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A portfolio approach to climate investments: CAPM and endogenous risk

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Abstract Is there a role for investments in climate change mitigation despite low expected return? We use a model of intertemporal expected utility maximisation to analyse this question. Similar to the capital asset pricing model (CAPM) the rate of return depends on the correlation of risk between the return on investments in climate change mitigation and the market portfolio, but in contrast to the classical CAPM we admit the fact that economic and environmental systems are jointly determined, implying that environmental risk is endogenous. Therefore, investments in climate change mitigation may reduce risk via self-protection and self-insurance. If risk reduction is accounted for in cost–benefit evaluations, climate investments may be justified despite low expected return. These aspects of climate investments are not, however, communicated via standard cost–benefit analyses of climate policy. Optimal climate policy may therefore be more ambitious than previously considered.

Keywords CAPM · climate change · endogenous risk · climate investment · risk management

JEL Classification D81 · G12 · Q28

Introduction

Uncertainty and time increase the complexity of monetary benefit evaluation of environmental investments. The relevant time period for assessing costs and benefits, particularly for investments in climate change mitigation, may often be 50–100 years or more, in contrast to the conventional 15 years. The long-term and

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uncertain impacts of climate investments have given rise to a dispute over the appropriate discount rate. The dispute has to a large extent been triggered by the well-known study of optimal emission rate by Nordhaus (1994), which was one of the first attempts to combine science models with economic growth models. To the disappointment of many environmentalist and ecologically oriented economists, Nordhaus' study only recommended minor abatement policies for greenhouse gasses. As a result some papers have argued in favour of a lower discount rate based on inter- (or intra) generational fairness—which would bring out higher emission abatement, c.f. Cline (1991, 2004) and Howarth and Norgaard (1993).

In this paper we explore an alternative argument for a lower discount rate for environmental investments that is not using fairness to attack the rate of time preference of individuals as a basis for the discount rate. Using a model of optimal portfolio selection, we argue that if benefits in the form of risk reduction are accounted for in the standard cost–benefit evaluation, environmental investments such as climate change mitigation may be justified despite relatively low expected return. The basis for our argument is twofold. First, we assume that global warming is negatively correlated with society's income.[1] The negative correlation implies a low rate of return since the environmental investment in effect insures against portfolio risk. Second, we assume that environmental risk is endogenous, implying that investments in climate change mitigation will reduce the probability of severe environmental hazards. Other things equal, an environmental asset that reduces the probability of environmental hazards needs a low rate of return. The first part of our argument is that an environmental investment may exploit the joint probability function of returns. The second part is that it may shift it in a favourable direction. Both parts of the argument contribute to lower portfolio risk and thus the required rate of return is low.

Although classical models of risk management assume that risks are exogenous, endogenous risks are well-known under the name of moral hazard in the recent microeconomic literature, c.f. Marshall (1976). In the literature on climate change and risk management, analyses incorporating endogenous risks are few. Fisher and Narain (2003) study the effect of learning about future damages on the optimal rate of investments in abatement capital. In their paper the risk of environmental damages is dependent on the stock of greenhouse gases; i.e., the risk is endogenous. Shogren and Crocker (1991, 1999) and Crocker and Shogren (2003) derive a conceptual framework for choosing environmental risk in a one-period setting. Their example is a person maximising utility in the face of exogenous health risk. One set of actions ("exercise") reduce the likelihood of health damage. Another set of actions ("medicine") reduce the consequences of health damage. Barbier and Shogren (2004) argue that investments in critical habitat areas pay off by lowering the risk of species extinction. Following Ehrlich and Becker (1972) investments that reduce specific risks, in terms of increasing the probability of favourable outcomes,

[1] Many environmental risks are uncorrelated with economic activities and economic risks. Environmental risks like exposure to radiation from natural sources, inflicting cancer from mobile phones, or accepting an unknown chemical substance are basically uncorrelated with economic risks (business cycles, stock-market volatility, property market disturbances). It is, however, plausible that climate change correlates negatively with economic activity. The net effect of global warming on economic activities will be negative; people will probably relocate, industries will see their resource base whither, and a host of industries and activities will close down. At the same time, investments in mitigating climate change will have paid off.

are labelled self-protection and investments that reduce consequences—or severity—of specific risks, are labelled self-insurance. In terms of endogenous risk and global warming, Kane and Shogren (2000) argue that climate change mitigation may lower the risk of global warming and therefore exemplifies protection, Climate change adaptation, on the other hand, mostly responds to a given risk by reducing its consequences, and therefore exemplifies insurance.

Our purpose in this paper is to analyse investments that may both change risks given consequences (protection) and change consequences given risks (insurance). We develop a new framework for compiling an investment portfolio of climate and ordinary investments. In this endeavour we are lead to a modified version of the Consumption Capital Asset Pricing Model (CCAPM): a CAPM with endogenous risk. The original CAPM was developed by Sharpe (1964), Lintner (1965) and Mossin (1966) to describe an efficient portfolio of financial assets and the associated structure of returns on assets. The Consumption CAPM was developed by Breeden (1979). For a recent critical review and extensions see Lettau and Ludvigson (2001).

In a well-known article Weitzman (2001) presents an argument for a low rate of return on long-term investments. Weitzman argues that when there is uncertainty about the future discount factors, a prudent procedure for a planner is to use a weighted average of possible discount factors in which the possible discount factors themselves combine to form the weights (see also Weitzman 1998). High discount rate/low discount factor states obtain low weights since in these states the future has little importance. Hence low rate/high factor states gain in importance and dominate in the weighted average. Weitzman's argument is different from ours, as his model addresses uncertainty regarding the future market interest rate. In our framework we take the market interest rate as given. We also take the risk-free interest rate as given. Uncertainty regarding the appropriate risk-free interest rate gives rise to yet another argument for a low discount rate, see Gollier (2001). Our argument for a low rate of return to environmental investments has similarities to Howarth (2003). Using a stochastic dynamic general equilibrium model he argues that climate change policies that reduce overall risks will enhance social welfare if benefits are discounted at the risk-free rate. Our argument sharpens this claim since we argue that when risk is reduced, whether by combining assets given a risk structure or by perturbing the risk structure, the rate of return requirement is lower than the risk free rate. Our argument is at the same time an extension of arguments put forward more than 20 years ago by Lind (1982). Lind points out that:

> "the returns from energy research and development in the future may be negatively correlated with the returns to all other investments so that public investments in this case would have the effect of insurance. If we were to account for this insurance effect by altering the rate of discount, we should use a lower rate of discount than the risk-free rate, not a higher one." (p. 70)

Given the time and circumstances Lind was particularly interested in energy research and energy investments, but the argument as such is still valid. However, Lind does not distinguish between exogenous and endogenous risk as we do in our modified CAPM.

The paper is organised as follows. The theoretical model framework is developed and analysed in the subsequent section. To illustrate the CAPM model with endogenous risk we also offer a numerical example, displayed in Sect. 3. Given assumptions on how climate change influences world economic activity our results indicate that greenhouse gas reducing investments require a discount rate that is slightly lower than the risk-free rate on comparable long-term investments. Thus, our intuition with respect to investments in climate change mitigation is confirmed by the data. The results are further discussed in the concluding Sect. 4.

Asset pricing and expected utility maximising

Consider a risk averse society that faces a two period problem of determining first-period savings $W–C_1$ and a portfolio of investments, $(x^a)_{a\in A}$, where A is a fixed and finite set, so that discounted expected utility is maximised. W and C_1 denote first-period wealth and consumption, respectively, and x^a denotes the fraction of wealth invested in asset $a\in A$. Second-period consumption is a random variable given by $\tilde{C}_2 = (W - C_1)\sum_{a\in A} x^a \tilde{R}^a$ where $\tilde{R}^a$ denotes one plus the return on asset a.

In our model there are three investment possibilities. There is one risk-free asset with return R^0 and two risky assets with return R^N and R^M. The two risky assets differ in a very specific manner. In fact we assume, using the consumption CAPM framework, that asset M is a portfolio consisting of all risky assets and that it correlates perfectly with consumption—in other words, it is a version of the market portfolio. Asset N is a climate friendly environmental asset. To fix ideas we think of asset N as correlating negatively with asset M. Moreover, we assume that investing in asset N affects ex ante expected return and variance via a change in the outcome probability, i.e., risk is endogenous.

We shall elaborate on the exact qualities of the environmental asset in the subsection below. First, we formally present the basic two-period problem. Assuming a social planner with von Neumann-Morgenstern utility function $u(C_1) + \delta Eu(\tilde{C}_2)$ which is additively separable over time with discount factor δ, the planner's objective is given by

$$\max_{C_1, x^0, x^M, x^N} u(C_1) + \delta E\left[u\left((W - C_1)\left(x^0 R^0 + \sum_{a\in\{M,N\}} x^a \tilde{R}^a\right)\right)\right] \tag{1a}$$

where E denotes the mathematical expectation operator. As it stands, this problem is just the standard asset-pricing model based on intertemporal expected utility maximisation. In the following we extend the standard asset-pricing model so that it incorporates endogenous risk.

Endogenous probability

We assume that an amount K^N of first-period savings invested in the climate stabilising asset N during the first-period, $K^N = x^N(W - C_1)$, implies both self-insurance and self-protection in the terminology of Ehrlich and Becker (1972). First, by assuming negative correlation with the market portfolio, investing $K^N > 0$ (up to a point) will reduce portfolio risk by exploiting the joint probability structure of returns. That is the self-insurance argument. Second, investing K^N in the climate stabilising asset during the first-period will alter the distribution function for returns. That is the self-protection argument.

More specifically, society's uncertain second-stage consumption level $\tilde{C}_2$ is defined by the distribution function $F(C_2;\ K^N,\xi)$, where ξ is an exogenous signal of climate change, for instance temperature change. Given an exogenous signal, climate damage and thus consumption is uncertain. Therefore an investment in reduced temperature change, such as methane collection from landfills, will only pay off if climate change is real. It is a self-insurance investment. However, whatever the signal, some environmental investments reduce the likelihood of damage and increase the likelihood of high consumption given the exogenous signal. These are investments in self-protection. Letting subscripts denote partial derivatives we have $F_{K^N} > 0$ in the sense of first order stochastic dominance. Since $\int \mathrm{d}F = 1$ we obtain $\int \mathrm{d}F_{K^N} = 0$.

Now we return to the two-period problem. Since portfolio weights sum to one, such that $x^0 = 1 - \sum_{a\in\{M,N\}} x^a$, the maximising problem in Eq. 1a can be rewritten as

$$\max_{C_1,x^M,x^N} u(C_1) + \delta E\left[u\left((W - C_1)\left(R^0 + \sum_{a\in\{M,N\}} x^a(\tilde{R}^a - R^0)\right)\right)\right]. \tag{1b}$$

Letting $\tilde{R} = (R^0 + \sum_a x^a(\tilde{R}^a - R^0))$ be society's uncertain portfolio return and $\tilde{C}_2 = (W - C_1)\tilde{R}$ the second-period consumption, we can write the first order conditions for the two-period problem defined in Eq. 1b as

$$\begin{aligned} u'(C_1) &= \delta E\left[u'(\tilde{C}_2)\tilde{R}\right] \\ 0 &= E\left[u'(\tilde{C}_2)(\tilde{R}^M - R^0)\right] \\ 0 &= E\left[u'(\tilde{C}_2)(\tilde{R}^N - R^0)\right] + \int u(\tilde{C}_2)\mathrm{d}F_{k^N}(\tilde{C}_2; K^N, \xi). \end{aligned} \tag{2}$$

To study the implications of endogenous risk on asset pricing, we focus on the third condition of Eq. 2. Making use of the covariance identity and the so-called Stein–Rubinstein lemma (Stein 1973; Rubinstein 1976), we can rewrite this condition as

$$E\tilde{R}^N = R^0 + \beta(E\tilde{R}^M - R^0) - \frac{\int u'(C_2)F_{K^N}(C_2; K^N, \xi)\mathrm{d}C_2}{Eu'(\tilde{C}_2)} \tag{3}$$

where $\beta = \mathrm{cov}(\tilde{C}_2, R^N)/\mathrm{var}(\tilde{C}_2)$, see the Appendix for technical details.

The term β of Eq. 3 is the (consumption) β of the climate investment. It captures the systematic correlation between the climate investment and consumption. In particular, if investing in the climate asset contributes to smoothing consumption, by paying off in states where consumption is low, the optimal expected rate of return on the environmental asset should be low. In fact, ignoring the last term of Eq. 3 it should be lower than the return on the benchmark risk-free asset. Another word for smoothing consumption would be insurance. If β is negative the environmental investment contributes to society's self-insuring against unhappy outcomes; we may say that β captures the self-insurance aspect of environmental investments.

The last term of Eq. 3 captures the self-protection impact of perturbing the probability structure. It says that if investing in the environmental asset increases the probability of high consumption, i.e., since $F_{K^N} > 0$,

$$\frac{\int u'(C_2) F_{K^N}(C_2; K^N, \xi) \mathrm{d}C_2}{Eu'(\tilde{C}_2)} > 0,$$

the optimal expected rate of return on the environmental asset should be low, in fact, ignoring the term containing β it should be lower than the risk-free return.

To emphasise the similarity between the numerator and denominator of Eq. 3 we write

$$E\tilde{R}^N = R^0 + \beta(E\tilde{R}^M - R^0) - \frac{\int u'(\tilde{C}_2) F_{K^N}(C_2; K^N, \xi) \mathrm{d}C_2}{\int u'(\tilde{C}_2) F_{C_2}(C_2; K^N, \xi) \mathrm{d}C_2} = R^0 + \beta(E\tilde{R}^M - R^0) - \frac{E_{K^N} u'(\tilde{C}_2)}{Eu'(\tilde{C}_2)} \tag{4}$$

where the subscript K^N of E in the numerator is added to distinguish it from the ordinary expected value displayed in the denominator. Numerator and denominator differ only by being derivatives in different directions. In the case of a negative correlation between asset N and asset M our model gives *two* reasons why the rate of return requirement is lower than the risk-free rate: one is that β is negative, corresponding to the insurance aspect of the investment. The other is that $(-E_{K^N} u'(\tilde{C}_2)/Eu'(\tilde{C}_2))$ is negative, corresponding to the protection aspect of the investment.

The Stein–Rubinstein lemma requires that the distribution of $\tilde{C}_2$ is normal. When normal it takes the form $F(C_2; K^N, \xi) = \Phi((C_2 - \mu)/\sigma; K^N, \xi)$ with μ the mean and σ the standard deviation of C_2. Φ is the distribution function of the standard normal variable. We assume $F(C_2; K^N, \xi) = \Phi(\{C_2 - \mu(K^N/\mu_0)\}/\sigma)$ with $\mu_0 = \mu(K_0^N)$ and $\mu' > 0$, i.e. the environmental investment has the capacity to increase the mean of the density function for C_2. K_0^N is chosen such that μ_0 equals expected period two consumption. Assuming $\mu' > 0$, in the normal distribution is a simple way of capturing the idea that environmental investments move probability mass in a favourable direction. The probability that $\tilde{C}_2$ takes on a low value is lower for high μ distributions. It turns out that the normal specification significantly simplifies the term $(E_{K^N} u'(\tilde{C}_2)/Eu'(\tilde{C}_2))$. We obtain

$$E\tilde{R}^N = R^0 + \beta(E\tilde{R}^M - R^0) - \frac{E(\tilde{C}_2')}{E(\tilde{C}_2)} \tag{5}$$

see the Appendix for details on the derivation of Eq. 5.

As it now stands, the fraction $E\tilde{C}_2'/E\tilde{C}_2$ is the percentage change in expected period two consumption with respect to environmental investments via endogenous probabilities. In other words the optimal rate of return that comes out of the portfolio diversification argument, which relies on the insurance property of environmental investments, should be reduced by the percentage increase in expected consumption due to endogenous probability. The percentage increase in expected consumption is the net outcome of the protection property of environmental investments.

Example: a climate model

In this section we give an example of the modelling framework in action. In the example there are two assets, an investment in potential GDP—which we assume is perfectly correlated with aggregate consumption—and a climate asset. Investment in the climate asset reduces the likelihood of climate damage.

Setting up the model

Inspired by the DICE-model of Nordhaus (1994) and Nordhaus and Boyer (2000) the simple structural model of the global economy reads as follows:

Period 1:

$$W - C_1 = K^0 + K^M + K^N = (x^0 + x^M + x^N)(W - C_1). \tag{6}$$

Period 2:

$$Y^* = AK^{M\alpha} \tag{7}$$

$$\tilde{D} = \tilde{\theta}_1 T^{\theta_2} \tag{8}$$

$$\tilde{Y} = (1 - \tilde{D})Y^* \tag{9}$$

$$T = -\eta K^N + T^0 \tag{10}$$

$$Y^0 = R^0 K^0 \tag{11}$$

$$\tilde{C}_2 = Y^0 + \tilde{Y} + K^M + K^N. \tag{12}$$

We start with the first-period choice of whether to invest wealth (W) in the risk free asset (K^0), the consumption asset (K^M) or the climate asset (K^N); or whether to consume (C_1) (Eq. 6). The association between the K's and the x's of Eq. 2 is established. In Eq. 7 Y^* is potential world GDP, a Cobb–Douglas function of consumption capital (K^M). $\alpha \leq 1$ gives the returns to scale of an increase in consumption capital. D in Eq. 8 is unit damage to potential GDP from rising temperatures (T). Y is actual GDP, which is equal to potential GDP less damage (Eq. 9). Global temperature is in this simple model a function of climate capital N installed to keep temperatures down (Eq. 10). The relation between GDP, emissions and temperature is not modelled. We assume for simplicity that the temperature–capital relation is linear over the range studied and has a benchmark of T^0. To complete the model we assume that society has access to a risk-free storage technology (Eq. 11). Finally, in this two-period problem aggregate period two wealth, including capital stocks, is consumed at the end (Eq. 12).

α, $\tilde{\theta}_1$, θ_2 and η are the parameters of the model. We assume that $\tilde{\theta}_1$ is stochastic $\sim N(\bar{\theta}_1(K^N), \sigma_\theta)$, $\bar{\theta}_1' \leq 0$. In other words the constant of proportionality θ_1 between temperatures and unit damage is stochastic and normally distributed. The reason we choose the normal distribution for θ_1 is that many of the results of Sect. 2 depend on it. If θ_1 has a high value, the damage factor significantly pulls down potential GDP with the consequence that temperature increases create significant damage. If θ_1 has a low value, temperature increases do not create significant damage. For negative θ_1, "damage" of climate change turns out to be a gain—a possibility that cannot be a priori excluded.

Having explained the equations we examine how climate investments influence the model. Consistent with the framework in Sect. 2 there are two channels of influence. The first is via restraining temperature increases, c.f. Eq. 10. The idea here is that society in the course of economic development might spend resources in order to bring down temperatures relative to its baseline level. Clear-cut examples of climate investments that restrain temperatures are carbon sequestration and methane collection from mines, landfills etc.

Besides restraining temperature increases, we assume through the term $\bar{\theta}_1' \leq 0$ that climate investments reduce the probability of temperatures causing economic damage, and reduce the probability of high damage outcomes. The risk of damage from a certain temperature increase depends on a chain of events. For instance it depends on the risk that rising temperatures increases storms, droughts and floods, and the risk that storms, droughts and floods reduce GDP. The risk that storms, droughts and floods reduce GDP is endogenous in view of our paper, since this type of risk depends on environmental investments and other human actions.

Examples of investments that reduce the risk of damage from rising temperatures include improved standards for construction (buildings, roads, railways, dams), especially for long-term infrastructure investments; irrigation systems and reforestation to protect against droughts, and much more. Reforestation is a good example of an investment that both reduces temperatures and the risk of damage from a temperature increase. For other examples of climate investments with both consequences it is perhaps best to think in portfolio terms.

Solving the model

The model is solved by acknowledging that Eq. 12 can be turned into Eq. 2. We have

$$\tilde{C}_2 = Y^0 + \tilde{Y} + K^M + K^N = (W - C_1)\left(x^0 R^0 + \sum_{a \in \{M,N\}} x^a R^a\right) \tag{13}$$

c.f. the Appendix for details. Maximising $u(C_1) + \delta E u(\tilde{C}_2)$ subject to the model (6)–(12), therefore, is equivalent to maximising Eq. 1b:

$$\max_{C_1, x^0, x^M, x^N} u(C_1) + \delta E\left[u\left((W - C_1)\left(x^0 R^0 + \sum_{a \in \{M,N\}} x^a \tilde{R}^a\right)\right)\right].$$

Equation 1b leads to Eq. 3, and eventually to Eq. 5.

Applying Equation (5)

We may use the structural features of the model to calculate an explicit solution to Eq. 5. We obtain

$$E\tilde{R}^N = R^0 + \beta(E\tilde{R}^M - R^0) - E\tilde{C'}_2/E\tilde{C}_2 \approx R^0 - \frac{\theta_2\eta}{T}(E\tilde{R}^M - R^0) - E\tilde{D}\frac{-\bar{\theta}'_1}{\bar{\theta}_1} \quad (14)$$

see the Appendix for details. A couple of interesting features emerge from Eq. 14. One, β is negative. The negative β emerges because climate abatement investments often pay off in states where ''normal'' investments do not pay off. Climate investments in this model act as societal insurance to risk in the manner that we have argued earlier in the paper. Second, the percentage increase in consumption is here specified as expected unit damage $(E\tilde{D})$ times the percentage reduction in expected unit damage resulting from endogenous probabilities.

Equation 14 has a form that we can take to the data. The data we assume is collected in Table 1.

With these assumptions we obtain a negative β, but ''only'' $\beta = -0.004$. We see that it depends on η, which empirically dwarfs the other two parameters, θ_2 and T. We further obtain $E\tilde{D}(-\bar{\theta}'_1)/(\bar{\theta}_1) = 0.00009$. Note that this result is obtained given the relative change in unit damage set as high as 10%. Inserting in Eq. 14 with assumptions for R^0 and R^M gives $ER^N = 0.01 - 0.004 * 0.06 - 0.00009 = 0.97\%$.

One should in other words treat a climate investment similarly to an investment in a safe, risk-free asset and require approximately the same return.

Since ours is a model of optima it is interesting to insert not only current levels, but optimal ones as well. Nordhaus and Boyer (2000) suggest that an optimal temperature change over the current century is approximately 2.5°C. Using 2.5°C instead of the current 0.5°C in Eq. 14 gives $\beta = -0.0008$, $E\tilde{D}(-\bar{\theta}'_1)/(\bar{\theta}_1) = 0.0022$ and $ER^N = 0.78\%$. The conclusion we draw is that even in the long-term one should treat

Table 1 Data used in numerical example

Variable	Value	Remark
R^0	0.01	Intermediate value of real return on safe assets. The average return to US short-term nominally risk free bonds over the period 1889–1978 equalled 1%, see Mehra and Prescott (1985). Ibbotson and associates (2001), quoted in Howarth (2003), have recently estimated the 1926–2000 rate to be 0.7%
R^M	0.07	Intermediate value of real return on real investment. Equal to the average return on US stocks 1889–1978, see Mehra and Prescott (1985). Ibbotson and associates (2001), quoted in Howarth (2003), have recently estimated the 1926–2000 rate to be 7.9%
θ_2	2	From Nordhaus and Boyer (2000)
η	0.001	Derived from Nordhaus and Boyer (2000). Nordhaus and Boyer find that the discounted cost of the optimal policy is 98 $billion. This policy reduces temperature growth by 0.1°C in 2105. Inefficient policies are much more costly. Assuming that 1,000 $billion reduces temperatures by 1°C we assume η = 0.001 in the best case
T	0.5°C, 2.5°C	0.5°C is the approximate change in temperatures from preindustrial times to the present. 2.5°C is an expected value a few decades from now
ED	0.001, 0.022	0.001 is the approximate unit current damage. Apply the unit damage function $\theta_1 T^{\theta}$ with θ_1=0.0035 and θ_2=2 (Nordhaus and Boyer 2000). 0.022 is the approximate unit current damage at 2.5°C
$-\theta'_1/\theta_1$	0.10	Assumption. One of the highest percentage changes from a marginal investment that we find conceivable

a climate investment similarly to a safe investment and require approximately the same return.

Conclusion

We have shown that the portfolio approach to climate investments motivates low discount rates on such investments. The self-insurance property of climate investments exploits the fact that climate investments may provide returns in otherwise unfavourable states. The self-protection or endogenous risk property increases the probability of favourable states. Both properties contribute to the motivation for a low discount rate. In the case of normal distributions we derive in the two-period case a simple formula describing the discount rate as a consumption CAPM-equation supplemented by the expected percentage consumption increase due to endogenous risk mitigation.

Applied to a simple model of climate change the optimal discount rate under the portfolio approach is 0.2% points below the risk-free rate. With a risk-free rate of 1% the optimal discount rate on a greenhouse gas mitigation or adaptation asset would be 0.8%.

It may be interesting to compare this discount rate with the rates employed by actual climate models. Nordhaus and Boyer (2000) is a case in point. They assume certainty, logarithmic utility and a rate of pure time preference of 2–3%. Assuming consumption growth of 3% this works out to an interest rate of 5–6%. In a certainty framework such a rate is a reasonable description of the opportunity cost of saving and investment.

Given that 6% represents certainty and 0.8% represents uncertainty the difference between certainty and uncertainty is huge. At 6% a dollar 100 years from now is currently worth 0.3 cent. At 0.8% a dollar 100 years from now is worth 45 cent. The incentive to investing in greenhouse gas mitigation or adaptation is more than a hundred times higher at 0.8% compared to 6%.

The comparison with actual climate models is of course limited by the fact that we reason in a two-period model. The two-period model allows us to analyse the insurance and protection aspects of climate investments in a relatively simple way. But the assumption of a constant, risk-adjusted interest rate is unrealistic when modelling many time periods, since it assumes that risk grows exponentially over time. In our case that would be risk *reduction* growing over time. Much more research on multi-period settings is needed to conclude on the appropriate multi-period discount rate for climate investments from a portfolio perspective. Yet, the portfolio approach under endogenous probability seems to us a promising approach to understanding the particular trade-offs involved in determining the appropriate levels of climate investments and environmental investments.

As an alternative to adjusting the discount rate for risk, risk might be internalised in the benefit and cost streams, i.e. certainly equivalents. It is our impression, however, that at least in climate change economics the income, benefit and cost streams are not processed this way. To the extent that risk aspects are internalised by taxes and subsidies the present analysis should be interpreted as a supplement and not an as an alternative to environmental policy instruments offered by standard public economics, such as taxes, subsidies or tradable permits, see e.g. Sandmo (2000) for a comprehensive review.

Although our theoretical model is linked to the climate issue, the results may be generalised to other environmental issues such as biodiversity, and indeed to consider how much to invest in any real asset that influences the return on the market portfolio both directly and indirectly via endogenous probabilities. This model is relevant for economic problems where the interplay of protection and insurance is important. Health, traffic accidents and conflict management are examples that come to mind where both protection, often in the form of prevention, and insurance play a role. The model could also be useful for risk management within companies. The synergies that arise when company branch A increases the success ratio of branch B might be analysed in our framework, or the synergies, in terms of increased probability of success, between ideas in a successful R&D lab.

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Appendix

This Appendix explains the derivations of the main equations presented in the text, i.e. Eqs. 3, 5, 13 and 14, respectively.

Derivation of Equation (3)

For convenience, first recall the third condition of Eq. 2:

$$E\left[u'\left(\tilde{C}_2\right)\left(\tilde{R}^N - R^0\right)\right] + \int u(C_2)\mathrm{d}F_{K^N}\left(C_2; K^N, \xi\right) = 0.$$

Then, using integration by parts over C_2, the last term of Eq. 2 can be written

$$\begin{aligned}\int u(C_2)\mathrm{d}F_{K^N}(C_2; K^N, \xi) &= u(\infty)F_{K^N}(\infty) - u(-\infty)F_{K^N}(-\infty) \\ &\quad - \int u'(C_2)F_{K^N}(C_2; K^N, \xi)\mathrm{d}C_2 \\ &= -\int u'(C_2)F_{K^N}(C_2; K^N, \xi)\mathrm{d}C_2\end{aligned}$$

since $F_K(-\infty) = F_K(\infty) = 0$ and we assume that $u(C_2)$ is bounded above and below or, more generally, $\lim_{C_2 \to -\infty} u(C_2)F_K(C_2) = \lim_{C_2 \to \infty} u(C_2)F_K(C_2) = 0$. Now we can rewrite Eq. 2:

$$E\left[u'\left(\tilde{C}_2\right)\left(\tilde{R}^N - R^0\right)\right] - \int u'(C_2)F_{K^N}(C_2; K^N, \xi)\mathrm{d}C_2 = 0. \tag{15}$$

By applying the covariance identity and rearranging, we get

$$E\tilde{R}^N = R^0 - \frac{\mathrm{cov}(u'(\tilde{C}_2), \tilde{R}^N)}{Eu'(\tilde{C}_2)} - \frac{\int u'(C_2)F_{K^N}(C_2; K^N, \xi)\mathrm{d}C_2}{Eu'(\tilde{C}_2)}. \tag{16}$$

Then we make use of the Stein–Rubinstein lemma (Stein 1973; Rubinstein 1976), which requires that joint distribution of $\tilde{R}^N$ and $\tilde{C}_2$ is bivariate normal and $u(\cdot)$ is twice differential. Invoking the result on the numerator of the first fraction of Eq. 16, such that $\mathrm{cov}(u'(\tilde{C}_2),\tilde{R}^N) = Eu''(\tilde{C}_2)\mathrm{cov}(\tilde{C}_2,\tilde{R}^N)$ we can rewrite Eq. 16 as

$$E\tilde{R}^N = R^0 + \left(-\frac{Eu''(\tilde{C}_2)}{Eu'(\tilde{C}_2)}\right)\mathrm{cov}(\tilde{C}_2,\tilde{R}^N) - \frac{\int u'(C_2)F_{K^N}(C_2;K^N,\xi)\mathrm{d}C_2}{Eu'(\tilde{C}_2)}. \tag{17}$$

Returning to the second condition of Eq. 2, $E\lfloor u'(\tilde{C}_2)(\tilde{R}^M - R^0)\rfloor = 0$, and rearranging it, we get

$$E\tilde{R}^M = R^0 - \frac{\mathrm{cov}(u'(\tilde{C}_2),\tilde{R}^M)}{Eu'(\tilde{C}_2)}. \tag{18}$$

Invoking again the Stein–Rubinstein lemma and recalling the assumption of perfect correlation between $\tilde{C}_2$ and $\tilde{R}^M$, we can write Eq. 18 as

$$-\frac{Eu''(\tilde{C}_2)}{Eu'(\tilde{C}_2)} = \frac{E\tilde{R}^M - R^0}{\mathrm{var}(\tilde{C}_2)}. \tag{19}$$

Equation 19 allows us to rewrite Eq. 17 to obtain Eq. 3 in the text.

Derivation of Equation (5)

We have

$$F(C_2;K^N) = \Phi\left(\frac{C_2 - \mu(K^N/\mu_0)}{\sigma}\right)$$

and

$$\begin{aligned} F_K(C_2;K^N) &= -\frac{\mu'/\mu_0}{\sigma}\Phi'\left(\frac{C_2 - \mu(K^N)/\mu_0}{\sigma}\right) \\ &= -\frac{\mu'/\mu_0}{\sigma}\phi\left(\frac{C_2 - \mu(K^N)/\mu_0}{\sigma}\right) = -\mu'/\mu_0 f(C_2)\cdot \end{aligned}$$

ϕ is the standard normal density function. Consequently,

$$\frac{\int u'(C_2)F_{K^N}(C_2;K^N)\mathrm{d}C_2}{Eu'(\tilde{C}_2)} = \frac{\int -u'(C_2)\mu'/\mu_0 f(C_2)\mathrm{d}C_2}{Eu'(\tilde{C}_2)} = -\frac{\mu' Eu'(\tilde{C}_2)}{\mu_0 Eu'(\tilde{C}_2)} = -\frac{\mu'}{\mu_0}. \tag{20}$$

By definition $\mu = \mathrm{E}\tilde{C}_2$. We assume μ_0 is chosen such that it equals actual period two expected consumption, i.e., $\mu_0 = \mu$. Inserting this in equation (20) now yields equation (5).

Derivation of Equation (13)

The production function of this model is not constant returns to scale and will therefore generate profit income. We assume that profit is related to investment of K^M and define

$$\begin{aligned}\tilde{R}^N &= \frac{\mathrm{d}\tilde{Y}}{\mathrm{d}K^N} + 1 = \frac{\theta_2\eta}{T}\tilde{D}Y^* + 1\\ \tilde{R}^M &= \frac{\tilde{Y}}{K^M} - \frac{\mathrm{d}\tilde{Y}}{\mathrm{d}K^N}\frac{K^N}{K^M} + 1.\end{aligned} \tag{21}$$

In words, K^N gets its marginal return and K^M gets the rest. With these assumptions we obtain

$$\begin{aligned}\tilde{C}_2 &= Y^0 + \tilde{Y} + K^M + K^N\\ &= R^0K^0 + \frac{\tilde{Y}}{K^M}K^M + K^M + K^N\\ &= R^0K^0 + \left(\frac{\tilde{Y}}{K^M} - \frac{\mathrm{d}\tilde{Y}}{\mathrm{d}K^N}\frac{K^N}{K^M}\right)K^M + \frac{\mathrm{d}\tilde{Y}}{\mathrm{d}K^N}K^N + K^M + K^N\\ &= R^0K^0 + \left(\frac{\tilde{Y}}{K^M} - \frac{\mathrm{d}\tilde{Y}}{\mathrm{d}K^N}\frac{K^N}{K^M} + 1\right)K^M + \left(\frac{\mathrm{d}\tilde{Y}}{\mathrm{d}K^N} + 1\right)K^N\\ &= R^0K^0 + R^MK^M + R^NK^N\\ &= (W - C_1)(R^0x^0 + R^Mx^M + R^Nx^N).\end{aligned} \tag{22}$$

Derivation of Equation (14)

Since $\tilde{\theta}_1 \sim N(\bar{\theta}_1(K^N), \sigma_\theta)$ and $\tilde{C}_2 = (1 - \tilde{\theta}_1 T^{\theta_2})Y^* + Y^0 + K^M + K^N$ (equations 8, 9, 12) it follows that $\tilde{C}_2$ is normal and we may apply Eq. 5:

$$E\tilde{R}^N = R^0 + \beta(E\tilde{R}^M - R^0) - E\tilde{C}'_2/E\tilde{C}_2, \quad \text{where } \beta = \frac{\mathrm{cov}(\tilde{C}_2, R^N)}{\mathrm{var}(\tilde{C}_2)}.$$

First, we develop a simpler expression for β. With $\tilde{C}_2 = Y^0 + \tilde{Y} + K^M + K^N$ (Eq. 12) we have

$$\frac{\mathrm{cov}(\tilde{C}_2, R^N)}{\mathrm{var}(\tilde{C}_2)} = \frac{\mathrm{cov}(\tilde{Y}, R^N)}{\mathrm{var}(\tilde{Y})}. \tag{23}$$

From Eq. 21 we know that $\tilde{R}^N = (\theta_2\eta/T)\tilde{D}Y^* + 1$, which from Eq. 9 is $\tilde{R}^N = (\theta_2\eta/T)(Y^* - \tilde{Y}) + 1$. Inserting in Eq. 23 gives us

$$\frac{\operatorname{cov}(\tilde{Y}, R^N)}{\operatorname{var}(\tilde{Y})} = \frac{\operatorname{cov}\left(\tilde{Y}, \frac{\theta_2 \eta}{T}(Y^* - \tilde{Y}) + 1\right)}{\operatorname{var}(\tilde{Y})} = \frac{-\frac{\theta_2 \eta}{T}\operatorname{var}(\tilde{Y})}{\operatorname{var}(\tilde{Y})} = \frac{\theta_2 \eta}{T}. \tag{24}$$

Then we develop an expression for $E\tilde{C}'_2$. By definition $E\tilde{C}_2 = \int C_2 \mathrm{d}F_{C_2}(C_2; K^N, \xi)$. Since, as stated, $\tilde{C}_2 = (1 - \tilde{\theta}_1 T^{\theta_2})Y^* + Y^0 + K^M + K^N$, it follows that

$$\begin{aligned} E\tilde{C}'_2 &= \int \frac{\mathrm{d}[(1 - \theta_1(K^N)T^{\theta_2})Y^* + Y^0 + K^M + K^N]}{\mathrm{d}K^N} \mathrm{d}F(C_2; K^N, \xi) \\ &= \int -\theta'_1 T^{\theta_2} Y^* \mathrm{d}F(C_2; K^N, \xi) = \frac{-E\tilde{\theta}'_1}{E\tilde{\theta}_1} E\tilde{\theta}_1 T^{\theta_2} Y^* = -\frac{\bar{\theta}'_1}{\bar{\theta}_1} E\tilde{D} Y^*. \end{aligned} \tag{25}$$

From Eq. 25 we get

$$-\frac{E\tilde{C}'_2}{E\tilde{C}_2} = \frac{\bar{\theta}'_1}{\bar{\theta}_1} \frac{Y^*}{E\tilde{C}_2} E\tilde{D}. \tag{26}$$

Assume that $E\tilde{C}_2 \approx E\tilde{Y}$. This is an assumption in the spirit of the model since it only disregards the somewhat artificial storage technology Y^0 and initial wealth K^M and K^N, consumption of which is an artefact of the two-period setup. Now

$$\frac{\bar{\theta}'_1}{\bar{\theta}_1} \frac{Y^*}{E\tilde{C}_2} E\tilde{D} = \frac{E\tilde{D}}{1 - E\tilde{D}} \frac{\bar{\theta}'}{\bar{\theta}_1} \approx E\tilde{D} \frac{\bar{\theta}'}{\bar{\theta}_1}. \tag{27}$$

Expression (27) underestimates the effect to the extent that $Y^* > E\tilde{C}_2$. Combining results we have

$$E\tilde{R}^N = R^0 + \beta(E\tilde{R}^M - R^0) - E\tilde{C}'_2 = R^0 - \frac{\theta_2 \eta}{T}(E\tilde{R}^M - R^0) - E\tilde{D} \frac{-\bar{\theta}'_1}{\bar{\theta}_1}. \tag{28}$$

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