Education and Information Technologies (2022) 27:11089–11113
https://doi.org/10.1007/s10639-022-11070-z

Check for updates

The mathematical and technological nature of tasks containing the use of dynamic geometry software in middle and secondary school mathematics textbooks

Fadime Ulusoy[1] · **İsmail Batuhan Turuş**[1]

Received: 12 January 2022 / Accepted: 21 April 2022 / Published online: 29 April 2022

Abstract
This study investigates the quality of tasks containing the use of dynamic geometry software (DGS) in the middle (5th–8th grade) and secondary school (9th–12th grade) mathematics textbooks in terms of mathematical and technological aspects. The DGS-related tasks in twenty-seven Turkish mathematics textbooks, approved by the Ministry of National Education, were analyzed according to the Dynamic Geometry Task Analysis Framework (Trocki & Hollebrands, *Digital Experiences in Mathematics Education,* 4(2), 110-138, 2018). Data analyses were conducted by using both qualitative and quantitative (descriptive statistics, independent samples t-test, and ANOVA) methods. The findings showed that DGS-related tasks were more common in the secondary school mathematics textbooks than in middle school mathematics textbooks. The mathematical depth level of DGS-related tasks in the middle school textbooks was significantly different from the mathematical depth level of DGS-related tasks in the secondary school textbooks. The mathematical depth levels of DGS-related tasks are quite low in middle school mathematics textbooks, and these tasks mostly cannot go beyond the practice of "drawing a shape according to the given steps". In terms of technological actions, most of the DGS-related tasks often required only drawing. Sliding and dragging, which are required to see invariant relationships within geometrical obje cts, were uncommon in textbook DGS-related tasks. The quantitative results also showed that DGS-related tasks with a high level of mathematical depth have a high number of technological actions. Based on the results of this study, recommendations are given for improving the use of DGSs in textbooks as well as for further research on this topic.

Keywords Dynamic geometry software · Mathematics textbook analysis · Mathematical depth level · Technological action

✉ Fadime Ulusoy
fadimebayik@gmail.com; fadime@kastamonu.edu.tr

1 Mathematics and Science Education Department, Kastamonu University, Kastamonu, Turkey

1 Introduction

There are numerous technologies accessible for mathematics instruction today. A form of technology that has garnered much attention in mathematics education is dynamic geometry systems such as Cabri, Geometer's Sketchpad, and GeoGebra. Dynamic geometry systems (DGSs) offer students tools that enable them to generate drawings, make measurements, and drag elements of a drawing (Hollebrands, 2007) for the learning and teaching of various geometric paradigms. Researchers have argued that the strategic use of DGSs has a great role in promoting student understanding of mathematical ideas (e.g., Hollebrands & Dove, 2011; Sherman et al., 2020; Zbiek et al., 2007). When compared to the time-consuming paper-and-pencil methods that have traditionally been utilized in mathematics instruction, DGSs provide opportunities for students to create geometric objects. However, the advantages of using a DGS go beyond the creation of objects easily. However, citing specifically the time-saving quality of digital resources is not, in and of itself, valid support for using that technology. According to numerous studies, DGSs allow students to analyze invariant relationships by dragging and to reach important conjectures and conclusions (e.g., Arzarello et al., 2002; Baccaglini-Frank & Mariotti, 2010; Christou et al., 2004; Hollebrands, 2007; Laborde, 2001). Students may explore and justify geometric relationships when engaging in activities in DGS environments. Hence, they have opportunities to develop their reasoning and proof skills (Trocki & Hollebrands, 2018).

Teachers need to make decisions about how and when to incorporate DGS-related tasks to support students' mathematical thinking (e.g., de Villiers, 1998; Mariotti, 2012). However, showing why students think a conjecture can be true using empirical data obtained from the use of a DGS is a new and challenging endeavor. In this regard, some mathematics teachers have the most difficulty planning and implementing classroom activities using dynamic systems (Cayton, 2012; Sherman, 2014). For this reason, it is necessary to offer teachers examples of tasks that include effective guidance that leads students to utilize DGSs for improving argumentation by exploring, conjecturing, and justifying conclusions. It is insufficient to just ask students to study a theorem in a DGS without providing appropriate prompts.

The characteristics of curricular materials become crucial to exemplifying how and when DGSs may be used in instruction since some mathematics teachers regulate their instruction based on the objectives and suggestions included in curricular resources (Grouws et al., 2004, 2013; Tarr et al., 2013). Standard documents (Common Core State Standards Initiative [CCSSI], 2010; National Council of Teachers of Mathematics [NCTM], 2000) and many curricula recommend the use of DGSs to improve student learning. The Turkish Ministry of National Education has been implementing innovative reforms to improve the educational system's quality. As part of these reforms, significant progress has been achieved in integrating technology at all levels of education (Bayazıt, 2013). In the Turkish middle and secondary school curricula (Ministry of National Education [MoNE], 2018a, 2018b), there are clear statements about the use of DGSs in the geometry content (e.g., "dynamic geometry software can be used to find angle bisector (p. 68),…, to prove triangle inequality (p.74)). As a result, tasks requiring the use of DGSs are included in mathematics textbooks because both such programs provide many opportunities for students to experience high-quality mathematical tasks and curricula require them (e.g., Hollebrands, 2007; Hölzl, 2001; Laborde, 2001; Mariotti, 2012; Sinclair, 2004).

Textbooks are an important learning resource because they allow students to study for classes and exams, do homework, and work on projects (van Zanten & van den Heuvel-Panhuizen, 2018). Moreover, textbooks are a major teaching resource since they allow teachers to plan lessons and produce tests and exams (Kajander & Lovric, 2009). Numerous studies have found that mathematics teachers spend the majority of their time in class performing textbook tasks (Ulusoy & İncikabı, 2020; Pepin & Haggarty, 2001; Roth & Givvin, 2008). In this regard, textbooks not only provide the learning opportunities available to students in general but also reveal the role of technology in learning mathematics (Sherman et al., 2020). For this reason, it is important to determine the mathematical and technological characteristics of tasks utilizing DGSs in mathematics textbooks to provide significant mathematical learning opportunities to students (de Villiers, 1998; Mariotti, 2012; Heid & Blume, 2008; Hollebrands & Dove, 2011; Zbiek et al., 2007). However, research on the characteristics of technology-oriented tasks in curricular resources is lacking (e.g., Jones et al., 2016; Sherman et al., 2020). In the study, we report the characteristics of tasks utilizing DGSs in terms of mathematical and technological aspects in Turkish middle and secondary school mathematics textbooks.

2 Background of the Study and Research Questions

2.1 Analytic Framework for Analyzing DGSs Tasks

In the present study, we defined *a mathematical task* as a problem or a sequence of problems in a textbook that is designed to highlight a mathematical idea (Stein & Smith, 1998). We used the following definition: “a task was defined as any example or activity, including a solution if one is given, which (a) contains one or more problems centered on a particular mathematical idea that the reader could reasonably be expected to engage in and (b) has a separate marker in the text” (Sherman et al., 2020, p. 365). To define a DGS task, we used the following definition:

> “A DGS task is defined as a combination of geometric objects (the sketch on the screen at various points in the task completion process) and the associated written directives or prompts used to accomplish particular learning goals. The sketch may be pre-constructed by the task writer, partially constructed with the expectation that the student will make additional constructions or not pre-constructed whereupon the student constructs the entire object (i.e. student-constructed)” (Trocki & Hollebrands, 2018, p.122-123).

While conducting the textbook analysis (Fan, 2013), an analytic framework is important. There are various frameworks developed for analyzing the use of technology in tasks in textbooks. For example, Sherman et al. (2020) grouped (a) the type of technology used, (b) the use of technology as superficial or substantial, and (c) the use of technology as an *amplifier* and *reorganizer* in secondary school textbooks. They defined the use of technology as an *amplifier* if it is used to perform calculations or quickly create representations.

Technology use was considered to be a *reorganizer* if the aim of offloading such calculations to technology was to support a shift in learners' cognitive focus by providing a novel representation or by allowing for some form of dynamic manipulation (Sherman et al., 2020). Moreover, they made a distinction between *superficial* and *substantial* use of technology. If the tasks use technology as an amplifier or reorganizer, these tasks are grouped as a subset of the tasks that use technology substantially. However, superficial use of technology generally consisted of verifying or providing a numerical approximation of a solution previously obtained by manual techniques. Their study revealed that calculators were the predominant technology utilized. According to the results, 15% of the activities in the textbooks included technology integration. They also found that technology was used as a reorganizer in only 21% of them.

Trocki and Hollebrands (2018) also document the Dynamic Geometry Task Analysis Framework to show the relative quality of tasks produced for dynamic geometry software (see Table 1). The main purpose of this framework is to assist curriculum writers

Table 1 The Dynamic Geometry Task Analysis Framework

Allowance for Mathematical depth*	
Levels	Descriptions
N/A	Prompt requires a technology task with no focus on mathematics
Level 0	Prompt refers to a sketch that does not have mathematical fidelity
Level 1	Prompt requires student to recall a math fact, rule, formula, or definition
Level 2	Prompt requires student to report information from the sketch. The student is not expected to provide an explanation
Level 3	Prompt requires student to consider the mathematical concepts, processes, or relationships in the current sketch
Level 4	Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch
Level 5	Prompt requires student to go beyond the current construction and generalize mathematical concepts, processes, or relationships
Types of Technological action	
Affordances	Descriptions
N/A	Prompt requires no drawing, construction, measurement, or manipulation of current sketch
Action A	Prompt requires drawing within current sketch
Action B	Prompt requires measurement within current sketch
Action C	Prompt requires construction within current sketch
Action D	Prompt requires dragging or use of other dynamic aspects of the sketch
Action E	Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s)
Action F	Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise that may be based on testing extreme cases

and teachers in assessing and producing dynamic geometry tasks. The framework includes two dimensions: i) *mathematical depth* and ii) *technological actions* (see Table 1). In the framework, *a prompt* is defined as "a written question or direction related to a sketch that requires a verbal or written response. It may require technological action, such as in the form of a drawing, construction, measurement, or manipulation of a sketch" (Trocki & Hollebrands, 2018, p. 123). A prompt frequently necessitates both a written and technological response.

Trocki and Hollebrands (2018) proposed the mathematical depth component by utilizing the cognitive framework developed by Smith and Stein (1998) and some studies (Baccaglini-Frank & Mariotti, 2010; Stylianides, 2008; Zbiek et al., 2007). In the mathematical depth component, researchers identified the cognitive actions expected from the students. They added code zero to the framework to recognize the importance of mathematical fidelity (Zbiek et al., 2007) before productive mathematical engagement can take place in a DGS. Based on Smith and Stein's lower-level task group, they identified codes one and two that have lower-level mathematical depth. For example, in a DGS-related task where the depth is coded 1, the student is expected to carry out a simple mathematical action such as performing an operation. Codes three and five are related to conjecture generating and testing tasks in DGSs (e.g., Baccaglini-Frank & Mariotti, 2010; Christou et al., 2004; Laborde, 2001; Sinclair, 2003; Stylianides, 2008). In a DGS-related task at the depth level 5, students are expected to discover mathematical relationships and reach generalizations rather than perform operations. As an indicator of doing mathematics, tasks "require students to explore and understand the nature of mathematical relationships" (Smith & Stein, 1998, p. 348). This situation can be seen in mathematical depth level four (the prompt requires students to explain the mathematical concepts, processes, or relationships in the current sketch). Code Four also emphasizes the need for learners to explain what they recognize while using a DGS.

Technological action types refer to the use of technological features of dynamic geometry software in a task. They identified technological action codes based on the literature. For example, codes A, B, and C reflect how DGSs may be used to mimic actions that have traditionally been done using paper and pencil and a measuring device. Code D focuses on the use of dragging when students interact with geometric objects (e.g., Arzarello et al., 2002; Hollebrands, 2007; Hölzl, 2001). Codes E and F are related to the potential for students to explore invariant relationships (e.g., Arzarello et al., 2002; Baccaglini-Frank & Mariotti, 2010; Christou et al., 2004; Hollebrands, 2007) and consider extreme cases (Sinclair, 2003). According to the theoretical framework, a task may include more than one technological action. Furthermore, Trocki and Hollebrands (2018) suggest that high-quality tasks are expected to include a collection of prompts requiring a combination of technological actions and high levels of mathematical depth. In the present study, we chose Trocki and Hollebrands's (2018) Dynamic Geometry Task Analysis Framework to analyze how DGS-related tasks are represented in mathematics textbooks for two reasons. It is mainly related to DGS task quality rather than the integration of any technology (e.g., calculators) in textbooks (e.g. Sherman et al., 2020). It is also possible to understand the coordination of mathematical depth with technological actions in this framework. Therefore, it may be possible to discriminate among the quality of mathematical tasks utilizing DGS.

2.2 Research on Technology Integration in Textbooks

According to Stein and Smith (2010), textbooks in mathematics education play a significant role in determining what teachers teach and what students learn. In general, if a topic is not covered in the textbook, it is unlikely to be discussed in the classroom (Stein et al., 2007). As a result, what is written in textbooks has a significant impact on the learning opportunities available to pupils. As a result, understanding the content of textbooks is critical. Many studies about textbook analyses have been published in the last ten years (Grouws et al., 2013; Otten et al., 2014; Sherman et al., 2016; Tarr et al., 2013; Thompson et al., 2012). However, very few studies have focused on the integration of technology into textbooks (Erbas et al., 2012; Jones et al., 2016; Lew & Jeong, 2014; Mersin & Karabörk, 2021; Sevimli & Kul, 2015; Sherman et al., 2020).

In the studies, researchers mostly focused on the integration of technology according to the types of instructional technologies in the grade levels, learning domains, and/or other specific purposes. For example, Jones et al. (2016) examined the technology integration in the mathematics content of six popular textbooks, written for prospective elementary teachers, in terms of location within the textbook, the role of technology, and type of technology. Their analysis showed that the technologies most frequently used were calculators, websites, and e-manipulatives. In another study, Sevimli and Kul (2015) examined the technological integration in middle school (5–8 grade) mathematics textbooks. They stated that the most frequently used technological component is the calculator, which is used to ease calculations. In addition, the researchers noted that the integration of dynamic geometry software is scarce in the textbooks. In terms of technology integration in learning areas and grade levels, Sevimli and Kul (2015) found that integration is mostly in data processing in 7th grade.

Some researchers also conducted comparative studies about the use of technology integration in mathematics textbooks. For example, Erbas et al. (2012) compared Turkish, Singaporean, and American sixth-grade mathematics textbooks in terms of technology integration. They discovered that Turkish and Singaporean textbooks have less technological content than American textbooks. Moreover, the use of the calculator is the technology that is most often suggested in textbooks. Similarly, Mersin and Karabörk (2021) examined technology integration in both Turkish and Singaporean mathematics textbooks at the middle school level. The results revealed that technology integration was quantitatively more intense in Singapore mathematics textbooks due to the frequent use of the calculator, but the technologies used were similar in both countries' textbooks. In terms of learning areas, Singaporean textbooks show a more homogeneous distribution of technology integration. Technology integration in Turkish textbooks focuses on "numbers and operations" and "geometry and measurement" (Mersin & Karabörk, 2021). It is noteworthy that these studies focused on technology integration in mathematics textbooks, but they mostly did not examine the mathematical depth levels and technological properties of the tasks. It is important to examine the mathematical and technological characteristics of technology integration rather than focusing on the distribution of technology integration in learning areas or grade levels.

2.3 Research Questions and Significance of the Study

The integration of DGSs into curriculum resources has received little attention in related literature, but there appears to be an increasing need to understand both the quality and quantity of such integration so that the use of DGSs becomes more widespread in classrooms since the quality of DGS-related tasks in the textbooks may directly or indirectly affect teaching and learning processes. Therefore, in-depth analysis of tasks related to the DGSs in the geometry learning area, which has critical components such as proof, reasoning, definition, and spatial thinking that students have difficulty understanding, may give ideas to mathematics educators, technology experts, and textbook writers that may help to design high-quality tasks that use DGSs. Therefore, the following research questions guided this study:

1) How were the DGS-related tasks in geometry distributed across the grade levels in mathematics textbooks, and what types of DGSs are used?
2) What is the nature of the mathematical depth level of the DGS-related tasks in middle and secondary school mathematics textbooks distributed by the government of Turkey?

   Is there a significant difference in the mathematical depth level scores for DGS-related tasks in middle and secondary school mathematics textbooks?

3) In middle and secondary school mathematics textbooks, what kinds of technological actions were used most often to present the tasks related to the DGSs?
4) What is the connection between mathematical depth levels and technological actions of DGS-related tasks in mathematics textbooks?

It is also crucial to emphasize why a local study, such as the analysis of Turkish mathematics textbooks, is timely and pertinent for international audiences. The reason is the need for mathematics educators to see the big picture. Turkey has a centralized education system, which means that in the elementary (Grades 1–4), middle (Grades 5–8), and secondary (Grades 9–12) levels, all students and teachers follow the same curriculum in every subject area. Moreover, Turkey is one of the few countries (e.g., Korea (Ju et al. 2016), Mexico (Aguilar & Castaneda, 2020)) in the world where textbooks are massively distributed for all grade levels. To be used in schools, any mathematics textbook needs to be approved by the Turkish Ministry of National Education (MoNE). Among the approved mathematics textbooks, MoNE decides which textbooks can be used by which public schools and distributes them free of charge to students and teachers.

Because textbooks in Turkey are government-authorized, Turkish mathematics textbooks adhere to the national curriculum's educational visions (MoNE, 2018a, 2018b). Turkish teachers depend on textbooks for mathematics instruction, lesson aims, and learning activities as well as for homework. As a result, millions of students are in close and frequent contact with their mathematics textbooks because they utilize them both at home and in the classroom. Without a doubt, this is an intriguing scenario in terms of the mathematics textbook research area. In such an educational system, major changes can be made in school curricula and textbooks, assuming that innovations in the curricula would have positive effects on every component of the educational process, resulting in quality improvements in a short time. We also think that an analysis of the current Turkish mathematics textbooks can be used to develop new policies and practices for a global curricular movement on technology integration.

3 Methodology

We carried out a textbook analysis in order to answer research questions related to middle and secondary school mathematics textbooks presently in use in Turkey. According to Weninger (2018), there are three broad frameworks or orientations that researchers generally employ in textbook analysis: *content analysis*, *critical discourse analysis,* and *multimodal analysis*. In particular, we used content analysis in this study. Content analysis is a research technique that entails identifying units for analysis in a well-defined textual sample, coding those units based on a priori criteria established by the researcher, then reducing the data by quantifying the results, and finally inferring the significance of the results (Krippendorff, 2013, p. 84). We identified each DGS-related task as a unit of analysis and coded them according to the Dynamic Geometry Task Analysis Framework (Trocki & Hollebrands, 2018).

3.1 Context and the Selection of the Textbooks

The Turkish school system is organized in terms of primary school (ages 7–10, grades 1–4), middle school (ages 11–14, grades 5–8) and secondary school (ages 15–18, grades 9–12). For each school subject and grade, there is a national curriculum accompanied by prescribed textbooks determined by the Educational Policy Institute with the authorization of the Ministry of Education. In Turkey, textbooks must be approved by a commission of six to eight people, made up of teachers with at least five years of experience, field specialists with doctoral degrees, and visual experts chosen at random by the Ministry of National Education from among volunteer teachers and educational scientists (Ministry of Education Textbooks & Education Tools Regulation, 2012). After the evaluation process, the MoNE distributes the highest-rated course books to all students free of charge. The official gazette Regulations about Textbooks and Educational

Materials (Ministry of Education Textbooks & Education Tools Regulation, 2012) stated that textbooks or other educational tools not approved by the MoNE cannot be used for teaching in schools. In all primary and secondary schools in Turkey, only the textbooks approved by MoNE are used and distributed to students and teachers. Hence, in Turkey, textbooks are compulsory, and students are required to have the textbook during lessons. In this study, we examined all middle and secondary school mathematics textbooks that were published in 2018 or 2019 (see Table 2).

The Ministry allows these books to be used as textbooks for 5 years from the date of publication. These textbooks were the highest-rated mathematics textbooks, as identified by the Ministry of National Education on their official Website. Therefore, we analyzed twenty-six middle (grades 5–8) and secondary (grades 9–12) school mathematics textbooks approved by the MoNE to characterize DGS-related tasks. We examined all DGS-related tasks in 27 mathematics textbooks. The MoNE publishes textbooks at every grade level in its own publishing house, that is called MEB. In addition, some textbooks from private publishing houses that have been approved by the MoNE can also be used as mathematics textbooks in schools. There is no significant variation in content among the mathematics textbooks from different publishers. As seen in Table 2, we coded textbooks according to grade levels (G5-G12) and publishers (MEB or Private).

Table 2 Analyzed textbooks

School-level	Textbooks			
	Textbooks of MEB publisher	# of DGSs task	Textbooks of private publishers	# of DGSs task
Middle school (Grade 5–8)	G5-MEB	2	G5-Private	2
	G6-MEB	1	G6-Private	0
	G6-MEB2	1		
	G7-MEB	3	G7-Private	1
	G8-MEB	17	G8-Private	14
	G8-MEB2	6		
	Total	30	*Total*	17
Secondary school (Grade 9–12)	G9-MEB(SHS)	8	G9-Private	10
	G9-MEB	4		
	G9-MEB2	14		
	G10-MEB(SHS)	7	G10-Private	0
	G10-MEB	3		
	G11-MEB(SHS)	5	G11-Private	7
	G11-MEB	3		
	G11-MEB2	0		
	G11-MEB3	2		
	G12-MEB(SHS)	7	G12-Private	3
	G12-MEB	4		
	G12-MEB2	5		
	G12-MEB3	0		
	Total	62	*Total*	20

3.2 Data Analysis Procedures

Based on our research questions, we used both qualitative and quantitative approaches when analyzing the data. To answer the first research question, we analyzed each mathematics textbook in terms of the use of DGSs in the geometry and measurement learning areas. We coded a mathematical task in the textbooks as the unit of analysis. Hence, we identified tasks that made use of DGSs. We were interested in all DGS-related tasks in any part of the textbooks (e.g., main narrative sections, extensions of narrative sections, stand-alone labs, and stand-alone activities, all exercise sections, review sections, summary sections, and practice assessments). Activities or questions that build on one another are considered a single task (e.g., 2a-2 h). We determined the DGS-related content of a task in two ways: (a) either the task explicitly refers to the use of DGSs (Pea, 1987) or (b) it contains a screenshot or visual reference to dynamic geometry software (Sherman et al., 2020). After we identified all DGS-related tasks, we labeled them as T1-…-T129 in an Excel document, respectively. Then, we identified each task in terms of grade level, publisher of the book, mathematical content, page number of the task, and the type of DGSs used (e.g., GeoGebra, no limitation, other) (see Fig. 1). Therefore, 47 tasks were found in middle school math textbooks, and 82 tasks were found in secondary school math textbooks.

To answer the second research question, we used a deductive approach to code each task in terms of its *mathematical depth level*. Hence, we used Trocki and Hollebrands' (2018) dynamic geometry activity analysis theoretical framework (Table 1). We coded the mathematical depth level of each DGS-related task in the Excel document, as seen in Fig. 1. A second researcher also coded all DGS-related tasks for reliability, separately. The inter-coder reliability between two coders was computed for the mathematical depth of the tasks. Cohen kappa value was found 82%. At the end of the coding process, the two coders discussed and resolved discrepancies together. For the statistical analyses, we analyzed the data in the SPSS package program. For the normality test, skewness and kurtosis values were examined. It was observed that the skewness value ranged between -1.441 and 0.545, and the kurtosis value varied between -1.323 and 1.471. Skewness values ranged from -393 to 0.209 and Kurtosis values ranged from 1.06 to -0.416. When Kurtosis and Skewness values are between -1.5 and + 1.5, it is considered to be a normal distribution (Tabachnick & Fidell, 2013). For the sub-question of the second research question, we performed an independent-sample *t*-test to compare the mean mathematical depth level scores of DGS-related tasks for middle school and secondary school textbooks.

Task Code	Grade Level	Textbook	Page no	Subject	Mathematical depth							Technological action							DGS type
					NA	0	1	2	3	4	5	NA	A	B	C	D	E	F	
T1	5. Sınıf	MEB	218	Geometric concepts			x						x						No restriction
T2	5. Sınıf	MEB	238	Triangles and Quadrilaterals			x						x	x					GeoGebra
T3	5. Sınıf	TUNA	210	Triangles and Quadrilaterals		x							x	x					GeoGebra
T4	5. Sınıf	TUNA	281	Geometric solids			x						x						GeoGebra
T5	6. Sınıf	MEB 2	343	Geometric solids			x						x						GeoGebra

Fig. 1 Coding of DGS-related tasks in an Excel document

For research question 3, we used a deductive approach. In this sense, we coded technological actions in all DGS-related tasks based on Trocki and Hollebrands' (2018) dynamic geometry activity analysis theoretical framework (Table 1). We calculated the types and frequencies of technological actions in the DGS-related tasks in both middle and secondary school mathematics textbooks. Because a DGS task can include more than one technological action, we grouped the use of technological actions in the tasks into four subgroups: (i) no action, (ii) single action, (iii) double actions, and (iv) multiple actions. A second researcher also coded the technological actions of all DGS-related tasks for reliability. Inter-coder reliability between two coders was computed for (i) the types of technological actions in the tasks and (ii) the use of technological actions in terms of frequency in the tasks. Cohen kappa values for these components were 86% and 88%, respectively. The two coders discussed and resolved these discrepancies.

For the fourth research question, we carried out a one-way ANOVA. Before conducting the analysis, the main assumptions, including the level of measurement, independence of observations, normality, and homogeneity of variance, were checked. In the analysis, we used the number of technological actions in each DGS task as the continuous dependent variable. Hence, the level of measurement assumption was assured. Thus, the technological action score of a DGS task can have a minimum of 0 and a maximum of 5. Then, as the categorical dependent variable, we grouped the mathematical depth level (MDL) of the tasks into three categories based on Trocki and Hollebrand's framework (2018): (i) the tasks with no depth (N/A), (ii) the tasks with lower MDLs (Level 0–1-2), and the tasks with higher MDLs (Level 3–4-5). In the present study, there was no interaction between DGS-related tasks during the data collection. Hence, it was assumed that independence of observations was also assured. As mentioned before, the skewness and kurtosis values indicate that there was no violation of the normality assumption. In order to determine whether homogeneity of variance was ensured, Levene's Test of Equality was examined. Results revealed that the variance within each population was equally distributed and the homogeneity of variance assumption was met. After that, we examined whether there is a statistically significant difference in technological action scores for the tasks with no depth, the tasks with lower mathematical depth levels, and the tasks with higher mathematical depth levels. Figure 2 shows how much time we spent analyzing the data.

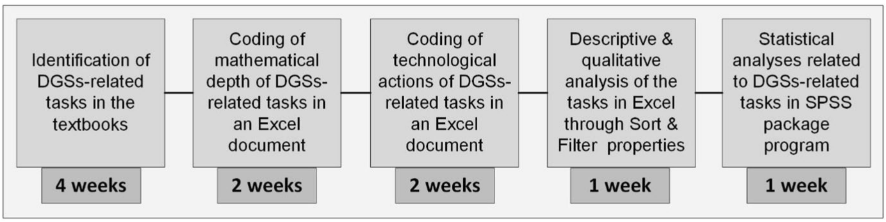


Fig. 2 Duration in the coding of DGS-related tasks in the textbooks

3.2.1 Coding examples

We present examples of tasks that are categorized as low, medium, and high quality according to the framework (Trocki & Hollebrands, 2018). For example, Task 43 in Fig. 3 is categorized as low-quality because it does not contain a collection of prompts that coordinate mathematical depth and technological actions in such a way as to require the student to make generalized conclusions based on invariant relationships that go beyond a static sketch.

The task requires students to draw a vertical prism according to the given steps in GeoGebra. The task is presented in the textbook after the formal definition and visual properties of the right prism are explained. The task requires students to reproduce previously learned facts about the definitions of a right prism. Since the task aims to recall a mathematical concept, its mathematical depth level is 0. The task includes only Action A since students are only supposed to draw the right prism.

The prompts in Task 48 ask students to examine the relationship between the areas of quadrilaterals in Fig. 4 by using the dragging tool. The prompts require students to consider a mathematical relationship in the current sketch. Although the task contains a collection of prompts that coordinate mathematical depth and technological actions in such a way that may encourage but not necessitate that the student makes generalized conclusions based on emergent invariant relationships. The task does not contain a prompt that asks to make a statement describing the relationship between the area of two quadrilaterals by providing reasons. For this reason, the mathematical depth of the task is classified as Level 2.

The prompts in Task 48 also received codes for technological actions of A, B, and D. Students are required to draw quadrilaterals and midpoints of the sides of the quadrilaterals, measure the areas of two quadrilaterals, and use

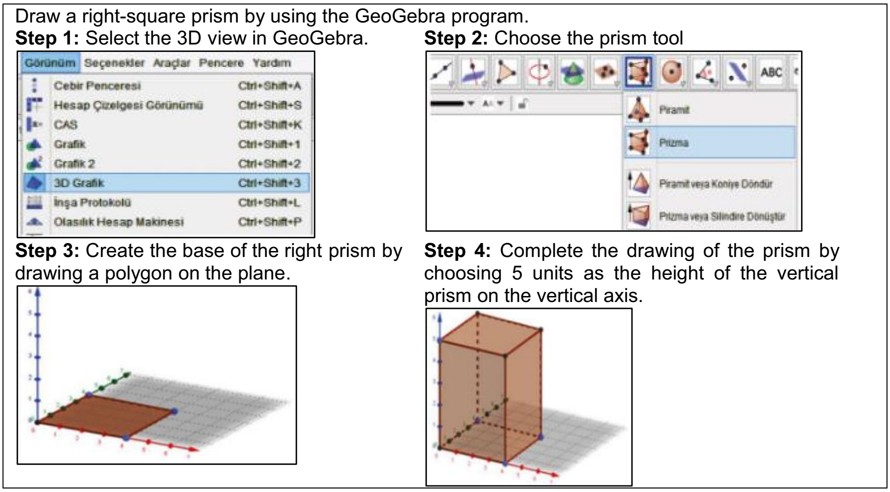


Fig. 3 Task 43 with MDL-0 and Action A (G8-MEB, p.199)

Technology Application

GeoGebra software has been used to look at the quadrilateral that can be formed by joining together the midpoints of two sides of a quadrilateral, as shown in the table.

	Select the polygon tool. Construct a rectangle ABCD.
	Select the midpoint tool. Identify the midpoints of the sides of the quadrilateral ABCD.
	Select the polygon tool. Connect the midpoints marked on the sides of the quadrilateral ABCD.
	Select the area tool. Find the areas of the quadrilaterals ABCD and EFGH.
	Select the angle tool. Determine the interior angles of the quadrilateral EFGH.

When you move the corners of ABCD with the mouse, examine the relationship between the areas of the two rectangles. Note that the rectangle EFGH is a parallelogram.

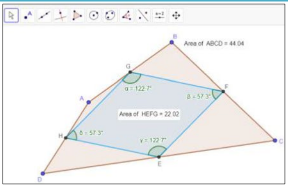


Fig. 4 Task 48 with MDL-2 and Action A, B, and D (G9-MEB (SHS), p.245)

the dragging tool to recognize invariant relationships between the areas of two quadrilaterals. This task is considered a medium-quality task.

The prompts in Task 84 (see Fig. 5) ask students to determine the relationship between the sides and angles of a triangle by going beyond the current construction. This task can be solved with some degree of cognitive effort, which means students need to consider the meaning of their actions as they wrestle

2. Angle-Side Relations in a Triangle

Technology Application

Draw a triangle using the instructions below with a dynamic geometry program. Calculate the side lengths and interior angles of the triangle you drew.

	Activate the Polygon tool. To create triangle ABC, select three points on the plane, A, B, and C. (Start from point A and click again to the last point A.)
	Activate the Angle tool. Select the vertices to determine the interior angle measures of the triangle.
	Activate the Length tool. Select the sides to measure the side lengths of the triangle.
	Activate the Move tool. Create different triangles by moving the corners of the triangle.

(1) Order the lengths of the sides of the triangle and the angles inside the triangle from smallest to largest.

(2) According to the data you have obtained, is there a relationship between the lengths of the sides of the triangle and the measures of its angles? Examine and explain this relationship.

(3) Check your result for a few new triangles created by moving the corners**.** Is your conjecture from (2) true? Explain.

An example is drawn in GeoGebra where you can examine the relationship between the angles and sides of the triangle.

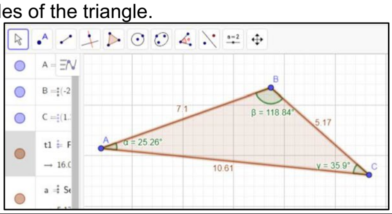


Fig. 5 Task 84 with MDL-5 and Action A, B, D, and E (G9-MEB(SHS), p.247)

with the task. Therefore, the mathematical depth of the task is classified as Level 5. The prompts also received codes for technological actions of A, B, D, and E. Students are expected to draw triangles (Action A). The task requires measuring angles and lengths of sides (Action B). Task 84 also includes Action D because the task includes a prompt that requires dynamic aspects of the sketch (e.g., dragging). Finally, the task requires revealing a mathematical relationship (Action E) between the length of sides and the measure of the angles of a triangle. In this regard, this task exemplifies the use of multiple technological actions. This task is considered to be of high-quality due to its coordination of mathematical depth with technological actions.

4 Results

4.1 The Use of DGSs in Mathematics Textbooks

For the first research question, descriptive statistics were obtained and presented in Fig. 6. As shown in Fig. 6, most DGS-related tasks were presented in secondary school textbooks (64%), whereas 36% ($n=47$) of them were found in middle school textbooks. Moreover, according to Fig. 6, it is noteworthy that there are no homogeneous percentage distributions of DGS-related tasks in mathematics textbooks at grade levels. The use of DGS-related tasks in the eighth- and ninth-grade textbooks is higher than those in the textbooks of other grades.

In terms of the dynamic geometry software used, GeoGebra was used in 117 DGS-related tasks out of 129. Thus, GeoGebra has been the most frequently used software in DGS-related tasks, with a rate of 91%. Moreover, there is no limitation or suggestion regarding the type of dynamic geometry software in 10 DGS-related tasks (8%) in mathematics textbooks. In the instructional part of these tasks, students were asked to use any suitable DGS.

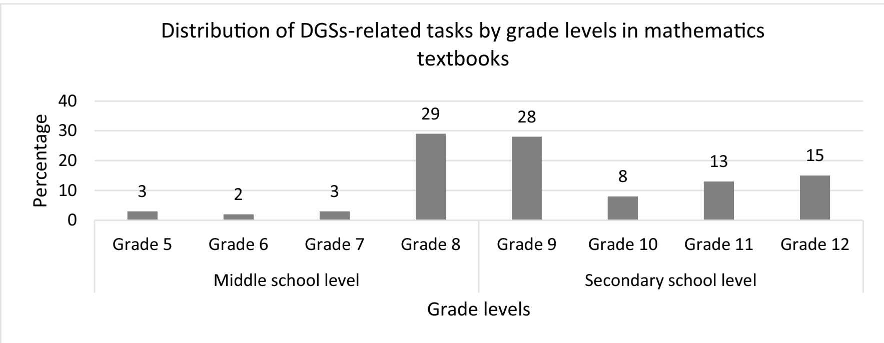


Fig. 6 Distribution of DGS-related tasks by grade levels in mathematics textbooks

4.2 Mathematical Depth Levels of the DGS-related Tasks

For research question 2, the descriptive statistics about the mathematical depth levels in tasks created with DGSs in the textbooks were given in Fig. 7. According to Fig. 7, the mathematical depth level of DGS-related tasks in middle school textbooks was mostly at Level 0 (53.2%) and Level 1 (40.4%). This result showed that the prompts in most of the DGS-related tasks were weak mathematically or only required students to recall a math fact, rule, formula, or definition. Moreover, DGS-related tasks in middle school textbooks did not include any prompts that required advanced levels of mathematical depth (e.g., students form conceptual ideas (Level 3), explain mathematical concepts, processes, and ideas (Level 4), and explore and generalize the nature of mathematical processes and relationships (Level 5)). [Note. In Fig. 7, mathematical depth levels were created by making separate percentage calculations over 82 tasks for the middle school level and 47 tasks for the secondary school level.]

In DGS-related tasks with low-MDLs, students are mostly expected to draw a sketch in A DGS by following the steps given in the textbook rather than guiding students to reason about geometric concepts. In such tasks, the use of DGSs could not go beyond the practice of drawing a static sketch according to the given steps.

In secondary school mathematics textbooks, some tasks reach the highest level of mathematical depth, but the number of these tasks is limited (see Fig. 7). The mathematical depths of DGS-related tasks are mostly at Level 1 and Level 2. An independent-sample *t*-test was conducted to compare the mean mathematical depth level scores of DGS-related tasks for middle and secondary school mathematics textbooks. According to Table 3, there was a statistically significant difference in the scores for mathematical depth levels of DGS-related tasks in middle school mathematics textbooks ($M=0.96$, $SD=0.208$) and secondary school mathematics textbooks ($M=1.17$, $SD=0.375$) in favor of secondary school textbooks [$t\ (127)=4.110$, and $p=0.000.05$]. These results showed that the mathematical depth level of DGS-related tasks differed significantly according to the grade levels of the textbooks (middle or secondary). In other words, these results show that the mathematical depth level of DGS-related tasks in secondary school math textbooks is higher than the mathematical depth level of DGS-related tasks in middle school math textbooks.

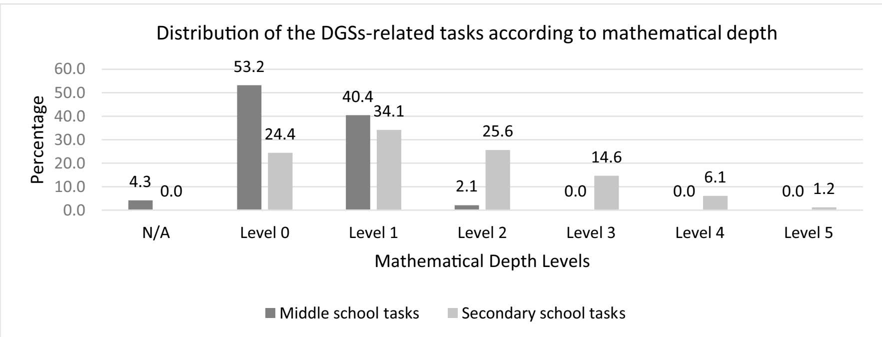


Fig. 7 Distribution of DGS-related tasks according to mathematical depth levels

Table 3 Results of *t*-test

DGS-related tasks in textbooks	*n*	*M*	*SD*	*df*	*t*	*p*
Middle level	45	0.96	0.208	127	4.110	0.000
Secondary level	84	1.17	0.375			

4.3 Types of Technological Actions in the DGS-related Tasks

Descriptive statistics related to research question 3 indicated that a DGS-related task in textbooks may include more than one technological action at the same time. We found 62 technological actions in DGS-related tasks in middle school mathematics textbooks and 163 technological actions in DGS-related tasks in secondary school mathematics textbooks). Figure 8 presents different types of technological action in the textbooks.

According to Fig. 8, Action A was used the most in DGS-related tasks in both middle (94%) and secondary (93%) school mathematics textbooks. Prompts in tasks involving Action A require students to make a drawing, but this can be done using paper, pencil, and a measuring device. The percentage of technological actions excluding Action A is lower in middle school textbooks than in secondary school textbooks. For example, while 46% of DGS-related tasks in secondary school textbooks require using the slider and dragging features (Action D), this rate is 15% in middle school textbooks. Specifically, the percentages of Action D and F that have the potential for students to uncover invariant relationships are low in DGS-related tasks. Figure 8 also indicates that the tasks that do not involve any technological action are more numerous in middle school textbooks than in secondary school textbooks. This means that the task doesn't ask learners to use any of the dynamic features of DGSs (e.g., drawing, measuring, dragging, and manipulating).

Since more than one technological action is used simultaneously in some of the DGS-related tasks, the use of technological actions in the tasks is divided into four subgroups: (i) no action, (ii) single action, (iii) double actions, and multiple actions (see Table 4). Thus, we examined the distribution of

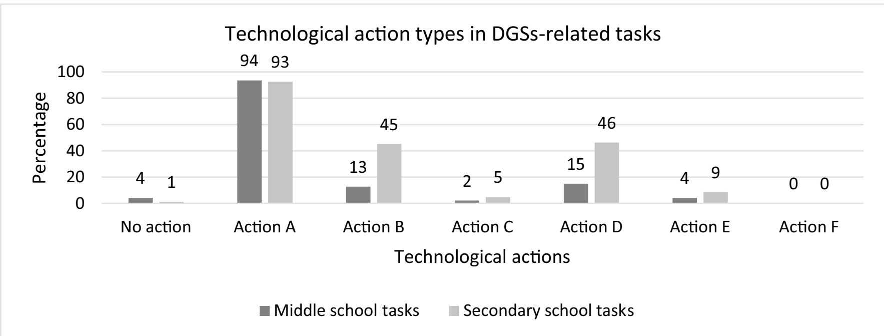


Fig. 8 Technological actions in DGS-related tasks

Table 4 Frequency of technological actions in DGS-related tasks

Technological actions	The number of tasks		Total frequency	
	Middle school textbooks	Secondary school textbooks		
No action	2	1	3	(2%)
Single action	31	29	60	(47%)
Double actions	13	28	41	(32%)
Multiple actions	1	24	25	(19%)
Total	47	82	129	(100%)

technological action frequencies in DGS-related tasks. According to Table 4, only one technological action was used in most of the DGS activities (47%). In middle school textbooks, there are a lot of tasks that only require one action. In secondary school textbooks, most DGS-related tasks require two or more technological actions.

4.4 The Use of Technological Actions for DGS Tasks with Different Mathematical Depth

To answer research question 4, we conducted a one-way ANOVA to see the technological action scores for the tasks at different mathematical depth levels (no depth, lower-level mathematical depth, and higher-level mathematical depth). One-way ANOVA results (see Table 5) showed that the technological action scores for the tasks differed significantly according to the mathematical depth levels of the tasks [F (2–126) = 15.594, $p < 0.05$]. Because we found a statistically significant difference, we performed a Post-Hoc analysis. Post-Hoc comparisons revealed that the mean score of technological actions for group A ($M = .50, SD = .707$) was significantly different from the mean score of technological actions for Group B ($M = 1.61, SD = .761$). The mean difference is −1.111 and the p-value is .048. Since $p < 0.05$, the difference between Group A and Group B is significant. Similarly, the other two comparisons (Group A vs. Group C and Group B vs. Group C) were also significant because of $p < .05$.

Taken together, these results suggest that when DGS-related tasks include a high level of mathematical depth, they also have more technological

Table 5 Results of ANOVA according to mathematical depth levels

Mathematical depth levels	*n*	*M*	*SD*	*df*	*F*	*p*	*Difference*
No depth (Group A)	2	0.50	0.707	2–126	15.594	0.000	Group B > Group A,
Low-MDLs (Group B)	113	1.61	0.761				Group C > Group A,
High-MDLs (Group C)	14	2.79	1.122				Group C > Group B

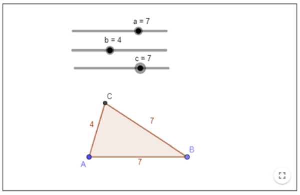
actions. When the mean differences were examined, a consistent increase was observed in the number of technological actions as the mathematical depth levels in the tasks increased. In other words, the number of technological actions is mostly high in DGS-related tasks with a high level of mathematical depth. Finally, we looked at descriptive data to understand the distribution of the use of technological actions in the tasks at different MDLs (see Table 6).

For example, there are various tasks with multiple technological actions and low-MDLs, as well as those with a single technological action and high-MDLs. To be more evident, we provide two examples: (1) a task with only one technological action and high-MDLs (see Fig. 9) and (2) a task with multiple technological actions and low-MDLs (see Fig. 10). Thus, DGS-related tasks with a high level of mathematical depth do not always necessitate more technological actions, or vice versa.

Table 6 The use of technological actions in different mathematical depth levels

Mathematical depth levels (MDLs)	Frequency				Total (%)
	The use of technological actions				
	No action	Single action	Double actions	Multiple actions	
No depth	2	0	0	0	2 (1)
Low-MDLs	1	58	37	17	113 (88)
High-MDLs	0	2	4	8	14 (11)
Total (%)	3 (2)	60 (47)	41 (32)	25 (19)	129 (100)

Creating triangles with GeoGebra



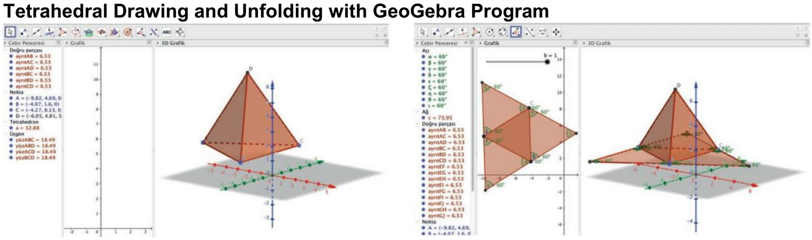
https://www.geogebra.org/m/gy5ah6xm

Students are asked to say the integers a, b, and c from 1 to 10. These values are written in the table below. Then, the students are asked whether the values they say form a triangle or not. The answers received are added to the table. Finally, it is observed whether the sliders will form a triangle by adjusting the values that the students have said.

a	b	c	Students' explanation	GeoGebra observation

Fig. 9 Task 58 with MDL-4 and Action D (G9-MEB, p. 161)

Tetrahedral Drawing and Unfolding with GeoGebra Program



Open the GeoGebra program. Click the “Appearance” tab and then the “3D Graphics” tab that opens. Click on the 9th box in the toolbar and then on the “tetrahedral” tab that opens. Then, when you click on any two points on the plane in the "3D Graphics" window, a regular tetrahedron with the vertex D, the base ABC equilateral triangle, and each side equal in length will be seen on the screen. Note that the edge lengths are equal in the algebra window.

Click on box 9 in the toolbar and then click on the “Open Flattening” tab. Then you can see the unfolding of this pyramid by clicking on the regular tetrahedron in the “3D Graphics” window. At the same time, looking at the graphic window, the expansion of this pyramid, whose base is the equilateral triangle ABC, can be seen. If the b slider on the screen is moved, it can be understood what the angle is. Click on the 11th box in the toolbar and then on the “Angle” tab that opens. When you click on the inner parts of the triangles in the graphics window, it is seen that the angles are 60◦.

Fig. 10 Task 93 with MDL-0 and Actions A, B, and D (G10-MEB, p. 325)

5 Discussion

This study examined the mathematical and technological nature of DGS-related tasks in middle and secondary school mathematics textbooks based on the Dynamic Geometry Task Analysis Framework (Trocki & Hollebrands, 2018). This study revealed a few important findings regarding the DGS-related tasks in the textbooks.

Our first research question was related to the distribution of DGS-related tasks across the grade levels in middle and secondary level mathematics textbooks. The results showed that secondary school textbooks included more DGS-related tasks than middle school textbooks. Moreover, it is found that DGS-related tasks are more frequently included in the textbooks written for the 8th and above-grade levels. At this point, textbook authors may argue that as the age levels of the students decrease, the process of using and adapting technology becomes more difficult (Sevimli & Kul, 2015). However, the use of DGSs in mathematics education is endorsed by professional organizations (CCSSM, 2010; NCTM, 2000). Moreover, numerous researchers have claimed that the use of DGSs supports students’ mathematical learning at all levels (e.g., Chan & Leung, 2014; Erbaş & Yenmez, 2011). While Turkish mathematics teachers are highly dependent on their textbooks to teach mathematics (Ulusoy & İncikabı, 2021), it would be beneficial if the DGS-related tasks in the textbooks were more equitable for both middle and secondary school levels. According to the results, GeoGebra has been the most frequently used dynamic geometry software in DGS-related tasks. It is possible that it was preferred more frequently because it is free and has a Turkish version (Ulusoy, 2019).

The results regarding the second research question, which was about the mathematical depth level of DGS tasks in the textbooks, indicated that the percentages of tasks at the low-MDLs are remarkable for both middle and secondary level textbooks. In these tasks, students are only encouraged to use the DGSs by following the steps to make a drawing. The results showed that there are no DGS-related tasks at high-MDLs (Level 3 and above) in middle school mathematics textbooks. Secondary school textbooks included DGS-related tasks with high-MDLs. However, such tasks were limited both in number and content compared to tasks with low-MDLs. The tasks at low-MDLs, which mostly require procedural skills, should not be viewed as ineffective or unnecessary. The tasks, ranging from practicing routine skills to improving conceptual understanding, should be incorporated into curricular papers (Stein et al., 1996). Furthermore, the inclusion of tasks at the low-MDLs may be motivated by a desire to avoid the perception that the textbooks only comprise high-level DGS tasks. This might be related to the fact that textbook authors intend to better assist learners and teachers by presenting tasks at different mathematical depth levels (Ubuz et al., 2010).

In this study, it was found that there was a statistically significant difference between the mathematical depth level of DGS-related tasks in middle and secondary school mathematics textbooks in favor of secondary level textbooks. This means that DGS-related tasks in secondary school mathematics textbooks require more abstract relations and complex processes than the tasks in middle school mathematics textbooks. This significant difference in the MDLs of the tasks in middle and secondary school mathematics textbooks may reflect the textbook authors' intention regarding the need to offer more higher-level tasks to support the geometric thinking of the students (e.g., Burger & Shaughnessy, 1986), who are in secondary school, where mathematical concepts begin to be taught at an increasingly abstract level. The results of this study conflict with the results of some other studies. For example, Lew and Jeong (2014) found that while technology in Korean middle school mathematics textbooks was mainly used in a conceptual role (conjecturing, verifying, and generalizing), technology in Korean secondary school mathematics textbooks was mainly focused on using it in a technical role (mechanical or procedural). These results suggest that it might be useful to conduct comparative studies to explore trends of DGS integration in various mathematics textbooks (e.g., mandatory and non-mandatory textbooks, or print textbooks and e-textbooks).

The third research question focused on the technological actions used in DGS-related tasks in middle and secondary school textbooks. The results showed that the types and number of technological actions in DGS-related tasks were higher in secondary school textbooks than in middle school textbooks. The most frequently used technological action in the tasks was Action A, which required students to make a drawing. However, Action A reflects how DGSs may be used to simulate and mimic actions that have traditionally been done using paper, pencil, and a measuring device. The availability of technological actions provides opportunities to change the way one interacts with geometric objects. For example, the use of the slider and dragging features of DGSs allows students to explore the invariant properties of geometric concepts and make inferences and proofs about these properties (e.g., Arzarello et al., 2002; Baccaglini-Frank & Mariotti, 2010; Christou et al., 2004; Hollebrands, 2007; Laborde, 2001). For example, when students change the shape of a triangle by dragging, they can discover that the sum of the measures of the interior angles in each triangle is 180° in a DGS. In this way, students develop mathematical reasoning skills related to geometric concepts and can understand the role of proofs in conceptual learning. However, the results of this research revealed that Action D, which requires using the dynamic features of DGSs, is quite limited, especially in middle school textbooks. Similarly, Action C, which requires the construction of geometric shapes, and Action E, which requires recognizing the relationship patterns related to concepts, are very few DGS-related tasks in mathematics textbooks. Trocki and Hollebrands (2018) stated that high-quality tasks included combinations of technological actions that were coordinated with mathematical depth. In terms of the frequency of technological actions in DGS-related tasks, almost half of the tasks included only a single technological action. The tasks containing more than one technological action were mostly found in secondary school textbooks.

In this study, it was determined that the technological action scores for the tasks differed significantly according to the mathematical depth levels of the DGS-related tasks. Accordingly, the technological action scores of the tasks with high-MDLs were found to be higher than those with low-MDLs. Hence, the statistical analysis revealed that when DGS-related tasks include prompts grouped as high-MDLs, these tasks have more technological affordances. This study revealed that the use of DGSs at higher mathematical depth levels in tasks was rare in the textbooks. Based on our findings, we recommend including high-quality DGS-related tasks in textbooks. The results of this study could be used in other countries where textbooks approved by the MoNE are compulsory to check the quality of DGS-related tasks, like in the Korea.

6 Limitations of the Study and Directions for Future Research

This study includes two important limitations and related suggestions for future studies. First, we examined DGS-related tasks in compulsory mathematics textbooks at middle and secondary school levels due to the mandatory use of these textbooks in Turkey. In future studies, researchers will compare how DGS-related tasks are incorporated into mathematics textbooks. Analyzing the characteristics of integration of DGSs in the textbooks of different countries according to the theoretical framework discussed in this study (Trocki & Hollebrands, 2018) may allow comparing the mathematical and technological features of DGS-related tasks. As for the second limitation, although this study is restricted to the geometry content, similar studies can be conducted to examine the mathematical and technological properties of DGS-related tasks in other strands (i.e., algebra, number sense) in the textbooks. DGS-related tasks might be easier to find if you compare and contrast the distribution of tasks in each strand of the textbooks. This could help with the revision of the textbooks.

Our study did not focus on the relationship between the quality of DGS-related tasks and the quality of teachers' implementation of DGS-related tasks in the classroom. Although textbooks might be used as the main teaching and learning resources in the classroom (e.g., Reys et al., 2004), it is clear that teachers also have an important role. If teachers select tasks requiring high cognitive demands and maintain the demand of the tasks in the implementation process, students' understanding and reasoning increase (Stein & Lane, 1996). However, choosing high-level tasks is not enough to maintain the demands of the tasks. Some teachers may tend to systematically decrease the mathematical depth of high-level tasks in curriculum resources. Besides, certain classroom settings may easily lead to a decrease in the demand for high-level tasks. Moreover, a teacher with good teaching skills can easily improve the quality of a low-level task (Smith & Stein, 1998; Stein & Smith, 1998). Trocki and Hollebrands (2018) stated that "findings do not imply any absolute guarantee of a match between task quality and the quality of student mathematical activity" (p.135). In this way, it might be interesting to look at how teachers use math and technology together in DGS-related tasks in the classroom.

Declarations

Conflict of Interest None.

References

Aguilar, M. S., & Castaneda, A. (2020). A Foucauldian Analysis of Representations of Mathematicians in Lower Secondary Mexican Mathematics Textbooks. *International Journal of Science and Mathematics Education, 18*(4), 753–770.

Arzarello, F., Olivero, F., Paola, D., & Robutti, O. (2002). A cognitive analysis of dragging practices in Cabri environments. *ZDM: The International Journal on Mathematics Education, 34*(3), 66–72.

Baccaglini-Frank, A., & Mariotti, M. (2010). Generating conjectures in dynamic geometry: The maintaining dragging model. *International Journal of Computers for Mathematical Learning, 15*(3), 225–253.

Bayazit, I. (2013). Quality of the tasks in the new Turkish elementary mathematics textbooks: The case of proportional reasoning. *International Journal of Science and Mathematics Education, 11*(3), 651–682.

Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education, 17*(1), 31–48.

Cayton, C. (2012). Examining the cognitive demand of tasks in three technology intensive high school Algebra 1 classrooms. In L. R. Van Zoest, J.-J. Lo, & J. L. Kratky (Eds.), *Proceedings of the Thirty-Fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 865–868). Western Michigan University.

CCSSM. (2010). Common core state standards for mathematics. Washington, DC: National Governors' Association Center for Best Practices, Council of Chief State School Officers.

Chan, K. K., & Leung, S. W. (2014). Dynamic geometry software improves mathematical achievement: Systematic review and meta-analysis. *Journal of Educational Computing Research, 51*(3), 311–325.

Christou, C., Mousoulides, N., Pittalis, M., & Pitta-Pantazi, D. (2004). Proofs through exploration in dynamic geometry environments. *International Journal of Science and Mathematics Education, 2*(3), 339–352.

de Villiers, M. (1998). An alternative approach to proof in dynamic geometry. In R. Lehrer & D. Chazan (Eds.), *New directions in teaching and learning geometry* (pp. 369–393). Lawrence Erlbaum Associates.

Erbas, A. K., Alacaci, C., & Bulut, M. (2012). A comparison of mathematics textbooks from Turkey, Singapore, and the United States of America. *Educational Sciences: Theory and Practice, 12*(3), 2324–2329.

Erbas, A. K., & Yenmez, A. A. (2011). The effect of inquiry-based explorations in a dynamic geometry environment on sixth grade students' achievements in polygons. *Computers & Education, 57*(4), 2462–2475.

Fan, L. (2013). Textbook research as scientifc research: Towards a common ground on issues and methods of research on mathematics textbooks. *ZDM-International Journal on Mathematics Education, 45*(5), 765–777. https://doi.org/10.1007/s11858-013-0530-6

Grouws, D. A., Smith, M. S., & Sztajn, P. (2004). The preparation and teaching practices of United States mathematics teachers: Grades 4 and 8. In P. Kloosterman & F. K. Lester (Eds.), *Results and Interpretations of the 1990 through 2000 Mathematics Assessment of the National Assessment of Educational Progress* (pp. 221–267). NCTM.

Grouws, D. A., Tarr, J. E., Chávez, Ó., Sears, R., Soria, V. M., & Taylan, R. D. (2013). Curriculum and implementation effects on high school students' mathematics learning from curricula representing subject-specific and integrated content organizations. *Journal for Research in Mathematics Education, 44*(2), 416–463.

Heid, M. K., & Blume, G. W. (Eds.). (2008). *Research on technology and the teaching and learning of mathematics: Syntheses, cases, and perspectives*. Vol. 1: Research syntheses. Charlotte, NC: Information Age.

Hollebrands, K. (2007). The role of a dynamic software program for geometry in the strategies high school mathematics students employ. *Journal for Research in Mathematics Education, 38*(2), 164–192.

Hollebrands, K. F., & Dove, A. (2011). Technology as a tool for creating and reasoning about geometry tasks. In T. P. Dick & K. F. Hollebrands (Eds.), *Focus in high school mathematics: Technology to support reasoning and sense making* (pp. 33–52). Reston, VA: NCTM.

Hölzl, R. (2001). Using dynamic geometry software to add contrast to geometric situations: A case study. *International Journal of Computers for Mathematical Learning, 6*(1), 63–86.

Jones, D., Hollas, V., & Klespis, M. (2016). The presentation of technology for teaching and learning mathematics in textbooks: Content courses for elementary teachers. *Contemporary Issues in Technology & Teacher Education, 17*(1), 53–79.

Ju, M. K., Moon, J. E., & Song, R. J. (2016). History of mathematics in Korean mathematics textbooks: Implication for using ethnomathematics in culturally diverse school. *International Journal of Science and Mathematics Education, 14*(7), 1321–1338.

Kajander, A., & Lovric, M. (2009). Mathematics textbooks and their potential role in supporting misconceptions. *International Journal of Mathematical Education in Science and Technology, 40*(2), 173–181.

Krippendorff, K. (2013). *Content analysis: An introduction to its methodology* (3rd ed.). Sage.

Laborde, C. (2001). Integration of technology in the design of geometry tasks with CabriGeometry. *International Journal of Computers for Mathematical Learning, 6*(3), 283–317.

Lew, H. C., & Jeong, S. Y. (2014). Key factors for successful integration of technology into the classroom: Textbooks and teachers. *Electronic Journal of Mathematics & Technology, 8*(5), 336–354.

Mariotti, M. (2012). Proof and proving in the classroom: Dynamic geometry systems as tools of semiotic mediation. *Research in Mathematics Education, 14*(2), 163–185.

Mersin, N., & Karabörk, M. A. (2021). The comparison of math textbooks in turkey and singapore in terms of technology ıntegration. *International Journal of Curriculum and Instruction, 13*(1), 552–573.

Ministry of Education Textbooks and Education Tools Regulation. (2012). T.C. Resmi Gazete (28409, 12 Eylül 2012). Retrieved fromhttps://www.resmigazete.gov.tr/eskiler/2012/09/20120912-2.htm. Accessed 27 April 2022.

Ministry of National Education [MoNE] (2018a). Mathematics teaching program (Primary and Middle School Grades 1, 2, 3, 4, 5, 6, 7 and 8) MEB: Ankara.

Ministry of National Education [MoNE] (2018b). Mathematics teaching program (Secondary School Grades 9, 10, 11, and 12), MEB: Ankara.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.

Otten, S., Gilbertson, N. J., Males, L. M., & Clark, D. L. (2014). The mathematical nature of reasoning-and-proving opportunities in geometry textbooks. *Mathematical Thinking and Learning, 16*(1), 51–79. https://doi.org/10.1080/10986065.2014.857802

Pea, R. D. (1987). Cognitive technologies for mathematics education. In A. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 89–122). Lawrence Erlbaum Associates Inc.

Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French and German classrooms. *ZDM, 33*(5), 158–175.

Reys, B. J., Reys, R. E., & Chavez, O. (2004). Why Mathematics Textbooks Matter. *Educational Leadership, 61*(5), 61–66.

Roth, K., & Givvin, K. B. (2008). Implications for math and science ınstruction from the TIMSS 1999 Video Study. *Principal Leadership, 8*(9), 22–27.

Sevimli, E., & Kul, Ü. (2015). Evaluation of the contents of mathematics textbooks in terms of compliance to technology: Case of secondary school. *Necatibey Faculty of Education Electronic Journal of Science & Mathematics Education, 9*(1), 308–331.

Sherman, M. (2014). The role of technology in supporting students' mathematical thinking: Extending the metaphors of amplifier and reorganizer. *Contemporary Issues in Technology and Teacher Education, 14*(3), 220–246.

Sherman, M. F., Cayton, C., Walkington, C., & Funsch, A. (2020). An analysis of secondary mathematics textbooks with regard to technology integration. *Journal for Research in Mathematics Education, 51*(3), 361–374.

Sherman, M. F., Walkington, C., & Howell, E. (2016). A comparison of symbol-precedence view in investigative and conventional textbooks used in algebra courses. *Journal for Research in Mathematics Education, 47*(2), 134–146.

Sinclair, M. (2003). Some implications of the results of a case study for the design of pre-constructed, dynamic geometry sketches and accompanying materials. *Educational Studies in Mathematics, 52*(3), 289–317.

Sinclair, M. (2004). Working with accurate representations: The case of pre-constructed dynamic geometry sketches. *The Journal of Computers in Mathematics and Science Teaching, 23*(2), 191–208.

Smith, M. S., & Stein, M. K. (1998). Reflections on practice: Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*(5), 344–350.

Stein, M. K., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50–80.

Stein, M., Grover, B., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488

Stein, M., Remillard, J., & Smith, M. (2007). How curriculum influences student learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 319–369). Information Age Publishing.

Stein, M., & Smith, M. (2010). The influence of curriculum on students' learning. In B. J. Reys, R. E. Reys & R. Rubenstein (Eds.), *Mathematics curriculum. Issues, trends, and future directions* (pp. 351362). Reston: National Council of Teachers of Mathematics.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268–275.

Stylianides, G. (2008). An analytic framework of reasoning-and-proving. *For the Learning of Mathematics, 28*(1), 9–16.

Tabachnick, B. G., & Fidell, L. S. (2013). *Using multivariate statistics*. Pearson.

Tarr, J. E., Grouws, D. A., Chávez, Ó., & Soria, V. M. (2013). The effects of content organization and curriculum implementation on students' mathematics learning in second-year high school courses. *Journal for Research in Mathematics Education, 44*(4), 683–729. https://doi.org/10.5951/jresematheduc.44.4.0683

Thompson, D. R., Senk, S. L., & Johnson, G. J. (2012). Opportunities to learn reasoning and proof in high school mathematics textbooks. *Journal for Research in Mathematics Education, 43*(3), 253–295. https://doi.org/10.5951/jresematheduc.43.3.0253

Trocki, A., & Hollebrands, K. (2018). The development of a framework for assessing dynamic geometry task quality. *Digital Experiences in Mathematics Education, 4*(2), 110–138.

Ubuz, B., Erbas, A. K., Çetinkaya, B., & Özgeldi, M. (2010). Exploring the quality of the mathematical tasks in the new Turkish elementary school mathematics curriculum guidebook: The case of algebra. *ZDM Mathematics Education, 42*(5), 483–491.

Ulusoy, F. (2019). Prospective mathematics teachers' geometric constructions using compass-straightedge and dynamic geometry software. *Turkish Journal of Computer and Mathematics Education,10*(2), 336–372.

Ulusoy, F. & İncikabı, L. (2020). Middle school teachers' use of textbooks in instruction of mathematics. *International Journal for Mathematics Teaching and Learning*, *21*(1), 1–18.

Ulusoy, F. & İncikabı, L. (2021). Preservice mathematics teachers' selection of curriculum resources in individual and group lesson planning processes, *International Journal of Mathematics Education in Science and Technology*. https://doi.org/10.1080/0020739X.2021.1958944.

Van Zanten, M., & van den Heuvel-Panhuizen, M. (2018). Opportunity to learn problem solving in Dutch primary school mathematics textbooks. *ZDM Mathematics Education, 50*(5), 827–838.

Weninger, C. (2018). Textbook analysis. In Chapelle, C.A. (Ed.), *The Encyclopedia of Applied Linguistics.* Hoboken, NJ: Wiley & Sons. https://doi.org/10.1002/9781405198431.wbeal1489

Zbiek, R., Heid, K., Blume, G., & Dick, T. (2007). Research on technology in mathematics education. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1169–1207). Information Age Publishing.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.