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Effect of use of GeoGebra on achievement of high school mathematics students

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Abstract

A quasi-experiment was conducted to determine the effects of using the software GeoGebra as teaching aid on the achievement of Form Two students. The experimental study was conducted on 80 Form Two students, 40 of whom comprised the treatment group and 40 formed the control group. Data were analysed using the software ANATES 4 and SPSS 24.0. Significant differences were found in student achievements in relation to the topics of functions and limit functions according to their group type. Research outcomes also showed that teachers and students approved the use of GeoGebra in the teaching and learning of mathematics. GeoGebra can illustrate mathematical concepts and procedures well through visuals and graphs, which considerably aid students in mastering and understanding concepts and procedures pertaining to functions and limit functions. This software is user-friendly and can relieve the teachers' burden in explaining functions. Even though the use of GeoGebra is time consuming, teaching by using the software can render the students' learning process increasingly active. Moreover, learning mathematics with the help of GeoGebra allows for an active interaction between teachers and students. This study provides suggestions as interventions for increasing student achievement.

Keywords Acceptance · Achievement · Function and limit function · GeoGebra · Quasi-experiment · Traditional method

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1 Introduction

A promising way to encourage students to understand mathematical concepts is to use information and communication technologies (ICT) in the teaching and learning process. When applying new technology, students encounter distinct learning environments and cognitive barriers will change. A meta-analysis by Fabian et al. (2016) found that the use of technology for learning produced a random effect size of 0.48 for elementary school students and a positive effect for secondary school students; however, mixed results were obtained. Therefore, the use of technology in mathematics education has been emphasised by numerous researchers to enhance students' mathematics achievement in different topics (Eyyam and Yaratan 2014; Fabian et al. 2016; Higgins et al. 2017). However, Cheung and Slavin (2013) indicated that the effects of technology application differ and depend on the sort of educational technology adopted in the classroom. Hence, educators should be wise in selecting the latest technology and appropriate topics in the classroom.

A feature of technology-based learning is its capability to provide support services for a diverse set of students through the free sharing of ideas and a certain learning focus. One tool for such idea sharing is the learning software GeoGebra. This software was developed by Markus Hohenwarter in 2001 at Salzburg University and funded by the Florida Atlantic University. Most GeoGebra users contribute valuable ideas and tips through a Wiki website, the content of which may be supplemented and modified by users through GeoGebra. This software is a learning aid for geometry, algebra and calculus, which are displayed simultaneously through windows of texts and graphics. Many previous studies have shown that GeoGebra can be adopted in teaching and learning mathematics. Hohenwarter et al. (2008) reported that GeoGebra was useful in teaching calculus. Haciomeroglu et al. (2009) demonstrated that using GeoGebra could facilitate the teaching of geometry, algebra and calculus. Rincon (2009) also investigated the use of GeoGebra in teaching geometry, algebra and calculus and confirmed that GeoGebra could clearly explain the sub-topics of the three subjects. This research finding aligned with an article written by Antohe (2009), which explained that GeoGebra was efficient and could be used to explore arithmetic, geometry, algebra and calculus. This software allows students to gain an improved understanding of abstract concepts and to correlate various mathematical concepts with their personal experiences. The features of the software, such as its Menu and Construction Tools, are user-friendly. Teachers or users can personally explore the software and independently apply it in mathematics. Antohe (2009) also shared his experience in using the GeoGebra software and provided tips for mathematics teachers in applying the GeoGebra software as a new and creative teaching approach.

GeoGebra is the best software amongst extensive software packages tailored for the study of calculus. Integrating such fundamental software into teaching calculus, algebra and geometry is deemed favourable. Given that this free software can be easily downloaded by students, it can be used as an inexpensive, computer-based homework for students. Other support materials for the software are readily available, extensive and effective as teaching resources. In particular, GeoGebra can be adopted in teaching and learning geometry, algebra and calculus (Antohe 2009; Haciomeroglu et al. 2009; Hutkemri and Zakaria 2010; Rincon 2009). The use of GeoGebra in teaching and learning mathematics is one method of creating a meaningful learning experience for teachers and students. Most people agree that the software is best used to explain various concepts and procedures in mathematics through the use of graphics, visuals, images and symbols. GeoGebra is also an interactive software that interacts with students in a two-way manner. This software is student-centred and thus enables students to control their own learning pace through sample questions, practices/ exercises and steps for solving mathematical problems. GeoGebra provides detailed steps and guidance on concepts and calculation in the process of helping students master every topic. Students or users are given sufficient time to answer the questions in the exercises.

2 Literature review

Calculus is a challenging subject for students (Tall 1996). Extensive research has revealed that teaching and learning limits (Denbel 2014; Nasab et al. 2014) and limit functions (Zulnaidi and Oktavika 2018; Zulnaidi and Zamri 2017) pose difficulties in comprehending the basic concepts underpinning calculus for secondary school (Zulnaidi and Oktavika 2018) and even for higher education students (Denbel 2014). In Indonesia, teachers also have difficulties in teaching the concept due to their limitations on mastering the subject (Desfitri 2015). Previous studies have emphasised that students have difficulty regarding limit concepts, which involve isolated facts, routine calculations and memorising algorithms and procedures; moreover, their conceptual understanding of limits, continuity and infinity is inadequate. Winarso and Toheri (2017) revealed that 40.38% of Indonesian high school students do not understand the concept of limit functions because of incorrect mathematical operations, inadequate procedural knowledge and erroneous conclusions. The fundamental reason for such finding may be the inappropriate and weak mental links between knowledge of limits and that of other calculus concepts, such as continuity, derivatives and integrals (Denbel 2014), or even because of incorrect assumptions in understanding lessons (Zulnaidi and Oktavika 2018). Therefore, teachers need to help these students by understanding their difficulties and mathematical skills in terms of limits and limit functions.

One of the best methods of enhancing student achievements in various mathematical topics is the use of technology in teaching and learning (Eyyam and Yaratan 2014; Fabian et al. 2016; Higgins et al. 2017), and GeoGebra is an example of such technology (Arbain and Shukor 2015; Bakar et al. 2015; Saha et al. 2010; Zulnaidi and Oktavika 2018; Zulnaidi and Zamri 2017). This free and platform-independent software, which can be downloaded from www.geogebra.org, assists educators in describing mathematical concepts and procedures by providing mathematical representations (symbols and graphs), particularly for geometry, algebra, calculus (Antohe 2009; Hutkemri and Zakaria 2010), graphing, spreadsheets, statistics, regressions, matrices, complex numbers, differential equations and programming (Hall and Lingefjärd 2017). Moreover, this software has different constituents which work together in a dynamic, interactive and intuitive manner.

Zulnaidi and Zamri (2017) regarded the use of GeoGebra in the mathematical classroom as an approach to creating a valuable learning environment. Therefore, educational researchers have suggested the application of GeoGebra to improve students' mathematics achievement. To date, various studies have investigated the effect of this software on students' mathematics outcomes (Arbain and Shukor 2015; Bakar et al. 2015; Martinez 2017; Saha et al. 2010; Zulnaidi and Oktavika 2018; Zulnaidi and Zamri 2017). Extant research has also verified that using GeoGebra as a tool in all kinds of mathematics (including mathematical modelling) can assist in solving real-life situations and answering interesting and difficult problems (Hall and Lingefjärd 2017). Takaci et al. (2015) found that students who used the GeoGebra package could check whether each step in the process of solving a task was correctly done or not, and they usually worked collaboratively. Furthermore, with use of GeoGebra, a real-world problem scenario is well modelled and interpreted via a simulator; the need for algebraic symbolic manipulations can be eliminated (Dayi 2015). This software not only improves students' mathematics results but can also enhance teachers' attitudes towards proof and proving (Zengin 2017).

3 Methodology

3.1 Research design

This quasi-experimental study used non-equivalent pretest and posttest control group designs. GeoGebra was introduced and used by the teachers and students for 20 h over 4 weeks. In the early stages of teaching, teachers used GeoGebra to explain the topic of functions. Students in the control group were taught using the traditional method whilst their counterparts in the treatment group utilised GeoGebra. After completing the session of teaching and learning using GeoGebra, interviews were conducted with teachers and student for 15 min per participant. The interview was meant to identify their perceptions of the use of GeoGebra in teaching and learning the topic of mathematical functions.

3.2 Participants

The sample consisted of students from one high school in Riau, Indonesia. A total of 80 students were involved in this study. Forty students were assigned to the treatment group, and the 40 other students were assigned to the control group. The same teachers taught both groups. Moreover, the use of GeoGebra in this study involved two teachers who were selected and trained to teach the topic of functions to both groups. This research used the qualitative method to determine the teachers' and students' perceptions of the use of GeoGebra in teaching and learning the topic of mathematical functions. The qualitative method involved two teachers and four secondary school students selected by purposive sampling. The two teachers chosen taught the treatment and control groups. Students were selected according to mathematical abilities: high ability (one student), moderate ability (two students) and low ability (one student).

3.3 Measurement

Ten items (questions) in the instrument were used to measure the students' achievement in relation to functions and limit functions. The pilot study was conducted on 60 Form

Three students to determine the validity of discrimination and the difficulty index of each question. The software Anates was utilised to determine the discriminant index and the difficulty level index of the achievement test. The discriminant index of each item (for testing the achievement) ranged from 35.94% to 76.56%, which indicated a good level of the discriminant index for each item. Therefore, the difficulty and discriminant indexes of the questions were balanced and near perfect (To 1996). The reliability value of the questions for testing the achievement regarding functions was .83, and the corresponding achievement for limit functions was .82, which indicated good levels of reliability of the questions (Lim 2007).

3.4 Data analysis

Research data were analysed using the software SPSS 23.0. Descriptive analysis involving frequency, percentage and mean was used to identify any difference in the achievements of the experiment and control groups. ANCOVA was performed for inferential analysis. Then, interview questions were validated by three experts to measure the teachers' and students' perceptions of the use of GeoGebra in teaching and learning mathematical functions.

4 Results

4.1 Difference in achievements regarding the topic functions between the treatment and control groups

With the pretest as the covariate, ANCOVA was performed to identify the difference between the achievements of the treatment and the control groups. Prior to ANCOVA, certain requirements for the test needed to be met, such as the normality and homogeneity of the variance between the groups. The normality test showed that the Kolmogorov–Smirnov values for the pretest and posttest achievements of the treatment and the control groups were (.089, .134), respectively. For the posttest achievement, the treatment and the control groups yielded (−.106, .105), respectively, which were more than .05 and therefore significant values. Hence, the normality requirement was satisfied. Pallant (2007) stated that data are considered normal if the significant value of the Kolmogorov–Smirnov is more than .05. Levene's test was performed with values of F = .195, sig = .660 ($p > .05$) to determine the homogeneity of variance between the groups. Data were found to have similar variance between the groups. Therefore, ANCOVA could be performed to identify the difference in the achievements between the treatment and the control groups, as shown in Table 1.

The ANCOVA test results (Table 2) showed a significant difference in the students' achievements regarding functions between the treatment and the control groups with the pretest as a covariate [F (1,77) = 12.030, sig = .001 ($p < .05$)]. However, students in the treatment group (mean = 74.69) obtained higher achievement than those in the control group (mean = 69.94) (Table 1). Therefore, GeoGebra had a better effect on the increase in the student achievement regarding functions compared with the traditional method. However, such differential effect was small (partial eta square = .083) (Cohen 1988).

Table 1 Descriptive statistics of student achievement regarding functions between the treatment and control groups

Group	N	Mean	Std. deviation
Treatment	40	74.69	9.25
Control	40	69.94	10.15
Total	80	72.31	9.94

4.2 Difference of achievements regarding the topic limit functions between the treatment and control groups

The normality test showed that the Kolmogorov–Smirnov values for the pretest and posttest achievement in terms of limit functions for the treatment and control groups were (.103, .131), respectively. For the posttest achievement regarding limit functions, the treatment and the control groups had (−.112, .141), respectively, which were more than .05 and therefore significant values. Thus, the normality requirement was satisfied. Levene's test was performed with values of F = 1.084, sig = .301 ($p > .05$) to determine the homogeneity of variance between the groups. Data were found to have similar variance between the groups. Therefore, ANCOVA could be performed to identify the difference in the achievements regarding limit functions between the treatment and the control groups, as shown in Table 3.

The ANCOVA test results (Table 4) showed a significant difference in the limit function achievements between the treatment and the control groups with the pretest as a covariate [$F\ (1,77) = 9.030$, sig = .004 ($p < .05$)]. Students in the treatment group (mean = 76.50) obtained higher achievement than those in the control group (mean = 71.94) (Table 3). Therefore, GeoGebra had a better effect on the increase in the student achievement regarding limit functions compared with the traditional method. However, such differential effect was small (partial eta square = .105) (Cohen 1988).

Table 2 ANCOVA: difference of achievements regarding functions between the treatment and control groups

Source	Type III sum of squares	df	Mean square	F	Sig.	Partial eta squared
Corrected Model	1445.545[a]	2	722.772	8.745	.000	.185
Intercept	78,154.879	1	78,154.879	945.599	.000	.925
Pre	994.295	1	994.295	12.030	.001	.135
Group	572.376	1	572.376	6.925	.010	.083
Error	6364.143	77	82.651			
Total	426,137.500	80				
Corrected Total	7809.688	79				

Table 3 Descriptive statistics of student achievements regarding limit functions between the treatment and control groups

Group	N	Mean	Std. deviation
Treatment	40	76.50	6.74
Control	40	71.94	7.63
Total	80	74.22	7.51

4.3 Teachers' acceptance of the use of GeoGebra software in the teaching and learning of mathematical functions

The first teacher taught 20 students from the experimental group and 20 from the control group. This teacher's acceptance of teaching and learning using GeoGebra was positive in general. They argued that GeoGebra prioritises graphing and provides an active learning situation in the classroom. Interview responses from the first teacher are as follows.

Teacher 1:

> My job in explaining functions becomes easier when using GeoGebra in teaching, especially in creating graphics. I do not need a ruler or pen to explain how to create function graphics. By using GeoGebra, I can create graphics by putting the linear function into the input, and the software automatically creates the graphic from the given function. In addition, students become more active in learning about mathematical functions when we use GeoGebra. They also have an independent learning process when they use the software and become active in expressing their ideas freely as the software allows them to instil their ideas into it. I did not encounter any major problem when using GeoGebra in my teaching process. The only matter of concern is that we need a long time to become accustomed to applying GeoGebra in the teaching and learning of mathematical functions. This [circumstance] is because teachers and students must master two skills simultaneously, namely, the use of GeoGebra in the teaching and learning of mathematics and the achievement of the learning objective.

Table 4 ANCOVA: difference of achievements regarding limit functions between the treatment and control groups

Source	Type III sum of squares	df	Mean square	F	Sig.	Partial eta squared
Corrected Model	687.003[a]	2	343.502	7.015	.002	.154
Intercept	64,818.533	1	64,818.533	1323.733	.000	.945
Pre	270.675	1	270.675	5.528	.021	.067
Group	442.176	1	442.176	9.030	.004	.105
Error	3770.419	77	48.966			
Total	445,131.250	80				
Corrected Total	4457.422	79				

The second teacher also taught 20 students from the experimental group and 20 from the control group. Overall, this teacher's acceptance of teaching and learning using GeoGebra was also good. They stated that GeoGebra prioritises graphing and can motivate students in mathematical learning. Answers from the second teacher's interview are as follows.

Teacher 2:

> I think GeoGebra is interesting. The use of this software can encourage students to be increasingly active in the learning process. Students are given the opportunity to express their ideas and input them into the software. Furthermore, an effective study environment can be achieved. The motivations of the students are also enhanced as they can try many things using GeoGebra in explaining functions. For instance, they can give an example of a function that is based on their daily lives. Students can provide that example using GeoGebra. I did not encounter any problem in using this software in teaching mathematical functions. The only problem that I had was that students lack time in calculating and visualising function graphics manually. When they created graphics, they focused more on creating the graphics through GeoGebra than on creating them manually. However, this [situation] is not a major problem because when they practice the creation of graphics, they can do so manually. All in all, I am interested and agree that GeoGebra should be used in my teaching and learning process.

4.4 Students acceptance of the use of GeoGebra in the teaching and learning of mathematical functions

Students with a variety of mathematical abilities had the same acceptance of the use of GeoGebra in teaching and learning mathematics. They claimed that the use of GeoGebra can illustrate the concept of functions clearly and accurately represent function graphs. Additionally, the students reported having no problems in their use of the software.

Student 1 (High ability):

> Learning by using GeoGebra is interesting. However, students need a long time in learning whilst using the software. Nevertheless, students can easily acquire knowledge by using GeoGebra because it provides direct visuals and enables the drawing of lines. The software helps students identify the dots in the produced graphics, and they can easily input pictures into the software. GeoGebra and the traditional method have their benefits. Teachers who use GeoGebra in their teaching and learning process will have an interesting class because it integrates the use of technology, which has not been accomplished to date. By contrast, in the traditional method, students must solve problems without help from technology.

Student 2 (Moderate ability):

Learning by using GeoGebra is easy and interesting. We are now in the globalisation era. Instead of being good in mathematics alone, we must master technology too. Using GeoGebra in learning mathematics is easy. The use of the software can directly visualise diagrams shaped by the function concept. Students' understanding can also be increased by using GeoGebra. Apart from receiving what the teacher provides in class, students can learn and apply the knowledge until they can memorise it within a sufficient time. By contrast, learning by using the traditional method renders the process monotonous. Students receive whatever materials are given by teachers without the effort to apply and memorise them.

Student 3 (Moderate ability):

Learning by using GeoGebra is good. Students will become accustomed to the use of technology in the modern era. The software is special because it provides an application through which the students can solve problems in a short time. They do not have to calculate manually, which is time consuming. Teachers can also save time in explaining the materials given because students can understand them by using the software. GeoGebra and the traditional method are both interesting. However, the use of GeoGebra is a new and easy practice for students to explore.

Student 4 (Low ability):

Learning by using GeoGebra can help students understand functions easily. Amongst the [notable] facilities is the easy manner of creating function graphics. Learning by using the software is clear in terms of creating graphics, whereas learning by using the traditional method is not as clear. I encountered a problem when I used the software for the first time because I was not familiar with it. However, it can be learned in a short time because GeoGebra is user-friendly. I hope that the learning process of mathematics involving the use of GeoGebra can be maintained.

5 Automatic question and answer of mathematical functions using GeoGebra

The students were given two questions to show the concepts and procedures in the mathematical topic of functions associated with their daily lives. The students had the freedom to use their own ideas in using GeoGebra to explain the concepts and procedures of function topics in daily life. Teachers in this section acted as facilitators who controlled the students' work in meeting the desired concepts and procedures. By using GeoGebra, the students provided examples of concepts from function topics, such as the process of making coffee milk drinks. In this part, teacher acts as a facilitator to ensure that each student follows the steps given to produce the concept and graph for the topic of function. Steps to describe the concept by applying GeoGebra are as follows.

Question 1: Explain the concept of composition function using GeoGebra software?

Answer: The students are use the GeoGebra software to answer the question and teacher also acts as a facilitator to ensure that students use appropriate steps as a bellow.

5.1 Input

Find the tool Insert Image in GeoGebra. Click anywhere in the drawing pad to place the image. You can place the image at a point, like a point on the ladder. After clicking on the point, choose an image from your files. You can also drag an image from your computer to the drawing pad of GeoGebra.

5.1.1 Process

Step 1. Eliminate axes in the GeoGebra software view because the axes are not required in explaining the above functions. Actions: Under Menu, select View and then click the Axes button.
Step 2. Place the picture of hot water, sugar and coffee that is stored in the file. Actions:

(i) Click on Insert Image.
(ii) Find where the image file is stored (Desktop).
(iii) Click once on the image to be inserted.
(iv) Click Open.

Step 3. Name the image generated in GeoGebra. Actions:

(i) Under Construction Tools, click on Insert Text and a text box will appear.
(ii) Enter the image name into the text box.
(iii) Click Ok.

Step 4. Insert the arrow icon as a process indicator of input, engine and output by clicking the Construction Tools section and by then clicking on Vector between Two Points.
Step 5. Enter a picture of the arrow as a pointer to the process from input, machine and output by clicking on the Construction Tools section and clicking on Vector between Two Points.

These steps were also used by the students to explain other function concepts, such as the process of cooking omelettes, baking cakes and producing traditional medicine. One of the results shown by the students using GeoGebra to explain the concept of functions is as follows.

5.2 Product

The result from steps above as a bellow (Fig. 1):

Inserted pictures on GeoGebra obtained from the following:

http://www.austfood.com/shop/coffee/coffee-powder-instant
https://www.gta5-mods.com/scripts/hot-coffee
http://www.vegetariansrecipes.org/category.php?page=3&category=drink
http://i.huffpost.com/gen/1432571/images/o-SUGAR-CUBE-facebook.jpg

Students also demonstrated proficiency in answering procedural questions for mathematical function topics such as creating graphs of types of functions, including linear, constant, identity, modulus and quadratic functions. The steps are as follows.

Question 2: Paint the equation of $f(x) = x^2 - 4x$

Answer: The students answer question with the following steps. Teachers escort the students to ensure student activity using the correct steps.

5.3 Input

In the  Algebra View you can directly enter algebraic expressions using the integrated Input Field (GeoGebra Web and Tablet Apps) or the Input Bar at the bottom

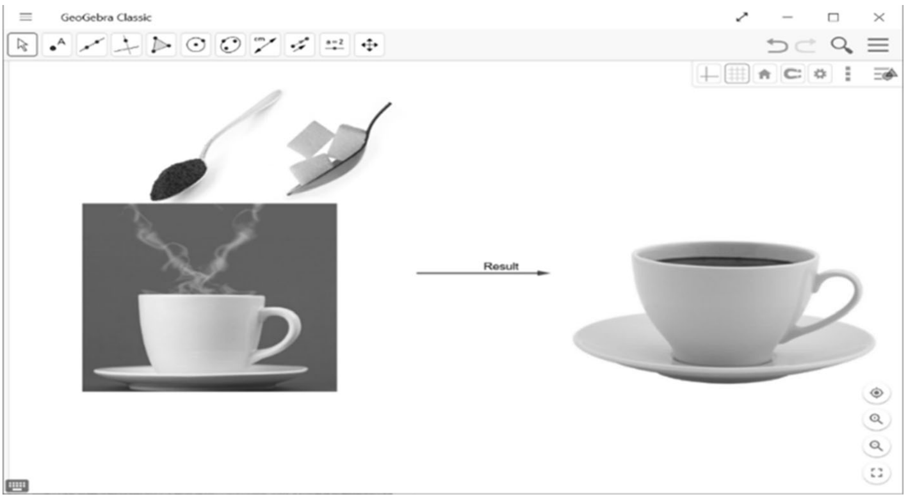

Fig. 1 Concept of function

of the GeoGebra window (GeoGebra Desktop). After hitting the Enter key your algebraic input appears in the Algebra View while its graphical representation is automatically displayed in the Graphics View. Example: The input $F(x) = x^2 - 4x$ gives you the linear equation in the Algebra View and the corresponding line in the Graphics View.

5.4 Process

Step 1: Under Construction Tools, click on Axes and Grid.
Step 2: Input the equation $y = f(x) = x^2 - 4x$ into the Input, and press Enter.
Step 3: Click on Move and New Point to make a point on the graphs generated by GeoGebra.
Step 4: Name the line. Actions:

a. Right-click the mouse so that the picture appears as the picture on the side.
b. Click on Setting so that Box setting will appear.
c. Under the Show Label section, select Name & Value.
d. Click Close.

5.5 Product

The result from steps above as a bellow (Fig. 2):

This study found that all students could use GeoGebra to explain procedures, especially in describing graph functions.

Question 3: Find the answer of situation bellow.

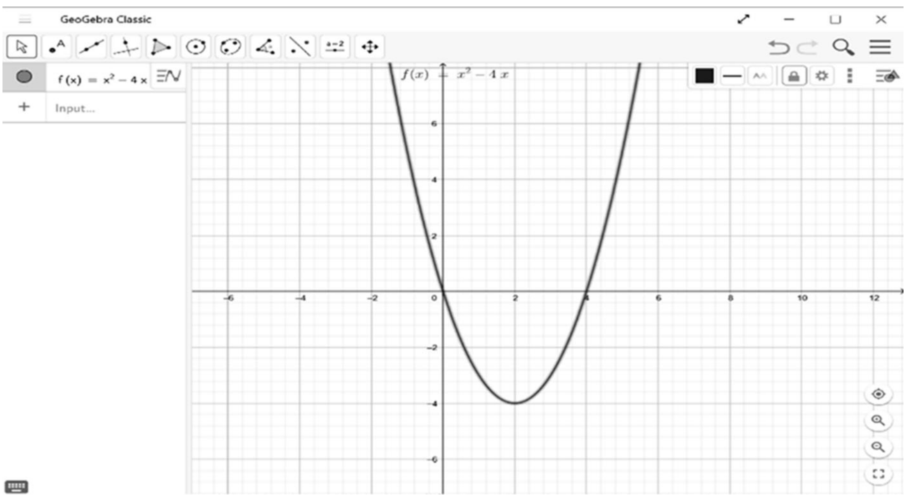

Fig. 2 Procedure of function

"Stairs are mounted on the wall at point A. Given the angle between the horizontal floor and the stairs is 60^0. The horizontal distance from the wall to the stairs is 3m. Find the high of point A from that floor"

Answer: The students answer the question with the following steps. Teachers escort the students to ensure students activity using the correct steps.

5.6 Input

Line  angle , intersect  and perpendicular line  were used to create of triangle. To make sure the answer of the question is correct, in the input, add the formula of Tangent.

5.7 Process

Step 1. Eliminate axes in the GeoGebra software view. Actions: Under Menu, select hide the grid button.
Step 2. Select line and build A to B line
Step 3. Select perpendicular line and press point B and A to B line.
Step 4. Select angle with given size and press point B and point A. Add 60^0 size angle and press OK
Step 5. Select line and press A and B^1point.
Step 6. Select Intersect and press in cross AB^1 line and perpendicular line
Step 7. Select Segment and press in CB line, BA line and AC line.
Step 8. Hide and rename the line. Right Click in the line and press show object or Rename the line. Change name of the line to Opposite (O), Hypotenuse (H) and Adjacent (A).
Step 9. Add the formula of Tangent in part of Input. Press ENTER and Check your answer.
Step 10. Select distance length to check the distance between point of line.

5.8 Product

The result from steps above as a bellow (Fig. 3):

This study found that all students could use GeoGebra to explain procedures, especially in describing of angle.

Question 4: If length of BC is 9 cm, find the length of OC (Fig. 4).

Answer: The students answer the question with the following steps. Teachers escort the students to ensure students activity using the correct steps.

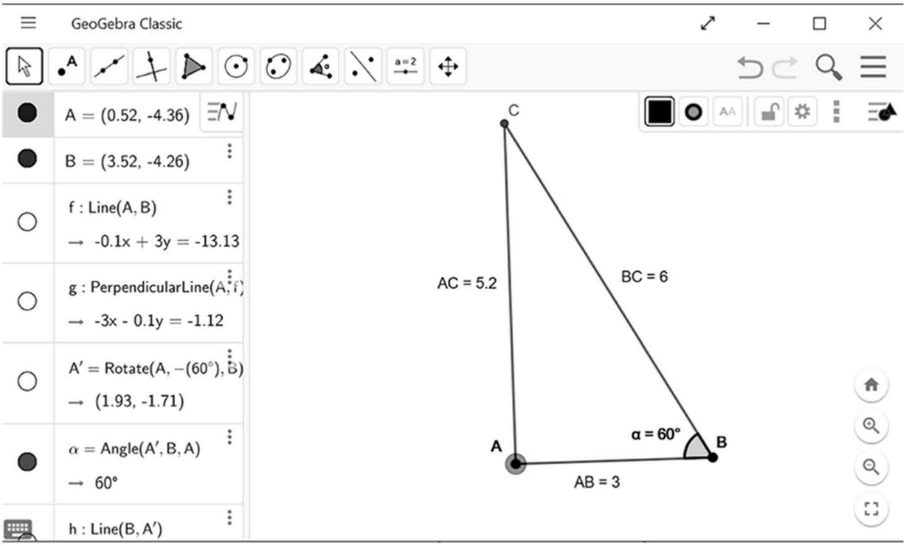

Fig. 3 Procedure of trigonometry

5.9 Input

Line, angle, segment and circle with centre though point were used to create of triangle and circle.

5.10 Process

Step 1. Eliminate axes in the GeoGebra software view. Actions: Under Menu, select hide the grid button.
Step 2. Select Circle with Center through Point and one point (C) in the line of Circle

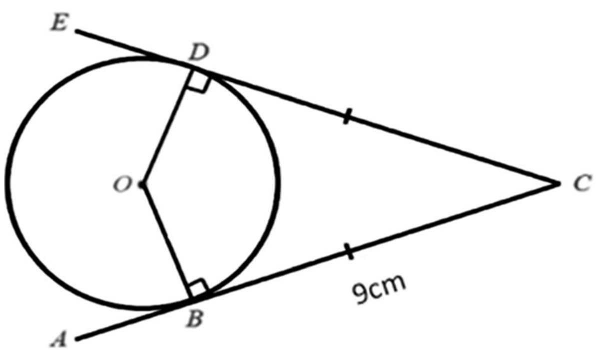

Fig. 4 Properties of tangent

Step 3. Select Tangents and press in point B and line of circle and point C and Line of Circle
Step 4. Manipulate the line of Tangents (make sure intersect)
Step 5. Select segment and press point AB, AC, BD, CD and AD.
Step 6. Select angle and press point DCA, ABD, CAB and CAD.
Step 7. Move it

5.11 Product

The result from steps above as a bellow (Fig. 5):

This study found that all students could use GeoGebra to explain procedures, especially in describing of properties of tangent.

Question 5: The use of GeoGebra software in Science, Technology, Engineering and Mathematics (STEM)

Answer: The students answer the question with the following steps. Teachers escort the students to ensure students activity using the correct steps.

5.12 Input

Insert image and move were used to create of STEM situation.

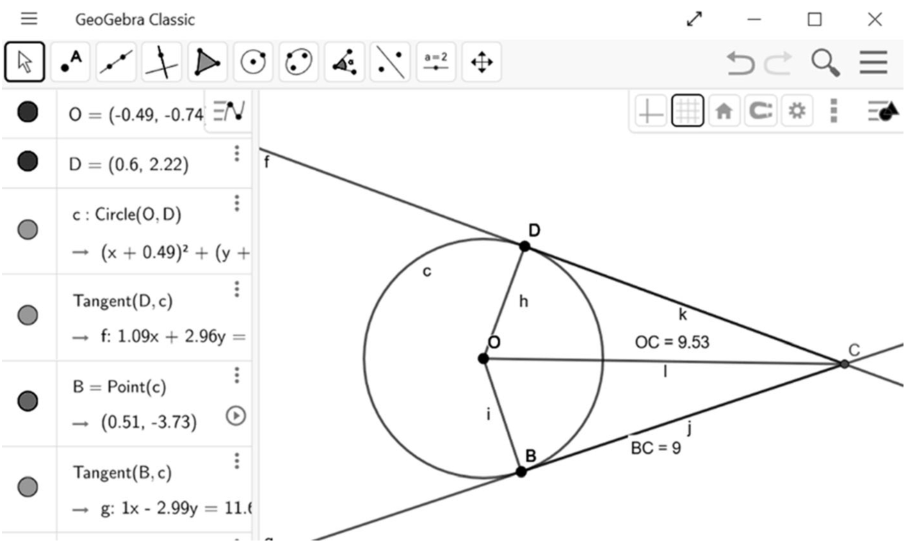

Fig. 5 Procedure of properties of tangent

5.13 Process

Step 1. Eliminate axes in the GeoGebra software view. Actions: Under Menu, select hide the grid button.
Step 2. Place the picture. Actions: (i). Click on Insert Image, (ii) Choose from File, (iii) Click OK.
Step 3. Adjust the point of picture (use Move)
Step 4. Insert Slider and tick in the screen. Put the value Min = 0, Max = 17 and Increment = 0.5 and press OK
Step 5. In Left side, Right Click in A point, Setting, Basic [Definition add (0 + a,0)]
Right Click in B point, Setting, Basic [Definition add (3 + a,0)]

Step 6. Hide Point Label
Step 7. In left side, Right click in Slider, Setting, Slider [Increasing (Once)] and close with press x
Step 8. Play the slider.

5.14 Product

The result from steps above as a bellow (Fig. 6):

This study found that all students could use GeoGebra to explain procedures, especially in describing of STEM.

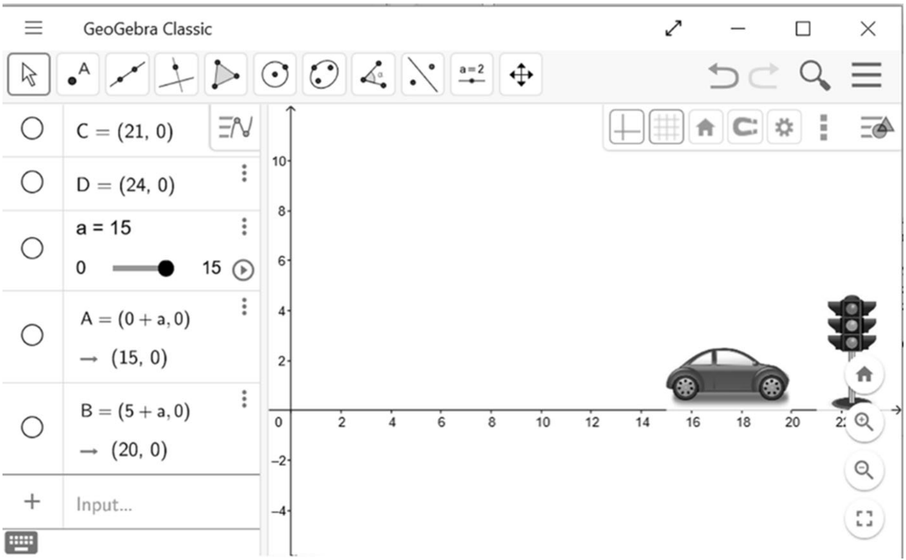

Fig. 6 Procedure of STEM

6 Role of teacher in automatic question and answer of mathematical functions using GeoGebra

In this research, teacher acts as a facilitator to ensure that each student follows the steps given to produce the concept and procedure for the topic of function. Teacher ask the student to answer the question given and use GeoGebra to find the correct answer. The design model of the Role of teacher in Automatic Question and Answer of mathematical functions using GeoGebra as bellow (Fig. 7).

Teacher questioning is a prominent feature of classroom discourse including teaching and learning using GeoGebra software. Questions given should be stimulate student conceptual and procedural knowledge. In this study, teacher used open questions that merely require students to recall the steps of using GeoGebra and find difference answer based on they are experience in real life. Teacher introduce the question content to real life situation and highlight the value of the content. Start the discussion by asking one of the questions and asking the students which of the questions they found most challenging. Start the activities and the process of writing down the answers will enable students to generate new ideas as well as question. After the have finished writing the answer and using GeoGebra, ask for volunteers or call on students to share their ideas.

7 Discussion

This study examined differences in achievements regarding functions and limit functions between the treatment and control groups of students of upper secondary schools. Given the important role of ICT (Eyyam and Yaratan 2014; Fabian et al. 2016; Higgins et al. 2017) in improving mathematics achievement, research is surprisingly limited on the effect of GeoGebra in relation to the function and limit function topics in upper secondary schools. We found a significant difference in the achievements regarding the aforementioned topics between the treatment and the control groups with pretest as a covariate. Students who used GeoGebra exhibited a better effect on the increase in the achievements regarding limit functions and functions than those who used the tradi-

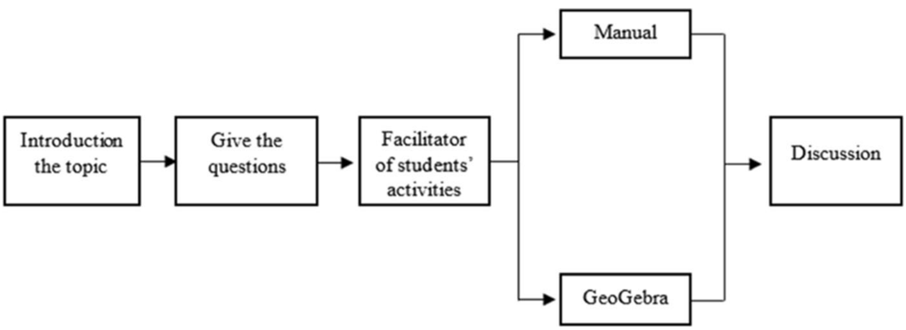

Fig. 7 The design of the role of teacher in Automatic Question and Answer

tional method. The expected positive effects of GeoGebra on the students' mathematics achievement resonated with several previous research findings (Arbain and Shukor 2015; Bakar et al. 2015; Saha et al. 2010; Zulnaidi and Oktavika 2018; Zulnaidi and Zamri 2017). This outcome can be clarified by prior research which confirmed a significant difference in the procedural and conceptual knowledge of mathematical functions between students who used GeoGebra and those taught using the conventional method (Zulnaidi and Zamri 2017).

Another possible reason for the disparity is the distinct learning environments. Zulnaidi and Zamri (2017) emphasised that GeoGebra helps students understand the concept of functions and directly relates the concepts with their daily lives. Teaching and learning mathematical concepts through GeoGebra assists students in prioritising mathematical concepts which are interconnected with their learning in and out of the classroom. Learning environments with the use of ICT can be used as real examples and regarded as visible links that relate mathematical ideas with daily life experiences. This finding confirms the opinion of Hall and Lingefjärd (2017) that the use of GeoGebra as a tool in all kind of mathematics (including mathematical modelling) can assist in solving real-life situations and answering interesting and difficult problems. Hence, this finding implies that the presence of GeoGebra in a mathematics classroom will strengthen students' mathematics achievement.

Students who used GeoGebra had better mathematics achievements. Hence, those taught using the conventional method have more misconceptions regarding limit functions and functions. This finding confirms those of Zulnaidi and Oktavika (2018), which stated that the use of GeoGebra as a teaching aid could lessen misconceptions about limit functions better than the traditional method. This outcome can be explained by Takaci et al. (2015), who found that students who used GeoGebra could check whether each step in the process of solving a task was correctly performed or not. Moreover, using GeoGebra can be beneficial as a simulator for modelling and interpreting real-world problems, aside from eliminating the need for algebraic symbolic manipulations (Dayi 2015). Therefore, the importance of the use of GeoGebra as a tool in mathematical classrooms involves the reduction of students' misconceptions about various mathematical topics, including limit functions and functions.

Research findings also showed that teachers gave positive feedback towards the use of GeoGebra in the teaching and learning of mathematical functions. They also did not have any problem in using the software and instead found it easy to use because it allowed them to explain functions and create function graphics. The use of GeoGebra can produce correct and exact pictures and graphics, and the software can help students in mastering concepts and solving problems related to procedures. This outcome is in line with that of Bu et al. (2011a), who stated that dynamic learning environments, such as GeoGebra, can serve as a technological tool and dynamic representation which can ease students' problems in calculating, constructing, modelling and reflection. All these benefits render GeoGebra a helpful tool in creating an effective teaching and learning process. Furthermore, teachers can make such process increasingly creative with use of GeoGebra.

Teachers can also explore and use various teaching methods by applying GeoGebra to improve the teaching and learning process. As Bu et al. (2011b) stressed, teachers can make their own teaching materials that are suitable according to their backgrounds as educators. Furthermore, GeoGebra can increase the learning environment with various representations, calculation utilities, documentation tools and friendly website characteristics to widen the teaching and learning scope outside the classroom. The software can likewise upload abstract concepts in mathematics that are related to the students' daily lives. Thus, students can have better understanding as they can view the concept directly when using GeoGebra. According to Antohe (2009), when students use GeoGebra, they can visualise abstract concepts and relate them to their mathematical knowledge. The ability to evaluate solutions by using electronic devices can increase students' interests in learning mathematics and enhancing their cognitive abilities. This software can also maintain the students' memory towards the learning topics in that they do not merely receive information but also become familiar with the application of such information by using GeoGebra.

The students strongly agreed that the use of technology in teaching and learning mathematics should be encouraged. They stated that by using the technology, they can learn the latest information from time to time. Moreover, students can enjoy a new learning method because they favour the learning process that involves GeoGebra than the traditional method. The statement from Stojanovska and Stojanovski (2009), which explains that the use of GeoGebra software can help students in exploring mathematics at home and school should be supported. This software allows for the interaction between students and between students and teachers online or offline without any legality involved. This facility can also assist in tackling mathematical solutions in an easier and understandable manner. The use of GeoGebra in the teaching and learning of mathematical functions can also increase procedural knowledge because teaching and learning by using GeoGebra can enable students to express their ideas in developing their knowledge on functions.

Students stated that when using GeoGebra in part of Automatics and Answer the question, they can visualise the function concept being studied and change the concept in the form of pictures. Moreover, the developed graphic in learning the function topic can be drawn and understood in a short time. Students can also draw the function graphic correctly by using the software. Although some students claimed that their ability to manually create function graphics will decrease with their use of GeoGebra, they can still have a complete explanation of how to visualise graphics perfectly. Hohenwarter et al. (2008) highlighted that GeoGebra can increase few implications which are from the free software and user friendly such as GeoGebra in the integration of technology into the teaching and learning of calculus. Examples of using GeoGebra software in this article also involve trigonometric and STEM. Students are able to produce the correct steps and answers as well as producing STEM examples in mathematics. Students are also able to produce STEM in mathematics learning at school. The ability of GeoGebra to make it easier for students to find the right answers and helps the students in mastering the topics taught, especially students who are able to learn themselves in anywhere.

Students became more interested in using GeoGebra when they used it in the teaching and learning process of mathematical functions. Activities that were conducted by the teachers by using the software were dynamic and could increase the interaction between teachers and students. When students performed the activities using GeoGebra, they became increasingly active in solving questions or presenting their assignments. Conversely, students using the traditional method only received information from the teachers rather than conducting activities that can increase their knowledge. In the traditional method, the students are more exposed to the problems solving under the procedural method. As suggested by Zakaria et al. (2007), procedural knowledge can only be meaningful if we relate it to the conceptual basics. Therefore, teaching and learning in the classroom should be parallel between the presentation of concept and the procedure. The advantage of GeoGebra is that the students can own the concepts and procedures after the teaching and learning process.

The limited time given to the use of this software is not a major problem in the teaching and learning of mathematics. Harper (2007) stated that the use of computer technology in teaching and learning can decrease students' procedural ability. However, teachers must play the main role in controlling the classroom to ensure balance between procedure and concept presentation until the learning objectives are achieved completely. Findings also showed that the teachers encountered problems with time, but this issue is not a major concern as the students can review the information provided once in their homes. The use of GeoGebra successfully explained the concept and procedure of functions. However, students needed additional time to do this task manually at home.

Teaching is easy with GeoGebra. The mathematics teachers suggested that GeoGebra should be used in the integration of technology in the teaching and learning process. GeoGebra gives teachers freedom to use their own teaching method and deliver information that is suitable to the teaching topic. For this research, GeoGebra was used to upload a picture and graphic in explaining a function topic. GeoGebra software can also enhance the teaching and learning process. Moreover, the interaction between teachers and students also becomes increasingly active. Teachers should consider using this software to improve the teaching quality in their classes. In addition, on behalf of teachers, GeoGebra is able to ease the work of teachers in the delivery of the topics taught. Student Center learning can also be generated through the use of GeoGebra. Through the given of activities, teacher assignments are more focused on controls to ensure students' learning works well.

Students became more active in learning when they used GeoGebra. They were given the opportunity to use GeoGebra in producing their own knowledge about the teaching topic. They also expressed their ideas and presented their work individually. The students became more dynamic in solving the problems and finding the answers whether by using the GeoGebra software or writing on paper. By contrast, the students under the traditional learning process were more passive as they merely waited for the teachers to give them the information without expressing their own ideas. As a result, the interaction between teachers and students were more active with the use of GeoGebra rather than the traditional method.

8 Conclusion

This study successfully improved and increased the students' mathematics achievement by utilising GeoGebra. We found a significant difference in the achievements regarding limit functions and functions between the treatment and the control groups with the pretest as a covariate in upper secondary school students. In summary, we argue that GeoGebra is a powerful software that can influence students' mathematics achievement and reduce misconceptions. The software is an easy-to-use application that provides students with the opportunity to visualise their ideas using certain concepts and even graphic illustrations. At the same time, it can enable students to inspect whether every single step in the process of tackling a complicated task is appropriate or not.

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