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WGL, a web laboratory for geometry

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Abstract The role of information and communication technologies (ICT) in education is nowadays well recognised. The *Web Geometry Laboratory*, is an e-learning, collaborative and adaptive, Web environment for geometry, integrating a well known dynamic geometry system. In a collaborative session, teachers and students, engaged in solving collaboratively a given set of problems, can exchange geometrical and textual information between them. In a normal work session (stand-alone mode), all the geometric steps done by the students are recorded, allowing, in a latter stage, their teachers to "play back" the students sessions. This information, alongside the navigation and chat information, can be used, later on, to assert the students level of geometric knowledge, adjusting the teaching strategies to each individual student. Teachers can register and begin using the public servers, defining students, preparing materials to be released to the students, open collaborative sessions, etc. Students can work in *WGL*, defining his/her own working space, sharing geometric constructions between themselves. From the case studies already conducted it was possible to conclude that, using *WGL*, the students improved their achievement in mathematics, in the classroom and doing homework. In this paper an in-depth, full description of the

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WGL system in its current version, is made, covering all the features and functioning modes, from the perspective of teachers and students.

Keywords Collaborative learning tools · Dynamic geometry systems · E-learning tools · Homework support systems · Personalised adaptive learning tools

1 Introduction

In this paper we discuss the development of an information and communication technology (ICT) learning environment for geometry. The long term goal that drives the development of WGL is the development of an e-learning Web-platform that is collaborative, adaptive, integrating a dynamic geometry system (DGS) and geometric automated/interactive theorem provers (G(A|I)TP). We also want to assess if it makes a good learning resource for the teaching and learning of geometry.

When designing a learning environment for geometry, including a DGS is a natural choice. The advantages of using DGSs in a learning environment for geometry are multiple: they are easy to use and they stimulate the creativity and the discovery process (De Villiers 2006; Laborde 2002).

Collaborative work can be an effective method to motivate students and encourage active learning, to develop essential critical-thinking, communication, as well as to improve decision making skills. Collaborative work promotes school performance and socialisation between students (Gillies 2003; Summers et al. 2005). Given the strong visual impact of geometry, a collaborative environment for geometry should allow the exchange of geometric information, alongside the exchange of textual information (Quaresma et al. 2013; Santos and Quaresma 2013, 2015).

A learning environment should have adaptive characteristics in order to cope with their students different learning needs and different learning characteristics, providing support to the specific needs, knowledge and background of each individual student (Chrysafiadi and Virvou 2013; Shute and Psotka 2001; Triantafillou et al. 2003). In a learning environment for geometry the authors claim that the environment should be able to infer the van Hiele levels of geometric reasoning (Usiskin 1982).

The synchronous and asynchronous interactions allow different modes of accessing *WGL*. In Mendicino et al. (2009) and in Fish (2013) it is shown that the students who used a Web based systems for homework in mathematics exhibited a significant improvement over students who did their homework traditionally.

With the inclusion of a G(A|I)TP it will be possible to fill the gap between the informal and formal proofs. It will became possible to check the soundness of the constructions, e.g. if two given lines are parallel, and also to make formal proofs of geometric conjectures with a natural language output (Janičić and Quaresma 2007; Quaresma et al. 2008; Santos and Quaresma 2012).

As said above, the *Web Geometry Laboratory*, *WGL*, is an e-learning, collaborative and adaptive, Web environment for geometry. It integrates a well known DGS, the GeoGebra JavaScript applet (Hohenwarter 2002) and it possess a database where each user can save geometric constructions produced using the DGS.

There are two main types of users in *WGL*: teachers and students. Students should always be connected to a given teacher. Teachers are able to work independently in the environment, but are also able to create student accounts and to organise them in classes and/or groups, and also to engage them in individualised work sessions (stand-alone sessions) or collaborative work sessions.

The change between a stand-alone session or a collaborative session is decided by the teachers, they are able to "`begin`" and, later on, "`end`", a collaborative work session. Whenever they do it the students interface change, changing the way the students access *WGL*.

In the stand-alone mode the students are working alone, accessing the environment using a computer, or tablet (or even a smartphone), through a Web-browser, having a DGS to work with. This mode can be used by teachers in a classroom, providing geometric problems to be solved by the students, to be solved with the help of the DGS, as a very powerful substitute of the physical rules and compass. The stand-alone mode can also be used in the context of homework, again to solve geometric problems given by the teachers, or for the students own work.

The collaborative mode is aimed to be used by groups of students, solving geometric problems given by their teachers in a collaborative way, again in a classroom, or at home. In the collaborative mode each student has two DGS windows at his/her disposal. In one of them he/she develops his/her own work, the other is the group window where the task is being done, collaboratively, by all the students. Besides the exchange of geometric information among the students, a chat channel is also available to allow the exchange of short textual messages between the students (and also with the teacher).

In the stand-alone mode the access to the environment is individualised. Each user has his/her own work-space, with a scrapbook of geometric constructions that can be shared, or not, among a group of colleagues. To be able to construct an adaptive learning environment *WGL* collects all the information about the students' navigation, and also all the steps done during each student's stand-alone work session. In its current version (v. 1.4) all this information can be visualised by the students' teachers, the visualisation of the geometric steps done by the students is done in "video mode", i.e. play, fast-forward, stop, and pause, buttons are available.

To assess the usefulness, and to certify the correction of the objectives formulated at the beginning, a series of case studies are being done in Portugal and Serbia. In Santos et al. (2016) some of the already concluded case studies are reported. The Portuguese case studies were conducted in a classroom (synchronous) setting, the Serbian case studies focus on homework (asynchronous).

Using an action research methodology to software development (Baskerville and Wood-Harper 2016), the *WGL* development cycles intertwine implementation and testing stages with evaluation stages.

The *WGL* is a Web client/server application. The database (constructions; users' information; constructions; permissions; etc.); the DGS JavaScript applet; the synchronous and asynchronous interaction, are all implemented using free cross-platform software. The *WGL* is an internationalised system with the English language as the default language and already localised to the Portuguese and Serbian languages. It is an open-source system,[1] versions of the server are available to be installed on Linux systems (or other systems through virtual machines).

Related systems There are several DGS available (see Wikipedia (2016) for a comprehensive list) but none of them defines an environment where the DGS is integrated into a learning environment with collaborative and adaptive features. In (Quaresma and Janičić 2006, 2007; Santos and Quaresma 2008) accounts of DGSs and G(A|I)TP integration and the integration of those tools in learning environments can be found, but always partial integrations not building any kind of collaborative, adaptive e-learning environment. Some learning environments in the area of geometry have been developed, e.g. Tabulae (Moraes et al. 2005) and GeoThink (Moriyón et al. 2008). There are also some tutoring systems in the field of geometry, e.g. Advanced Geometry Tutor (AGT) (Matsuda and VanLehn 2005), AgentGeom (Cobo et al. 2007) and geogebraTUTOR (Richard et al. 2007). The *WGL* distinguishes itself by relying on an external, full fledged DGS, well known by its users and supported by its developers. The two modes, collaborative and stand-alone, the well grounded permissions system and the capability that this opens for a personalised contact with the environment are also points in favour of *WGL*. The internationalisation, i.e. the ability to receive translations into different languages (Tabulæ lacks this feature) is also a positive point for *WGL*. Several case-studies already conducted have validated the *WGL* goals.

Overview of the paper This paper is organised as follows: first, in Section 2 we describe the global characteristics of *WGL*. In Section 3 the collaborative module is described. In Section 4 the adaptive module is described. Section 5 describe the Case studies already performed. The description of the access to the system is done in Section 6. Finally in Section 7 some conclusions are drawn and the future developments are foreseen.

2 The web geometry laboratory learning environment

With the development of *WGL* our aim is to build an e-learning environment for geometry with collaborative, adaptive and automatic reasoning features. An environment to be used in a classroom, in synchronous interactions, mediated by a teacher, but also at home, in synchronous or asynchronous interactions.

The main features of the *Web Geometry Laboratory* (v. 1.4) are:

- An integrated DGS, the GeoGebra JavaScript applet;
- A user's management module for: administrator(s), teachers and students, allowing the definition of classes and groups;
- A repository of geometric problems: each user has his/her own list of constructions;
- A permissions system allowing the sharing (or not) of each construction between users and groups;

[1] webgeometrylab.sourceforge.net/

- A collaborative module, where a given geometric task can be worked collaboratively by a group of users;
- An adaptive module, allowing the capture of all the information regarding the students interactions with the system. Later this information can be viewed and analysed by teachers;
- A chat, to allow the exchange of short textual messages between users engaged in collaborative sessions;
- A forum to allow the exchange of messages between users about different subjects regarding the *WGL*.

The *WGL* is a client/server application (see Fig. 1). The *WGL* server is the place where all the information is kept: the log-in information; the groups definitions; the geometric constructions of each user; the users activity logs; etc. The users will access the server through a Web browser, each loading an instance of the DGS applet, and using the server for all necessary information exchange. For a remote access to the *WGL* servers,[2] a normal bandwidth ($\geq$ 4Mbps) should be enough for a good interaction with the system.

2.1 Type of users in *WGL*

Apart from teachers and students the *WGL* type of users include, administrators and anonymous visitors.

The administrator(s) main role is the administration of teachers. They have also access to the log-in information of all users, information that can be used to streamline the server.

The teachers are privileged users in the sense that they are capable of defining other users, their students. In the beginning of each school year the teachers should define all their classes and the students belonging to those classes. At any time the teachers could also decide to aggregate their students into groups (see Fig. 2).

The students, each linked to a given teacher, are able to work in the environment, solving geometric problems created by their teachers and/or pursuing their own work. The students are unable to create other users.

Finally, the anonymous user is a student-type user, not linked to any teacher and because of that, unable to participate in collaborative sessions. The purpose of this type of user is solely to allow unregistered users to test *WGL*.

2.2 Types of interaction within the *WGL*

There are two distinct modes for the students to interact with the *WGL* system. The collaborative sessions (see Section 3) and the regular (stand-alone) sessions (see Section 4). The change between these two distinct modes is controlled by the teachers. In a collaborative session the students, working in groups, have some specific assignment to fulfil and they will do it in a collaborative way, exchanging geometric

[2] hilbert.mat.uc.pt/WebGeometryLab; http://jason.matf.bg.ac.rs/wgl

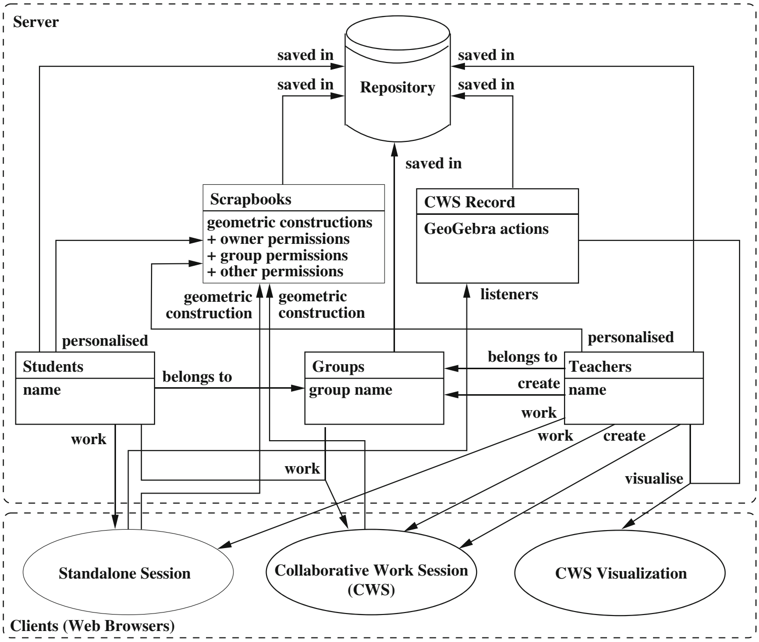

Fig. 1 *WGL* system schema

and textual information to reach the common goal. In a regular session the students will be working alone, they can share constructions with the other users of the environment, but all this exchange of information will be asynchronous.

2.3 Individualised scrapbook

Each user (teachers/students) has access to a "scrapbook", kept in the server, where he/she can keep all the geometric construction produced using the DGS integrated in the *WGL* environment. Each user will have full control over this personal scrapbook, having the possibility of saving, modifying, deleting, and change the access permissions of each construction produced.

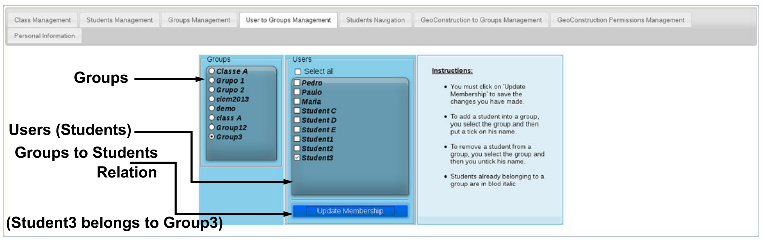

Fig. 2 *WGL* group membership relation

To allow sharing geometric constructions among users, a permission system was implemented. This permission system is similar to the "traditional Unix" file permissions system. The users will own the geometric construction defining the reading, writing and visibility (`rwv`) permissions per geometric construction. For example, given a geometric construction `cnst01`, created by the user `stdN`, belonging to the group `grpX`, the construction could have a permission `rwvr-v---`, meaning that the construction's owner, the first three positions, will have all the permissions, the members of the group, the middle three positions, will be able to **r**ead and see (**v**isibility) but not to **w**rite (modify), and all the other users, the three last positions, will have no permission over this construction, not even being aware that the geometric construction exists. By default, the teacher will belong to all the groups he/she had created, giving him/her the group access privilege to the students' constructions.

During the last three years these systems have been intensively used, e.g., for testing collaborative learning in teaching geometry.

3 The collaborative module

Planning a collaborative working session the teacher has to decide how to group the students and has also to conceive the geometric problems to be solved collaboratively, i.e., prepare a set of geometric constructions, starting points for problems to be completed during the class; illustrative cases; etc.

In a *WGL* collaborative session the students will solve the geometric problems proposed by their teachers, being able to exchange geometric and textual information, producing the geometric constructions in a collaborative fashion.

The students engaged in a collaborative session will always be in working groups, with access to the material prepared by the teacher and with access to two DGS applets. One of those DGS applets is for their own work, the other is where the group construction is being done. The *group-construction* is shared by all the members of a given group, one of the students will have the lock over the construction, all the other group members will see the work being done (synchronised every 20s). At any given moment the student can release the lock, which can be claimed by any student in the group.

At the same time, the students have their own work-space that can be used to: follow the work that is being done by the group representative; develop their own constructions; to anticipate the group construction; to develop auxiliary constructions. In this work-space the saving (to his/her scrapbook) of the work being done is the responsibility of the student.

The students have the possibility to exchange constructions between DGS work-space windows. The students without the lock are able to "import" the group construction to his/her own work-space. The student with the lock adds to that, the possibility to "export" the construction to the group work-space. A chat is provided to allow the exchange of short messages between all the members of the group, including the teacher (see Figs. 3 and 4).

Apart from being responsible for setting the collaborative session, and being able to assess its results at the end, the teacher has also access to a DGS work-space

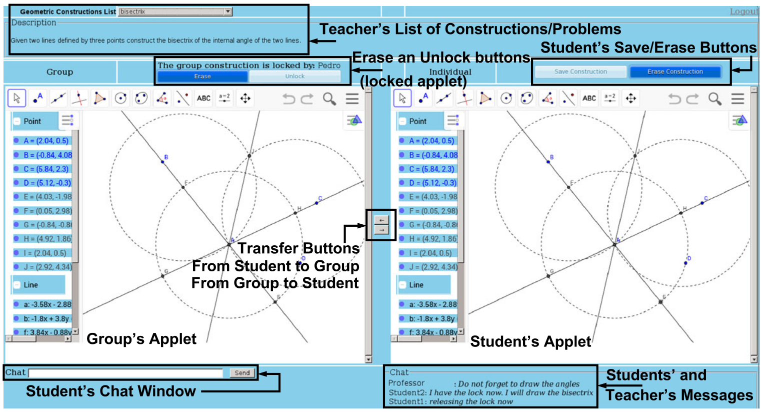

Fig. 3 Collaborative session – student's view

window where he/she can follow the work of all the students in each group (see Fig. 4).

3.1 A collaborative session

For a better grasp of the roles of teachers and students, a collaborative session is now described. A simple problem, "Given two lines, defined by three (non-collinear) points, construct the bisector of the internal angle of the two lines", will be asked to be solved, collaboratively, by the students.

3.1.1 Session preparation

When the teachers plan a collaborative lesson they begin to create a new collaborative work session (CWS), give it a name, a description, and the goals to attain in the session. At the end of the CWS the teacher can save a text record of the session (see Fig. 5). After the CWS creation its status is "`Open`". From this point on, up-to the moment where the teacher decides to change the status of the CWS to "`Start`", only the teachers are aware that the CWS is created, the students' interface remain unchanged, in stand-alone mode (see Fig. 6).

The next step is to decide how to distribute the students in groups and from those groups which will be engaged in the collaborative session. This amounts to keep the existing groups, change them, or create new groups

Next, it is necessary to prepare one or more problems to be solved by the students, collaboratively, in groups (see Fig. 7). If needed, the permissions, namely the visibility permission, can be manipulated in such a way that different problems can be given to different groups at different moments of the collaborative session (see Fig. 8).

When the preparation steps are completed the teachers can decide, at any given moment, to change the status of the CWS to "`Start`", starting the collaborative work session. From this point on, up-to the "`End`", the *WGL* students' interface change, from the stand-alone interface, to the collaborative interface where the students will

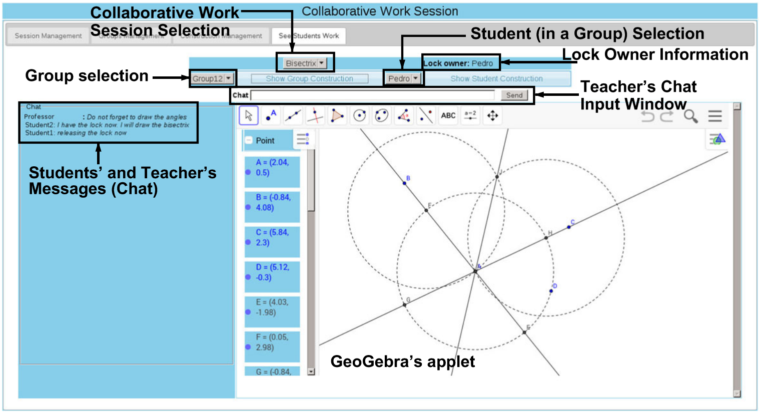

Fig. 4 Collaborative session – teacher's view

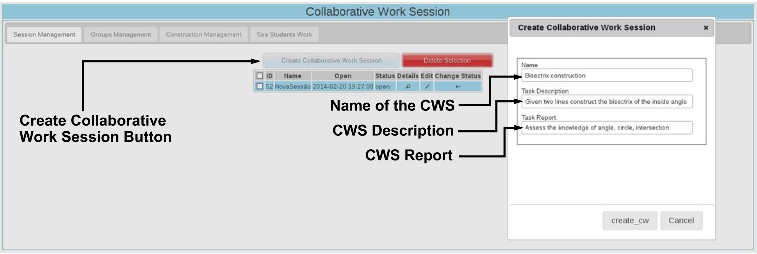

Fig. 5 Collaborative session creation

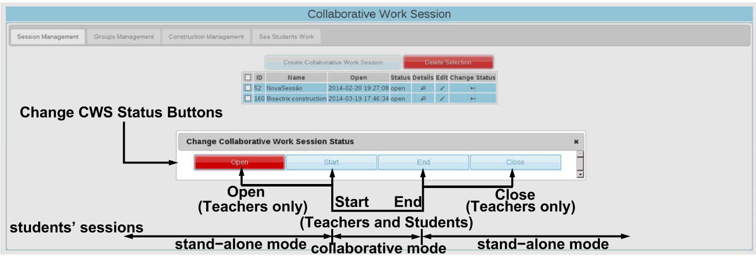

Fig. 6 Collaborative session – changing status

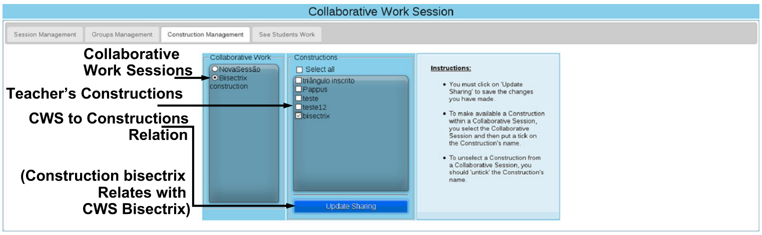

Fig. 7 Constructions and collaborative sessions relation

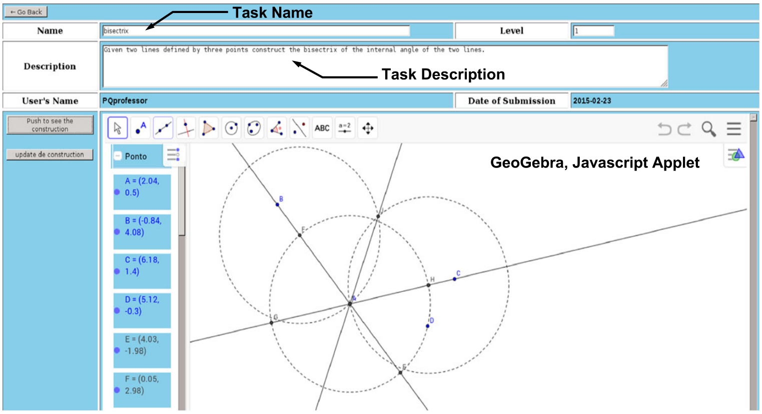

Fig. 8 Task planning (teacher)

be engaged in the CWS, with geometric problems to solve and two DGS windows to work with. The window on the left is where the group is developing the construction, the window on the right is the individual student work-space (Fig. 3).

After the problems are solved, in a classroom setting or in a remote homework setting, the teachers can change the status of the CWS to "`End`". From this point on the students will resume a normal workflow in the environment. The teachers still have access to the CWS, with all the work done by the students, and can use this period to write the CWS's report.

All the constructions made by the students, and saved by them, during the CWS are saved in the student's scrapbook, in this way the student can profit from the work done during the CWS, eventually exploring new ways to tackle the problems.

3.1.2 Students perspective

As was already mentioned above, when the teacher changes the status of the CWS to "`Start`" the students' interface change, from a stand-alone mode to a collaborative work mode.

In the new mode, the collaborative mode, the students are in groups and they have a specific set of problems to solve. To do that, they have a new interface with two DGS windows, the one on the left, is the group window, where all the work done there is broadcasted (synchronised every 20s) to all the members of the group. The window on the right is for the individual work of each student (see Fig. 3).

At the beginning of the CWS, a given element of the group gets the "`lock`" over the group construction, meaning that he/she will be working in the group construction, being the group representative, with his/her work being broadcast to the other members of the group. The "`unlock`" button is visible, meaning that any time he/she can release the lock, allowing others to take charge.

Whenever the student releases the lock (pushing in the "`unlock`" button), the "`lock`" button appears above the left window of all the students in the groups, meaning that the group construction belongs to nobody, it is necessary that someone pushes the "`lock`" button, to resume the work in the group construction. The chat windows allow the members of the group to interact, changing messages between the group members, and also with the teacher, and this is very useful to change ideas and also to arrange the unlock/lock status between the members of the group.

The DGS window on the right is the student individual working space, it should be use to follow the group representative work, to pursue the student own work, maybe anticipating the next steps of the group representative. The possibility of importing the group construction is very useful to allow to each student to save the latest version of the group construction to his/her working space, and, using the "`save`" button, to save that construction to the scrapbook.

The group representative can export his/her own construction to the group construction window. This could be useful, e.g. when the student was already some steps ahead when he/she claims the lock.

The saving of the student's construction in his/her scrapbook is not automatic, the student needs to push the "`save`" button, in order to transfer the construction from the CWS to his/her scrapbook (see Fig. 3).

3.1.3 Teachers perspective

Beside all the preparation stage and final report stage, teachers are also active players in a collaborative session. Teachers have access to a DGS window where they can follow the students work, and can also participate in their conversations, reading the students' chat messages, and sending messages to each, selected, group (see Fig. 4).

At any given moment teacher can have only one active collaborative session (in the "`Start`" status), so only for that session the constructions of the students are begin updated, and only for that session the chat messages are active. Nevertheless, the access to the final work done in all collaborative session is always available to the teachers, in this way teachers can access the work done by the students, using that information to write a final CWS report.

4 The adaptive module

The permission system allow teachers to differentiate groups' working plans. Teachers can prepare a set of problems alongside a definition of groups, setting the permissions in such a way that different construction will be available to different groups.

Besides this, and the individualised scrapbooks, the *WGL* adaptive features are implemented in the adaptive module, whose aim is the construction of student's profiles and individual learning paths. To be able to build individual student's profiles and individual learning paths, the system collects information about the student's interactions when in the stand-alone mode, i.e., in a regular work session (see Fig. 9).

The system records navigation and also geometric information for each student. The navigation information is a plain list of all the pages visited with enter and exit timestamps. The geometric information is recorded when the student is using the

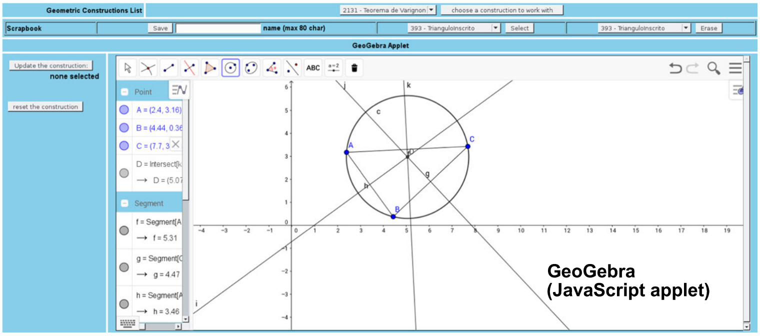

Fig. 9 *WGL* stand-alone session (students' perspective)

DGS applet, using JavaScript listeners of the DGS application programming interface (API). We record every step done by the students, every point, line, circle, etc. introduced, any deletion of any previously introduced object and even all the modifications done to them. Only the time spent in doing each and every step is not recorded.

At a later stage the student's teacher is able to see the work done by the student, play it step by step, play in a regular speed, play in a fast forward fashion (see Fig. 10). Given the fact that the *WGL* does not record the time spent in doing any given step, the reproduction as a regular motion. In this way the teacher can analyse all the steps done by any one of his/her students, to solve a given task, getting information that can be used to assert the student's van Hiele level of geometric knowledge (Crowley 1987) and also the student's learning style (Kolb and Kolb 2005; Felder and Silverman 1988).

In a collaborative session (see Section 3) the students are not developing the construction alone, so no recording of the geometric interactions is performed. Nevertheless the teachers have access to the 'chats' exchanged during those sessions and given the fact that every entry is tagged with the student (or teacher) identification, this could also be used to help the teachers to access his/her students levels of geometric knowledge.

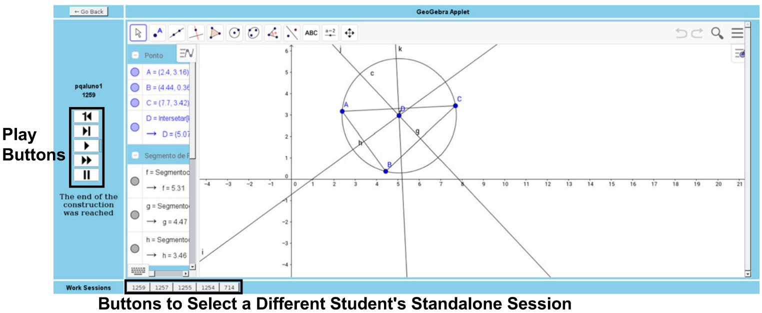

Fig. 10 *WGL* adaptive, 'playing' a student's session

5 Case studies

After the development of a first *WGL* prototype the need to evaluate the environment and its usefulness as a collaborative, synchronous and asynchronous, environment for the learning of geometry, lead to the preparation of case studies in Portugal and Serbia.

Two different set of studies were performed, one in Portugal, in a classroom setting, and another in Serbia, for homework. The first case studies on those sets where done using *WGL* version 1.2, still without the group-wise communication channel (chat). The second case studies were done using *WGL* version 1.3, already with the chat communication channel, among other improvements done in *WGL*. The case-studies are being used to improve the system but also to publicise the system, and training teachers in its use.

The first case studies where conducted in the school year of 2012/2013 in Portugal and Serbia, one in each country. In Portugal 22 students where engaged, using *WGL* in a classroom setting. In the Serbian study, 69 students where engaged, this time, using the platform to develop homework. These first case-studies served to refine the platform and also to prepare materials and methodologies for the main studies presented in Santos et al. (2016). The Portuguese main case study was conducted in a classroom setting, the Serbian main case study focus on homework (Rogers et al. 2003).

In 2016 a pilot study with a 15 year old student, in a stand-alone session, was conducted in Portugal. The goal was to analyse the student's interactions, the student behaviour, to find his level of geometric knowledge and learning style.

6 Access to the system

The *WGL* public servers can be used by any teacher willing to incorporate such an e-learning environment in his/her teaching practice. After registration (subject to validation) a teacher can create classes and use the system as a geometry laboratory, or as an e-learning environment for homework tasks, in stand-alone or collaborative sessions.

7 Conclusions & future work

From the case studies already conducted it can be concluded that:

- a Web learning environment should have an easy and simple interface. The teachers and students involved in the case studies were pleased by the *WGL* environment;
- the collaborative module was validated with the students being highly motivated in its use.
- the asynchronous interaction allowing the resolution of homework with the help of *WGL* was validated;

- it was possible to conclude that, using *WGL*, the students improved their achievement in mathematics, in the classroom and doing homework. The inclusion of a full-fledged DGS in *WGL* revealed itself a very important asset, in accordance with the findings in Botana and Abánades (2012) and Ruthven et al. (2008).
- analysing the student's interaction it was possible to perceive the level of geometric knowledge and the student's learning style.

The case-studies gave the necessary information to make improvements in the environment, e.g. the "chat" window, from version 1.2 to the version 1.3. Many other small changes were made and also some more changes to be done where identified. One of those, identified needs, is the change in the teachers' participation in the CWS, they should be allowed to exchange also geometric information with the students, not only textual information.

The case studies conducted in two different countries where also useful to validate the internationalised character of the environment with the Portuguese and Serbian languages already supported.

More case-studies, with a significant number of students, must be prepared and conducted to validate the adaptive features.

In a collaborative session the lock management is being felt to be too restrict. It is planned that the lock could be claimed also by teachers, to, at any given point in time, help students complete their tasks. Even between students, the lock mechanism can be improved, at any given time, any student should be able to claim the lock, that would be released, or not, by the student owning it.

At the moment the adaptive module only collects the student's information and allow the teachers to "play" that information. To be able to know the time took in each step the recording of timestamps will be implement in a future version. Another improvement, already planned, will give the teachers the possibility to use all the collected information alongside with other additional information, e.g. questionnaires, to determine students' learning styles and build individualised learning paths, adjusted to those learning styles. A second, more ambitious, step would provide the system with the capabilities of automatic management of those learning styles and the construction of the learning paths.

Another development planned is the integration of a G(A|I)TP. To be able to provide a formal validation of geometric properties, e.g. "*two lines are perpendicular, because* ...", formal reasoning, visual proofs and also to support the automatic or semi-automatic adaptive features, e.g. one-step guidance.

The case studies and their findings allowed us to conclude that, using *WGL*, our students improved their achievement in mathematics, in the classroom and doing homework. The cases studies also gave us good clues to future research in the context of using *WGL* to improve the classroom, homework or mixed learning, on secondary schools' geometry.

The *Web Geometry Laboratory* is an e-learning, collaborative, adaptive, Web environment for geometry already being used by teachers in Portugal and Serbia and it is expected that its user base can grow not only in those countries but also in other countries.

Compliance with Ethical Standards

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References

Baskerville, R.L., & Wood-Harper, A.T. (2016). *A critical perspective on action research as a method for information systems research*. Springer International Publishing, Cham (pp. 169–190). doi:10.1007/978-3-319-29269-4_7.

Botana, F., & Abánades, M. (2012). Automatic deduction in dynamic geometry using sage. In *THedu'11, CTP components for educational software (postproceedings)*.

Chrysafiadi, K., & Virvou, M. (2013). Student modeling approaches: a literature review for the last decade. *Expert Systems with Applications*, *40*(11), 4715–4729. doi:10.1016/j.eswa.2013.02.007. URL http://www.sciencedirect.com/science/article/pii/S095741741300122X.

Cobo, P., Fortuny, J., Puertas, E., & Richard, P. (2007). AgentGeom: a multiagent system for pedagogical support in geometric proof problems. *International Journal of Computers for Mathematical Learning*, *12*, 57–79. doi:10.1007/s10758-007-9111-5.

Crowley, M.L. (1987). The van Hiele model of the development of geometric thought. In M. M. Lindquist (Ed). *Learning and teaching geometry, K12, Yearbook of the national council of teachers of Mathematics, chap 1* (pp. 9–23). Reston, VA, USA: National Council of Teachers of Mathematics.

De Villiers, M. (2006). Some pitfalls of dynamic geometry software. *Learning and Teaching Mathematics*, *4*(46), 52.

Felder, R.M., & Silverman, L.K. (1988). Learning and teaching styles in engineering education. *Engineering Education*, *78*(7), 674–681.

Fish, L. (2013). A comparison of student perceptions of traditional versus online homework. *The BRC Academy Journal of Education*, *3*(1), 135–160.

Gillies, R.M. (2003). Structuring cooperative group work in classrooms. *International Journal of Educational Research*, *39*(1–2), 35–49. doi:10.1016/S0883-0355(03)00072-7. http://www.sciencedirect.com/science/article/pii/S0883035503000727.

Hohenwarter, M. (2002). *GeoGebra – a software system for dynamic geometry and algebra in the plane*. Master's thesis, University of Salzburg, Austria.

Janičić, P., & Quaresma, P. (2007). Automatic verification of regular constructions in dynamic geometry systems. In F. Botana, & T. Recio (Eds.) *Automated deduction in geometry, Lecture notes in computer science,* (Vol. 4869, pp.39–51*)*. Berlin: Springer.

Kolb, A.Y., & Kolb, D.A. (2005). *The kolb learning style inventory—version 3.1 2005 technical specifications*. Lsi Technical Manual, Experience Based Learning Systems, Inc.

Laborde, C. (2002). *Integration of Technology in the Design of Geometry Tasks with Cabri-Geometry*. *International Journal of Computers for Mathematical Learning*, *6*(3), 283–317. doi:10.1023/A:1013309728825.

Matsuda, N., & VanLehn, K. (2005). Advanced geometry tutor: An intelligent tutor that teaches proof-writing with construction. In G. C. K. Looi, B. McCalla, Bredeweg, & J. Breuker (Eds.) *Proceedings of the 12th international conference on artificial intelligence in education* (pp. 443–450). Amsterdam: IOS Press.

Mendicino, M., Razzaq, L., & Heffernan, N.T. (2009). A comparison of traditional homework to computer-supported homework. *Journal of Research on Technology in Education*, *41*(3), 331–359. doi:10.1080/15391523.2009.10782534.

Moraes, T.G., Santoro, F.M., & Borges, M.R. (2005). Tabulæ: Educational groupware for learning geometry. In *5th IEEE international conference on advanced learning technologies, 2005. ICALT 2005* (pp. 750–754). doi:10.1109/ICALT.2005.251.

Moriyón, R., Saiz, F., & Mora, M. (2008). In J. Sánchez (Ed.) *GeoThink: an environment for guided collaborative learning of geometry, Nuevas Ideas en Informática Educativa* (Vol. 4, pp. 200–2008). Santiago de Chile.

Quaresma, P., & Janičić, P. (2006). Integrating dynamic geometry software, deduction systems, and theorem repositories. In J. M. Borwein, & W. M. Farmer (Eds.) *Management, Mathematical Knowledge. Lecture Notes in Computer Science* (Vol. 4108, pp. 280–294). Berlin: Springer. doi:10.1007/11812289_22.

Quaresma, P., & Janičić, P. (2007). GeoThms—a Web system for euclidean constructive geometry. *Electronic Notes in Theoretical Computer Science*, *174*(2), 35–48. doi:10.1016/j.entcs.2006.09.020.

Quaresma, P., Janičić, P., Tomašević, J., Vujošević-Janičić, M., & Tošić, D. (2008). Communicating mathematics. In *The digital era*, A. K. Peters, Ltd., chap XML-Bases Format for Descriptions of Geometric Constructions and Proofs (pp. 183–197).

Quaresma, P., Santos, V., & Bouallegue, S. (2013). The Web geometry laboratory project. In J. Carette, D. Aspinall, C. Lange, P. Sojka & W. Windsteiger (Eds.) CICM 2013, lecture notes in computer science (Vol. 7961, pp. 364–368). Springer. doi:10.1007/978-3-642-39320-4_30.

Richard, P., Fortuny, J.M.H., & Gagnon, M. (2007). Geogebratutor: Une nouvelle approche pour la recherche sur l'apprentissage compétentiel et instrumenté de la géométrie à l'école secondaire. In T. Bastiaens, & S. Carliner (Eds.) *Proceedings of e-learn: world conference on e-learning in corporate, government, healthcare, and higher education 2007, association for the advancement of computing in education (AACE) (pp. 428435). Chesapeake, VA* (pp. 428–435).

Rogers, P.C., Graham, C.R., Rasmussen, R., Campbell, J.O., & Ure, D.M. (2003). Blending face-to-face and distance learners in a synchronous class: Instructor and learner experiences. *Quarterly Review of Distance Education*, *4*(3), 245–51.

Ruthven, K., Hennessy, S., & Deaney, R. (2008). Constructions of dynamic geometry: A study of the interpretative flexibility of educational software in classroom practice. *Computers in Education*, *51*(1), 297–317. doi:10.1016/j.compedu.2007.05.013.

Santos, V., & Quaresma, P. (2008). eLearning course for Euclidean geometry. In *Proceedings of the 8th IEEE international conference on advanced learning technologies, July 1st- July 5th, (pp.387–388).* Santander, Cantabria, Spain. doi:10.1109/ICALT.2008.156.

Santos, V., & Quaresma, P. (2012). Integrating DGSs and GATPs in an adaptative and collaborative blended-learning Web-environment. In *First workshop on CTP components for educational software (THedu'11), EPTCS* (Vol.79, pp. 111–123). doi:10.4204/EPTCS.79.7.

Santos, V., & Quaresma, P. (2013). Collaborative aspects of the WGL project. *Electronic Journal of Mathematics & Technology, 7*(6).

Santos, V., & Quaresma, P. (2015). Online Experimentation: Emergent Technologies and IoT, International Frequency Sensor Association Publishing, Barcelona, Spain, chap A Collaborative Environment for Dynamic Geometry Software (pp. 33–46).

Santos, V., Quaresma, P., Marić, M., & Campos, H. (2016). Web Geometry Laboratory: Case Studies in Portugal and Serbia. *Interactive Learning Environments* (online). doi:10.1080/10494820.2016.1258715.

Shute, V., & Psotka, J. (2001). The Handbook of Research for Educational Communications and Technology, 1st edn, The Association for Educational Communications and Technology, chap Intelligent Tutoring Systems: Past, Present, and Future (pp. 570–600).

Summers, J., Beretvas, S., Svinicki, M., & Gorin, J. (2005). Evaluating collaborative learning and community. *Journal of Experimental Education*, *73*, 165–188.

Triantafillou, E., Pomportsis, A., & Demetriadis, S. (2003). The design and the formative evaluation of an adaptive educational system based on cognitive styles. *Computers & Education*, *41*(1), 87–103. doi:10.1016/S0360-1315(03)00031-9, URL http://www.sciencedirect.com/science/article/pii/S0360131503000319.

Usiskin, Z. (1982). *van Hiele levels and achievement in secondary school geometry*. Tech. rep., University of Chicago.

Wikipedia (2016). List of interactive geometry software URL http://en.wikipedia.org/wiki/List_of_interactive_geometry_software, last accessed, 2016-09-29.