASESA (1997)

consumers through public spending. Under this scenario, the net loss in social welfare is:

$$\nabla W^{p} = \int_{p}^{\mathrm{pv}} Q\left[p, p_{-i}, y, s; f\left(c=0, y\right)\right] \mathrm{d}p - \left(\mathrm{pv} - p\right) \cdot Q_{R}. \tag{4}$$

Comparing (2) and (4) using Fig 1, it can be seen that the welfare loss under price rationing is always lower than that corresponding to an interruption. This result is due to the fact that under a price rise consumers only acquire units of the good which they value more than pv. This implies that consumers forego the marginal units of water. Under an interruption, on the other hand, they acquire units which they value more than p as long as they are available. However, an interruption may prevent consumption of certain units of the good valued by the consumer above pv which are substituted by other units valued between p and pv which are available. This implies that the consumer foregoes units of water which are not necessarily the marginal ones, implying a further loss in consumer welfare.

3 The drought in Seville in the early 1990s

Seville is a city in the south of Spain with just over 700,000 inhabitants (INE 2003) which has been severely affected by drought at various times. In recent decades, water supply and sanitation in the city and some surrounding municipalities has been the responsibility of the Seville Municipal Water Company (EMASESA), and during this time three water emergencies have arisen. The first two occurred in the periods 1974–1976 and 1981–1983. The third, and most recent, was experienced during the first half of 1990s, and will be the focus of our attention. Table 1, which shows the status of reserves in reservoirs, illustrates the gravity of the situation caused by this latest shortage in the Guadalquivir basin:[4]

The shortages appeared at the end of 1991 and the situation worsened the following year, which coincided with Expo 1992.[5] The first restrictions on consumption, formally introduced through the publication of municipal edicts,[6] were put into place at the beginning of 1992. These edicts specified the conditions under which water would be supplied as well as the response which was expected from users. The edicts served the

[4] This is the water basin to which Seville belongs.

[5] The Universal Exposition which took place in the city in 1992.

[6] Although supply cuts are legally permitted under certain circumstances, it is obligatory to notify users in advance about when the cut will take place, as well as the intensity and the estimated duration (Molina 2001).

purpose of attracting attention to the problem so that users would become aware of it and act in an appropriate manner while shortages lasted.

The restrictive measures put into practice comprised cuts in supply and reductions in water pressure. These were merely recommendations at first, but they were later turned into impositions and prohibitions as water conditions worsened. Hour restrictions were particularly intense during the years 1993 and 1995. During this period, water cuts sometimes lasted for up to 8 h per day, although these restrictions were not applied to users, such as health centers, providing services deemed to be essential to the public interest.

Apart from restrictions on the hours of water availability, the quality of the resource was also affected. Despite the efforts of the supplier to guarantee water quality, the special circumstances of the time and the consequent decision to diversify the sources of provision led to a considerable deterioration of the resource. The health authorities found themselves with no other option but to implement a provisional relaxation of water standards using the argument that this was a period of "exceptional conditions."

4 Empirical analysis

In order to estimate the marshallian demand in Eq. 1, we propose the following empirical specification:

$$Q_{it} = \left[\beta_0 + \beta_1 p_{it-2} + \beta_2 y_i + \beta_3 \mathrm{TEMP}_t + \beta_4 \mathrm{NPER}_i + \beta_5 \mathrm{QUAL}_t\right] \left[1 + d_h\left(\alpha_0 + \alpha_1 c_t + \alpha_2 (c_t)^2 + \alpha_3 (c_t)^3 + \alpha_1 y_i\right)\right], \quad (5)$$

where $p_{it-2}$is the price that the ith consumer pays for water in period t-2; y_i is the consumer's income; TEMP_t is the average temperature in the period t; $\mathrm{NPER}_i$is the number of persons in the consumer's household; QUAL_t is a dummy variable indicating the existence of drops in pressure and/or chemical parameters which determine the quality of the product; d_h is a dummy variable which takes a value of 1 when there has been an interruption; $c_t$is the daily duration of the interruption measured in hours; and β_j and α_j are parameters to be estimated.

The demand function in Eq. 5 is specified as the product of two components. The first component represents the marshallian water demand in the absence of supply cuts, modeled as a linear relationship. The second component, f(·), captures the effect of the interruption on the quantity of water acquired. The specification, which is non-linear in the duration of the cut, provides the function with a flexibility which allows the marginal reduction in consumption to vary with the duration of the interruption. The aim here is to adapt the function as best as possible to the characteristics of the cuts introduced by the local government in Seville. When the interruptions were of a shorter duration, it can be observed that they usually began at 2 a.m., so their impact on consumption would be minor. Cuts of a longer duration, on the other hand, began at 7 p.m. It thus seems appropriate to assume that the effect of the marginal hour on consumption varies with the duration of the cut.

5 Data

In this section we describe the principal characteristics of the data base and the variables used to estimate Eq. 5. The data set comprises a balanced panel consisting of quarterly observations corresponding to 208 Sevillian households with individual meters, covering the period from the fourth quarter of 1991 to the third quarter of 2000.

5.1 Quantity (Q)

This is expressed in cubic meters and measures total household consumption per period. The data have been supplied by EMASESA, the municipal water supply firm.

5.2 Price of water (p)

The two-period lag in the price is introduced to capture the fact that users only observe the price when they receive the bill. The complexity of the water rate structure means that the bill thus becomes a crucial informative tool for users, The bill is received, however, during the quarter following consumption, so consumers can only react to variations in the price after two periods, hence the two-period lag.[7] The price itself is calculated as the average quantity paid per cubic metre, which is regarded as the most appropriate indicator in a framework of imperfect information (Charney and Woodard 1984; Opaluch 1984). The data have been provided by EMASESA.

5.3 Family income (y)

We use a proxy based on the location of the home within the municipality. The Revenue Office of the Municipality of Seville provides information on the fiscal category of each street, and for each category an interval of disposable income has been assigned. The estimates for disposable income are provided by a research study carried out by La Caixa for the year 1998 (La Caixa 2000). There are eight income intervals in total, and the mean of each interval is used to approximate family disposable income.

5.4 Temperature (TEMP)

We use the arithmetic mean of the maximum daily temperatures during the period under consideration. This information has been supplied by the Spanish National Meteorological Institute.

5.5 Number of users per household (NPER)

The variable has been obtained by dividing the total number of people in each building who appear in the census by the number of households in each building. Difficulties in getting information prevent us from determining how the number of people in each household varies over time. Hence, the variable only exhibits variation across the different households and does not change over time. The information on this variable is provided by Seville City Council.

5.6 Quality (QUAL)

Details on this variable have been provided by EMASESA.

[7] Thus, unless a price rise is accompanied by a publicity campaign by the City Council, the effect on water usage will not be immediate.

Table 2 Variables used in the estimations : descriptive statistics

Variable	Units	Mean	SD	Maximum	Minimum
Q	M^3	108.69	150.48	527.84	1.90
p	Pesetas/m^3	140.82	36.04	215.53	71.21
y	Pesetas/Quarter	235,987	45,808	359,149	162,672
TEMP	Celsius Degrees	25.54	5.32	32.6	18.1
NPER	Persons/House	3.78	2.11	11	1
c^a	Hours :Minutes	4:50	2:03	7:00	0:40

[a] When calculating the statistics referring to the duration of the interruption we exclude periods for which there are no interruptions

Table 3 Demand function : parameter estimates

Parameter	Coefficient	SE	t-statistic
α_0	−1.048	0.117	−8.989
α_1	0.851	0.138	6.146
α_2	−0.226	0.041	−5.487
α_3	0.016	0.003	4.852
α_4	0.0000011	0.0000003	3.629
β_0	−107.213	8.702	−12.320
β_1	−0.282	0.039	−7.230
β_2	0.000034	0.000024	1.439
β_3	0.854	0.199	4.288
β_4	62.145	0.605	102.672
β_5	−20.678	2.736	−7.557
R^2	0.695		

5.7 Interruptions (c)

The duration of interruptions has been made available by EMASESA.

Some descriptive statistics relating to the variables and the quantities consumed are presented in Table 2.

6 Results

To estimate the demand function in Eq. 5, an additive error term was introduced. The estimation was carried out using a non-linear least squares estimator[8] using the TSP package. The parameter estimates are presented in Table 3.

From the above table it can be seen that the results correspond with the consumption pattern expected a priori.[9] Thus, in the function which captures the influence of the interruption we find that all the parameters are significantly different from zero. This confirms, in line with our comments above, that the influence of the duration

[8] See Greene (2002), Chap.9, for details.

[9] In order to test the assumption maintained in the model that planned water consumption does not depend on the duration of the cut, we estimated the marshallian water demand function for the complete sample of households and for the subsample which did not face supply cuts. We carried out a Wald test to check the hypothesis that the parameters common to each estimation were equal. The chi-squared statistic was $\chi^2_{(6)} = 1.501$, so the assumption in the model appears to be valid.

of the interruption on consumption follows a non-linear pattern. It is also confirmed that household income increases the proportion of water acquired when a supply cut takes place. Thus, as was postulated above, high income users seem to have more possibilities to acquire higher storage capacity which cushions the impact of supply cuts on consumption. This result shows that the management of water shortages by implementing supply interruptions is a policy which has clearly regressive effects.

With regard to the part of the function corresponding to water demand in the absence of interruptions, the estimated parameter values also comply with expectations. The (own) price elasticity of demand of water, calculated at the arithmetic mean of price and quantity demanded, is 0.31, implying relatively little reaction on the part of consumers to changes in price. This result is consistent with existing studies on water demand. For example, Foster and Beattie (1979) find that the elasticity of demand for water varies between 0.27 and 0.76, Hanke and Maré (1982) calculate an elasticity of 0.15 for water in Sweden, while Billings and Day (1989) obtain an elasticity of 0.7 for water in AZ, USA.[10] Generally, studies show that water demand has a low-price elasticity, which is unsurprising given that it is a necessity good with few substitutes.

The value of the parameter which captures the influence of income on water demand is positive but not statistically significant. This result is in line with other studies on water demand in developed countries which find that the influence of income on consumption is a weak one. Billings and Day (1989), for example, find an income elasticity of 0.36, while Renwick and Green (2000) obtain a value for the income elasticity of water demand in CA, USA, of 0.25.

The influence of the climatic temperature on water consumption also has the expected sign, showing that increases in temperature lead to increases in water consumption. It can also be seen that consumers react quite strongly to changes in temperature. This result is to be expected as temperatures during the summer in Seville frequently rise to over 40○C, triggering off large increases in the population's water consumption.

Finally, it can be seen from the estimates that reductions in the quality of the water cause a notable reduction in consumption.

Based on the results of the estimation, two alternative scenarios are then considered: the first is where there are no restrictions on the quality of water provided (QUAL = 0), and the second where there are restrictions on quality (QUAL = 1). From the estimates, and taking the average values of the variables during the reference period, Table 4 shows the quantity demanded of water in the absence of interruptions (q) and the quantity demanded when the interruption reaches the average duration observed (in periods where interruptions take place), which is 4 h and 50 min. It can be seen that the average interruption reduces consumption by 12.33%.

For each of the scenarios mentioned above, we calculate (1) the virtual price which would give rise to an equivalent reduction in consumption (pv), (2) the corresponding reduction in consumer surplus (∇E^P), (3) the social welfare loss associated with the virtual price (∇W^p), (4) the loss of surplus caused by the average interruption (∇W^c), and (5) the welfare loss associated with the interruption calculated in accordance with

[10] For an exhaustive review of studies on water demand and a brief summary of the main results, see Arbués et al. (2003).

Table 4 Residential water demand : mean levels

	$\overline{q}$	q
QUAL = 0	105.34	120.16
QUAL = 1	87.28	99.56

Table 5 Quarterly welfare losses : rationing through prices versus interruptions (1992 pesetas)

<table>
<tr><th></th><th>p</th><th>∇W^{c}</th><th>pv</th><th>∇E^{p}</th><th>∇W^{p}</th><th>$\nabla W^{C}_{\text{WOO}}$</th></tr>
<tr><td>QUAL = 0</td><td rowspan="2">132.64</td><td>3,161.04</td><td>185.26</td><td>5,932.18</td><td>389.90</td><td>430,123.48</td></tr>
<tr><td>QUAL = 1</td><td>2,170.33</td><td>176.24</td><td>4,072.96</td><td>267.70</td><td>356,402.88</td></tr>
</table>

the methodology proposed by Woo $\left(\nabla W^{c}_{\text{WOO}}\right)$.[11] These calculations, along with the average price per cubic meter of water over the period, are shown in Table 5.

From our calculations it can be seen that the welfare loss is larger under a supply interruption than with price rises. However, the difference is of the order of 2,200–3,200 pesetas[12] per consumer per quarter, depending on the scenario. Given the modesty of these gains, one should be cautious when recommending one policy over the other. In particular, it is difficult to imagine that the modest gain in social welfare represented by applying a rationing system based on price rises would compensate the additional water savings generated by a supply interruption which, insofar as it removes pressure from the water circuit, would reduce losses arising from leakages during transportation.[13]

In contrast, the value of the welfare loss due to an interruption using the Woo (1994) methodology is very large, rising to 430,000 pesetas (2,584 ć7) which represents 60% of average consumer income. It seems difficult to believe that an average reduction in water consumption of little over 12% could generate such a large value for the utility loss. Whereas the Woo methodology would strongly recommend the authorities to replace supply interruptions with price rises, our calculations show that the differences in welfare losses between the policies are of a much lesser magnitude and that a policy of supply interruptions may well be more appropriate if we take leakages into account.

11 Considering the linear demand function within expression (5), the compensating variation associated with the hourly cuts derived by Woo (1994) is

$$\nabla W^{c}_{\text{woo}} = \frac{Q_{\text{NR}} - Q_{\text{R}}}{\beta_2}$$

12 Which corresponds to 13.22–19.23ć7.

13 During the period under consideration, water losses from leakages in Seville amounted to 23% of the quantity consumed.

7 Conclusions

Water resources must be managed well in order to guarantee its supply to the population, not only for development of economic activity but also to ensure a good level of public health. A key issue for the optimal management of the resource is the administration of water in periods where scarcity repeatedly impedes the provision of the normal supply. In these circumstances, the most common policy implemented by the authorities is to interrupt the provision. However, this policy has recent begun to be criticized (Woo 1994), raising the possibility of introducing rationing through price increases which may lead to a smaller loss in social welfare. It is therefore essential to have a method which allows the impact on social welfare of different policies to combat scarcity to be evaluated. This information will permit the authorities to choose the policy which is less damaging to consumer welfare.

Our analytical results show that the only method in the literature which has been used to measure the loss in welfare arising from water supply interruptions (Woo 1994) suffers from certain limitations. We then propose an alternative method to measure these welfare effects. We show that the segmentation of the marshallian water demand function into two components allows the consumer surplus to be used to evaluate the welfare losses caused by interruptions.

In an empirical application, we study the effects of the interruptions imposed by the Sevillian authorities during the drought in the mid-1990s. We find that supply cuts are regressive in the sense that the higher the level of household income, the lower the effect of the cut. This is consistent with the fact that higher income households have easier access to water storage technologies. We evaluate the welfare impact of interruptions, by comparing the loss in consumer surplus under price rises and interruptions. Our results show that price rises lead to lower welfare losses than interruptions but that the difference between these losses is relatively small. We then compare our results with the losses calculated according to the Woo (1994) method and find the policy recommendations are substantially different.

Appendix

In this appendix we show that dual analysis is not capable of determining the value of the compensating variation needed to return the consumer to his initial utility level when there are changes in variables which do not affect either consumer income or prices.

To simplify the analysis, assume that there are only two consumption goods, q_1 and q_2, with the price of the second good normalized to unity. The Marshallian demand functions then depend on the price of water (p), income (y) and another variable (s) which affects consumer behavior without affecting the consumer's budget set, and take the following form:

$$\begin{aligned} q_1 &= q_1(p,y,s), \\ q_2 &= y - pq_1(p,y,s). \end{aligned} \tag{a.1}$$

The expenditure function of the consumer is given by:

$$G(p,u,s) = pq_1^h(p,y,s) + q_2^h(p,y,s), \tag{a.2}$$

where q_i^h is the Hicksian demand for the i^{th} good and u is utility. Differentiating the expenditure function we have:

$$\frac{\partial G\left(p,u,s\right)}{\partial s}=p\frac{\partial q_1^h\left(p,u,s\right)}{\partial s}+\frac{\partial q_2^h\left(p,u,s\right)}{\partial s}. \tag{a.3}$$

In equilibrium (Varian 1992) it must be the case that:

$$q_i^h\left(p,u,s\right)=q_i\left(p,y,s\right) \tag{a.4}$$

Differentiating (a.4):

$$\frac{\partial q_1^h}{\partial s}=\frac{\frac{\partial q_1}{\partial s}\cdot\left(1-\frac{\partial q_2}{\partial G}\right)+\frac{\partial q_1}{\partial G}\frac{\partial q_2}{\partial s}}{\left(1-p\frac{\partial q_1}{\partial G}\right)\cdot\left(1-\frac{\partial q_2}{\partial G}\right)-\left(p\frac{\partial q_1}{\partial G}\frac{\partial q_2}{\partial G}\right)},$$

$$\frac{\partial q_2^h}{\partial s}=\frac{\frac{\partial q_2}{\partial s}\cdot\left(1-p\frac{\partial q_1}{\partial G}\right)+\frac{\partial q_2}{\partial G}\frac{\partial q_1}{\partial s}}{\left(1-\frac{\partial q_2}{\partial G}\right)\cdot\left(1-p\frac{\partial q_1}{\partial G}\right)-\left(p\frac{\partial q_2}{\partial G}\frac{\partial q_1}{\partial G}\right)}. \tag{a.5}$$

Substituting (a.5) in (a.3):

$$\frac{\partial G}{\partial s}=\frac{P\frac{\partial q_1}{\partial s}+\frac{\partial q_2}{\partial s}}{\left(1-\frac{\partial q_2}{\partial G}\right)\cdot\left(1-p\frac{\partial q_1}{\partial G}\right)-\left(p\frac{\partial q_2}{\partial G}\frac{\partial q_1}{\partial G}\right)}. \tag{a.6}$$

Now, differentiating q_2:

$$\frac{\partial q_2}{\partial s}=-p\frac{\partial q_1}{\partial s},$$

$$\frac{\partial q_2}{\partial G}=1-p\frac{\partial q_1}{\partial G}. \tag{a.7}$$

Substituting (a.7) in (a.6):

$$\frac{\partial G}{\partial s}=\frac{0}{\left[1-\left(1-p\frac{\partial q_1}{\partial G}\right)\right]\cdot\left(1-p\frac{\partial q_1}{\partial G}\right)-\left[p\cdot\left(1-p\frac{\partial q_1}{\partial G}\right)\cdot\frac{\partial q_1}{\partial G}\right]}\Rightarrow\frac{\partial G}{\partial s}=\frac{0}{0}. \tag{a.8}$$

Q.E.D.

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