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Geometry learning with dynamic software in pre-service mathematics teacher education: A systematic review

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Abstract
Providing an overview of the accumulated scientific knowledge regarding the learning of geometry with dynamic software by pre-service mathematics teachers is currently a pertinent issue in Mathematics Education. The interest in this topic within the field is reflected in the significance attributed to the role of the teacher as a pivotal actor in their students learning. Considering this, we present a systematic review that seeks to understand how the learning of school geometry content with dynamic software has been promoted and studied during pre-service mathematics teacher education. To do so, we have reviewed a group of journal articles, published between 2017 and 2023, following the principles of PRISMA declaration. The results reveal how researchers understand such learning, the pre-service teachers, the teacher educator, the tasks that promote it, and the production and analysis of data. Given these findings, we identify gaps in the research on this topic, among which stand out the view of pre-service teachers as autonomous subjects, the instrumentalist conception of dynamic software, and methodological approaches to data analysis that do not fully consider the role of the body in the act of knowing. These gaps reveal that the promotion of and study of learning of school geometry content with dynamic software have been explored little from sociocultural perspectives. Because of this, we believe it necessary to develop studies regarding learning from theoretical and methodological perspectives that demonstrate pathways for future explorations on the addressed topic and thus allow for the reduction of these gaps.

Keywords Teacher education · Learning · Geometry · Technology

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1 Introduction

In recent decades, recognition of the teacher as a decisive actor in the learning of students (Lerman, 2001) has implied the consolidation of mathematics teacher education as an emerging area of study and practice within the field of Mathematics Education. In a broad sense, this area of study encompasses all phenomena regarding processes of learning and professional development of mathematics teachers in any of their stages or phases (pre-service, novice and experienced), through diverse and dynamic training activities (Even & Ball, 2009; Krainer & Llinares, 2010; Sánchez, 2009).

Regarding mathematics teacher education in geometry, the International Program Committee (IPC) for the 26th ICMI Study, dedicated to the theme Advances in Geometry Education, sustains that there is a wide range of professional learning focuses related to geometry education, for both pre-service and in-service teachers (International Congress on Mathematics Education [ICME], 2023). Such focuses have led to repercussions in researchers' work, especially when they seek to understand the processes of learning of school content for teaching (in particular, of geometry) and promoting such processes in the pre-service mathematics teacher education (Liljedahl et al., 2009).

In this ICMI study, the incorporation of dynamic software in pre-service mathematics teacher education has become a focus of attention to researchers. We use the term *dynamic software* to refer to computer programs that feature tools designed to dynamically interact with, construct and manipulate mathematical objects in their algebraic, graphical, and spreadsheet representations simultaneously (Chan & Leung, 2014; Tatar, 2013). GeoGebra is an example of this kind of software which provides a combination of the graphical, numerical, and algebraic perspectives of the mathematical entities (Hohenwarter et al., 2008; Preiner, 2008). Given the possibilities that the dynamic software offers the teaching–learning processes of geometry in the classroom, it is understood that the incorporation of this artifact in pre-service mathematics teacher education is a matter of concern, particularly about the development of teaching skills for the teaching of geometry with dynamic software (International Congress on Mathematical Education, 2023).

For its part, the incorporation of dynamic software into the pre-service mathematics teacher education in geometry has represented a challenge for researchers in recent decades (Herbst et al., 2020; Laborde et al., 2006; Sinclair et al., 2017). This challenge can be seen reflected in, for example, research interested in bringing pre-service mathematics teacher education closer to different ways of leveraging dynamic software to enrich mathematics activity in the classroom (Camargo et al., 2010; Gellert et al., 2009; Gómez-Chacón et al., 2016; Hodge & Frick, 2009; Isotari & Brandão, 2013; Koyuncu et al., 2015). One of the ways in which pre-service teachers can leverage the dynamic software with this purpose in mind is, for example, to abstract and generalize properties of geometric objects from the visualization and direct dragging of figures in the software workspace (Bretscher, 2017, 2023; Mavani et al., 2018). According to the IPC (2023), many of the studies in this direction have been guided by a focus on design research

(The Design Based Research Collective [DBRC], 2003), characterized by the study of the learning of pre-service teachers from the promotion of this phenomenon *within the training activities themselves*. In some way, this focus highlights the importance of social interaction and cultural contexts in the learning of pre-service mathematics teachers.

In this regard, advances surrounding this issue are accompanied by important differences in the way in which researchers seek to understand the learning of pre-service mathematics teachers about school geometry content with dynamic software as a culturally and historically situated process, as well as how said researchers produce and analyze data to study this phenomenon within the training contexts themselves. Nevertheless, we note a scarcity of information in the field of Mathematics Education that exhaustively gathers, synthesizes and analyzes the differences in the ways in which researchers tackle the previous questions, in a way that offers a more complete and precise understanding of the learning of pre-service teachers regarding school geometry content with dynamic software, which contributes to the development of scientific understanding on this topic.

Considering this reality, the aim of this article is to systematically review the research in Mathematics Education that highlights the multitude of ways in which researchers study and promote such learning in the training activities of mathematics teachers. Recognizing the close relationship between activity and learning, we seek to answer the following question: According to specialized literature, how has the learning of school geometry content with dynamic software in pre-service mathematics teacher education been promoted and studied within the training activities themselves, in recent years?

In the following section we describe in detail the method we adopted to complete the systematic review. Then, we present the main results from the analysis of the information collected and present a discussion of these results, with attention to the ways in which the learning of school geometry content with dynamic software is promoted and studied in pre-service mathematics teacher education. We close the article with the conclusions, in which we include perspectives on future studies in line with the topic discussed.

2 Methods

To answer this question, we conducted a systematic review of the relevant literature. In general terms, a systematic review is a systematic way of collecting, critically evaluating, and integrating and presenting findings regarding a determined topic of study to respond to a research question in a transparent and reproducible manner (Lame, 2019; Pati & Lorusso, 2017). The review we conducted considered the principles that make up the PRISMA declaration (Moher et al., 2009) and was carried out in three stages.

2.1 Stage 1: Identification of sources

In this stage, we identified the sources of the articles. Given the level of specificity of the topic of review within the field of Mathematics Education, we chose to seek out articles directly from *academic journals* rather than doing so via databases. To identify journals of interest, we consulted three international rankings: on the one hand, the rankings of Toerner and Arzarello (2012) and Williams and Leatham (2017), whose journals are considered quality within the field and have English as their lingua franca; and, on the other hand, the ranking of Andrade-Molina et al. (2020), which presents a comprehensive list of journals in Spanish, Portuguese, and other languages, that are considered of high quality for Ibero-American academics.

To decide which journals to consult within these rankings, we established the following exclusion criteria: (a) language of publication; (b) publication date; (c) focus and scope of the journal; (d) process for publication of the study; and (e) online consultation availability. In this way, we excluded journals that: (a) do not publish in English, Spanish or Portuguese; (b) were published before 2017 or after 2023; (c) do not have Mathematics Education as their primary focus; (d) do not follow a double-blind review process; and (e) are unavailable online at the time of enquiry.

Regarding criterion (a), though the selection of journals that publish in English is justified by this being the dominant language of education research worldwide, we decided to consider journals that publish in Spanish and Portuguese because these are the most widely used languages in the geographic areas of this study's authors. Regarding criteria (b), we chose the period 2017–2023 given that, in 2016, during the activities of the 13th ICME-13, the Survey Team Geometry (including technology) presented eight primary lines of research in teaching and learning of geometry at primary and secondary education levels, as well as in mathematics teacher education (Sinclair et al., 2016). Based on this, we considered adequate and pertinent the review of articles published from 2017 to 2023, as this period permits us to see the advances that have been made regarding the study of and promotion of learning of school geometry content with dynamic software in the training activities of pre-service teachers. Regarding criterion (c), we chose to consult journals in the field of Mathematics Education due to the specificity of the topic we studied in our review. Finally, with respect to criterion (d), we chose to consult journals that use double-blind peer review processes given that these processes guarantee the publication of articles of high scientific quality.

Upon applying these criteria, we identified 55 journals of interest, of which 23 belong to the rankings of Toerner and Arzarello (2012) and Williams and Leatham (2017), while the remaining 32 journals belong to the ranking of Andrade-Molina et al. (2020).

2.2 Stage 2: Selection and eligibility of articles

This stage was carried out in two phases. First, we selected potentially relevant articles for analysis by reading the title and abstract of each work directly on the websites of the 55 identified journals. To select these articles, we used the inclusion and exclusion criteria identified in Table 1. We formulated these criteria based on the research question and our interest in the study of the learning of pre-service teachers in their own initial training contexts, under a qualitative approach. In the case that a journal did not have enough information to decide whether any a criterion was met or not, we chose to keep the article for evaluation at the next phase of the stage. As a result, we selected 35 articles.

Next, we studied the eligibility of the 35 selected articles. To do so, we carefully read each in its entirety, paying special attention to the methodology and results, in search of *false positives* (Codina, 2018), that is to say, those articles that, despite meeting the criteria of inclusion and exclusion in Table 1, are not relevant to the review. In effect, the articles that we considered false positives were those whose: (a) content did not concern Euclidean Geometry (e.g., analytic geometry content); (b) tasks were not construction/exploration of geometric objects with dynamic software (e.g., tasks to define these objects); or (c) produced data come from individual interviews, the application of questionnaires, or narratives of participants' experiences. As a result of this phase of the stage, we selected 15 articles which constitute the *corpus* of the systematic review.

In Fig. 1, we outline the process we have followed to develop stages 1 and 2. Additionally, in Table 2 we list the 15 articles selected for review.

2.3 Stage 3: Critical review of articles

This stage took place in three phases. First, we defined categories and descriptors for the review of the articles, which we organized in two classes derived from our research question: *promotion* and *study* of the learning of school geometry content with dynamic software. The categories of each class account for different aspects of the promotion and study of learning as an object of research. To define these categories, we situate ourselves in a historical-cultural perspective of learning (that we share as researchers), which places attention on *activity* as a fundamental methodological category to promote and study this phenomenon (Goos, 2013; Moura, 2016; Radford, 2015; Radford & Sabena, 2015). For their part, the descriptors express

Table 1 Inclusion and exclusion criteria for the selection of articles

Inclusion criteria	Exclusion criteria
The content deals with Euclidean geometry with dynamic software	The content deals with other types of geometry with or without dynamic software (e.g., analytic geometry)
The articles present empirical studies	The articles present theoretical studies, literature reviews or other types of non-empirical studies
The articles are available online	The articles are not available online
The studies have the training of pre-service teachers as their context	The studies were carried out in contexts other than that training of pre-service mathematics teachers
The studies follow a qualitative methodology	The studies follow a quantitative methodology

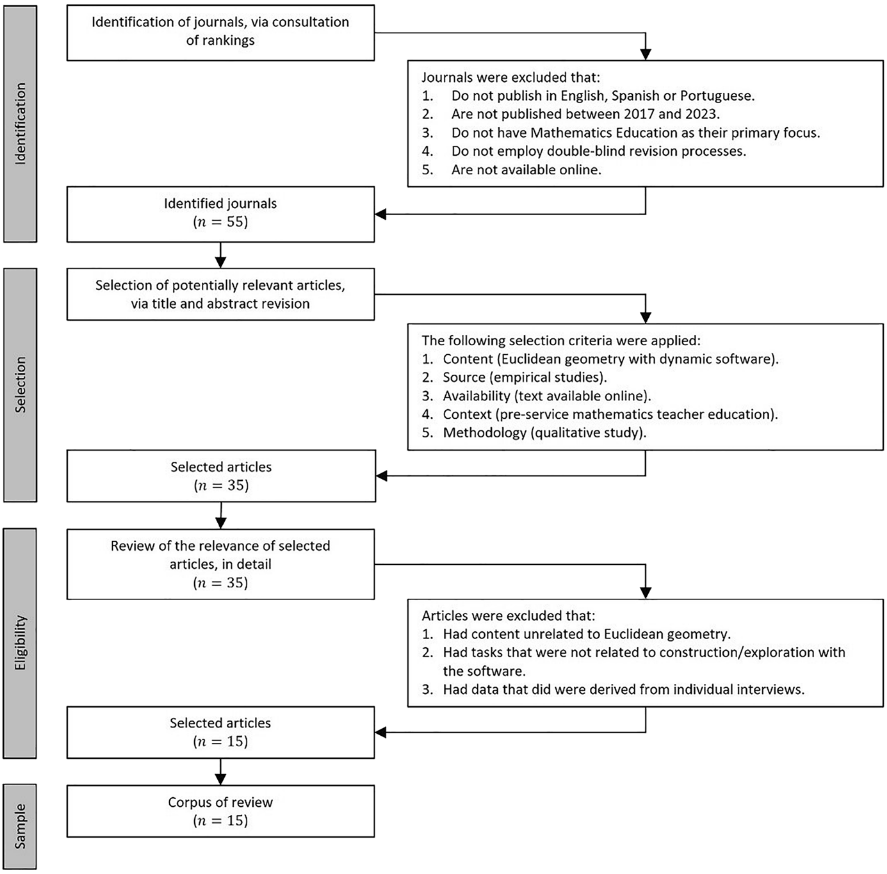

Fig. 1 Flowchart of the development of stages 1 and 2 of review

specific characteristics of each category, which we use to gather detailed information about these categories in the articles under review. The classes, categories and descriptors used in this stage of the review are indicated in Table 3.

In the second phase, we gathered information about the articles for their critical review, based on the classes, categories, and descriptors in Table 3. We used a study matrix to organize the information extracted from the articles. To guarantee the validity and trustworthiness of information gathered from the articles, we first completed the study matrix independently. Later, we came together to compare gathered information and resolve any discrepancies that arose throughout this process. The data in our review comes from the final version of the study matrix.

Finally, in the third phase, we critically analyzed the gathered information for each category of the review. This led us to compare the information of each descriptor in all articles, in order to recognize similarities and differences between the elements characteristic to the promotion or study of learning. For example, regarding the descriptor *type of task* (category C2), comparison allowed us to recognize

Table 2 Selected articles for review

Selected articles	References
RA1	Cruz and Mantica (2017)
RA2	Brunheira and Da Ponte (2018)
RA3	Mantica and Freyre (2018)
RA4	Pinheiro (2018)
RA5	Ruiz-López (2018)
RA6	Brunheira and Da Ponte (2019)
RA7	Cruz and Mantica (2019)
RA8	De Almeida et al. (2019)
RA9	Arnal-Bailera and Oller-Marcén (2020)
RA10	Zambak and Tyminski (2020)
RA11	Freyre and Cavatorta (2021)
RA12	Prieto and Arredondo (2021)
RA13	Gutiérrez et al., (2022a, 2022b)
RA14	Bairral and Silvano (2023)
RA15	Brito and Bairral (2023)

RAn (Research Article number n)

different types of tasks characteristic to the promotion of learning, namely: construction tasks, exploration tasks, definition tasks, etc. This analysis allowed us to offer an explanation of this phenomenon and organize the presentation of the results.

3 Results and discussion

This section presents and discusses the results from the review, with attention to the two aspects presented in the research question.

3.1 The way in which learning is promoted

The first aspect presented has to do with the promotion of learning of school geometry content with dynamic software on the part of pre-service mathematics teachers. In this regard, we refer to categories C1, C2 and C3 to describe the way in which this aspect is addressed in the studies reviewed.

3.1.1 Conception of learning (C1)

In the review, two aspects related to this category stand out: the content of learning and the way in which researchers understand the learning of this subject matter.

Regarding the first aspect, the content of learning in the reviewed articles is of two types: (a) geometric objects, relations, and properties ($n=8$); and (b) mathematical processes inherent in these objects ($n=7$). Regarding type (a), the focus is on polyhedrons (RA3), plane figures (RA8; RA9; RA10; RA14; RA15) and geometric locus (RA4; RA7). Regarding type (b), the focus is on the processes of construction

Table 3 Classes, categories, and descriptors for the review of articles

Classes	Categories	Descriptors
Promotion of learning	Conception of learning (C1)	• The object of learning
		• The process of process of learning of the object
	Designed tasks to promote learning (C2)	• Type of task
		• Structure of the task
		• Worksheet
	Implementation of tasks (C3)	• Moments of implementation
		• Conditions of implementation
		• Role of the pre-service teachers and teacher educator
		• Rol of the software
		• Support materials
Study of learning	Data production (C4)	• Techniques and instruments for information gathering
		• Research data
	Data analysis (C5)	• Analysis unit
		• Analysis procedure
		• Analysis tools

of geometric figures (RA12; RA13), formulation and validation of conjectures (RA1; RA11), definition of geometric objects (RA2), hierarchical classification (RA6) and appropriation of instrumented action schemes (RA5).

Concerning the second aspect, we observed the influence of certain theories and constructs about the way in which the researchers understand the learning of the aforementioned content. Regarding the theories, those that stand out include the *theory of didactic situations* by Brousseau (2007) (RA1; RA5; RA8; RA9), the *theory of communities of practice* by Wenger (1998) (RA2; RA6) and the *theory of objectification* by Radford (2021) (RA12; RA13). Regarding the constructs, the notions of *instrumental genesis* (RA9), *collaborative groups* (RA4; RA14; RA15) and *figural concept* (RA3) stand out. Only in some studies ($n=3$) is the meaning of learning assumed by the researchers left unspecified (RA7; RA10; RA11).

According to the results above, researchers resort to both theories of a general nature (theory of communities of practice) and theories specific to the field of Mathematics Education (theory of didactic situations and theory of objectification) to interpret the learning of school geometry content with dynamic software by pre-service teachers. Among the mentioned theories, the theory of didactic situations is the theory that has had the most influence among the reviewed studies ($n=4$), being used to promote and study the learning of different types of content (geometric objects and processes, as well as cognitive processes not necessarily of a mathematical nature).

Concurrently, other authors resort to constructs of a general nature (collaborative groups and instrumental genesis) as well as those that are characteristic to Mathematics Education (figural concept) to interpret the learning of school geometry content with dynamic software by pre-service teachers. These results highlight that, on the one hand, interest in collaboration within the reviewed studies ($n=3$) is important, particularly in formative contexts developed online in which the interactions among participants in crucial to the promotion of learning. This finding corresponds to the assertions of Goos (2008, 2013) regarding a growing interest in recent decades for the social, cultural, and institutional dimensions of teacher learning in mathematics. On the other hand, we noted that the instrumental genesis and figural concept are also emphasized in the studies based on the use of constructs ($n=2$), showing the influence of the cognitive on the promotion and study of the learning of pre-service teachers. This means that, in some way, research about the learning of geometry with dynamic software continues to evolve from a perspective of this phenomenon as "a form of cognitive adaptation" (Radford, 2021, p. 11).

3.1.2 Designed tasks (C2)

Regarding the tasks designed by the researchers, three aspects stand out in the articles: the type of task, the structure of the task, and the use of worksheets.

With respect to the type of task, the majority of the studies ($n=11$) resort to tasks of construction. One particularity of these studies, with the exception of RA9, is that the task presented is framed as a series of actions that include explain, justify, analyze, or investigate. Additionally, in the reviewed works we found other types of tasks (n = 4), including tasks of definition (RA2), classification (RA6), exploration (RA4), and modeling (RA10).

Regarding the structure of the designed tasks, the authors highlight three aspects. On the one hand are the *demands of the task,* which reveal four trends in the reviewed articles. The first trend has to do with the scope of the task in function of the required actions: (1) construct the figure (RA9); (2) construct the figure and communicate the construction steps (RA5); and (3) construct the figure, communicate the construction steps, and explain/justify the consistency of the employed process (RA1; RA12; RA13).

These results (regarding the aforementioned type of tasks and their scope) show a clear recurrence of researchers to the design of construction tasks that maintain, so to speak, their links with the classical method of solving geometric construction problems with a ruler and compass, historically related to the description of the construction procedure of the desired object and the demonstration that said procedure delivers said object with certain properties (Scriba & Schreiber, 2015). Actions, such as explaining, justifying, analyzing, exploring, and investigating, demanded in the tasks present in the reviewed studies are an example of these links. This fact contrasts with the tendency to teach constructions with ruler and compass (or with dynamic software) at school based on the precision of geometric drawings to the detriment of the validity of geometric constructions (IPC, 2023) and the understanding of the fundamental properties of the geometric objects that are constructed (Kuzle, 2013). Within this trend, the activity of geometric construction is reduced to an act of memorization and repetition of techniques necessary for the representation of the desired geometric object (Esonov et al., 2023).

The second trend refers to a demand that includes the recognition of the conditions necessary for the construction of the figure and the uniqueness of the response (RA3). The third trend is related to the demand for investigation processes regarding the drawing constructed for the recognition of geometric locus (RA7) and the verification of conjectures (RA8; RA11; RA14) or properties (RA15). Concerning these last articles, we noted that research processes with dynamic software have been incorporated in the articles under the premise of exploring geometric objects on the computer screen, making, and testing conjectures, and generalizing, discussing, justifying, and testing, constitute key elements in classroom work (Gutiérrez-Araujo et al., 2022). This does not mean that research processes in geometry are without their difficulties. For example, according to various authors (Abrantes, 1999; Da Ponte et al., 2016; Stein & Smith, 1998), with mathematics activities of a research nature, one cannot expect a significant evolution of students in the short term.

The fourth trend accounts for tasks of definition (RA2), classification (RA6), exploration (RA4) and modeling (RA10). From the tasks of the fourth trend, we observe that RA2 and RA4 converge in promoting the definition of geometric objects. In the case of RA4, the study leads pre-service teachers to go through a definition process whose previous actions include dragging points in the software, discussing what is observed on the screen and recording in writing what is observed and discussed after dragging. However, the software's dragging functionality is not exclusive to definition tasks. For example, in research tasks (RA15) the dragging modality of the software plays an important role in the production and analysis of study data, by revealing discursive aspects of individuals' interactions with the technology.

On the other hand, with respect to the *geometric object of the task*, we observe that quadrilaterals (RA2; RA5; RA6; RA8; RA11; RA14) and triangles (RA12; RA13; RA15) have the greatest presence in the statements of the designed tasks, regardless of the demands they require. Only in the case of RA9 are both geometric figures considered in the task's instructions. Finally, regarding *the conditions imposed for the resolution of the task*, we observe that in the majority of the articles ($n=12$), the researchers show the tasks implemented to the pre-service teachers. In the cases of RA6, RA10 y RA15 the designed tasks are not clearly shown or sufficiently defined. With respect to the construction tasks identified here, we find that the conditions for their resolution are related to the characteristics and/or properties of the object to be constructed (RA1; RA3; RA5; RA7; RA8; RA9; RA11; RA12; RA13) and to how to use the software for this purpose (RA8; RA14). These results are an example of how important it is for researchers to present the proposed tasks' instructions to pre-service teachers with clarity and precision. In other words, clarity in the formulation of a mathematical problem is essential to ensure that it is correctly understood and can be addressed accurately.

Finally, as to the use of worksheets (software files with the initial conditions), we found that there is a tendency for researchers to not provide these inputs to solve the proposed tasks. In fact, only in the cases of RA4, RA9, RA12, and RA13, do the authors consider it necessary to provide worksheets to pre-service teachers. Furthermore, six of the seven tasks that involve exploration or investigation processes (e.g., in RA4) demand the construction of the figures that will be the object of exploration, starting from a blank sheet. Despite this finding, studies such as those by Dove and Hollenbrands (2014) and Mavani et al. (2018) reveal that research processes with dynamic software starting from blank worksheets can be hindered by this decision.

3.1.3 Implementation of tasks (C3)

Concerning the implementation of tasks, the following aspects are emphasized: moments of implementation, conditions of implementation, the roles of pre-service teachers and the teacher educator, and the roles of software and support materials.

In the review, we observed two issues related to the moments of the implementation of the tasks. First of all, it is worth highlighting the fact that, in some studies ($n=5$), the authors assert that they account for the moment of the introduction of the task (RA2; RA5; RA6; RA12; RA13). Secondly, we noticed a trend in the studies ($n=9$) of considering, as a whole, the moments of solving the task and collective discussion. An exception to this is in RA1, in which only the moment of resolution of the task was considered. Regarding this moment, we noticed a clear trend in which the researchers promote group work to solve the task ($n=13$), which contrasts with the way in which some understand learning (see findings in C1). For example, in RA3, we recognize a contradiction within the study's theoretical and methodological assumptions, since a construct of a cognitive nature (individual) is used to analyze the (collective) work of the pre-service teachers in solving the proposed task.

Regarding the conditions of implementation, we found that, with the exception of RA4, the implementation of the tasks was carried out within the framework of courses or subjects that constitute part of the curriculum of some initial training program for mathematics teachers. In most cases ($n=13$), the tasks were part of the training activities of a course (RA2; RA6; RA8; RA9; RA10; RA11; RA12; RA13; RA14; RA15), including those deemed practical workshops (RA3; RA5; RA7). Only in the cases of RA1 and RA7, were the tasks implemented to assess the learning of pre-service teachers.

These results show that the study the learning of pre-service teachers, promoting such learning in their own contexts of professional performance, favors the development of researchers' capacities to more deeply understand the educational challenges of the specific contexts in which they operate, strengthen their teaching approaches by putting the theoretical assumptions that inspire them into dialogue with findings when carrying out their research, and overcome a perspective of the learning of pre-service teachers, supported by the evaluation of the results at the end of an instruction period (Llinares, 2014; Valverde-Soto, 2014).

Of the studies reviewed, some studies ($n=6$) do not specify the role of pre-service teachers at the moments of implementation of the task. In another number of studies ($n=7$), we found information that suggests a tendency for researchers to promote independent work of the pre-service teachers (RA1; RA4; RA7; RA8; RA11; RA14; RA15). An exception to this trend is the studies conducted in RA12 and RA13, in which the pre-service teachers and the teacher educator work together in seeking to solve the tasks.

In agreement with the findings of C1, these results suggest that researchers are inclined to conceive of pre-service teachers as *autonomous subjects*, that is, as subjects who must think and reason independently for themselves (Radford, 2021). From this perspective, autonomy becomes a necessary condition for the learning of pre-service teachers to occur. For example, in the studies based on the theory of didactic situations ($n=4$), the researchers make sure that the pre-service teachers assume the responsibility of solving the proposed tasks by themselves. As an autonomous subject, the pre-service teacher constitutes an already given entity, that is, "someone already endowed with her own intellectual capacities, who, in order to develop them, simply needs a stimulating social environment" (Radford, 2021, p. 184).

On the other hand, in some articles ($n=4$) the role of the teacher educator in the implementation of the tasks is not made explicit. On the other hand, in other studies ($n=5$) there is a tendency for researchers to grant a more instrumental role to the teacher educator in the sense that he or she is seen as a mediator of learning (RA4), a guide (RA5) or the person responsible for generating spaces for the mathematical work of pre-service teachers (RA11; RA14; RA15). In the same way, in other works ($n=3$) the researchers understand that the role of the teacher educator involves returning the responsibility of carrying out the task to the pre-service teachers (RA1; RA7; RA8) in accordance with the assumed theoretical tradition (French didactics). Finally, in three articles a different vision of the teacher educator as an instrumental subject is revealed: in RA10 a *pastoral vision* of the teacher educator is revealed, while in RA12 and RA13 a conception of the teacher educator as someone who works together with pre-service teachers in the task resolution is suggested. In any case, we believe that these roles of the teacher educator are closely related to the roles of pre-service teachers in the reviewed studies, in terms of getting involved in training activities guided by principles of a pedagogy centered on the learning subject. According to Radford (2016), this pedagogy prevents pre-service teachers and the teacher educator from entering into dialogue with humanity in its historical and cultural aspects. Consequently, the training activity of these subjects "departs from its proper function of humanly mediating in the subject-object relationship between man (sic) and nature" (Mészáros, 1972, p. 82).

Regarding the role of software, we observe a variety of ways in which researchers understand said role in the resolution of tasks. In most of the studies ($n=7$), the software is considered an environment for the formulation and justification of conjectures (RA1; RA3; RA4; RA6; RA8; RA11; RA15). In other studies ($n=5$), the researchers understand the software as: (a) an instrument that intermediates between a subject and a geometric object (RA5); (b) an artefact that facilitate the representation of this type of objects (RA9); (c) a cognitive tool to improve understanding (RA10); and (d) a cultural artifact in which the construction tools hold underlying conceptual contents (RA12; RA13). The remaining studies ($n=3$) do not specify any conception of the software (RA2; RA7; RA14).

As these results suggest, the role attributed to the dynamic software in the reviewed articles is primarily one of an environment favorable to the solving of proposed tasks. In correspondence with the idea of the autonomous subject that underlies the majority of the reviewed studies, we consider that the technology as a "stimulating environment" is what supports the decisions made by the authors regarding the role of software as an environment favorable to solving tasks.

Finally, in relation to the support materials for resolution of the tasks, we found that the majority of the studies reviewed ($n=12$) do not speak to these materials, neither in the task instructions (RA2; RA5; RA6; RA8; RA14) nor in the researchers' discussion (RA4; RA7; RA9; RA11; RA12; RA13; RA15). The remaining studies ($n=3$) offer materials that include physical objects (RA10) and information gathered from books, lecture notes, etc. (RA1; RA3).

3.2 The way in which learning is studied

The second aspect of the review refers to the study of learning of school geometry content with dynamic software in the consulted studies. To describe how this aspect is addressed by researchers, we use categories C4 and C5 relative to the production and analysis of data.

3.2.1 Production of data (C4)

Concerning the production of data, both the techniques and instruments of data collection as well as the data produced in the studies stand out in the articles.

The data collection techniques are reported in only some of the works ($n=3$), including participant observation (RA3; RA9) and non-participant observation (RA10). For their part, the data collection instruments used in the majority of the studies ($n=10$) were written responses from participants, audio and/or video recordings of the moments of task implementation, and worksheets (RA1; RA3; RA7; RA8; RA9; RA11; RA12; RA13; RA14; RA15). Regarding worksheets, the authors of RA1 and RA7 assert to turn to the software construction protocol as a source of data. In two other studies, the instruments include only written answers and recordings (RA2; RA6). Finally, only one study fails to inform the employed collection techniques and instruments.

Regarding data, only in some studies ($n=6$) is the source of the data stated. In these cases, the data comes from the transcription of audio and/or video recordings (RA4; RA9; RA12; RA13), or simply from recording of the software's workspace with participants' interactions (RA14; RA15). Although not specified with respect to methodology, we inferred that the data from the remaining studies ($n=9$) was sourced from transcriptions (RA1; RA3; RA7; RA8; RA10), written responses (RA11) or both (RA2; RA5; RA6).

These results suggest that, on the one hand, although data collection techniques are not reported in the majority of the studies reviewed, the instruments used suggest the researchers' preference for the technique of participant observation in order to approach and deeply understand the phenomenon of learning school geometry content with dynamic software by pre-service teachers. However, none of the articles reviewed provided clues as to how the cultural diversity of the participants might have influenced the data collection procedures. It is crucial to highlight the interest in this cultural diversity, especially in countries such as Chile, Brazil, and Venezuela, where university classrooms have opened their doors to members of indigenous and Afro-descendant populations, among other groups, who come from geographical areas where access to digital technology is limited and restricted.

3.2.2 Data analysis (C5)

With regard to data analysis, the articles highlight the units, procedure and tools of analysis.

Regarding the units of analysis, we observed a clear trend in the studies ($n=8$) to consider as units of analysis the interactions between the pre-service teachers around the resolution of the tasks (RA1; RA3; RA4; RA6; RA7; RA9; RA10; RA14). Two issues stand out in these studies. On the one hand, in two of them, researchers decided to constitute specific units called *scenes* or *events*, to examine the interactions of the participants in greater detail. On the other hand, in three of the eight papers the authors combined the interactions with *written records*, as a way of triangulating information. Beyond interactions, we found that in two articles (RA11; RA15), the unit of analysis was determined by the written responses of the pre-service teachers to the implemented tasks. Similarly, only two studies (RA12; RA13) considered the *activity* of the participants in a broader sense as the unit of analysis. Finally, in the remaining studies ($n=3$) the units of analysis are not explicitly stated.

Regarding interactions as units of analysis, we highlight that the majority of cases report the communication of the participants only at the level of oral discourse, leaving aside the multimodal character of human thought that manifests itself in the activation and articulation of various semiotic resources (e.g., cognitive, physical and perceptual) in the communication of mathematical meanings in classroom activity (Radford et al., 2009). The problem of analyzing the interactions of pre-service teachers solely from oral language lies in the devaluation that the body has historically had in the act of knowing and, consequently, in the analysis of this act within the field of Mathematics Education. For its part, the analysis based on written productions has the problem of ignoring the role of activity as a space of human production from which the learning of pre-service teachers can be understood and explained as an educational phenomenon. From a historical-cultural perspective, school geometry content becomes an *object of consciousness* for pre-service teachers given that they display an activity in which said content acquires meaning, within the limits imposed by the culture to which they belong (Roth & Radford, 2011).

As to the analysis procedure, we observed that some studies ($n=4$) provide a detailed description of the way in which the research data was analyzed. Two of these studies have in common the fact that they consider the data production process as part of the analysis (RA4; RA10). However, we observed differences in the ways in which the two studies analyzed the data. In the case of RA10, the analysis conducted is characterized by the coding of the information as well as comparing and contrasting the coded data. For its part, the analysis described in RA4 is based on the development of descriptions and interpretations of the interactions between study participants. In this last form of analysis, we noticed a certain similarity to the procedures followed in RA12 and RA13 with respect to the elaboration of interpretations of the data according to the objective of the investigation. In these last studies (RA12; RA13), interpretations derive from the use, on the data, of certain methodological categories linked to the assumed theoretical framework. Despite stating that a mixed analysis of the data was carried out, the study in RA9 does not report in detail how this analysis was carried out.

Regarding the analysis tools, most of the studies ($n=9$) offer a description of the analytical framework used in the research, while some do not ($n=6$). This framework comprises three types of analysis tools: categories, levels and constructs. Some of these investigations ($n=4$) use *categories* to group the data according to the type of elaborated definition (RA2) or the type of construction provided (RA11), as well as to organize the analysis according to different aspects related to the research theme (RA5; RA13). For their part, other studies ($n=2$) use *levels* to examine the progression, at different scales, of the perception of geometric objects (RA2) or of the understanding of classifications of this type of objects (RA6). Finally, other works ($n=2$) resort to theoretical or methodological *constructs* to account for the formation of geometric concepts (RA3) or the semiotic activity displayed by the participants and their relationship with learning (RA12). Given the type of mixed analysis used, in RA9 a $\chi 2$ test of independence was carried out.

With regards to these last results, the fact that the majority of the studies reviewed do not explain how the data were examined may be due, to a large extent, to the fact that the authors implicitly assume that the analysis procedures used are known to their readers, to the theoretical-methodological complexities associated with the development of these procedures, or simply to the space limitations imposed on scientific articles, which lead authors to sacrifice this information. On the contrary, we observe that the researchers in the reviewed articles seem more concerned with describing the analysis tools of their studies (e.g., the idea of a semiotic node in RA12), in correspondence with the type of research they carry out.

4 Conclusions and perspectives on future work

In this article, we describe the results of a systematic review of 15 research articles with the intent to answer the following question: According to specialized literature, how has the learning of school geometry content with dynamic software in pre-service mathematics teacher education been promoted and studied within the training activities themselves, in recent years?

Within the findings of our review, we discovered that the articles reflect an important effort by researchers to understand this learning by studying the interactions between pre-service teachers and a variety of geometric objects, different technological artifacts (dynamic software and environments of online learning), themselves and the teacher educator. The review has revealed to us that the choice to study interactions in contexts of teacher training in geometry with dynamic software responds to certain theoretical-methodological principles of learning, which do not necessarily take into account the intertwining of social, historical, and cultural conditions of the learning subjects.

Specifically, we observe that the interactions examined in the reviewed articles, assumed as the unit of analysis of learning geometry school content with dynamic software, do not sufficiently explore the crucial role of the body in the act of knowing, as suggested by mathematics educators that adopt historical-cultural educational perspectives (Maffia & Sabena, 2015; Mariotti, 2009; Radford, 2021). Following the ideas of Radford (2021), we believe that becoming aware of how the corporeal, sensory and artifactual activity employed by pre-service teachers and the teacher educator during their interaction with geometric content with dynamic software is an aspect fundamental to understanding the phenomena that accompany the learning and teaching of geometry.

The aforementioned has implications for the analysis procedures conducted in studies related to the topic of our review. In general, research on the teaching and learning of geometry shows progress with regard to the consideration of the semiotic and embodied nature of geometric thinking in the analysis procedures of the geometric activity of schoolchildren, to know their capacities of visuospatial reasoning as well as the role of semiotic processes, gestures and technological artifacts in the teaching–learning of geometry (Bartolini & Mariotti, 2008; Bautista & Roth, 2012; ICME, 2023; Ng & Sinclair, 2015; Sinclair et al., 2016). However, this progress is not fully reflected in the articles that constitute our review, which represents an opportunity to carry out further studies that contribute to the understanding of how multimodal analysis (with its different nuances) can be conducted in research. Recent works such as that of Prieto et al. (2024) show how a multimodal analysis approach can be conducted to account for the learning of school geometry content with dynamic software by pre-service teachers.

Regarding our methodological choices for conducting the systematic review, we recall that the articles reviewed were taken from academic journals included in the top journal rankings in the field of Mathematics Education. Although this decision seems correct to us, we want to share a reflection that we have been making about undertaking systematic reviews through rankings and not only through academic databases. The issue of the specificity of the review topic was decisive when making this decision. However, it was not until after using certain databases and obtaining results inconsistent with our research interests, that we became aware of this fact.

We know that academic databases are not infallible because, as Schimmer et al. (2015) warn, these tools present limitations in terms of coverage, restricted accessibility and geographical bias that considerably affect the visibility and recognition of the growing scientific production in our field in Latin American countries. To a large extent, relying on the ranking of Andrade-Molina et al. (2020) has allowed us to face these limitations and contribute, through our research, to other ways of carrying out systematic reviews on very specific topics.

In summary, our review has allowed us to identify gaps in the research on the topic we have addressed in the period of 2017–2023. Among these gaps, the following stand out: (i) the view of pre-service teachers as autonomous subjects, which keeps them distant from their historical, social and cultural contexts; (ii) the instrumentalist conception of dynamic software, which does not recognize the ontological role of artifacts in the geometric activity of pre-service teachers; and (iii) methodological approaches to data analysis that do not fully consider the role of the body in the act of knowing. In light of this, we suggest that future research on this topic should take into account the following aspects: (a) the relationships between the geometric meanings underlying dynamic software tools and the semiotic activity employed by the pre-service teachers and the teacher educator; (b) how these subjects deal with the tensions and conflicts that arise during geometry training activities, especially with the introduction of dynamic software; and (c) the promotion and study of learning geometry school content with software in training activities that include pre-service teachers with diverse cultural and social realities.

Data Availability The data generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest None.

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