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Sustainability with Uncertain Future Preferences⋆

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Abstract. A feature of the sustainability problem is that the preferences of future generations are uncertain. In this paper, we put forward a fairness-based definition of sustainability that takes this uncertainty into account. We analyze the implications of this definition in the context of a model of project evaluation. We show that our definition encompasses the concepts of non-declining welfare and of weak and strong sustainability. Furthermore, we show that preference uncertainty has a substantial influence on the implications of sustainability.

Key words: intergenerational justice, project evaluation, substitution possibilities, sustainability, uncertainty

JEL classsification: Q56, Q20, D81, D63, D99

1. Introduction

Sustainability has become one of the major normative frameworks for evaluating environmental policy. Common to all definitions of sustainability is the concern about the well-being of future individuals. This well-being depends on two things: the conditions in which future individuals will live and the individuals' preferences. Therefore models of sustainable development need to make assumptions on how present actions will influence future conditions and on what conditions future individuals will prefer.

The economic analysis of sustainability has developed sophisticated models to assess the effects of present actions on the situation of future individuals. In contrast, the preferences of future individuals are usually depicted with the model of an infinitely lived agent which implies that these preferences are constant and known today. Only few exceptions exist, like Heal et al. (1998), Ayong Le Kama (2001), and Ayong Le Kama and Schubert (2004).

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But future preferences cannot be known today because the respective individuals are not yet in existence. At best, the present generation can have expectations about future preferences. But it has to take into account that almost surely the actual future preferences will differ from these expectations. So from the perspective of the present, future preferences are uncertain.

In this paper, we put forward a method to model uncertain future preferences and a concept of sustainability that can be used on the basis of uncertain preference information. We use a simple model of project evaluation to illustrate our concept and to highlight the effects of preference uncertainty.

The main difference between our study and the literature on sustainability under uncertainty (see Section 5) is that we argue that preference uncertainty makes intergenerational welfare comparisons impossible. Therefore we use a normative framework that does not need such comparisons.

The definition of sustainability that we introduce is based on the concept of fairness. We show that it is not a further definition of sustainable development but a generalization of several existing definitions to a setting with preference uncertainty. Furthermore, we show that in our modeling context, preference uncertainty has an influence on the implications of sustainability that is comparable to that of the elasticity of substitution between natural and man-made capitals.

The paper is organized as follows. In the next section, we present our model. In Section 3, we put forward our concept of sustainability and derive several results that relate our definition to existing definitions and that highlight the consequences of uncertainty about future preferences. Section 4 provides an example of how our definition can be applied. In Section 5, we discuss the relation of our definition to the literature on sustainability and intergenerational justice.

2. The Model

Assume that there is an infinite number of generations indexed by $t=0, 1, \ldots$ All generations are of the same size, that is, there is no population growth.[1] Each generation lives for one period and acquires well-being from consuming goods, the consumed quantities being denoted by a vector $c \in \mathbb{R}^n$. The goods are produced by using two stocks: capital K and nature N. These stocks can be changed by their use in production. In addition, N can change due to regeneration, and K due to investment and depreciation. We denote the overall changes by ΔK and ΔN. The changed stocks $K+\Delta K$ and $N+\Delta N$ are passed on to the following generation.

The decision problem that we analyze is which changes ΔN and ΔK meet the requirement of sustainability. We refer to such changes as a project. We assume that the sustainability of a project is evaluated with respect to a fixed time horizon T, that is, a generation takes the project's consequences on its first T successors into account. By allowing for the possibility of an infinite T, we cover the case where all future generations are taken into consideration.

Let $\mathcal{P}(K, N)$ be the set of technological possibilities, that is, exactly the $(c, \Delta K, \Delta N) \in \mathcal{P}(K, N)$ are feasible production plans given the stocks K and N. Note that although we refer to this set as a set of technological possibilities, it includes the regeneration of the natural capital N as well as the depreciation of and investment into man-made capital K. We assume that for all $K, N > 0$, the set $\mathcal{P}(K, N)$ is a non-empty compact subset of $\mathbb{R}^n_+ \times \mathbb{R}^2$ and that the preservation of the stocks is feasible, that is, for all $K, N > 0$, there is a $c \geq 0$ so that $(c, 0, 0) \in \mathcal{P}(K, N)$.

The welfare of each generation is depicted by the utility function $\tilde{U}(c)$ of a representative individual.[2] To account for preference uncertainty, we introduce a stochastic type parameter into this function, that is, we have $\tilde{U}(c, \tau)$ with $\tau \in \mathbb{R}$ being the type parameter.[3] A probability distribution over the type parameter of a generation thus leads to a probability distribution over the welfare that this generation derives from a given bundle of goods. So we have modeled uncertainty with respect to which bundle of goods a future generation will choose and thus, if the production processes of the goods differ with respect to their factor requirements, uncertainty about the relative amounts of K and N it will prefer.

This concept introduces preference uncertainty in a quite general way. Especially, it can model dwindling information about future preferences by using a probability distribution whose variance increases with time.

For the following analysis it is helpful to directly depict the dependency of the welfare that a generation can achieve on the stocks that it has inherited. Assume that every generation maximizes its welfare with respect to $(c, \Delta K, \Delta N)$, but that, as in Asheim and Brekke (2002), all generations are obliged to pursue only sustainable changes $(\Delta K, \Delta N)$. We define this sustainability constraint in detail in the next section, for now we only assume[4] that it implies that the changes to the stocks are restricted to being an element of a closed set $\mathcal{Z}(K, N, \tau)$ that contains (0, 0).[5] Given this set, we can define a function $U(K, N, \tau) := \max_{c,\Delta K,\Delta N} U(c, \tau)$ subject to $(c, \Delta K, \Delta N) \in \mathcal{P}(K, N)$ and $(\Delta K, \Delta N) \in \mathcal{Z}(K, N, \tau)$.[6] This function describes the maximal welfare that a generation of type τ can achieve, when this generation inherits the stocks K and N and is obliged to effect only sustainable changes to these stocks.

Note that a change of K or N affects $U(K, N, \tau)$ in two ways: via the production possibilities $\mathcal{P}(K, N)$ and via the sustainability constraint $\mathcal{Z}(K, N, \tau)$. Thus our setup takes into account that a change in the stocks not only influences the production possibilities of a future generation but also its possibilities to enact further changes to the stocks.[7]

Furthermore, our model can depict not only preference uncertainty but also technological uncertainty.[8] Given that $U(K, N, \tau)$ has been derived by using information about production possibilities, we could interpret changes in τ not only as preference changes but also as changes of the production possibilities. However, for presentational simplicity, we refer to τ as representing only preference uncertainty in this paper.

Altogether, we now have a compact description of the uncertain future welfare effects of changes to the stocks. In addition, we need a description of the technological possibilities of the present generation to enact such changes. For this, we define the projection $\mathcal{T}(K,N)$ of $\mathcal{P}(K,N)$ by $\mathcal{T}(K,N) := \{(\Delta K, \Delta N) \in \mathbb{R}^2 | \exists c \in \mathbb{R}^n_+ \text{ with } (c, \Delta K, \Delta N) \in \mathcal{P}(K,N)\}$. Note that $\mathcal{T}(K,N)$ and $U(K, N, \tau)$ are not compatible, as they are defined for different values of consumption, and thus cannot be used simultaneously w.r.t. the same generation.[9] But since we use $\mathcal{T}(K,N)$ only to depict the technological possibilities of the present generation, whereas we use $U(K, N, \tau)$ only to depict the preferences of future generations, no problems arise from this incompatibility in the following analysis. We assume that the endowments of the present generation are given and fixed, so that we depress the dependency of $\mathcal{T}$ on K, N whenever no ambiguities arise.

For the following analysis, we use several assumptions on $U(K,N,\tau)$, $\mathcal{T}(K,N)$, and τ.

- A1 For all finite $(K,N,\tau) \in \mathbb{R}_{++} \times \mathbb{R}_{++} \times \mathbb{R}$, the function $U(K, N, \tau)$ is quasi-concave w.r.t. (K, N) and twice differentiable with U_N, $U_K > 0$.
- A2 For all finite K, $N > 0$, the set $\mathcal{T}(K,N)$ is a compact and convex subset of $\mathbb{R}^2$.
- A3 $(0,0) \in \mathcal{T}(K,N)$ for all $(K,N) \in \mathbb{R}^2_{++}$, that is, a preservation of the stocks is feasible.
- A4 For all $0 < t \leq T, \tau_t \in \mathbb{R}$ is a continuous stochastic variable with non-zero variance.
- A5 $\tau_0 = 0$.
- A6 For $\tau \to -\infty$, we have $U_N/U_K \to \infty$ for all $(K,N) \in \mathbb{R}^2_{++}$, that is, N becomes indispensable or K becomes worthless.
- A7 For $\tau \to \infty$, we have $U_N/U_K \to 0$ for all $(K,N) \in \mathbb{R}^2_{++}$, that is, N becomes worthless or K becomes indispensable.
- A8 U_N/U_K is a monotone function of τ for all $(K,N) \in \mathbb{R}^2_{++}$.
- A9 The stocks K, N available to the present generation are strictly positive and finite.

The assumption A1 on the utility function states that inheriting more of a stock benefits a generation but this effect decreases with a larger stock.[10] The assumption A2 on the technological possibilities is standard; the compactness of $\mathcal{T}(K,N)$ implies that regeneration and production are constrained by the available stocks. Assumption A3 implies that it is possible to keep the *status quo*, which is necessary to model the concept of strong sustainability. Furthermore it ensures that a change to the stocks at one time does not necessarily entail an additional change at a later time; all future generations have the possibility to preserve the changed stocks. Note that the assumptions on $\mathcal{T}(K, N)$ are consistent (although being special cases) with those imposed on $\mathcal{P}(K, N)$ in the derivation of $U(K, N, \tau)$.

The assumptions on the type parameter imply that this parameter influences the marginal rate of substitution between N and K. According to A6–A8, the whole range between a worthless and an indispensable natural asset is theoretically possible, and the relevent part of this range depends on the choice of the probability distribution of τ. For a distribution that is zero outside a bounded set, the present generation expects τ to remain in this set almost surely. In this way, it is possible to exclude the cases of a stock becoming worthless or indispensable. With an unbounded variance (e.g., for $T \to \infty$), this is not feasible, at least one of the extreme cases $\tau \to \infty$ or $\tau \to -\infty$ is possible.

We provide a detailed example of specifications of $U(K, N, \tau)$ and a probability distribution for τ_t in Section 4. For now, we only define classes of expectations. Let $P(\tau_t)$ for $t = 1, \ldots, T$ denote the probability distribution that describes the expectation of the present generation concerning the type of generation t.

Definition 1. $P(\tau_t)$ is *driftfree*, if $E(\tau_t) = E(\tau_0) = 0$ for all $0 < t \leq T$.

Definition 2. $P(\tau_t)$ is *symmetric*, if $P(E(\tau_t)) - P(E(\tau_t) - \varepsilon)) = P(E(\tau_t) + \varepsilon)) - P(E(\tau_t))$ for all $0 < t \leq T$ and for all $\varepsilon \in \mathbb{R}$.

Definition 3. $P(\tau_t)$ has an *unrestricted domain* at $\bar{t}, 0 < \bar{t} \leq T$, if there is no $\underline{\tau} \in \mathbb{R}$, so that $P(\tau_{\bar{t}}) = 0$ for all $\tau < \underline{\tau}$, and no $\bar{\tau} \in \mathbb{R}$, so that $P(\tau_{\bar{t}}) = 0$ for all $\tau > \bar{\tau}$.

Definition 4. $P(\tau_t)$ is *informationless* at $\bar{t}, 0 < \bar{t} \leq T$, if for all fixed $\varepsilon > 0, P(\tau_{\bar{t}} + \varepsilon) - P(\tau_{\bar{t}})$ is a constant w.r.t. $\tau_{\bar{t}}$, that is, the possible types of generation $\bar{t}$ all have the same probability.

Note that the random walk, which is a common tool for modeling expectations in economics, is driftfree and symmetric, has an unrestricted domain and is informationless for $t \to \infty$.

3. Defining Sustainability under Preference Uncertainty

Sustainability is usually seen as the obligation to keep some measure of well-being non-declining over time.[11] Given that an individual's well-being depends on his/her preferences and that future preferences are uncertain, some care is necessary to define sustainability. Even given expectations about future preferences are certain, we cannot simply compare the well-being of future and present individuals. For such a comparison, we would have to assume at least interpersonal level comparability of utility. This assumption is commonly seen as problematic even within one generation, and it seems rather implausible when applied to generations living some hundred years apart.[12] Indeed, it amounts to assuming that all future generations share a basic concept of well-being. Assuming such certain knowledge about future generations, would not be compatible with our view that no sure knowledge about future generations should be taken for granted.

Therefore we need a welfare-based definition of sustainability that does not rely upon interpersonal welfare comparisons. A useful starting point is the concept of fairness or envy-freeness as it is used in Varian (1974) and Daniel (1975). This concept evaluates allocations without relying on interpersonal comparisons by demanding that an allocation should be envy-free, in the sense that no person prefers the bundle of goods of others to his/her own.

We transfer this concept to our setting by defining a development as sustainable if no future generation (which by itself is subject to the same sustainability constraint w.r.t. its successors) will envy the situation of the present generation. Thus no future generation shall prefer to live under the present conditions, given that both present and future possibilities are restricted by the demand of sustainability. So a project that changes the stocks N and K is sustainable if future generations will not prefer to have it undone. A similar fairness-based definition of sustainability is used in Woodward (2000), see Section 5 for a comparison.

Our definition circumvents the problem of interpersonal utility comparisons, since it relies only upon the comparison between two states (the original state and the state after the implementation of a project) by each future generation, that is, the comparison of the values of one utility function at different states.

But in a stochastic setting, it cannot be determined with certainty whether a future generation is worse off due to a project or not. Using the expected utility of future generations is not a satisfying solution because the probability for each future generation to be worse off could amount to 50% (or more for strongly skewed distributions of future types). Another possibility is to restrict the probability that a future generation will object to a proposed action. Such an approach is, for example, used in the literature on social targeting, see Naga (2003) as well as Cornia and Stewart (1995) for applications, and Sen (1995) for a discussion. Applying this statistical approach to the problem of sustainable planning under preference uncertainty leads to a flexible concept of sustainability.

Definition 5. A change to the stocks N and is K *sustainable on the level* α, if for every generation $0 < t \leq T$ the probability that this generation would like to reverse the change is at most α, given that it by itself is subject to the same sustainability constraint with respect to its T successors.

In this paper, we consider only choices of α that lie in the range [0, 0.5] because $\alpha = 0.5$ already implies a rather undemanding form of sustainability.

Note that imposing the obligation of sustainability not only on the present generation but also on future generations is essential for Definition 5 to be meaningful. Otherwise, we would compare the effects of a change to the stocks on the welfare of a future generation in a setting in which this generation by itself is free to enact all technically feasible changes to the stocks. Given that once it is in existence, a future generation will be constrained by the obligation to act sustainably w.r.t. its successors, such an analysis would not be connected to the welfare effects of a project and would thus have no ethical foundation.

To work out the implications of Definition 5, it is helpful to formalize it in the context of our model. Consider a project consisting of a marginal change $(\Delta K, \Delta N) \in \mathcal{T}$ to the stocks K and N that is undertaken by the present generation. By our assumptions on U and $\mathcal{T}$, a larger change can only be sustainable if this marginal change is sustainable. The welfare consequence of this marginal change for generation t can be expressed as

$$\Delta U(K, N, \tau_t) = U_K(K, N, \tau_t)\Delta K + U_N(K, N, \tau_t)\Delta N. \tag{1}$$

The definition of $U(K, N, \tau)$ already takes into account that generation t by itself is obliged to act sustainably and that the set of sustainable actions available to this generation might be changed by $(\Delta K, \Delta N)$. So generation t is worse off due to the change, and thus wanting to reverse it, if $U_K(K, N, \tau_t)\Delta K + U_N(K, N, \tau_t)\Delta N < 0$. For fixed K, N, ΔK, and ΔN this is an inequality in τ_t that splits $\mathbb{R}$, the domain of τ, into the set $\mathcal{S} := \{\tau \in \mathbb{R} | U_K(K, N, \tau)\Delta K + U_N(K, N, \tau)\Delta N \geq 0\}$ and its complement $\bar{\mathcal{S}} := \{\tau \in \mathbb{R} | U_K(K, N, \tau)\Delta K + U_N(K, N, \tau)\Delta N < 0\}$.

Thus according to our definition of sustainability, the change $(\Delta K, \Delta N) \in \mathcal{T}$ undertaken by the present generation is sustainable at the level α if $P(\tau_t \in \bar{\mathcal{S}}) \leq \alpha$ for all $0 < t \leq \mathcal{T}$, where $P(\tau_t)$ describes the present expectations concerning generation t's type. If T is finite this is equivalent to

$\max_{t\in\{1,\dots,T\}} P(\tau_t \in \bar{\mathcal{S}}) \leq \alpha$. To allow for an infinite T, we replace the maximum with the supremum:

$$\sup_{t\in\{1,\dots,T\}} P(\tau_t \in \bar{\mathcal{S}}) \leq \alpha. \tag{2}$$

This is the formal version of our definition. If, given the present stocks K and N, there is a feasible change $(\Delta K, \Delta N) \in \mathcal{T}(K,N)$ so that (2) holds, then this change is sustainable on the level α.

By Definition 5, this sustainability constraint shall hold both for the present and for all future generations. Thus the set $\mathcal{Z}(K,N,\tau_k)$, which we have used for defining the welfare $U(K, N, \tau_k)$ of some future generation $k>0$, is the set of all $(\Delta K, \Delta N) \in \mathcal{T}(K,N)$ that meet (2) with $P(\tau_t)$ representing the expectations of generation k w.r.t. the type of its successors and with t ranging from k to $k+T$ (instead of 1 to T). This definition complies to the assumptions used in Section 2.[13] Furthermore, it assures that the function $U(K, N, \tau)$ contains exactly the sustainability requirements spelled out in Definition 5.

However, since sustainability constrains both the present generation and all future generations, $\mathcal{Z}(K,N,\tau)$ and consequently $U(K, N, \tau)$ will depend on the level of sustainability α. Thus, in general, α has two effects on our model. First, the changes of α influence the maximal utility that future generations can achieve for given levels of K and N by acting sustainably and thus they change the function $U(K, N,\tau)$. Second, the value of α determines which stocks of K and N have to be preserved. The first effect is only relevant for changing values of α. For constant values, α is simply a fixed parameter of $U(K, N, \tau)$. In the following analysis, we analyze the obligations arising from sustainability for a given α. Therefore only the second effect is relevant, and we suppress the dependency of $U(K, N, \tau)$ on α to shorten our notation.

Finally, note that by A1 and Definition 5, a change $(\Delta K, \Delta N)$ to the stocks can only be sustainable if it lies on the boundary of $\mathcal{T}$; a reduction in one stock cannot be compensated by an increase in the other stock if this increase is smaller than technically feasible (which might, e.g., allow for a higher present consumption). Thus, using up a stock for consumptive purposes is not sustainable. In contrast, forgoing consumption to allow for the growth of both stocks is always sustainable, but no generation is obliged to do so.

In the following sections, we derive some general insights into Definition 5 and analyze whether preference uncertainty has relevant consequences.

3.1. THE RELATION TO EXISTING DEFINITIONS

As a first step to characterize our definition of sustainability, we show that it encompasses the concepts of weak and strong sustainability. Both concepts see sustainability as the obligation to keep some measure of the stocks of natural and man-made capitals non-declining over time. The proponents of weak sustainability hold that natural and man-made capitals are substitutes, so that it suffices to keep some suitable aggregate of these stocks non-declining. An aggregate is suitable if it is strongly connected to a concept of welfare, and if it allows for substitution between the types of capital. In contrast, the proponents of strong sustainability hold the view that natural and man-made capital cannot be substituted against each other. They have to be kept non-declining separately.

The following propositions show that both concepts can be reproduced from our definition by special choices of α.

Proposition 1. Let A1 to A9 hold, and let $P(\tau_t)$ be a driftfree and symmetric probability distribution**.** Then for $\alpha = 0.5$, Definition 5 specializes to the concept of weak sustainability: exactly those changes $(\Delta K, \Delta N) \in \mathcal{T}(K, N)$ that leave the aggregate $A(K, N) := U(K, N, 0)$ at least constant are sustainable on the level 0.5.

Proof. Since $P(\tau_t)$ is driftfree, the expected value of τ_t is zero for all $0 < t \leq T$. By the symmetry of $P(\tau_t)$, the probability that $\tau_t \leq 0$ is exactly 0.5. Together with A6–A8, this implies that the set $\bar{\mathcal{S}}$ for $\alpha = 0.5$ must be the set of negative real numbers. By A6 and A8, the value of τ at which $U_K(K, N, \tau)\Delta K + U_N(K, N, \tau)\Delta N$ becomes equal to zero is therefore $\tau = 0$. Since this is true for all $0 < t \leq T$, we can set $\tau_t = 0$ for all $0 < t \leq T$ to obtain the critical case, where a transfer between the assets will first become unsustainable according to our definition. But this implies that a transfer $(\Delta K, \Delta N) \in \mathcal{T}(K, N)$ is sustainable if and only if it does not decrease the aggregate $A(K, N) := U(K, N, 0)$, which is obviously connected to welfare and which, by A1, allows for substitution of N and K. □

Proposition 1 shows that weak sustainability can be seen as sustainability on the level 0.5, if it is not expected that the marginal rate of substitution between N and K has a time trend. So weak sustainability results from our definition of sustainability, if the probability that a future generation will be worse off is only restricted to 50%. In our context, weak sustainability is thus a rather undemanding form of obligations to future generations.

The next proposition shows that strong sustainability can be seen as the other extreme.

Proposition 2. Let A1 to A9 hold, and let $P(\tau_t)$ be a probability distribution that has an unrestricted domain for at least one $t \in \{1, \ldots, T\}$. Then for $\alpha = 0$, Definition 5 specializes to the concept of strong sustainability: only changes $\Delta K, \Delta N \geq 0$ are sustainable on the level 0.

Proof. That $P(\tau_t)$ has an unrestricted domain for some $t \in \{1,\ldots,T\}$ implies that at least for one generation there is no $\bar{\tau}$ with the property that the probability that τ is smaller than this bound is zero. Since $\alpha = 0$, equation (2) together with assumption A6 implies that we cannot exclude $\tau \to -\infty$. But this corresponds to the case where N is indispensable (or K worthless). So $\Delta N < 0$ cannot be sustainable for a finite ΔK, and an infinite ΔK is excluded by the assumptions A2 and A9. A similar argument for $\tau \to \infty$, excludes $\Delta K < 0$, leaving only ΔK, $\Delta N \geq 0$ as sustainable changes. □

So under the assumptions of Proposition 2, keeping the stocks at least constant is necessary for $\alpha = 0$. Indeed, it is sustainable for all $\alpha \geq 0$ and by A3 it is also always feasible. Note however that doing so might reduce the welfare of most or even all (since an infinite τ might never actually occur) generations and could thus be inefficient. So this case should rather be seen as an extreme version of our definition that allows to cover the concept of strong sustainability than as a particularly attractive setting.

Our definition can also be seen as one possible way to extend the concept of non-declining welfare to a setting with preference uncertainty.[14] In the commonly used deterministic setting, that is, without preference uncertainty and with level comparability of utility, our concept reduces to non-declining welfare for all values of $0 \leq \alpha < 1$.

So our definition is not a new definition of sustainability but rather a generalization of the most frequently discussed existing concepts. It includes the concepts of weak and strong sustainability as well as that of non-declining welfare, and it extends these concepts to cover uncertainty about future preferences. Weak and strong sustainability can be seen as polar concepts concerning the extent of the obligation not to lower the utility of future generations: strong sustainability holds that this obligation shall always be met, weak sustainability implies that it should only be met with a probability of 50%. Our concept includes these extremes, but it is not restricted to them. As we show in Section 4, we can define possibly more reasonable intermediate concepts by choosing an $\alpha \in (0, 0.5)$.

3.2. THE RELEVANCE OF PREFERENCE UNCERTAINTY

In the next step of our analysis, we inquire whether preference uncertainty has consequences that warrant the effort of including it in the analysis of sustainability.

We begin by considering a case in which there is no information about the preferences of a future generation. This case could, for example, arise for an unlimited planning horizon, that is, for $T \to \infty$.

Proposition 3. Let A1 to A9 hold and let $P(\tau_t)$ be a probability distribution that is informationless at some $\bar{t}, 0 < \bar{t} \leq T$ and that has an unrestricted domain for $t = \bar{t}$. Then for $0 \leq \alpha < 0.5$, Definition 5 specializes to the concept of strong sustainability, whereas for $\alpha = 0.5$, we get the concept of weak sustainability.

Proof. That $P(\tau_t)$ is informationless and has an unrestricted domain for $t = \bar{t}$ implies that all $\tau_{\bar{t}} \in \mathbb{R}$ have the same probability. Thus for $\alpha < 0.5$, we cannot find a finite $\bar{\tau}$, so that $P(\tau_{\bar{t}} \leq \bar{\tau}) \leq \alpha$ or $P(\tau_{\bar{t}} \geq \bar{\tau}) \leq \alpha$. Therefore we cannot exclude the cases $\tau_{\bar{t}} \to -\infty$ and $\tau_{\bar{t}} \to \infty$. As in the proof of Proposition 2, this leaves only $\Delta K, \Delta N \geq 0$ as sustainable changes, corresponding to the concept of strong sustainability. That $\alpha = 0.5$ implies weak sustainability follows from Proposition 1. □

This result highlights the role of uncertainty in our concept. Without information about the preferences of at least one generation within the planning horizon, there is only the choice between weak and strong sustainability; from the infinite number of sustainability concepts that are included in our definition, only these two polar cases remain. Given that the former concept is extremely restrictive and the latter ethically objectionable, as it limits the probability of harming future generations only to 50%, some information about future preferences is essential for defining a reasonable and ethically attractive concept of sustainability.

If preference information is important, the question arises how different levels of such information influence the sustainability constraint. Therefore we now analyze the effects of varying values of the variance of the expectations concerning the type of future generations. Especially, we inquire how such changes in the uncertainty compare to changes of the elasticity of substitution between the two capital stocks, which have been found to be of considerable importance for sustainability, see Gutès (1996).

To highlight the effects of preference uncertainty, we focus on the substitution possibilities as they are given by the preferences of the future generations. For this, we set $\mathcal{T} = \{(a, -b) \in \mathbb{R}^2 | a \leq b\}$, that is, we assume that each unit of N can be transfered into one unit of K but that free disposal of the stocks is allowed for. This implies an infinite elasticity of substitution on the production side and consequently focuses the analysis on the preference side. For presentational simplicity, we restrict our attention to the case $\Delta K \geq 0$, $\Delta N \leq 0$; the case $\Delta K \leq 0$, $\Delta N \geq 0$ can be analyzed in the same way and will lead to similar results.[15]

Our choice of $\mathcal{T}$ implies that the sum of the stocks K and N cannot be increased. We first calculate the maximal stock of N at which a further reduction is not sustainable for a given value of α and then analyze how this stock depends on the uncertainty about future preferences. Denote this stock by N^* and the corresponding stock of K by K^*. Given that at these stocks the sustainability constraint has to be binding (or N could be reduced further), there exists a $\bar{t} \in \{1, \ldots, T\}$ and a τ_{crit} defined by $P(\tau_{\bar{t}} \leq \tau_{\text{crit}}) = \alpha$ and by $U_K(K^*, N^*, \tau_{\text{crit}})\Delta K + U_N(K^*, N^*, \tau_{\text{crit}})\Delta N = 0$, the latter being evaluated for a marginal change $(\Delta N, \Delta K) \in \mathcal{T}$. Furthermore we need to have $\Delta K = -\Delta N$. Together this implies

$$U_N(K^*, N^*, \tau_{\text{crit}}) = U_K(K^*, N^*, \tau_{\text{crit}}). \tag{3}$$

Now let the variance of the expectations about the type of the future generations increase. For a continuous distribution of the type variables and a fixed $\alpha < 0.5$, this implies that τ_{crit} decreases (note that in the considered case $\Delta N \leq 0$, A5–A8 imply $\tau_{\text{crit}} < 0$). Both, N^* and K^*, depend on the value of τ_{crit}. Thus differentiating (3) with respect to τ_{crit} yields

$$\begin{aligned} U_{N,\tau}(K^*, N^*, \tau_{\text{crit}}) &+ U_{NN}(K^*, N^*, \tau_{\text{crit}})\frac{\partial N^*}{\partial \tau_{\text{crit}}} + U_{N,K}(K^*, N^*, \tau_{\text{crit}})\frac{\partial K^*}{\partial \tau_{\text{crit}}} \\ &= U_{K,\tau}(K^*, N^*, \tau_{\text{crit}}) + U_{K,N}(K^*, N^*, \tau_{\text{crit}})\frac{\partial N^*}{\partial \tau_{\text{crit}}} \\ &\quad + U_{K,K}(K^*, N^*, \tau_{\text{crit}})\frac{\partial K^*}{\partial \tau_{\text{crit}}}. \end{aligned} \tag{4}$$

Our specification of $\mathcal{T}$ implies $\frac{\partial K^*}{\partial \tau_{\text{crit}}} = -\frac{\partial N^*}{\partial \tau_{\text{crit}}}$. To simplify (4), we use the elasticity of substitution between N and K, which is defined as $s_{N,K} := \frac{d(N/K)}{d(U_N/U_K)}\frac{U_N/U_K}{N/K}$ and which can be calculated from the utility function by

$$s_{N,K} = -\frac{U(K,N)}{KN}\left(U_{N,N}\frac{U_K}{U_N} + U_{K,K}\frac{U_N}{U_K} - 2U_{K,N}\right)^{-1}. \tag{5}$$

With this definition, we can rearrange (4) to

$$\begin{aligned} \frac{\partial N^*}{\partial \tau_{\text{crit}}} = &\frac{K^*(\tau_{\text{crit}})N^*(\tau_{\text{crit}})}{U(K^*, N^*, \tau_{\text{crit}})} s_{N,K}(K^*, N^*, \tau_{\text{crit}}) \\ &\times (U_{N,\tau}(K^*, N^*, \tau_{\text{crit}}) - U_{K,\tau}(K^*, N^*, \tau_{\text{crit}})). \end{aligned} \tag{6}$$

Equation (6) has several interesting consequences.

Proposition 4. Let A1 to A9 hold, let $\mathcal{T} = \{(a, -b) \in \mathbb{R}^2 | a \leq b\}$, and let α be fixed at a value strictly smaller than 0.5. Then for $s_{N,K} > 0$, an increase in the variance of the distribution of the type variables (for all $0 < t \leq T$) leads to an increase in N^*. For given values of N^* and K^*, this effect is the stronger, the higher $s_{N,K}$.

Proof. Since $\alpha < 0.5$ and τ is continuously distributed, an increase in the variance of the distribution of the type variables for future generations t with

$0 < t \leq T$ leads to a decrease of τ_{crit}. According to the assumptions A6–A8, we have $U_{N,\tau}(K^*, N^*, \tau_{\text{crit}}) < U_{K,\tau}(K^*, N^*, \tau_{\text{crit}})$ and, by (6), therefore $\frac{\partial N^*}{\partial \tau_{\text{crit}}} < 0$.

Also by (6), for given values of N^* and K^*, an increase in the elasticity of substitution $s_{N,K}$ increases the absolute change of N^* that is necessary to counter a decrease of τ_{crit}. □

So as in the literature on project evaluation under uncertainty and irreversibility, see, for example, Arrow and Fisher (1974) and Fisher and Krutilla (1974), more uncertainty provides a rationale for more preservation.

Furthermore, in our model, this effect is amplified by better substitution possibilities. Knowing less is worse with abundant than with limited substitution possibilities. The reason for this seemingly counter-intuitive result gets clear from the definition of the elasticity of substitution. Holding N/K and U_N/U_K constant, $\frac{dU_N/U_K}{dN/K}$ is smaller for a high than for a low value of $s_{N,K}$. Thus a decrease of U_N/U_K that is due to a decrease of τ_{crit} has to be compensated by a stronger change of N/K than in the case of more limited substitution possibilities. Less technically, better substitution possibilities make the wants of future generations more dependent on their type and, since this type is a stochastic variable, thus more volatile from the perspective of the present generation. Therefore, for a given fixed value of α, the present generation can affect only smaller changes to the stocks.

Equation (6) has a second interesting implication. Substitution possibilities, as displayed by $s_{N,K}$, and uncertainty about future preferences, as displayed by τ_{crit}, are substitutes in the sense that an increase in the uncertainty about future preferences can be compensated by a lower elasticity of substitution, and vice versa.

To see this, it is helpful to consider N^* as a function of τ_{crit} for different values of $s_{N,K}$. By Proposition 4, such a function $N^*(\tau_{\text{crit}})$ has a negative slope for $0 < s_{N,K} < \infty$ and by (6), it is a constant for $s_{N,K} = 0$. Given that by equation (6), the slope of N^* depends continuously on $s_{N,K}$ for a finite $s_{N,K}$ and that $N^*(\tau_{\text{crit}})$ becomes discontinuous for $s_{N,K} \to \infty$ at some point $\bar{\tau}$, the function $N^*(\tau_{\text{crit}})$ has to have the form displayed in Figure 1.

Figure 1 implies the following relation between $s_{N,K}$ and τ_{crit}.

Proposition 5. Let A1 to A9 hold, let $\mathcal{T} = \{(a, -b) \in \mathbb{R}^2 | a \leq b\}$, let $P(\tau_t)$ be driftfree with a continuous density, and let α be fixed at a value that is strictly smaller than 0.5. Then any change in the elasticity of substitution $s_{N,K}$ can be compensated by a change of the variance of the distribution of the type variables (for all $0 < t \leq T$) so that the amount of natural capital that has to be preserved remains constant. The converse also holds, as long as the change in the variance does not imply that τ_{crit} crosses the point of discontinuity $\bar{\tau}$.

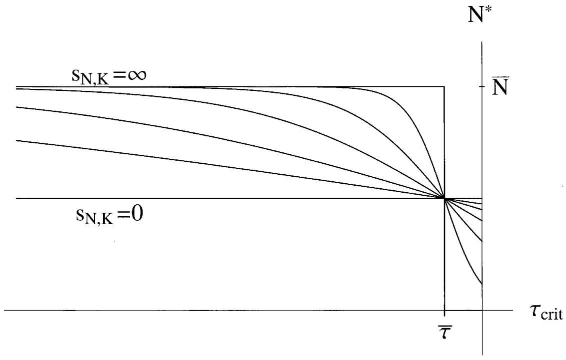

Figure 1. The amount of natural capital (N^*) at which a further reduction of N is not sustainable as a function of the critical value of the type parameter (τ_{crit}) for different values of $s_{N,K}$, given a technology that implies a fixed total amount of capital ($\bar{N}$). The point $\tau = \bar{\tau}$ is the point of discontinuity of $N^*(\tau_{crit})$ for $s_{N,K} \to \infty$.

Proof. Since $\alpha < 0.5$ and τ is continuously distributed, a change in the variance σ_t of the distribution of the type variables for all generations $0 < t \leq T$ leads to a change of τ_{crit}. Let σ be the variance of the future generation that is most likely to want to reverse a marginal transfer between the assets. Since the density of τ is a continuous function, τ_{crit} changes continuously with σ. Furthermore, $\sigma \to \infty$ leads to $\tau_{crit} \to -\infty$, because $P(\tau_t)$ is driftfree, and by A5, $\sigma \to 0$ leads to $\tau_{crit} \to 0$. So $\tau_{crit}(\sigma)$ is a continuous function that maps $\mathbb{R}_+$ surjectively to $\mathbb{R}_-$.

Now consider a fixed $\bar{N}^*$ that results for some $\bar{s}_{K,N}$ and $\bar{\tau}_{crit}$. By Figure 1, the same $\bar{N}^*$ can result for all $s_{N,K}$ for an appropriate choice of τ_{crit}. Similarly, the same $\bar{N}^*$ can result from a different τ_{crit} for an appropriate choice of $s_{N,K}$ as long as τ_{crit} remains either greater or smaller than $\bar{\tau}$. Since there exists the surjective map $\tau_{crit}(\sigma)$ between σ and τ_{crit}, we can transfer this result to compensating changes of $s_{K,N}$ and σ. □

Proposition 5 provides a compelling argument for the relevance of preference uncertainty. Changes with respect to the uncertainty of future preferences have a similar effect as changes of the elasticity of substitution between natural and produced capitals. Since the latter effect is central to the economic discussion of sustainability, see, e.g., Gutès (1996), the former should not be neglected.[16]

4. An Example

We now give an example that highlights several aspects of our definition of sustainability. To focus on the effects of preference uncertainty, we again use $\mathcal{T} = \{(a, -b) \in \mathbb{R}^2 | a \leq b\}$. As in the last section, we consider only the case $\Delta N \leq 0$, $\Delta K \geq 0$. Let the preferences be given by a CES utility function

$$U(K, N, \tau) = (\delta_1(\tau)N^{\varrho} + \delta_2(\tau)K^{\varrho})^{1/\varrho}. \tag{7}$$

For A5–A8 to hold, we have to set $\delta_1(\tau) = \delta_0 e^{-\tau/2}$ and $\delta_2(\tau) = (1 - \delta_0)e^{\tau/2}$. The marginal rate of substitution between N and K is given by $\frac{U_N}{U_K} = \frac{1-\delta_0}{\delta_0} e^{-\tau}(\frac{K}{N})^{1-\varrho}$. The elasticity of substitution is $s_{N,K} = 1/(1 - \varrho)$. Concerning the expectations of the present about the preferences of generation t, we depict these by a normal distribution for τ_t with mean zero for all t and with a variance that increases linearly with t, that is, $\tau \sim N(0, \sigma\sqrt{t})$. Furthermore, we hold that all generations have such expectations with respect to the type of their successors.

Obviously, $P(\tau_t)$ is symmetric, driftfree, and has an unbounded domain for all $t > 0$, and is informationless for $t \to \infty$. Thus from Propositions 1 and 2, we know that for $\alpha = 0.5$, we get the concept of weak sustainability, whereas for $\alpha = 0$, we get the concept of strong sustainability. Proposition 3 implies that for $T \to \infty$, only these polar cases remain.

But for a finite planning horizon T and for $0 < \alpha < 0.5$, our definition interpolates between weak and strong sustainability. From the specification of $P(\tau_t)$, it is clear that the supremum in (2) is attained for $t = T$. Thus to decide whether a project is sustainable, we have to analyze its effect on the generation T, which is the generation that is most likely to object to the project.

Consider first the special case $s_{N,K} = 1$, that is, the case of Cobb–Douglas preferences. Using $\delta_0 = 0.5$ and $T = 100$, we have computed the relative amount of N compared to K (denoted by $(N/K)_{\text{crit}}$) that is necessary for an infinitesimal transfer from N to K to be sustainable at given values of σ and α. Figure 2 shows this amount as a function of α for different values of σ.[17] As is apparent from Figure 2, for each value of (α, σ), there is a unique value of $(N/K)_{\text{crit}}$ at which a further decrease in N is not sustainable. For small

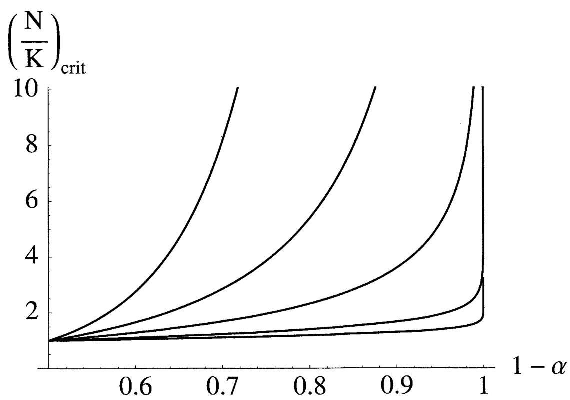

Figure 2. The lowest value of N/K at which a marginal decrease in N is sustainable for a Cobb–Douglas utility function, plotted in dependency on α for $\delta_0 = 0.5$, for $T = 100$, and for several values of σ. The sequence of the curves (from the upper left to the lower right curve) corresponds to $\sigma = \{0.4, 0.2, 0.1, 0{,}04, 0.02\}$.

values of σ, that is, with relatively precise information about future preferences, our definition remains close to the concept of weak sustainability (which implies $(N/K)_{\text{crit}} = 1$ in this example) for a large range of values of α; only if α gets very close to zero, the sustainability criterion changes to strong sustainability (which implies $(N/K)_{\text{crit}} = \infty$). In contrast, if σ is large, that is, if there is only poor information about future preferences, our concept tends to strong sustainability even for moderate values of α, and it only gets close to weak sustainability, if α approaches 0.5. For middle values of both σ and α, we get a form of sustainability that differs from these extreme concepts: there is the possibility of decreasing N relative to K, but the boundary below which N/K cannot fall is higher than weak sustainability would imply.

Now consider the more general case of CES preferences. We have calculated the amount of N relative to K that is necessary for an infinitesimal transfer from N to K to be sustainable for $T = 100$ and for various values of α, σ, and $s_{N,K}$. Figure 3 shows a contour plot of $(N/K)_{\text{crit}}$ in dependency of σ and $s_{N,K}$ for two values of α. It highlights several aspects.

First, as in the Cobb–Douglas case, our concept of sustainability interpolates between weak and strong sustainability, coming closer to weak sustainability for small values of σ and closer to strong sustainability for high values of σ. However, the elasticity of substitution $s_{N,K}$ also has an influence. The lower $s_{N,K}$, the closer we get to weak sustainability, whereas with good substitution possibilities, we get close to strong sustainability even for moderate values of σ.

This leads to the second point, namely that there is a correspondence between σ and $s_{N,K}$. Given a level of α, the same level of $(N/K)_{\text{crit}}$ is needed for an infinitesimal transfer from N to K to be sustainable for different

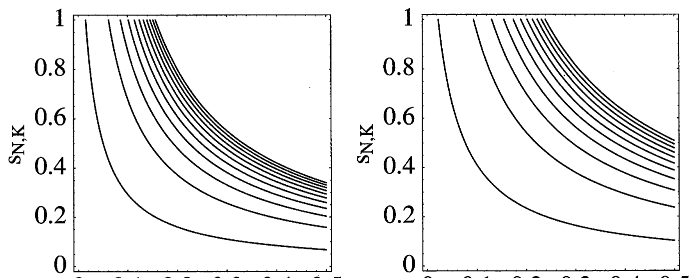

Figure 3. Contour plot of the lowest value of N/K at which a marginal decrease in N is sustainable for a CES utility function, plotted in dependency on the elasticity of substitution $s_{N,K}$ and the variance of the expectations σ for $\delta_0 = 0.4$, for $T = 100$, and for $\alpha = 0.025$ (left plot), respectively $\alpha = 0.1$ (right plot); the contours are equidistant in terms of N/K.

combinations of σ and $s_{N,K}$: the higher σ, the lower $s_{N,K}$ has to be, if at the same relative amount of N a transfer is to be sustainable, and vice versa.

Also, Figure 3 shows that the effect of an increase in the uncertainty about future preferences is stronger, the higher the elasticity of substitution is. For high values of $s_{N,K}$, the contours in the Figure get very close, implying that even a small increase in the uncertainty leads to a large increase in the value of N/K that is necessary for a transfer to be sustainable.

5. Discussion and Conclusions

In this paper, we have modeled uncertain future preferences and proposed a definition of sustainability that can be applied in this setting. In the context of a simple model of project evaluation, we have shown that our definition encompasses the definitions of weak and strong sustainability and can be seen as a possible generalization of the concept of non-declining welfare. Furthermore, uncertainty about future preferences has a substantial influence on the implications of sustainability; an influence that is comparable to that of the well-discussed substitution possibilities between natural and man-made capital.

Our definition of sustainability is based on the concept of fairness. It takes the view that an irreversible action is only sustainable if it does not render a coming generation worse off, in the sense that this generation would like to reverse the action. This implies that no future generation will prefer the bundle of goods of the present generation to its own. Our definition is therefore a concept of one-way fairness. Given that the present generation can influence future conditions, but the future generations cannot affect the present generation, this one-way protection seems to be adequate.[18]

A possible objection to the use of fairness as a concept of justice is that it does not employ interpersonal welfare comparisons, which might be essential for a decision about a just distribution, see Roemer (1996). However, we see sustainability only as a minimal requirement for intergenerational justice and not as a complete concept of justice in itself. It may be that welfare comparisons have to be made to define intergenerational justice. But to derive a minimal obligation to future generations, this does not seem necessary.

Our analysis is connected to several strands of the economic literature on sustainability. It is also related to the fairness-based sustainability concept of Woodward (2000), to the stochastic sustainability concept of Asheim and Brekke (2002), and there is a close relation to the literature of optimal resource use with uncertain future preferences.

Woodward (2000) presents a fairness-based definition of sustainability that is similar to ours. He analyzes a stochastic programming problem, in which uncertain shocks enter the objective function, and uses sustainability, defined by a no-envy condition, as a constraint in this optimization procedure. The main difference to our approach is that Woodward focuses on uncertain outcomes, whereas our focus is on uncertain preferences. This has important consequences. First, Woodward assumes full comparability of the welfare of all generations. As we have discussed in Section 3, this rules out general forms of uncertainty about future preferences. But it allows to use an optimization approach, which is impossible in our setting. Second, Woodward uses the expected value of future utility in his measure of welfare, whereas we bound the probability of unsustainable outcomes. The preferable approach is a matter of dispute. The expected utility theory is well-founded and often used in positive economics, but its application in normative models raises important ethical questions, see Roemer (1996). Finally, Woodward analyzes the question whether growth is sustainable, whereas we focus on the problem of choosing the relative stock sizes of natural and man-made capital. Thus, although both approaches are based on the concept of fairness and both include uncertainty, the lines of investigation differ substantially.

Similar remarks hold for the relation of our approach to Asheim and Brekke (2002), who advance a concept of non-declining welfare that can handle stochastic influences on the evolution of the capital stocks. Again, the main difference is that Asheim and Brekke focus on defining sustainability for stochastic outcomes and thus use welfare comparisons as a basis of their definition.

Finally, our analysis is connected to Heal et al. (1998), Ayong Le Kama (2001), and Ayong Le Kama and Schubert (2004). Like these studies, we analyze a planning problem with uncertainty about the evaluation criterion. But the models of preference uncertainty differ substantially. These studies introduce uncertainty by using a single preference change in an uncertain direction that occurs at a possibly uncertain date. Therefore the uncertainty is resolved at some point in the future and the relative probabilities of different preference changes are the same for a close as well as for a distant generation. In contrast, we allow for uncertain preference changes between all generations so that the uncertainty is never resolved and there can be more uncertainty concerning the preferences of distant than of close future generations. Also, the above studies use an intertemporally aggregated measure of welfare, which, as discussed in Section 3, limits the scope of the considered preference uncertainty. Thus although our model of resource use is much more limited in scope than the models in this literature, our model of preference uncertainty is more general.

So the main contribution of our analysis is that we provide a welfare-based concept of sustainability that does not need interpersonal welfare comparisons and is therefore suitable for analyzing sustainability in the case of substantial preference uncertainty. Since interpersonal welfare comparisons are often seen as being problematic, even within one generation, this removes an important methodological obstacle from the sustainability discussion.

Furthermore, our approach provides some additional insights. First, our analysis highlights the importance of information about future preferences. Less information about future preferences, in the sense of a higher variance of the expectation about future preferences, results in less projects being sustainable for a given technology. Also, in the extreme case of a complete lack of information about future preferences, there remains only the choice between the polar concepts of weak and strong sustainability.

Second, some of the most frequently discussed economic concepts of sustainability can be understood as special cases of our definition. This opens the possibility of comparing the normative content of these definitions, which is often a problem, because they are usually based on different positive assumptions. Furthermore, our definition encompasses a range of sustainability concepts between weak and strong sustainability, which correspond to extreme ways of handling preference uncertainty. In many settings, this range might provide a more reasonable and ethically more attractive choice than either of these extremes.

Finally, we have built our definition upon the concepts that are used in hypothesis testing. Except for some applications in the analysis of social targeting, this is a new approach in normative economics. We propose to bound the probability of harming a future generation by a constant α. As in test theory, reducing the probability of this error increases the probability of the complementary error of not enacting a change that would benefit all generations. In similarity to hypothesis testing, we have to weigh these errors against each other. The difference is that in our setting choosing α is a normative decision. But the insights from test theory can at least provide guidance in the normative choice of α.

Notes

1. Our analysis could include population changes, because the model introduced below allows for changes in the utility function of a generation's representative agent; a changing population would simply imply that the same amount of inherited resources allows to reach a different maximal utility.
2. Although we use the concept of a representative agent for ease of presentation, this is not essential for our results. Our analysis is based on a distribution of preferences and could thus account for preference heterogeneity within one generation.
3. Thus we use the concept of a master preference. As Howe (1987) has shown, n arbitrary preference orderings can be described by one master preference and a sufficient number of type parameters. The introduction of more than one type parameter would be possible,

but this would complicate the following arguments without leading to substantially differing results.

4. We prove these conditions in the following section.
5. By assuming that $\mathcal{Z}(K, N, \tau)$ depends only on the actual value of τ and not on its past values, we exclude learning processes from our model. With the exception of Section 4, we could allow for learning throughout the paper but this would not influence our results, so that we avoid the implied notational complexity.
6. Assuming that $\tilde{U}(c, \tau)$ is at least continuous w.r.t. c, the maximum exists for all $(K, N, \tau) \in \mathbb{R}^2_{++} \times \mathbb{R}$, because $\mathcal{Z}(K, N, \tau)$ is closed, $\mathcal{P}(K, N, \tau)$ is compact, and we have $(0, 0) \in \mathcal{Z}(K, N)$ and $(c, 0, 0) \in \mathcal{P}(K, N)$ for some $c \geq 0$.
7. The set $\mathcal{Z}(K, N, \tau)$, and thus also the function $U(K, N, \tau)$, will depend on the planning horizon T and on the level of sustainability α that we introduce in Section 3. But since we work with a constant T and a constant α, we depress these dependencies.
8. We are indebted to an anonymous referee for pointing out this.
9. We owe this point to an anonymous referee.
10. These conditions could be derived from assumptions on $\tilde{U}(c, \tau)$ and on $\mathcal{P}(K, N)$. But since our assumptions on $U(K, N, \tau)$ are rather intuitive, such a derivation is tangential to our analysis and thus left out in this paper.
11. Resource-based policy rules, like weak and strong sustainability, are usually derived from a welfare-based criterion in economics. In this paper, we maintain this convention.
12. It could be argued that the problem also arises in the deterministic setting and is only veiled by the assumption of constant preferences implicit in the model of an infinitely lived representative individual.
13. We have assumed that $\mathcal{Z}(K, N, \tau)$ is closed and contains (0, 0). By Definition 5, we have $(0, 0) \in \mathcal{Z}(K, N, \tau)$. By (2), $\mathcal{Z}(K, N, \tau)$ is closed. These conditions follow solely from Definition 5 and do not depend on the existence or the characteristics of $U(K, N, \tau)$, so that using them in the derivation of $U(K, N, \tau)$ poses no logical problem.
14. See Pezzey (1992) for a definition.
15. By A1, the change to at least one stock has to be positive for sustainability and increasing both stocks is always sustainable in our setting. Thus only the cases $\Delta K \geq 0$, $\Delta N \leq 0$ and $\Delta K \leq 0$, $\Delta N \geq 0$ are of interest.
16. Note that the condition that τ_{crit} shall not cross $\bar{\tau}$ strengthens this argument. If τ_{crit} crosses $\bar{\tau}$ due to a change in σ, the minimal stock N^* has to change; no compensation by $s_{N,K}$ is possible.
17. As mentioned in Section 2, the sustainability constraint $\mathcal{Z}(K, N, \tau)$ and the function $U(K, N, \tau)$ depend on α. In contrast to the last sections, we vary α in this example, so that these dependencies might have to be accounted for. But due to the assumption that all generations have the same expectations about the type of their successors, all generations are subject to an identical sustainability constraint. So if the critical value of N/K is implemented by the present generation, no future generation can enact further changes. Therefore changes of α can have no indirect effect via changing $U(K, N, \tau)$ on the critical value of N/K.
18. Since our concept is based on the *status quo*, it does not pose excessive burdens on the present generation; future generations cannot demand changes to the *status quo* in the name of sustainability.

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