

Choosing Children's Environmental Risk[★]

DAVID W. ARCHER¹, THOMAS D. CROCKER²
and JASON F. SHOGREN^{2,*}

¹North Central Soil Conservation Research Laboratory, USDA-Agricultural Research Service, Morris, MN 56267, USA; ²Department of Economics and Finance, University of Wyoming, Laramie, WY 82071, USA; *Author for correspondence (e-mail: jramses@uwyo.edu)

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Abstract. A model of endogenous risk provides a foundation to study a parent's child care decisions when the child could be exposed to an environmental hazard (e.g., toxic substance, foodborne pathogen). The parent invests in childcare quality and quantity to reduce the likelihood of a hazard exposure occurring and to reduce its severity if the exposure is realized. We supply conditions to sign unambiguously the effects on a child's hazard exposure of an increased probability a parent fails to access or have command over a technique of exposure prevention or that a technique is ineffective in preventing exposure. Also, we consider these effects when the parent is unsure what a technique can do to reduce the child's probability of exposure. We conclude public policies designed to encourage use of a particular childcare technique, if childcare quality and quantity are stochastic substitutes, can reduce parental use of other techniques. The net impact of the policy could increase the chance the child suffers.

Key words: childcare quality, childcare quantity, stochastic substitution

JEL Classifications: D10, D80, J13

1. Introduction

Parents spend valuable time and effort to nurture, monitor, and teach their children. This care would be unnecessary if the world posed no risks to children's current well-being and future prospects or if children could take care of themselves. But young children are innocent of the world's ways. They know very little or nothing of the world's hazards or of how to defeat or to temper these hazards. A parent knows considerably more but the care the parent chooses to render varies with parental preferences and resources, and

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with her access to techniques of care, the effectiveness of these techniques, and the intensity of their use. Parents differ in their willingness and in their ability to cope with threats to their children. All parents combine their time, effort, and accessible goods of varying effectiveness to produce flows of child care services on which the well-being of their children depend but the combinations chosen differ with parental circumstances and tastes.

We treat children's health risks as endogenous—as susceptible to manipulation by the child's parents. These risks are endogenous both (i) because parents choose levels of care, and (ii) because they choose from a *portfolio of alternative risk reduction technologies or input mixes* having different quality attributes (also see Chambers and Quiggin 2000; Kumbhakar and Lovell 2000). Herein we describe an endogenous risk framework which captures parents' choices among alternative childcare technologies distinguished by quality and quantity attributes. We consider how risks to a young child's health and well-being from environmental and other hazards affect the childcare techniques parents choose.

Endogenous risk recognizes that a parent can invest resources in risk-reducing technologies which influence the expected consequences of the hazard of concern. She can undertake actions which modify events or which reduce the vulnerability to loss (Kates 1978). This recognition has profound implications for predicting the results of risk-reducing strategies, the formal economic evaluation of which has traditionally been driven by an assumption that risk is exogenous (Crocker and Shogren 1998). In accordance with Burton et al. (1979), we define 'risk' as the probability times the severity of damages, not as the variability of expected wealth. The endogenous risk approach opposes the mind-set that equates the probability distribution of outcomes with the probability distribution of states. Caregivers choose a child's level of environmental risk (Agee and Crocker 1994, 1996; Shogren 2001). The endogenous risk approach allows outcomes for the child and the lotteries defining these outcomes to depend on a parent's behaviors.

Since the seminal paper of Ehrlich and Becker (1972), several researchers have explored in a variety of settings the behavioral implications of endogenous risk, including Heibert (1983), Dionne and Eckhoudt (1985), Shogren and Crocker (1991, 1999), Sweeney and Beard (1992), Quiggin (1992), Haritchabalet (2000), Courbage (2001), and Fisher and Narain (2003) among others. One general conclusion of this literature is that the endogenous risk model expands the scope of expected utility theory to allow a broader range of observed behaviors under risk to be explained. This broader, more robust framework occurs because it presumes preferences over both outcomes and the lotteries which define the possible outcomes (Shogren 1991). Consequently, the endogenous risk concept directs attention to a primary factor of interest in child care strategies—the technologies used to reduce risks for the child.

Here we view child care as a set of private risk-reduction techniques aimed at decreasing a child's expected damages from exposures to an environmental hazard (e.g., ozone and particulate matter in the air, toxics and nitrates in the water, pathogens in the food). Our focus is on parental access to or command over these childcare techniques, their effectiveness, and the intensity with which parents choose to use them. A technique is a set of child care inputs whose relative prices are fixed in terms of child-specific commodity inputs and of the opportunity costs of parental time. Our results suggest that although an increased probability of damages to a child can reduce parents' use of a technique of childcare, this reduction is likely to be offset by a shift toward a more effective technique of care. For some plausible conditions, this result holds even if the probability of damages to the child is uncertain. If childcare technique quality and intensity of use are stochastic substitutes, public environmental policies to encourage a particular technique of childcare will likely reduce parental use of other techniques. The net result may be an increase in the child's exposure to the environmental hazard.

We consider the roles played by a parent's access to multiple techniques of caring for her child, the effectiveness of these techniques, and the flow of child care services from these techniques on the chance a child suffers damages from exogenous hazards, including environmental hazards. Childcare involves considerably more than merely selecting an effort level for a single technique and then waiting for the environment to act. We account simultaneously for the intensive (quantity) and the extensive (quality) margins of childcare. The next section presents a single period model of childcare quality and quantity when parents know with certainty the probability of a better child health state. Section 3 explores the ramifications for parental choices of childcare quality and quantity when these certain probabilities change. In Section 4, we relax the assumption of certain probabilities to probe the restrictions necessary to sign unambiguously the comparative statics of increased uncertainty regarding the efficacy of childcare techniques. We offer our conclusions in Section 5. Appendices provide details of the derivations of our key results.

For specificity throughout the paper, think of the exposures of a young child of a poor, single, never-married mother living in an old, rundown neighborhood of a congested urban area. To reduce the likelihood or the severity of lead-induced cognitive deficits for the child, the mother might remove leaded paint from the family residence, have the child drink bottled rather than tap water, not allow the child to crawl about whether inside or outside, or even investigate the consequences of having the child chelated. But these exposure-reducing techniques are costly, even when accessible. Also, some of them may not be accessible to the mother, or she may be unable to command them. For example, she may have no legal right to remove leaded paint from the residence she rents; or with the same result as no access, she

may not even know lead exposures harm her child. Moreover, even if she knows these harms, having the child drink bottled rather than tap water offers the child ineffective protection if the major source of the child's exposures is leaded paint in the residence. Finally, even if the mother can remove the leaded paint, she may remove any portion from only a small amount to all of it; or she may monitor where her child plays when outside part of the time.

Our description fits a large enough number of real-world cases to be policy-relevant. At any time, about one-quarter of US households with children have only a single parent (Bureau of the Census 1998). About half the children in the US spend part of their childhood in households headed by a single parent, typically the mother. However measured the frequency of poverty among these households is high.

2. Model

Our model adapts Archer and Shogren (1996), who construct an endogenous risk framework for production risk management, to a parent's decisions about her children's exposures to environmental risks. We restrict our analysis of a parent's choice of the quality and the quantity of childcare technologies by considering only the special case of parental risk neutrality, leaving open the question of whether the risk averse parent uses more or less or better or worse care than the risk neutral parent. Antle (1983) shows that risk can affect optimal production (i.e., childcare) decisions whether producers are risk neutral or risk averse. Consider this risk neutral parent who can use various combinations of own health, own time, home environment, and child-specific inputs to defend her child's health against a particular environmental hazard in a single period. For now, let this parent know the likelihood her child can suffer from this hazard and what the quality and quantity of her childcare efforts can do to reduce this likelihood.

The parent's chosen childcare technique can come to naught for two reasons—access or command failure and effectiveness failure. Access failure occurs when she cannot gain access to a childcare technique to control a particular source (air, water, soil, food, etc.) or pathway (respiration, ingestion, absorption) of exposure to an environmental hazard. She may not have the stamina, be unable to acquire medical care (Currie and Reagan 2003; Dafny and Gruber 2005), or there may be no market where she can acquire the right to remove neighborhood auto traffic or observe market prices, for example. Effectiveness failure occurs when exogenous conditions render a childcare technique completely ineffective in controlling a source or a pathway, as when bad weather prevents a parent from sending a child outside to prevent its exposure to toxic fumes or particulates inside its house. In addition, even if a chosen childcare technique is accessible and effective, the protection it provides is dependent on the degree of use it receives.

The rational parent or caregiver cares for her child by investing resources in childcare techniques to reduce the probabilities and severity of hazard exposures, thereby increasing her expected wealth. Let h be the parent's child harm probability reduction efforts directed at the specific environmental hazard, and H be her child harm severity reduction efforts. Assume she perceives two mutually exclusive and exhaustive states of the world: a *better state* implying less severe losses in her child's health and a *bad state* implying an utter failure of care. The probability of the better state, $g(h)$, is a function of childcare quality, h , where $g'(h) > 0$. The bad state occurs with probability $[1-g(h)]$.¹

Childcare quality of a technique, h , is defined by its *flexibility* and its *control capacity*. Flexibility measures access—referring to the weighted percentage of the number of sources and pathways of the specific child health hazard against which a care technique can offer full protection. The weights are the frequency of occurrence of a particular source and pathway of the threat during a given time interval. Regular vacuuming of the floor of a house is a more flexible technique than is making the child wash his hands. Floor vacuuming contracts both the respiratory and ingestion pathways to the child's uptake of household toxins while hand washing affects only the ingestion pathway. A convex combination of hand washing and vacuuming would be another technique yet more flexible because it offers protection against more sources and more pathways. Control capacity, which measures effectiveness, deals with the productivity of a technique of childcare in terms of the fraction of maximum damage abated by a given intensity of use of a technique applied to a given source or pathway. Greater flexibility or control capacity imply more quality in the child care technique as reflected by an enhanced probability of responding to and coping with the threat to the child's health and to the parent's wealth. The unit cost of a better quality childcare method is $c(h)$, where $c'(h) > 0$, i.e., a better quality childcare technique is more expensive. The cost comes from the setup costs (time, transportation, durable good outlays) when resources are devoted to adding a new child care means to an existing set.

Here, H , refers to the *quantity* or *intensity of use* of a childcare technique. Low use intensity of an accessible childcare technique can fade into child neglect. A parent may acquire a potentially useful technique for child care (e.g., own-health) but may choose not to devote it to that care. She may spend her time at the local tavern or workout club. Wealth losses *via* child health damages can occur in both the better and the bad states, with child care completely failing only in the bad state. Severity reduction lessens wealth losses *via* child health damages if the child care method is accessible and effective (i.e. in the better state). Greater intensity of use of an accessible and effective childcare technique drives the wealth positions of the two states further apart by increasing wealth in the better state.

The damage, $D(X)$, to the parent's wealth equivalent of the child's health in the better state is a positive function of the child's exposure to the hazard, X . Let X_0 be the value of the child's health loss to the parent in the absence of any parent-provided care against the threat from the hazard. If a childcare technique exists, the child's exposures to the health hazard are influenced by the use intensity of the child care method, $X(H)$, such that $D'(X) X'(H) < 0$, where $X'(H) < 0$. An increased flow of childcare services is not risk reducing in the traditional economic sense because more childcare increases wealth variability across the two states. Note $D(X(H))$ may take on a variety of functional forms depending on the nature of the threat to the child's prospects.

Total parent wealth consists of her predetermined money wealth, Y_0 , and her discounted expected value of the child's potential adult consumption support to her as well as the money equivalent of the child's current and future companionship and emotional support. As in Graham (1992), the simplifying assumption that these private goods are claims to dollars contingent on the occurrence of either one of the two states of nature supports our use of money equivalents. The parent's utility is a state-dependent positive function of her residual wealth—the wealth remaining to her after child health damages have been deducted [$Y_0[1-D(X)]$]. This residual wealth is devoted to immediate own consumption, savings, and investments other than in the child. The residual helps a poor single mother's household survive. Survival depends on her acquisition of a given prerequisite wealth level before she can indulge any aversion to risks to her child's health. Her chances of reaching this level depend on her rational response to exogenous market and other institutional forces which affect her wealth.

Consistent then with Grossman's (1999) investment model, the parent invests in her child's health by acquiring access to effective childcare techniques (including own-health), and by using these methods productively. Taking good care of one's child is easier if one takes good care of own-health. Child health losses are caused by the inability or the unwillingness to care for the child, whether due to own-health, income, taste, or technological reasons. We presume the parent's labor supply decisions to be separable from her childcare efforts. We focus on the conditional demands for h and H .

In reduced form, a parent maximizes her expected wealth, $E(Y)$, by selecting h and H :

$$\begin{aligned} \max_{h,H} E(Y) = & [g(h)\{Y_0[1 - D(X(H))] - c(h)H\}] \\ & + [(1 - g(h))\{Y_0[1 - D(X_0)] - \rho c(h)H\}]. \end{aligned} \quad (1)$$

This characterization of the parent's decision problem incorporates the behavioral motivation underlying her childcare choices and her technical substitution possibilities among childcare techniques.² For simplicity, we

consider access and effectiveness as separate cases. In reality, the parent may decide them jointly. The parameter ρ is a binary zero-one variable. Let $\rho = 1$ for an *effectiveness failure* in which an adopted childcare technique proves to be useless. In spite of this uselessness, the cost $c(h)H$ of gaining access to and of using the technique is nevertheless borne in both states. Let $\rho = 0$ for an *access failure* since the parent then incurs no cost of childcare in the bad state. The first term in expression (1) represents the parent's expected wealth if there is no failure of technique access or of effectiveness; the second term represents such failure. Note what distinguishes this endogenous risk approach from an exogenous risk approach to the childcare problem. Expression (1) focuses on the probability relevant to the parent (increasing the probability of the better state) and not on the probability that is independent of the parent's child care activities (the level of the environmental hazard).

Also, the parent may have the option of treatment of a realized health problem. The usefulness of this *ex post* treatment can be influenced by the *ex ante* child care the parent has adopted. We abstract from these complicating dimensions of jointness and of path dependence. Rather we assume that the selection of a lottery in a period affects the set of lotteries in subsequent periods only through its direct effect on the mother's wealth.

The first-order-conditions for expression (1) to have an interior maximum are:

$$\begin{aligned} \frac{\partial E(Y_0)}{\partial h} &= g'(h)\{Y_0[D(X_0) - D(X)] - (1 - \rho)c(h)H\} \\ &\quad - c'(h)H\{\rho + (1 - \rho)g(h)\} = 0 \end{aligned} \tag{2}$$

and

$$\frac{\partial E(Y_0)}{\partial H} = -g(h)Y_0D'(X)X'(H) - \{(1 - \rho)g(h)c(h) + \rho c(h)\} = 0 \tag{3}$$

Assume the second-order conditions to be fulfilled.

The first term on the right hand side of expression (2) is the parent's marginal expected benefit of childcare quality, h , in increasing the probability of successful childcare. For the access case, the benefits of reduced wealth damages with the successful childcare of the better state are partially offset by the added cost of gaining access to and of using this technique of childcare. The offset does not occur in the bad state since the technique is not accessible. For the effectiveness case, this offset occurs in both states. The cost of the technique is incurred whether or not the technique is effective. The second term in (2) represents the realized marginal cost of h for the effectiveness case and the expected marginal cost of h for the access case.

In expression (3) the first term represents the expected marginal benefit of childcare quality, H , in reducing wealth damages from child health losses. The second term is the realized unit price of the technique of childcare for the effectiveness case and the expected unit price of this means for the access case. For the access case, think of the decision about the intensity of use of a technique being made as it is being used. Then the parent chooses H so that the marginal benefit of H in the good state is equal to its marginal cost in this same state.

3. Certain Probabilities

Following Hiebert (1983), let $g(h)$ take the form

$$g(h) = g_0 + \gamma m(h) \quad 0 \leq m(h) \leq 1 \quad (4)$$

where g_0 and γ are positive constants, and $m'(h) > 0$. A decline in either g_0 or γ is an exogenous increase in the probability of the child's exposure to a hazard from a given source or along a given pathway. A decline in g_0 represents an increase in the baseline or background probability of the bad state (see Ehrlich and Becker 1972), independent of the flexibility or control capacity of the technique of childcare – a constant increase in the probability of the bad state for all levels of flexibility or control capacity, h . Our poor single mother can do little about the dangers, say, congested traffic patterns pose to her child's health. A poor mother may value collectively supplied reductions in background risks more than the wealthy mother because the poor mother lacks access to or is less effective in using a childcare technique (Shogren and Crocker 1991).

In contrast, a decrease in γ represents an increase in the probability of the bad state which increases in proportion to h . This parameter is a decrease in the efficacy of a technique of child care. For identical sources and pathways of exposure to the same hazard, children's health damages can differ with its developmental stage or "critical windows of exposure," diet, exposures to other hazards, and social factors (Tamburlini 2003). For example, suppose for a given home environment to contribute substantially to a child's health the mother must actively interact daily with the child. If one child has an unmarried mother who is worn down by her job, this physical home environment is less likely to contribute to the child's health than is the same environment with a married mother who is a homemaker. In this case, the unmarried, overwhelmed mother corresponds to a lower γ . Given mothers with identical work programs and marital status, the productivity of the home physical environments are identical; when her outside responsibilities overwhelm one mother, the same physical environment is less efficacious. Our mother's $m(h)$ in expression (4) is to be viewed as the arithmetic sum of a background probability determined by the child exposure consequences of,

say, local traffic congestion and by the efficacy of her chosen childcare technique. The traffic congestion does not affect the inherent efficacy of a technique and *vice versa*. It remains to explore the effects of changes in g_0 and in γ on the parent's choice of h and H .

For access failure ($\rho = 0$), the comparative static effects of a decrease in g_0 on the optimal (*) levels of h and H are $\partial h^*/\partial g_0 < 0$ always, and $\partial H^*/\partial g_0 > 0$ for stochastic substitutes (see Appendix A).³ We summarize the key result. Given access failure ($\rho = 0$), an increase in the background probability of the bad state always increases the use of childcare (h)—which implies more flexibility of the optimal technique of childcare, h . The intensity with which any technique is used, H , decreases since childcare quality and quantity are stochastic substitutes. In the days of leaded gasoline, increased neighborhood traffic congestion about which the mother could do nothing might cause the mother to clean the house more frequently but to clean it less intensely each time.

For effectiveness failure ($\rho = 1$), in general, $\partial H^*/\partial g_0 > 0$, but $\partial h^*/\partial g_0$ is ambiguous (Appendix A). Again we summarize this result. For effectiveness failure ($\rho = 1$), an increase in the background probability of the bad state, $[1 - g_0]$, always reduces childcare quantity, H . Assuming successful childcare is globally concave in H , the level of childcare quality, h , increases since flexibility and intensity are stochastic substitutes. If childcare is not globally concave, quality could decrease as well since the chance exists that quality and quantity are complements. For example, if feeding the child a better diet does little to counteract the cognitive impact of increased lead exposure on the child, the mother may reduce her insistence that the child always wash his hands, which may induce her to clean the house less frequently given hand washing and house cleaning are stochastic complements.

Given access failure, our result suggests an exogenous increase in the background probability of the child's exposure to a hazard always induces the parent to use a given technique with more intensity. Whereas given effectiveness failure, the results imply the parent uses more care quality, i.e., a more flexible technique. The technical relationship—stochastic complements or substitutes—determines how care quality then adjusts for access failure; or how care quantity changes for effectiveness failure. The point is that an exogenous change in the probability of child exposure from a given source and along a given pathway has an effect on both the quality and the quantity of the technique the parent chooses to protect the child from an environmental hazard. The endogenous risk approach allows us to model this effect. Child exposures are dependent not only on the quantity of care employed. The quality matters as well.

For access failure, greater background probability decreases the expected marginal cost of childcare quality, h , thereby increasing the optimal level of flexibility. Additionally, here h and H are stochastic substitutes, inputs in

which more of one has the indirect effect of decreasing the other. Since h and H are stochastic substitutes, greater background probability causes increasing optimal flexibility of the technique of childcare; this in turn reduces the optimal intensity of use of any technique. There is an indirect but no direct effect of this exogenous increase on the optimal intensity of use.

In contrast, for effectiveness failure, an exogenous increase in the background probability of child exposure has the direct effect of reducing the expected marginal benefit of H , thereby decreasing the optimal intensity of use of a technique of childcare. Since h and H can be stochastic substitutes under sufficient conditions of global concavity, this in turn increases the flexibility of the optimal technique. There is no direct effect of the exogenous increase on the optimal control capacity.

The effect of an exogenous increase in the probability of the bad state due to the impact of access or effectiveness failure on the net likelihood of child exposure is found by differentiating $g(h^*) = g_0 + \gamma m(h^*)$ with respect to g_0 :

$$\frac{\partial g(h^*)}{\partial g_0} = 1 + \gamma m'(h) \frac{\partial h^*}{\partial g_0}. \quad (5)$$

The two terms in expression (5) again represent a direct and an indirect effect. The first term, which is positive, is the direct effect. It shows that a decrease in g_0 directly reduces $g(h^*)$. The second term is the indirect effect. Our result implies this term is negative, indicating that a decrease in g_0 increases h^* and has the indirect effect of increasing $g(h^*)$. The net result depends on which term dominates. If the second term exceeds -1 , an exogenous increase in the probability of the bad state due to access failure increases the likelihood of child exposure, resulting in the less intensive use of a technique of childcare that has greater control capacity. An exogenous increase in the probability of the bad state due to access or effectiveness failure leads to use of a greater capacity or more flexible childcare technique which has the indirect effect of reducing the probability of the bad state. If this indirect effect is small (the second term in (5) is greater than -1), then the net probability of child exposure increases. If this indirect effect is large (the second term in (5) is less than -1), however, the net probability of child exposure decreases. This possibility is somewhat counter intuitive. It says an exogenous increase in background probability of a child's exposure to an environmental hazard may cause parents to shift to a technique of care having a degree of flexibility such that the probability of preventing exposure actually increases.

Now consider the comparative statics of decreased γ of the optimal levels of h and H . For the access case ($\rho = 0$), we know $\partial h^*/\partial \gamma > 0$ and $\partial H^*/\partial \gamma < 0$ (Appendix B), which is summarized as follows. Given an access failure ($\rho = 0$), an increase in the probability of the bad state due to a decrease in the efficacy of childcare (γ) always decreases the use of childcare quality, i.e., the

flexibility of the optimal technique decreases. If quality and quantity are stochastic complements, less efficacy then increases the intensity with which any technique of childcare is used.

This result suggests that a reduction in the efficacy of childcare results in using techniques of childcare more intensively than otherwise but the chosen technique has less flexibility and control capacity. As a child gets older and becomes more independent of the parent, the parent has less access to the child and less command over some care techniques. The parent may then admonish the child about the dangers inherent in some of its behaviors. But older children are harder to control and the child may not transfer admonitions about one hazard to other hazards. Reductions in g_0 and in γ have opposite effects on the optimal values of H and h . This is due to a combination of three factors. First, the first-order condition, expression (3), implies that the marginal benefit of H in the better state is equal to the unit price of H in the better state. This means that H^* is not directly affected by an increase in the probability of the bad state ($EY_{hg_0} = EY_{H\gamma} = 0$), so H^* depends only indirectly on g_0 or on γ through h^* . Conversely, h^* is directly affected by an increase in the probability of the bad state ($EY_{hg_0} \neq 0$, and $EY_{h\gamma} \neq 0$), but since H^* is not directly affected, there is no indirect effect of H^* on h^* . The net result is that h^* depends directly on g_0 or γ , while H^* depends only on h^* and on whether h and H are stochastic substitutes or complements.

Second, expression (A4) implies that h and H are stochastic substitutes. An increase in the probability of the bad state shifts H^* in the opposite direction from the shift in h^* . Intuitively, an increase in h^* increases the unit price of the optimal technique of childcare, causing the parent to use less of it; decreasing h^* reduces the unit price, thereby increasing H^* .

Third, when the first-order condition in Expression (2) is examined, a decrease in g_0 or in γ reduces the probability that a technique of child care eliminates exposure, thereby causing the expected marginal cost of h^* to fall. A decrease in g_0 , however, leaves the expected marginal benefit of h^* unchanged, so the parent finds it optimal to increase her expenditures on h . Then if h^* and H^* are stochastic substitutes, the parent then reduces her expenditures on H . In contrast, a decrease in γ reduces the expected marginal benefit of h^* . Furthermore, this expected marginal benefit declines faster than the expected marginal cost.

Our results reveal the importance of understanding the source of changed probabilities of failure to protect the child to in terms of the parent's choice of the qualities and the quantities of childcare techniques. Suppose we know that an exposure is highly dangerous to a child's health and that a shift in the technique of care has little effect on the probability of child exposures. In this case, increased probability of failure through a reduction in the efficacy of a given level of flexibility results in a childcare technique being used more

intensively. Although the probability of exposure increases, the potential for damages may decrease. But an increased probability of failure when the efficacy of a given level of flexibility is not altered results in a childcare technique being used less intensively, causing child exposure and the potential for damages to the child to increase.

The effect of a decrease in γ on the probability of successfully preventing exposures for the child is given by:

$$\frac{\partial g(h^*)}{\partial \gamma} = m(h^*) + \gamma m'(h^*) \frac{\partial h^*}{\partial \gamma}. \quad (6)$$

For access failure, both right-hand-side terms in expression (6) are positive, implying that a childcare technique has a lower probability of being used when γ decreases. This suggests less flexible techniques are successfully used less often but that when they are used they are used more intensively.

For the effectiveness case ($\rho = 1$), in general the results of the comparative statics are ambiguous. The lack of sharp results arises from the efficacy parameter showing up in both first-order conditions. We sharpen the comparative statics by adding more structure to the problem by presuming h^* and H^* are stochastic complements, which then implies $\partial h^*/\partial \gamma > 0$, and $\partial H^*/\partial \gamma > 0$ (see Appendix B). Again we summarize this result. For effectiveness failure ($\rho = 1$), a sufficient condition for an increase in the probability of the bad state due to a decrease in the efficacy of childcare quality to decrease the use of both the optimal flexibility and intensity of use of childcare is if quality and quantity are stochastic complements. Otherwise, if they are stochastic substitutes, decreased efficacy generates ambiguous impacts on flexibility and intensity of the childcare technique.

Intuitively, a decrease in the efficacy of childcare quality has the direct effect of reducing the marginal benefit of such quality, causing the parent to use a technique of childcare that has less control capacity. Also, a decrease in the efficacy of quality has the direct effect of reducing the expected marginal benefit of care quantity, causing the parent to use a technique less intensively. If there is a higher likelihood of the bad state because a technique is ineffective, the parent reduces her wealth loss by using this technique less intensively. If quality and quantity are stochastic complements, a decrease in one has the indirect effect of reducing the other, so both the direct and the indirect effects move in the same direction. In this case, a decrease in the efficacy of quality reduces the optimal levels of both quality and quantity. If they are stochastic substitutes, however, a decrease in one indirectly increases the other. Unless one can determine whether the direct or indirect effect dominates, the sign of decreased efficacy of quality on the optimum levels of quality and quantity is an empirical question.

4. Uncertain Probabilities

Our parent may be ambiguous about what her care can do for her child (Shogren 1991). She may not know with certainty the effect of childcare on the likelihood the child is exposed to the environmental hazard. She is unsure about the precise level of probabilities.⁴ This means she is ambiguous about the contribution additional flexibility, control capacity, or efficacy will make to reduce the probability of her child's exposure to the hazard.

In this section, we show that if a poor parent doubts the childcare a given technique provides, she may adopt a more flexible or greater capacity technique of care but use it less intensively than she would in the absence of doubt. Consider the rational parent's problem of choosing a level of childcare quality and quantity when she is uncertain about the probabilities of the better child health state and the bad state being realized. Her wealth-maximizing problem is:

$$\begin{aligned} \max_{h,H} E\tilde{Y} = & \int_a^b \{g(h, \varphi) [Y_0 [1 - D(Y)] - c(h)H] \\ & + (1 - g(h, \varphi)) \{Y_0 [1 - D(Y_0)] - \rho c(h)H\} \} dF(\varphi, \alpha) \end{aligned} \quad (7)$$

This problem is identical to that set forth in expression (1) except for the addition of the random variable φ which enters the probability function g . Assume a higher φ corresponds to greater flexibility or capacity such that $g_\varphi > 0$, and $g_{\varphi\varphi} < 0$.

Let $F(\varphi, \alpha)$ represent the parent's subjective cumulative distribution function for φ defined over the support $[a, b]$ where a and b are constants. The parameter α represents the level of riskiness as measured by second-order stochastic dominance

$$\int_a^b F_\alpha(\varphi, \alpha) d\varphi \geq 0, \text{ and } \int_a^\varphi F_\alpha(z, \alpha) dz > 0, \quad (8)$$

where the first term in expression (8) is the mean effect and the second term is the spread effect of α on the distribution. This representation of increased uncertainty includes a mean-preserving spread as a special case. An increase in α corresponds to a decline in the expected probability of successfully preventing the child's exposure to the hazard. Formally, the expected probability of no exposure *via* no access or effectiveness failure is:

$$Eg(h, \varphi) = \int_a^b g(h, \varphi) dF(\varphi, \alpha). \quad (9)$$

Differentiating expression (9) with respect to α , and integrating twice by parts, yields:

$$\begin{aligned} \frac{\partial E g(h, \varphi)}{\partial \alpha} &= -g_\varphi(h, \varphi) \int_a^b F_\alpha(\varphi, \alpha) d\varphi \\ &\quad + \int_a^b g_{\varphi\varphi}(h, \varphi) \left[\int_a^\varphi F_\alpha(z, \alpha) dz \right] d\varphi < 0 \end{aligned} \quad (10)$$

Increasing α also increases the variance of $g(h, \varphi)$, where this variance is given by:

$$\text{Var}[g(h, \varphi)] = \int_a^b g(h, \varphi)^2 dF(\varphi, \alpha) - \left[\int_a^b g(h, \varphi) dF(\varphi, \alpha) \right]^2 \quad (11)$$

Differentiating expression (11) with respect to α , integrating each term twice by parts, and rearranging gives:

$$\begin{aligned} \frac{\partial \text{Var}[g(h, \varphi)]}{\partial \alpha} &= 2g_\varphi(h, \varphi)[1 - g(h, \varphi)] \int_a^b F_\alpha(\varphi, \alpha) d\varphi \\ &\quad + \int_a^b \left\{ 2g_\varphi^2 - 2g_{\varphi\varphi}[1 - g(h, \varphi)] \right\} \times \left[\int_a^\varphi F_\alpha(z, \alpha) dz \right] d\varphi \end{aligned} \quad (12)$$

Intuitively, φ can represent the parent's uncertainty about the flexibility or the capacity of a technique of childcare. For these uncertainties, write $g(h, \varphi)$ as $g(h + \varphi)$. The parent believes flexibility, say, may be h but it may be a bit more or less. An increase in α implies that flexibility is more variable, while expected flexibility may be either constant or declining. Alternatively, φ can represent uncertainty about the efficacy of access or effectiveness. For uncertain efficacy, write $g(h, \varphi)$ as $g(h\varphi)$. The parent believes the flexibility or capacity efficiency of a technique is likely to contribute a set amount to the probability of not exposing the child but she is not exactly sure what this set amount is. Now increasing α implies an increase in the variability of efficacy, while the expected efficacy of the flexibility or the capacity may be constant or decreasing. After defining the model, we separately consider these two cases.

The first-order conditions for the general model of expression (7) are:

$$\begin{aligned} E\tilde{Y}_h &= \int_a^b \{g_h(h, \varphi) Y_0[D(X_0) - D(X)] \\ &\quad - (1 - \rho)H[g_h(h, \varphi)c(h) + g(h, \varphi)c'(h)] \\ &\quad - \rho c'(h)H\} dF(\varphi, \alpha) = 0, \end{aligned} \quad (13)$$

and

$$E\tilde{Y}_H = \int_a^b -g(h, \varphi) Y_0 D'(X) X'(H) - (1 - \rho)g(h, \varphi)c(h) - \rho c(h) dF(\varphi, \alpha) = 0. \quad (14)$$

Assuming fulfillment of the second-order conditions, the comparative statics of an increase in uncertainty for the access case ($\rho = 0$) are given by $\partial h^*/\partial \alpha > 0$ and $\partial H^*/\partial \alpha < 0$ (see Appendix C), which is summarized as follows. For access failure ($\rho = 0$), assuming $g_{h\varphi} \leq 0$ and $g_{h\varphi\varphi} > 0$, an increase in riskiness about the probability of the bad state increases childcare quality and quantity, i.e., the parent decreases the optimal use intensity of a technique and increases the flexibility of the optimal technique of childcare.

If $g_{h\varphi} \leq 0$, and $g_{h\varphi\varphi} > 0$, uncertainty about probabilities originating in access failure causes a parent to consider a technique of childcare with a lower probability of the bad state than she would use in the absence of this uncertainty. It also causes her to use a more costly technique of care than she would use in the absence of this uncertainty.

Similarly, for the effectiveness case ($\rho = 1$), if successful childcare is globally concave in H , then $\partial h^*/\partial \alpha > 0$, and $\partial H^*/\partial \alpha < 0$. We now summarize this result. For effectiveness failure ($\rho = 1$), assuming $g_{h\varphi} \leq 0$, and $g_{h\varphi\varphi} > 0$ and successful childcare is globally concave in quantity, H , an increase in riskiness about the probability of the bad state reduces the optimal intensity of use of a technique of childcare and increases the capacity of the optimal technique. Otherwise, the impact of more riskiness is ambiguous.

If $g_{h\varphi} \leq 0$ and $g_{h\varphi\varphi} > 0$, and successful care is globally concave in H , uncertainty originating in effectiveness causes the parent to use a technique of care having a lower probability of the bad state than the technique she would use in the absence of such uncertainty. In this case, although uncertainty tends to reduce the optimal intensity of use of a technique of childcare, it leads the parent to choose a more flexible and greater capacity technique. Note that these results are similar to our earlier findings. This is not surprising since an increase in riskiness originating in access or effectiveness reduces the expected probability of successful care, which is the effect we considered in the certain efficacy case. As in the problem of childcare quality and quantity with certain efficacy, an increase in uncertainty originating in access failure has no direct effect on intensity of use of a technique since, by assumption, this decision occurs with actual use. Another similar result is that an increase in uncertainty originating in effectiveness has the direct effect of increasing the chance a given use of a technique will have been wasted in retrospect, thereby decreasing the intensity of use of the technique. Both the certain and the uncertain efficacy cases also require restrictions on the curvatures of the probability functions to achieve this result.

The conditions $g_{h\varphi} \leq 0$ and $g_{h\varphi\varphi} > 0$ imply that g_h is decreasing or constant and is convex in φ . Intuitively, this means that care quality efforts are less effective at shifting the probability of success as φ increases, but this decrease in effectiveness diminishes as φ increases. This has different implications for the shape of $g(h, \varphi)$ depending on how φ enters the model. We now consider separately the implications of uncertain flexibility and capacity and of uncertain efficacy.

4.1. UNCERTAIN FLEXIBILITY AND CAPACITY

Consider the case for a technique of childcare of uncertain flexibility or capacity represented by $g(h, \varphi) = g(h + \varphi)$. Here $g_{h\varphi}(h + \varphi) = g''(h + \varphi)$ and $g_{h\varphi\varphi}(h + \varphi) = g'''(h + \varphi)$. If $g(h, \varphi)$ is seen as a cumulative distribution function, the condition $g_{h\varphi} \leq 0$ implies that the probability distribution function, $g'(h + \varphi)$ is non-increasing, which is guaranteed by our assumption that $g_{\varphi\varphi} < 0$. The condition $g_{h\varphi\varphi} > 0$ implies that the probability distribution is a convex function. Using an argument parallel to non-increasing Arrow-Pratt absolute risk aversion, Shogren (1991) shows $g'''(h + \varphi) > 0$ if one assumes non-increasing aversion to uncertain care quality efficacy. The marginal efficacy of the probability function is convex if a parent's willingness to pay a *protection premium* decreases monotonically with the level of child care quality. A protection premium is defined as the amount a person will pay to remove uncertainty about the efficacy of childcare quality (see Shogren, 1991, Proposition 2). For $g(h, \varphi) = g(h + \varphi)$, distributions that meet both $g_{h\varphi} \leq 0$ and $g_{h\varphi\varphi} > 0$ include the exponential and the Pareto distributions.

4.2. UNCERTAIN EFFICACY

Now consider uncertain efficacy for a technique of childcare. Uncertain efficacy can be represented by $g(h, \varphi) = g(h\varphi)$. In this case, $g_{h\varphi} = g''(h\varphi)h\varphi + g'(h\varphi)$, and $g_{h\varphi\varphi} = g'''(h\varphi)h^2\varphi + 2g''(h\varphi)h$. Now $g_{h\varphi} \leq 0$ if and only if $-g''(h\varphi)h\varphi / g'(h\varphi) \geq 1$. The term $\eta(h\varphi) = -g''(h\varphi)h\varphi / g'(h\varphi)$ is the elasticity of the marginal probability of the better state. Similar to Shogren (1991), one might think of η as a measure of the parent's relative aversion to uncertain care quality efficacy. For a simple problem, η indicates the size of a multiplicative protection premium. A sufficient condition for $g_{h\varphi} \leq 0$ is for the probability density function $g'(h\varphi)$ to be elastic. Lichtenberg and Zilberman (1986) suggest that this is a reasonable assumption for protection problems. For many common distributions $g'(h\varphi)$ can be shown to be elastic if $(h\varphi)$ is bounded sufficiently far from zero. Arrow (1984) shows that relative risk aversion tends to a limit below one as wealth approaches zero. Similarly, η approaches a limit below one as $(h\varphi)$ approaches zero, implying that $(h\varphi)$ must be restricted away from zero if $\eta > 1$ is to occur.

It is difficult to generalize about which distributions for $(h\varphi)$ always satisfy $\eta > 1$ and thus $g_{h\varphi} \leq 0$. One cannot readily predict the relationships that occur between flexibility, effectiveness, and intensity of use in the case of uncertainty about the efficacy of a technique of childcare quality. Prediction almost certainly becomes more difficult if parental risk aversion rather than risk neutrality is admitted.

5. Conclusions

At least for poor families whose household survival is at stake, the endogenous risk framework presented here captures some of the heretofore unexplored complexities involved in parental decisions about childcare. In particular, we model joint decisions involving qualities of multiple childcare technologies and the extent of use of these techniques. Endogenous risk implies the observed risks environmental hazards pose to young children are functions of natural science parameters and of the quality and the quantity of the preparation and forearming decisions the child's parent make against a hazard she perceives. Our concern has been how a parent's choice of the quality and quantity of childcare affects the probability of a child's exposure to a hazard, and how a change in probability of exposure affects the choice of quality and quantity.

The paper provides a cautionary tale that policy decisions about protecting children from environmental hazards requires attention to parental decisions, endogenous risk, and the structure of the substitution opportunities parents have available for childcare. Though we have not minimized the number and peculiarity of assumptions needed to support our results, our findings are consistent with simple empirical observations. USEPA (1998), for example, explains in detail the access to and likely effectiveness of the many techniques ranging from window-sill dust cleaning to soil removal that parents have to reduce their young children's exposure to lead. The same document stresses the variability in efficacy among these techniques and the stochastic features of the individual techniques.

Application of the endogenous risk concept to childcare reveals two key points—a clear tradeoff between the parent's intensity of use of a technique of care and parental effort devoted to acquiring access to a technique and the effectiveness of the technique; and recognition that the source and the pathway of a change in risk to the child affects this tradeoff. The first point implies that a policymaker concerned with reducing risks to the health of children must understand the structure of the tradeoffs a parent makes between the qualities and the quantities of the multiple risk-reducing techniques she might use. Policies aimed at encouraging a particular technique of child care from a given hazard can reduce the use of other techniques. The

net impact of the policy could increase the risk the child suffers. A technique which discourages uptake *via* ingestion may increase uptake via respiration or absorption. Or a technique which discourages ingestion from outside play in lead-laden dirt may encourage ingestion from lead paint chips inside the home.

Second, different sources of changes in risk for the child have different implications for child care. An increase in the exogenous probability of damages to the child—probability that cannot be ameliorated by childcare—reduces the intensity of use of childcare techniques and increase the capacities of the techniques used. Similarly, uncertain flexibility or capacity reduces use intensity and increases capacity. In contrast, a decrease in the efficacy of childcare reduces the capacity of the techniques parents choose and may increase intensity of use of the chosen techniques.

Our central point is that childcare quality and quantity can be viewed as stochastic substitutes. Public policies aimed at increasing one can reduce the other, just as the provision of public care can reduce incentives to provide private care. Furthermore, we show that increases in the probability of the bad state due to access or command failure leads to parents trading off the intensity of use of a technique of childcare for the effectiveness of the technique selected. The net effect on risks to the child depends on the relative contributions use intensities and effectiveness make to risk reduction. Econometric applications of this framework to a particular hazard must deal jointly with discrete (type of technique) and continuous (intensity of use) choices, (e.g., Lee and Frost 1978). They must also consider selection issues for parents who do not use particular modes of care, and the error structures for the overall discrete and continuous care process and for the stochastic nature of care techniques.

The endogenous risk framework as applied to childcare can be extended in two obvious directions. First, it seems worthwhile to work toward defining the compensating surplus measures of *ex ante* values for endogenous risk reduction when issues of access, effectiveness, and intensity of use involving multiple technologies of care simultaneously enter a decision problem. The on-going federal government demand for a better assessment of potential risks to children's health requires more understanding of the measures of value supporting their cost-benefit assessments of alternative policy options (see USEPA 1999). Currently, these assessments focus primarily on the intensity of use of predetermined single technologies: one technology per care problem and then focus solely on the intensity with the technology is used. Quiggin (2003) offers a framework for valuation which accounts for multiple technologies; but his framework does not explicitly account for those quality and quantity properties of these technologies which we posit matter to parents' technology choices.

Notes

1. We interpret the terms “better” and “bad” as subjective concepts dependent on the parent’s completely ordered preferences. Given the marginal utility of money is constant within a state, we follow the subjective utility framework of de Finetti (1974) such that the parent’s risk-neutral probabilities and the relative utilities of the two states coincide and are revealed by her acceptance of a money gamble. We use “bad” to refer to a state in which \$1 is worth more to her than the probability of the state and “better” to refer to a state in which the \$1 is worth less than the probability. Since the bad state involves something to be avoided, an additional dollar provides more utility in the “bad” state than in the “better” state.
2. A referee points out that our results could be obtained more parsimoniously and be made more robust by drawing on Topkis’s (1998) Monotone Maximum Theorem in mathematical lattice theory and the monotone comparative statics (e.g., Athey 2002) based on it. This theorem yields decision rules monotonically increasing in parameters. Its comparative statics do not require a continuous, differentiable, or concave objective function.
3. Let G represent the Hessian matrix of the problem and the G_{ij} are the associated minors of G ; Hiebert (1983) defines elements i and j as stochastic substitutes (complements) if $G_{ij} < 0$ (> 0).
4. This model considers behavior of childcare following the work on imprecise probability judgments and ambiguous probabilities (see for example the overview by Camerer and Weber 1992). Most models of this sort presume probabilities are ambiguous due to exogenous factors that create uncertainty about beliefs without considering notions of endogenous risk (e.g., Mukerji 1998). In contrast, we assume the ambiguity arises due to uncertain efficacy of endogenous childcare quality and quantity decisions.

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Appendix A

The comparative static effects of a decrease in g_0 representing increased probability of access or effectiveness failure are:

$$\frac{\partial h^*}{\partial g_0} = \left[\frac{EY_{hg_0}EY_{HH} + EY_{Hg_0}EY_{Hh}}{G} \right], \quad (A1)$$

and

$$\frac{\partial H^*}{\partial g_0} = \left[\frac{EY_{Hg_0}EY_{hh} + EY_{hg_0}EY_{hH}}{G} \right], \quad (A2)$$

where subscripts indicate derivatives and G is the Hessian matrix of the problem.

To determine the effect of a change in the probability of child health exposures and damages on the optimal flexibility, capacity, and intensity of use of childcare techniques, the signs of the terms in (A1) and (A2) must be established. From the first-order-condition set forth in expression (3) of the text, we know $EY_H = 0$. Then

$$EY_{Hg_0} = -Y_0D'(X)X'(H) - (1 - \rho)c(h). \quad (A3)$$

Additionally, using expression (2) in the text where $EY_h = 0$, we can write

$$\begin{aligned} EY_{hH} &= -g'(h)Y_0D'(X)X'(H) - (1 - \rho)[g'(h)c(h) + g(h)c'(h)] - \rho c'(h) \\ &= -(1 - \rho)g(h)c'(h) - \rho g'(h)Y_0 \left[\frac{D(X_0) - D(X)}{H} + D'(X)X'(H) \right]. \end{aligned} \quad (A4)$$

Given $c'(h) > 0$, and $EY_h = 0$, we know that

$$EY_{hg_0} = -(1 - \rho)c'(h)H. \quad (A5)$$

For the access case ($\rho = 0$), we know $EY_{Hg_0} = 0$ from the first-order conditions. Given the conjugate pairs result (duality) in comparative statics (see Silberberg 1978), we know $[\partial h^*/\partial g_0][-c'(h)H] > 0$. Since $[-c'(h)H] < 0$, then $\partial h^*/\partial g_0 < 0$. Assuming second-order conditions hold, and since $EY_{hg_0} < 0$ and in this case childcare quality and quantity are stochastic complements, $EY_{hH} < 0$, we have $\partial H^*/\partial g_0 > 0$.

For the effectiveness case ($\rho = 1$), we know $EY_{hg_0} = 0$, and from conjugate pairs we have $[\partial H^*/\partial g_0][-Y_0D'X'] > 0$; since $[-Y_0D'X'] > 0$, then $\partial H^*/\partial g_0 > 0$. Assuming second-order conditions hold and since $EY_{Hg_0} = [-Y_0D'X'] > 0$, signing $\partial h^*/\partial g_0$ depends on the sign of EY_{hH} , which is now ambiguous. The term $[D(X_0) - D(X)]/H$ represents average child health damages avoided and the term $-D'(X)X'(H)$ represents marginal child health damages avoided. In general, although we cannot always determine which of these two terms is larger, damages avoided are frequently concave in H . Average damages avoided exceed marginal damages avoided, implying $EY_{hH} < 0$, and therefore $\partial h^*/\partial g_0 < 0$.

Appendix B

A decrease in γ increases the probability of the bad state by reducing the efficacy of childcare such that:

$$\frac{\partial h^*}{\partial \gamma} = -[EY_{h\gamma}EY_{HH} + EY_{H\gamma}EY_{Hh}]/G \tag{B1}$$

and

$$\frac{\partial H^*}{\partial \gamma} = -[EY_{H\gamma}EY_{hh} + EY_{h\gamma}EY_{Hh}]/G \tag{B2}$$

Differentiating EY_H and EY_h with respect to γ yields:

$$EY_{H\gamma} = -m(h)Y_0D'(X)X'(H) - (1 - \rho)m(h)c(h), \tag{B3}$$

and using $EY_h = 0$, yields:

$$\begin{aligned} EY_{h\gamma} &= m'(h)Y_0[D(X_0) - D(X)] - (1 - \rho)H\{m'(h)c(h) + m(h)c'(h)\} \\ &= \left\{ \frac{(1 - \rho)g_0c'(h)H}{\gamma} \right\} + \rho\{m'(h)Y_0D(X_0) - D(X)\} > 0. \end{aligned} \tag{B4}$$

For the access case ($\rho = 0$), given $EY_H = 0$, then $EY_{H\gamma} = 0$. From the conjugate pairs result and the sign of (B4), we know $\partial h^*/\partial \gamma > 0$ for all cases. Now assuming second-order conditions hold, we can sign (B2): since $EY_{h\gamma} > 0$ from (B4) and $EY_{hH} < 0$ from (A4), then $\partial H^*/\partial \gamma < 0$.

For the effectiveness case ($\rho = 1$), expressions (B3) and (B4) imply $EY_{H\gamma} > 0$ and $EY_{h\gamma} > 0$, which by the conjugate pairs result does not provide guidance into the general signs of either $\partial h^*/\partial \gamma$ or $\partial H^*/\partial \gamma$. We return to traditional comparative static analysis and add additional structure to the problem since the sign of EY_{hH} is ambiguous, in general. If we assume h and H are stochastic complements such that $EY_{hH} > 0$, the signs of $\partial h^*/\partial \gamma$ and $\partial H^*/\partial \gamma$ remain ambiguous. But if instead we assume h and H are stochastic substitutes, which implies $EY_{hH} < 0$, then $\partial h^*/\partial \gamma > 0$ and $\partial H^*/\partial \gamma > 0$. Also assume child exposure reductions are globally concave in H . Note for access failure a decrease in the efficacy of childcare has a direct effect on h^* but no direct effect on H^* . In contrast, for effectiveness failure, a decrease in care efficacy has a direct effect on both h^* and H^* .

Appendix C

The comparative statics of an increase in uncertainty on h^* and on H^* are given by

$$\frac{\partial h^*}{\partial \alpha} = [-E\tilde{Y}_{h\alpha}E\tilde{Y}_{HH} + E\tilde{Y}_{H\alpha}E\tilde{Y}_{Hh}]/EG \tag{C1}$$

and

$$\frac{\partial H^*}{\partial \alpha} = [-E\tilde{Y}_{H\alpha}E\tilde{Y}_{hh} + E\tilde{Y}_{h\alpha}E\tilde{Y}_{hH}]/EG \tag{C2}$$

First, evaluate the term $E\tilde{Y}_{H\alpha}$ by integrating twice by parts, using expression (13), and assuming $Eg(h, \varphi) \neq 0$, which yields

$$\begin{aligned} E\tilde{Y}_{H\alpha} &= -\frac{\rho c(h)}{Eg(h, \varphi)} \left\{ g_\varphi(h, \varphi) \int_a^b F_\alpha(\varphi, \alpha) \right. \\ &\quad \left. - \int_a^b g_{\varphi\varphi}(h, \varphi) \left[\int_a^\varphi F_\alpha(z, \alpha) dz \right] d\varphi \right\} \leq 0. \end{aligned} \tag{C3}$$

For access uncertainty ($\rho=0$), we know $E\tilde{Y}_{H\alpha} = 0$ from (C3), which in turn allows us to sign (C1) if we can determine the sign of $E\tilde{Y}_{H\alpha}$. Differentiating $E\tilde{Y}_h$ with respect to α , integrating twice by parts, using expression (12), and assuming $Eg_h(h, \varphi) \neq 0$ yields:

$$E\tilde{Y}_{h\alpha} = c'(h)H \left[\frac{(1-\rho)Eg(\cdot) + \rho}{Eg_h(\cdot)} g_{h\varphi}(\cdot) + (1-\rho)g_{\varphi\varphi}(\cdot) \right] \int_a^b F_\alpha(\varphi, \alpha) d\varphi + \int_a^b \left[\frac{(1-\rho)Eg(\cdot) + \rho}{Eg_h(\cdot)} g_{h\varphi\varphi}(\cdot) - (1-\rho)g_{\varphi\varphi\varphi}(\cdot) \right] \times \left[\int_a^\varphi F_z(z, \alpha) dz \right] d\varphi \tag{C4}$$

In general, the sign of $E\tilde{Y}_{h\alpha}$ is ambiguous. But if sufficient conditions hold such that $g_{h\varphi\varphi} \leq 0$, and $g_{h\varphi} > 0$, then $E\tilde{Y}_{h\alpha} > 0$ for both access ($\rho=0$) and effectiveness ($\rho = 1$) uncertainty. Assuming these sufficient conditions hold, we can sign expression (C1) unambiguously due to the conjugate pairs result, $\partial h^*/\partial \alpha > 0$, i.e., an increase in riskiness increases child care.

The sign of (C2), however, still depends on the sign of the cross-effect term, $E\tilde{Y}_{Hh}$, which is

$$E\tilde{Y}_{Hh} = - \int_a^b c'(h)g(h, \varphi) dF(\varphi, \alpha) < 0. \tag{C5}$$

i.e., quality and quantity are stochastic substitutes. The sign of expression (C5), combined with $E\tilde{Y}_{H\alpha} = 0$, implies a change in riskiness regarding the probability of access causes child H^* to fall. Parents who face greater access uncertainty choose childcare techniques with greater flexibility and use them less intensively.

For effectiveness uncertainty ($\rho = 1$), the signs of comparative statics (C1) and (C2) are ambiguous, in general. We again impose some additional structure to determine if we can sign the results using traditional comparative statics. We know from (C3) and (C4) that $E\tilde{Y}_{H\alpha} < 0$ and $E\tilde{Y}_{h\alpha} > 0$. Using expression (13), we the cross-effect term is

$$E\tilde{Y}_{Hh} = Y_0 \left[\frac{D(X_0) - D(X)}{H} + D'(X)X'(H) \right] \left[\int_a^b g_h(h, \varphi) dF(\varphi, \alpha) \right] \tag{C6}$$

in which the second-order condition $E\tilde{Y}_{HH} < 0$ requires hazard exposure to be locally concave in H . If we assume it is globally concave in H such that average care exceeds marginal care, then we have $E\tilde{Y}_{HH} < 0$. Given expressions (C3), (C5), and (C6), if $g_{h\varphi} \leq 0$, $g_{h\varphi\varphi} > 0$, and $\rho = 1$, and hazard exposure is globally concave in H , then $\partial h^*/\partial \alpha > 0$ and $\partial H^*/\partial \alpha < 0$.