Upward and Downward Multifractality and Efficiency of Chinese and Hong Kong Stock Markets

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Abstract

This study examines the upward and downward multifractality, long-memory process, and efficiency of the Shanghai stock exchange composite index of mainland China and the Hang Seng index (HSI) of Hong Kong using the symmetric multifractal detrended fluctuation analysis (MF-DFA), asymmetric MF-DFA (A-MF-DFA), and the Hurst exponent. The results reveal significant differences in upward and downward multifractality, indicating asymmetric multifractality regardless of the frequencies. Moreover, we find evidence of excess asymmetry in multifractality for both markets and for all frequencies, which is more pronounced during downward stock price movements for Hang Seng Index (HSI) markets. The Hong Kong market is less inefficient than Chinese markets. Additionally, Bitcoin (BTC) volumes and BTC trading capitalizations affect the efficiency level across quantiles. Finally, robustness tests confirm our results are robust.

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1 Introduction

The application of fractal geometry theory solves the problem that the effective market theory cannot, and it can be used to examine the nonlinear behavior of stock markets (Zhicao et al., 2017). The multifractality and long-ranges memory are two important concepts in portfolio theory. Multifractal method is flexible to assess the scaling properties of temporal and spatial data in terms of a set of exponents (Sosa-Herrera & Rodríguez-Romo, 2019). The presence of multifractal scaling behavior indicates a dependence between two successive price dates, indicating long-range autocorrelations and evidence against random walk hypothesis (Cajueiro & Tabak, 2004). The presence of multifractality in a market indicates that the volatility of prices is clustering and reveals predictability, providing opportunity for investors to beat the market. Accounting for these two variables are important for optimal portfolio design and hedging purposes. However, a single fractal is restrictive and fails to describe the exact price dynamics. The multifractal theory is more flexible as it accounts for different fluctuations of stock prices on various time scales. In addition, it offers more analysis for unpredictable financial markets (Jingjing et al., 2012). More interestingly, an accurate modeling of market efficiency, multifractality, and long memory persistency improve the decision making process for investors and portfolio investors and enhances policy makers' understanding of market mechanisms and instability. A stock exchange market is defined as efficient in its weak form if the current share price instantaneously and rapidly reflects all its historical price information (including price, volume and short sale amount, etc.), suggesting evidence against multifractality, long-range dependency, and price predictability (Wei & Wang, 2008). The stock market experiences phases of recursive upward and downward trends explained by the law of demand and supply and more importantly, by the external shocks that may be explained by bad and good news. These news releases lead to both upward and downward trends that impact investment strategies. Thus, the multifractality and efficiency can vary not only over time but also under downward and upward market price trends. The risk appetite and reactions of investors are time varying and sensitive to the direction of market conditions.

A vast body of literature has documented the multifractality, long memory, and efficiency of stock markets. Andreadis and Serletis (2002) show evidence of random multifractality in the Dow Jones Industrial Average index. Zunino et al. (2009) show that the multifractality in developed stock markets is lower than that in emerging markets. Using a symmetric MF-DFA and Hurst exponent, Zhu and Zhang (2018) analyze the price dynamics and efficiency of the Chinese CSI 800 stock index and find evidence of multifractality. In addition, the results of the Hurst exponent reveal the presence of a larger market crash in summer 2015 than the one in 2008. Gu and Huang (2019) analyze the multifractality of the Shenzhen stock market using the MF-DFA and 5-min high frequency data and find evidence of multifractality. In addition, the values of the Hurst exponent vary across the order of the fluctuation function. Using the same methodology, Yan et al. (2020) examine the liquidity in

china's new OTC market. The authors show that liquidity has a non-linear and multifractal characterization. In addition, the multifractal degree of liquidity is lower than that of the large-cap stock market. Zhang and Li (2018) explore the multifractality in both the Shanghai and Hong Kong stock markets by accounting for the 2014 reform (connect program) and find that the degree of multifractality increased more after the reform than before. In addition, the sources of multifractality are fat-tailed distribution and long-range correlation. However, the MF-DFA assumes that the multifractality behavior is symmetric during upward and downward trends. Alvarez-Ramirez et al. (2009) indicate the presence of asymmetric correlations between stock returns. The authors developed the asymmetric MF-DFA method to analyze the multifractality of downtrends and uptrends in Chinese stock markets. The results reveal that the multifractality level with uptrends is higher than with downtrends. Moreover, the authors show that long-range correlations and fat-tailed distributions are the principal sources of asymmetric scaling. They attribute the asymmetry to the long-range memory for the Shanghai stock market and the fat-tailed distribution for the Shenzhen stock markets. As for Ruan et al. (2018), they use the MF-DFA and MF-DCCA (multifractal detrended cross-correlation analysis) methods to investigate the multifractality of the Shanghai and Hong Kong stock markets and the effect of financial liberalization on the correlations between them. The authors find evidence of multifractality in both markets. In addition, the efficiency of the Shanghai stock market was enhanced after the implementation of financial liberalization (the Shanghai-Hong Kong Stock Connect in China). The correlations increased after this financial liberalization. China is the second largest economy in the world. Cao and Zhou (2019) revise the methodology adopted by Ruan et al. (2018) by using both MF-ADCCA and DMF-ADCCA methods. The authors support evidence of long memory in downward and upward trends in A+H shares in China. Cao et al. (2013) overcome the limit of the symmetric MF-DFA by disentangling the upward and downward trend and considering the asymmetry in the MF-DFA approach. The authors show that the multifractality degree of Chinese stock markets with uptrends is stronger than that of Chinese stock markets with downtrends. Gajardo and Kristjanpoller (2017) investigate the asymmetric multifractal cross-correlations between oil and Latin-American stock markets. They find the presence of multifractality in the cross-correlations between considered markets. Mensi et al. (2021) analyze the asymmetric asymmetry of the stock markets in major oil-producer and oil-consumer economies (Brazil, Canada, China, India, Japan, KSA, Russia, and USA) using the asymmetric MF-DFA approach. The authors find higher upside multifractality for all markets except China. Moreover, the degree of efficiency of the stock markets of oil producers is lower than that of oil consumers. Oil price uncertainty is a predictor factor for stock prices during the oil crisis and the COVID-19 pandemic outbreak.

Since its creation in 2008, BTC has been weakly correlated with stock markers. This low correlation degree pushed retail and institutional investors to consider BTC as a diversifier, hedger, and safe haven asset against extreme negative market stock prices in the wake of the bankruptcy of Lehman Brothers investment bank (Dimitriou et al., 2020; Mariana et al., 2021; Wen et al., 2022). Given their properties, the BTC's demand has increased which as a result increased its correlation with stock markets. The strong dependence and spillovers between BTC and stock

markets represent a main factor in selecting BTC assets to explain the stock market efficiency. The literature shows a strong spillover between BTC and stock markets (Jiang et al., 2022; Khalfaoui et al., 2022). In contrast, Nguyen (2022) shows unidirectional return transmission from the US stock market to the BTC market. This result is confirmed by Li (2022) who finds a unidirectional spillover from Meme stocks to BTC. Li (2022) also shows that Meme stocks are much riskier than BTC in terms of more extreme returns, greater volatility. These mixed results require further analysis on the relationships between BTC and stock markets and as a result motivated as to explore the impact of BTC prices on the stock market efficiency. Therefore, shocks occurring in BTC market is transmitted to stock market, creating more instability and volatility of stock prices. We notice that BTC price reacts positively to negative shocks to stock prices (Dyhrberg, 2016). On the other hand, common factors (supply, demand, investor sentiment, economic conditions, and geopolitics) affect stock and BTC markets; therefore, traders treat both BTC and stocks in the same way. The efficiency of stock markets is therefore sensitive to BTC price instabilities.

To the best of our knowledge, this study is the first to examine the downtrends and uptrends of multifractality, long memory, and asymmetric efficiency of the Hong Kong and Chinese stock markets and to investigate the drivers of market efficiency across quantiles. To achieve these objectives, we apply Cao et al.'s (2013) symmetric multifractal fluctuation detrended analysis (MF-DFA) and the asymmetric multi-fractal fluctuation detrended analysis (A-MF-DFA) to account for downward and upward market conditions. A high frequency intraday data at four different frequencies is applied for robustness purposes. In contrast to the DFA and MF-DFA approaches, which consider respectively the monofractality and the symmetric multifractality during episodes of bear and bull markets, this study considers asymmetry as in an important stylized fact that affects the funds allocation and financial risk management, and results in optimizing the decision making process.[1] The quantile regression approach (QRA) is applied to examine the drivers of market efficiency across different statuses of efficiency level (from the lowest to highest level).

Using the MF-DFA method (Sect. 3.1), the results provide evidence of a crossover time scale for both markets and for all frequencies, which does not support the monofractal behavior hypothesis. Moreover, the multifractality of Hong Kong stock markets has a much larger width than China stock markets for all frequencies. For both markets, we find that the Hurst exponent is time varying and above 0.5 along the sample period regardless of the frequencies. In addition, the inefficiency is higher under small fluctuations than large fluctuations. Using the A-MF-DFA method (Sect. 3.2), the results reveal significant distinctions between uptrend and downtrend values throughout various time scales. The behaviors of the A-MF-DFA functions for different frequencies are quite similar. The Hurst exponent values decrease with scale increases for overall, downward, and upward trends. In addition, we find that the upward Hurst exponent values for Shanghai stock index (SSEC) markets are superior to downward Hurst exponent values for all scales. In contrast,

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[1] For more information on multifractality and long-range memory, see Kantelhardt et al. (2002), Lo (1991), and Lo and Mackinlay (1996).

the reverse is true for Hang Seng index (HSI) markets. We note that the deviation between upward and downward Hurst exponents is more significant for negative scales than for positive scales. Using the market deficiency measure (MDM), we find that the Chinese and Hong Kong stock markets are highly inefficient under downward trends compared to upward trends. In addition, the Hong Kong market is less inefficient than the Chinese market for all frequencies. The Chinese and Hong Kong markets are more inefficient under 60-min data than 5-min data. Using a QRA (Sect. 3.3), we find that BTC price returns have no impact on efficiency in both Chinese and Hong Kong markets for all quantiles. In contrast, BTC volumes negatively and significantly influence the efficiency level of Chinese stock markets in the lowest, normal, and highest quantiles and only in the lower quantiles for Hong Kong stock markets. The Bitcoin USD trading capitalizations contribute significantly to the efficiency level of Chinese stock markets under different quantiles. Finally, we find asymmetric relationships between BTC USD trading capitalizations and efficiency level for Hong Kong stock market.

This study makes four contributions to the existing literature. First, it examines the upward and downward multifractality in two important stock markets in the Asian region, namely the Shanghai stock index (SSEC) and the Hang Seng index (HSI) as the proxies of mainland China and Hong Kong stock markets, respectively. The selection of these stock markets is attributable to their significant cross-correlations (Cao & Zhou, 2019; Ruan et al., 2018). Launched in November 17, 2014, the Chinese government has implemented a new financial reform with Hong Kong, the Shanghai–Hong Kong Stock Connect Program (SHSCP), which allows investors from both markets to trade in both markets. This reform stimulates the development of Hong Kong stock markets and increases the trade volume of the Shanghai stock market. It provides global funds far easier access to shares in mainland China by removing the requirement for an investment license (Ruan et al., 2018). Zhang and Li (2018) document that this reform offers an expandable, controllable, and feasible channel for mutual market access between mainland China (Shanghai) and Hong Kong for a broad range of investors.

Second, we test and compare the degree of weak-form efficiency for the overall trend as well as for upward and downward trends. This decomposition provides useful information for investors in terms of investment strategies and asset allocations.

Third, this study relies on various high frequencies (i.e., 5-min, 10-min, 30-min, and 60-min) to help investors and portfolio managers obtain accurate information on the evolving Chinese and Hang Seng stock markets. High frequency data is a significant source of information in portfolio risk management and asset pricing. In addition, high frequency information is used to enhance our understanding of the market price behaviors and price dynamics. It allows a better measurement of market surprises and reactions due to specific news. Merton (1980) highlights the importance of high frequency in estimating market returns and volatility. Engle (2000) suggests that high frequency is important to measure the realized volatility by using the available information set with the added advantage of not having to estimate parametrical models commonly used generalized autoregressive conditional heteroscedasticity (GARCH) models. More importantly, high frequency information is also important

for policy makers to understand the monetary policy transmission and shocks caused by each news announcement.

Finally, we augment our analysis by examining the determinants of the dynamic Hurst exponent. To do so, we examine the effect of Bitcoin (BTC) 5-min prices, BTC volume, and BTC trading capitalizations. The choice of these determinants' variables is due to the growing importance of BTC trading in China and Hong Kong. Since its creation, BTC, which has changed the way the world looks at money, has attracted the attention of investors, the media, and policymakers. This innovative virtual product gained massive exposure when China entered in the cryptocurrency markets in May 2013 (Gloudeman, 2014). China surpassed all other countries in terms of the number of downloads of desktop BTC clients, or computer programs, that offer users the option to trade and store BTCs in digital "wallets." BTC promised to be the world's first decentralized, peer-to-peer cryptocurrency, and 65 percent of the total hash power resides in China.[2] In July 2018, the Chinese authorities identified 88 virtual currency trading platforms and 85 initial coin offering (ICO) platforms.[3] Hong Kong is the largest BTC mine. It is positioned to become a global hub for BTC entrepreneurs and businesses (Gloudeman, 2014). We have selected the QRA to examine the drivers of Hurst exponents to consider the nonlinearity of relationships across different quantiles. It provides new insights on the relationships between Hurst exponents and the determinant variables (BTC prices, changes in BTC volumes, and BTC trading capitalizations). Since the efficiency is time-varying and shows periods of upward and downward trends, it is important to consider the relationship between Hurst exponents and the driver variables under different efficiency conditions (from lowest inefficiency to highest inefficiency).

We follow the methodology of Lee et al. (2018) to compute the generalized Hurst exponent by simultaneously disentangling the overall long-range dependence for overall, downtrend, and uptrend. This asymmetric multifractal fluctuation detrended analysis (A-MF-DFA) technique explores the asymmetric efficient market hypothesis (EMH) by investigating the differences among the various trends in the movement of the asymmetric generalized Hurst exponent. The MF-DFA model quantifies the multiple scaling exponents within a nonstationary financial time series. The asymmetric multifractality is flexible to identify the long-range autocorrelations during upwards and downwards trends. It also detects the multifractality in equity markets for nonstationary time series (Eldrige et al., 1993). The literature documents the presence of asymmetric correlations in stock markets because investors respond asymmetrically to news releases (Bae et al., 2003; Longin & Solnick, 2001). To the best of our knowledge, this study is the first empirical work to employ the A-MF-DFA method to explore the stock price dynamics for overall, uptrend, and downtrend trends in both Shanghai Stock Exchange Composite Index and Hang Sheng Index.

The rest of this paper is organized as follows. Section 2 discusses the materials. Section 3 discusses the empirical results. Concluding remarks are presented in Sect. 4.

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[2] https://www.coindesk.com/highest-in-2-years-65-of-bitcoin-hash-power-is-in-china-report-finds.

[3] https://www.loc.gov/law/help/cryptocurrency/china.php.

2 Materials

2.1 MF-DFA Analysis

Multifractal detrended fluctuation analysis (MF-DFA) was developed by Kantelhardt et al. (2002) and is an extension of detrended fluctuation analysis. Let us assume that $\{x_t, t = 1, \ldots, N\}$ is a time series of length N.

We define the profile y_k:

$$Y(k) = \sum_{t=1}^{k} [x_i - \bar{x}], k = 1, \ldots, N, \tag{1}$$

where $\bar{x}$ is the average over the entire return series. We divide the profile $Y(k)$ into $N_n \equiv int(N/n)$ non-overlapping segments of equal length n. Since the length N of the series is often not a multiple of the considered time scale n, we repeat the same procedure starting from the opposite end to prevent being lost and obtain $2N_n$ sub-intervals.

The local trend for each Y of the $2N_n$ segments is calculated using the least squares fit of the series. Then the variance is determined as:

$$F^2(n, v) = \frac{1}{n} \sum_{i=1}^{n} \left\{ Y[(v-1)n + i] - y_v(i) \right\}^2 \tag{2}$$

for $v = 1, 2, \cdots, N_s$ and

$$F^2(n, v) = \frac{1}{n} \sum_{i=1}^{n} \left\{ Y[N - (v - N_n)n + i] - y_v(i) \right\}^2 \tag{3}$$

for $v = N_n + 1, \cdots, 2N_n$.

The qth order fluctuation function $F_q(n)$ is computed by averaging over all segments (sub-intervals):

$$F_q(n) = \left\{ \frac{1}{2N_n} \sum_{v=1}^{2N_n} [F^2(n, v)]^{q/2} \right\}^{1/q} \tag{4}$$

The q-order Hurst exponent can now be defined as the slopes $H(q)$ of regression lines for each q-order root-mean-square (RMS) $F_q(n)$ (Ihlen, 2012). The parameter q enhances the small fluctuations when $q < 0$, otherwise, the large ones when $q > 0$. Therefore, different q describes the effect of varying degrees of fluctuation on $F_q(n)$.

We determine the scaling behavior of fluctuation functions by examining log–log plots of $F_q(n)$ versus n for each value of q. If series x_i are correlated in the long-range correlation, $F_q(n)$ rises for wider values of n, as the power law:

$$F_q(n) \sim n^{H(q)}. \tag{5}$$

where $H(q)$ is the qth-order generalized Hurst exponents. For H(2) = 0.5, time series is a random process, whereas for $0.5 > H(2)$ and $0.5 < H(2)$, the time series is anti-persistent and persistent (long memory), respectively.

The MF-DFA is applied to calculate the H(q), which have a direct link with the classical multifractal scaling exponent by:

$$\tau(q) = qH(q) - 1 \tag{6}$$

Using the spectrum of generalized Hurst exponents H(q), the singularity strength and spectrum denoted by α and f(α) can be defined as:

$$\alpha = H(q) + qH\prime(q) \text{ and } f(\alpha) = q[\alpha - h(q)] + 1 \tag{7}$$

In multifractal methods, different values of the singularity strength α are used to characterize different parts of the structure, which lead to the existence of the spectrum $f(\alpha)$. The above statistical and analysis work was carried out by using the software MATLAB2014.

2.2 A-MF-DFA Analysis

We extend the traditional MF-DFA method with the asymmetric multifractal scaling behavior among different trends using the A-MF-DFA method of Cao et al. (2013). Similar to the traditional MF-DFA method in Eq. (1), based on the recommendations of Peng et al. (1994), we then divide the time series X and its profile Y into nonoverlapping sub-time series of length n that are selected from 5 to $N/4$. Since N may not be a multiple of n, the length of the last segment may be shorter than n. Thus, we obtain a $2N_n \left(N_n = \lfloor N/n \rfloor \right)$ sub-time series $\{X_j\}_{j=1}^{2N_n}$ for X. The sub-time series $\{Y_j\}_{j=1}^{2N_n}$ for Y can be obtained in the same manner. The jth sub-time series of X is denoted by $X_j = \{x_{j,k}\}_{k=1}^{n}$, where $x_{j,k}$ indicates the kth element of X_j.

For each sub-time series X_j and Y_j, we estimate the linear fit $\overline{X}_j(k) = a_{x_j} + b_{x_j} k$ and $\overline{Y}_j(k) = a_{y_j} + b_{y_j} k$, which represents the linear trends for the jth sub-time series. The liner fit $\overline{X}_j(k)$ is discriminated by using the sign of the slope b_{x_j}; that is, $b_{x_j} > 0 \left(b_{x_j} < 0 \right)$ indicates that the time series x(t) has a positive (negative) trend in the sub-time series x_j. The liner fit $\overline{Y}_j(k)$ is used to detrended the integrated time series X_j.

The fluctuation functions can be defined as follows:

$$F_j(n) = \frac{1}{n} \sum_{k=1}^{n} \left(y_{j,k} - \overline{Y}_j(k) \right)^2 \text{ for } j = 1, 2, \ldots, 2N_n \tag{8}$$

The directional q-order average fluctuation functions are calculated by:

$$F_q^+(n) = \left(\frac{1}{M^+} \sum_{j=1}^{2N_n} \frac{sign\left(b_{x_j} \right) + 1}{2} \left[F_j(n) \right]^{q/2} \right)^{1/q}, M^+ = \sum_{j=1}^{2N_n} \frac{sign(b_{x_j}) + 1}{2}, \tag{9}$$

$$F_q^-(n) = \left(\frac{1}{M^-} \sum_{j=1}^{2N_n} \frac{-[sign(b_{x_j})-1]}{2} [F_j(n)]^{q/2} \right)^{1/q}, M^- = \sum_{j=1}^{2N_n} \frac{-[sign(b_{x_j})-1]}{2}, \qquad (10)$$

where $F_q^+(n)$ and $F_q^-(n)$ denote the upward and downward q-order average fluctuation functions, respectively. Assuming that $b_{x_j} \neq 0$ for all $j = 1, \cdots, 2N_n$, then $M^+ + M^- = 2N_n$.

From the fluctuation functions of Eqs. (8–10), we calculate the scaling or power-law relationship, which is defined as:

$$F_q(n) \sim n^{H(q)}; F_q^+(n) \sim n^{H^+(q)}; F_q^-(n) \sim n^{H^-(q)}, \qquad (11)$$

where $H(q)$, $H^+(q)$, and $H^-(q)$ denote the overall, upward, and downward scaling exponents, respectively. The scaling behavior of the fluctuations in Eq. (11) is calculated by analyzing the log–log plots of $F_q(n)$, $F_q^+(n)$, and $F_q^-(n)$ versus n for each value q. Using the ordinary least squares method, $H(q)$, $H^+(q)$, and $H^-(q)$ can be estimated based on the logarithmic form. Furthermore, for $H^+(q) = H^-(q)$ and $H^+(q) \neq H^-(q)$, the correlation in time series is symmetric and asymmetric (Cao et al., 2013), respectively. The asymmetric scaling behavior indicates that the correlations in the time series are different positive and negative trends.

Following Cao et al (2013) to measure the degree of asymmetry in correlation, the variable $\Delta H(q)$ is defined as:

$$\Delta H(q) = H^+(q) - H^-(q) \qquad (12)$$

where the larger the $\Delta H(q)$, the asymmetric degree of correlation is stronger. If $\Delta H(q) > 0$, then the persistence of the correlation is stronger when the series is in upward trend than when it is in downward trend. By contrast, if $\Delta H(q) < 0$, then the persistence of the correlation is stronger when the series is in downward trend than when it is in upward trend.

2.3 Data

This study considers two stock indices, the Shanghai stock exchange composite index (SSEC) of mainland China and the Hang Seng index (HSI) of Hong Kong. We utilize the closing price of high frequency intraday trading at four different frequencies (i.e., 5-min, 10-min, 30-min, and 60-min). This sample data covers the period from December 4, 2013 to December 29, 2017. The sample period covers Chinese stock market turbulence. More precisely, the crash period began on June 2015 until February 2016. According to the Guardian, the Shanghai stock markets are tumbling by 30% in June 2015. Hundreds of Chinese companies have suspended dealings in their shares in a bid to arrest a frenzy of selling. Despite efforts by the government to reduce the fall, this continues until August 2015 when the market falls by 8.48%. Another important fall in the Shanghai stock market occurs in January 2016 by 8%. Therefore, the oversupply of stocks (herding behavior) has direct implications

for stock price dynamics and as a result for stock market efficiency.[4] The literature provides evidence of different degrees of efficiency before and during crisis periods. For example, Lim et al. (2008) show that the global financial crisis affects the efficiency of Asian stock markets. This result is confirmed later by Jin (2016) who find that the 2008 global financial crisis adversely affected the Hurst exponent of Asian stock markets, indicating a rise in inefficiency. It is also consistent with Mensi et al. (2017) who analyze the multifractality and efficiency of Islamic sector stock markets and show that the Islamic stock markets become more inefficient after the 2008 global financial crisis. The Chinese government took a sequence of measures to stem the tide of the turbulence like stopping the initial public offerings and providing cash to brokers to buy shares. High frequency data has the advantage of providing extra and accurate information rather than daily or weekly data about price dynamics and market efficiency.

The data for all series are extracted from DataStream. The continuously compounded intraday percentage returns at time n on day t are defined as $R_{t,n} = \ln\left[\frac{P_{t,n}}{P_{t,n-1}}\right] \times 100$, where n is the number of time intervals in the day (i.e., 5-min, 10-min, 30-min, and 60-min), and t is the number of trading days in the sample period. Figure 1 shows the dynamics of 5-min returns (a) SSEC and (b) HSI over the sample period. As shown in this figure, we observe volatility clustering and fats tails that are stronger for HSI than SSEC mainly during Chinese stock markets crash. Figure 2 shows the distributions of intraday returns in different time scales. It is observed that the distributions have the high peak with fat-tailed futures, as known as a leptokurtic distribution. These findings support the non-normality for all intraday returns.

Table 1 presents the preliminary analysis for intraday SSEC and HSI index returns. The results show that the average returns for both markets are positive and increase with frequencies for both markets. Moreover, the average SSEC index returns are higher than HSI index returns for all frequencies. Similarly, the unconditional volatility is higher for SSEC markets than HIC markets. The market risk also increases with frequencies. The skewness coefficient values for all cases are negative and the kurtosis values are superior to the value of three Gaussian distributions, indicating asymmetry and leptokurtic behaviors (fats tails) for intraday return series. The results of a Jarque–Bera test strongly reject the normal hypothesis. All intraday returns series are stationary according to the ADF and PP unit root tests of Augmented Dickey and Fuller (1979) and Phillips and Perron (1988) and KPSS stationary test of Kwiatkowski et al. (1992).

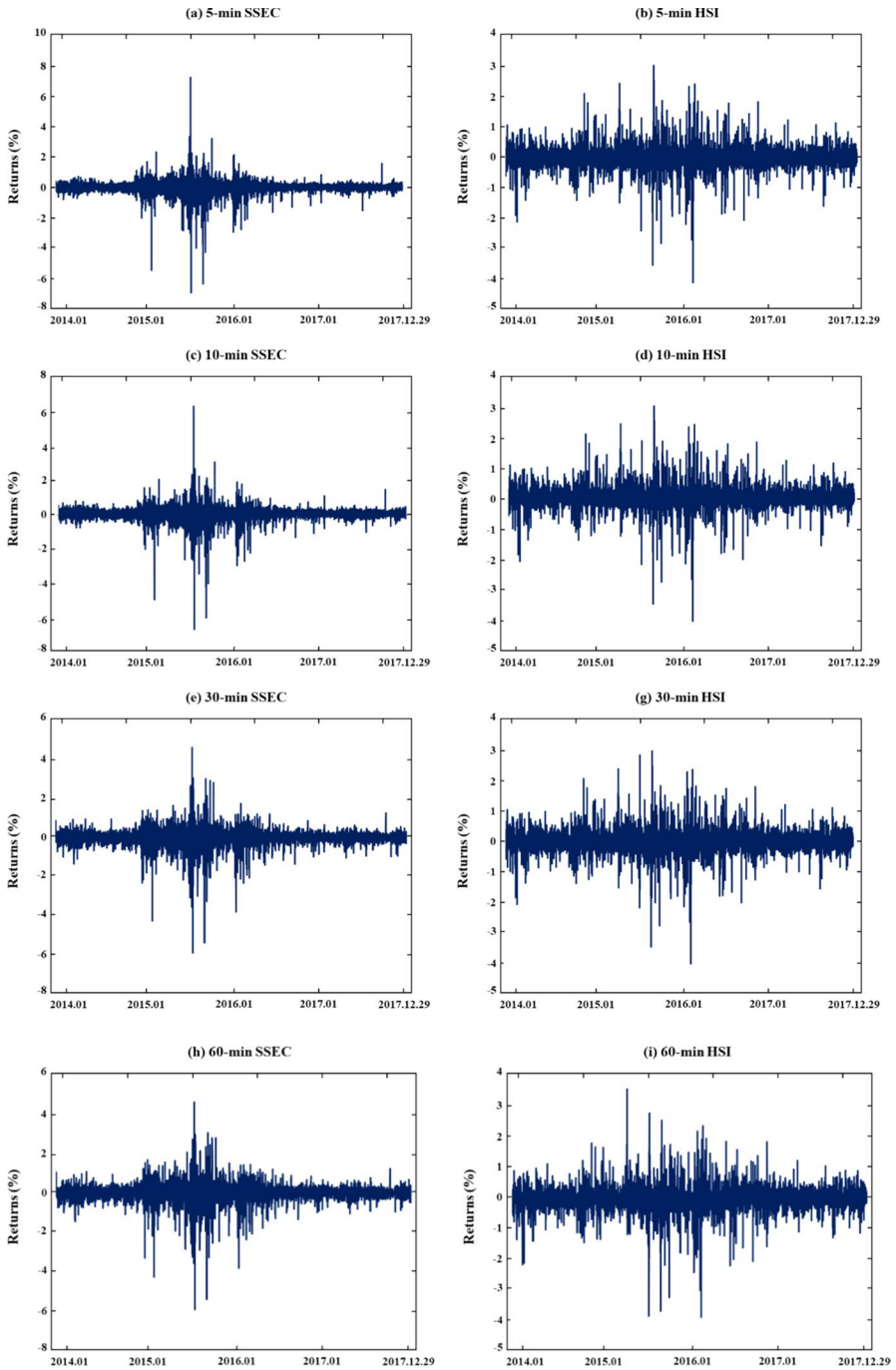[4] The Chinese government implemented a number of rescue programs like direct purchase intervention of stocks from firms.
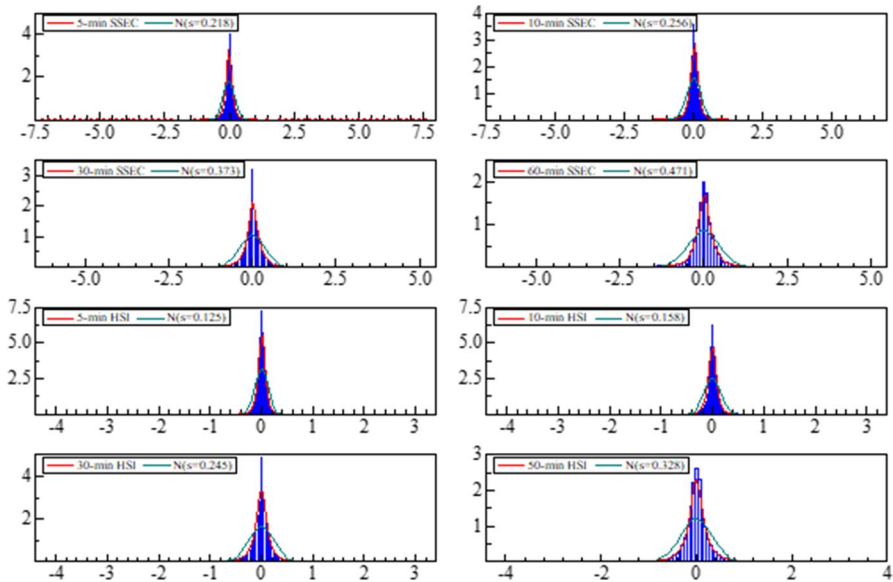
Fig. 1 The dynamics of intraday index returns

Fig. 2 The density of intraday index returns

3 Empirical Analysis Results

3.1 MF-DFA Analysis

Figure 3 shows a log–log plot of the MF-DFA functions $Fq(n)$ versus the time scale n. Note that fluctuation functions $Fq(n)$ are calculated with a range of q values from -5 to 5. For 5-min data, the local slope changes its trajectory for almost all colored lines. This result indicates the appropriateness of MF-DFA. Specifically, when q varies from -5 to 5, we find that the change of generalized Hurst exponents of two sub-series depends on q. The stock markets are characterized by multiple fractal and the monofractal scaling behavior hypothesis is therefore rejected. Regarding the rest of the frequencies (10-min, 30-min and 60-min), changes in the local slope of the plots occur in (log n^*=5). Figure 4 displays the slope of H(q) in different time scales. A se can see, the stock markets are more inefficient under small fluctuation than large fluctuation. Moreover, we observe that the Hurst exponent is less than 0.5 for large fluctuations, underlying an anti-persistence process (mean-reverting process). The HSI market is more (less) inefficient than SSEC particularly for 5-, 10-, and 30-min (60-min) data.

Figure 5 plots the multifractal spectrum of different time scales and shows that the multifractality of Hong Kong stock markets has a very large width compared to the Chinese stock markets for all frequencies. For example, the multifractal spectrum for 5-min data ranges between 0.15 to 0.8 for Hong Kong and 0.3 to 0.7 for China. This result implies that multifractality in Hong Kong market dynamics is much higher than in the Chinese market. The magnitude of the multifractal spectrum is higher in 5-min data than in 60-min data, indicating that

Table 1 Descriptive statistics and unit root tests for intraday returns

	SSEC				HSI			
	5 min	10 min	30 min	60 min	5 min	10 min	30 min	60 min
Mean	0.0008	0.0015	0.0036	0.0056	0.00037	0.00068	0.0017	0.0032
t-statistic	(0.8188)	(0.9513)	(1.0177)	(1.0096)	(0.7750)	(0.8334)	(0.8459)	(0.8576)
Maximum	7.5286	6.6380	4.8802	4.8802	3.1666	3.1666	3.1666	3.7579
Minimum	-7.2233	-7.1318	-6.2479	-6.2479	-4.2427	-4.2427	-4.2427	-4.0357
Std. dev	0.2184	0.2557	0.3729	0.4709	0.1250	0.1578	0.2448	0.3279
Skewness	-1.8845	-2.2806	-1.8616	-1.6255	-0.6053	-0.3562	-0.3100	-0.6153
Kurtosis	111.75	88.631	37.427	26.439	104.27	74.524	34.571	24.637
Jarque Bera	$2.4\text{e}{+}07^{***}$	$8.2\text{e}{+}06^{***}$	$5.4\text{e}{+}05^{***}$	$1.6\text{e}{+}05^{***}$	$2.8\text{e}{+}07^{***}$	$7.7\text{e}{+}06^{***}$	$6.1\text{e}{+}05^{***}$	$1.5\text{e}{+}05^{***}$
ADF	-80.55^{***}	-74.66^{***}	-29.10^{***}	-28.79^{***}	-187.4^{***}	-176.0^{***}	-110.4^{***}	-79.39^{***}
PP	-233.0^{***}	-155.1^{***}	-85.50^{***}	-66.93^{***}	-255.7^{***}	-175.8^{***}	-110.5^{***}	-79.39^{***}
KPSS	0.1747	0.1573	0.1576	0.1580	0.2128	0.2140	0.2118	0.2236

The intraday returns represent the percentage intraday returns. ADF and PP are the Augmented Dickey-Fuller and Philipps-Perron unit root tests, respectively. KPSS refers to the Kwiatkowski-Phillips-Schmidt-Shin test for stationarity

***indicates the rejection of the null hypothesis at the 1% significance level. (.) values indicate the t-statistics of H_0: mean $= 0$

the multifractality is higher as the frequency data rises. One possible explanation of this result is that the fear, irrationality and the strong herding behavior of equity investors are more pronounced in 5-min data than 60-min data. This result also justifies the reason behind selecting different frequencies. Time horizon is an important factor for investment strategies.

Figures 6 and 7 illustrate the time-varying dynamics of Hurst exponents for SSEC and HSI stock markets, respectively. We choose the minimum segment size $s = 8$ and maximum segment size $s = 128$ to compute the local Hurst exponents (H_t), shown in Figs. 6 and 7. Two scales were selected to analyze the spatial distribution of the Hurst exponent. Looking at the evolving efficiency of the SSEC market, we find that the Hurst exponent is time varying and positive among the sample period regardless of the frequencies for both scales. SSEC market experiences phases of persistence and anti-persistence behaviors. In addition, the Hurst exponent varies between 0.174 and 0.840 for scale 8 and between 0.189 and 0.861 for scale 128. The mode of the Hurst exponent reaches 0.588, indicating persistence and evidence against efficiency hypothesis. When the frequency data decreases, the Hurst exponent value rises from 0.617 for 10-min data to 0.638 for 60-min data, suggesting an increase in the persistence with decreasing frequency. The market share price deviates from its fair value (intrinsic value) with time lags. We note that the minimum Hurst exponent value occurs in June 2015 and its maximum is in October 2016 for scale 8 and in October 2017 for scale 128. Similarly, the Hurst exponent value of the Hong Kong market evolves over time and is above 0.5, indicating that the persistence of Hong Kong stock markets is not constant. The mode of the Hurst exponent at scale 8 is 0.599 at 5-min data, 0.589 at 10-min data, 0.555 for 30-min data, and

0.611 for 60-min data. Like the SSEC market, the persistence level of the HSI market increases as frequency decreases.

Overall, both the Chinese and Hong Kong markets are inefficient, as illustrated by the Hurst exponent. More importantly, the Chinese stock market is more inefficient than the Hong Kong market for 10-, 30-, and 60-min data. Financial investors can predict the futures prices in these markets to generate abnormal returns. In addition, the difference in the market inefficiency may be due to the microstructure for each market.

Table 2 shows the parameters of generalized Hurst exponents at different time scales (from q = −20 to q = 20) and different intraday price returns (5-min, 10-min, 30-min, and 60-min). The Hurst exponent values vary across scales, indicating multifractal behavior in Chinese and Hong Kong stock markets. In addition, we find that the Hurst exponents are superior to the 0.5 value of random walk hypothesis. Moreover, the Hurst values decrease as the scale increases. This implies that the persistence level decreases with scale rises. Furthermore, we note that the Hurst exponent values are higher at 60-min data than 30-min data. The market becomes more inefficient with time increases. Looking at the value of Hurst exponent for q = 2, the results reveal that both stock markets exhibit persistence behavior as the Hurst exponent value, which also increases with frequencies, is superior to 0.5. Specifically, the Hurst exponent of SSEC and HSI at 5-min data is 0.51 and 0.50, respectively, whereas it is 0.55 and 0.52 at 60-min data. The SSEC is more inefficient than the HSI market. This result is in line with the graphical evidence of Figs. 6 and 7.

Figure 8 displays the generalized Hurst exponent for different time scale intraday returns. The graphical evidence shows a significant persistence under negative scale and anti-persistence under positive scales. Furthermore, the gap in the degree of inefficiency of SSEC market is more pronounced under negative scales and 60-min data. Conversely, the difference in the inefficiency of HSI is high under positive scales.

However, markets experience phases of upward and downward trends. Investors' psychology, reaction, and risk appetite depend on market trends. Thus, asymmetry is an important variable that should be considered to improve the investment decision-making process. On the other hand, the descriptive statistics (Skewness test) show evidence of asymmetry. In the next subsection, we employ the A-MF-DFA to study the efficiency and multifractality during different market trends.

3.2 Asymmetric MF-DFA Analysis

Following Cao et al. (2013), Fig. 9 shows the A-MF-DFA functions $Fq(n)$ versus the time scale n in a log–log plot of the SSEC and HSI price returns. The results reveal significant distinctions between uptrend and downtrend values throughout various time scales. This result rejects the symmetric multifractality hypothesis. The deviations from the symmetry are significant at higher time scales (3 and above). The trajectories of the A-MF-DFA functions for different frequencies are quite similar. The
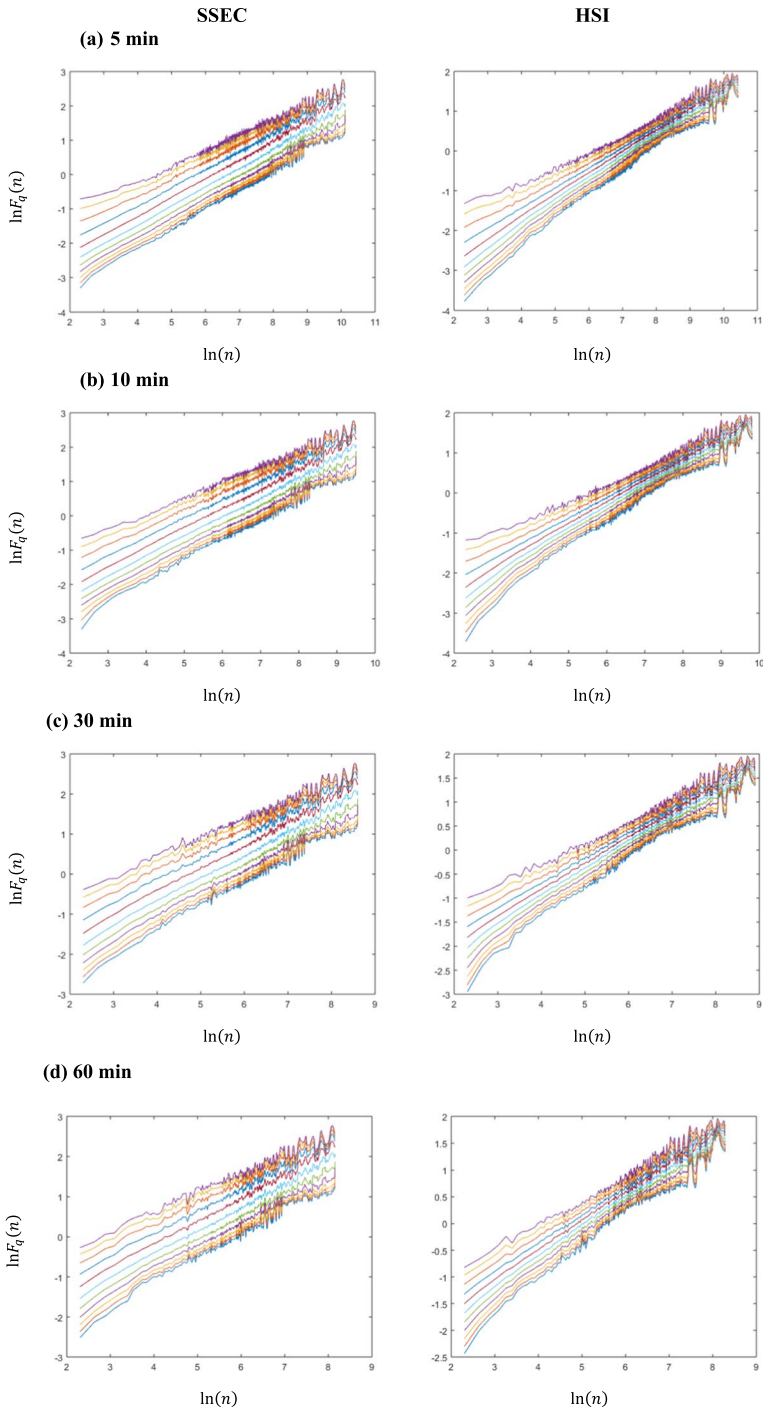
Fig. 3 The MF-DFA functions $Fq(n)$ versus the time scale n in log–log plot. *Note* The functions $Fq(n)$ are calculated with a range of q values from -5 to 5

SSEC   **HSI**

(a) 5-min

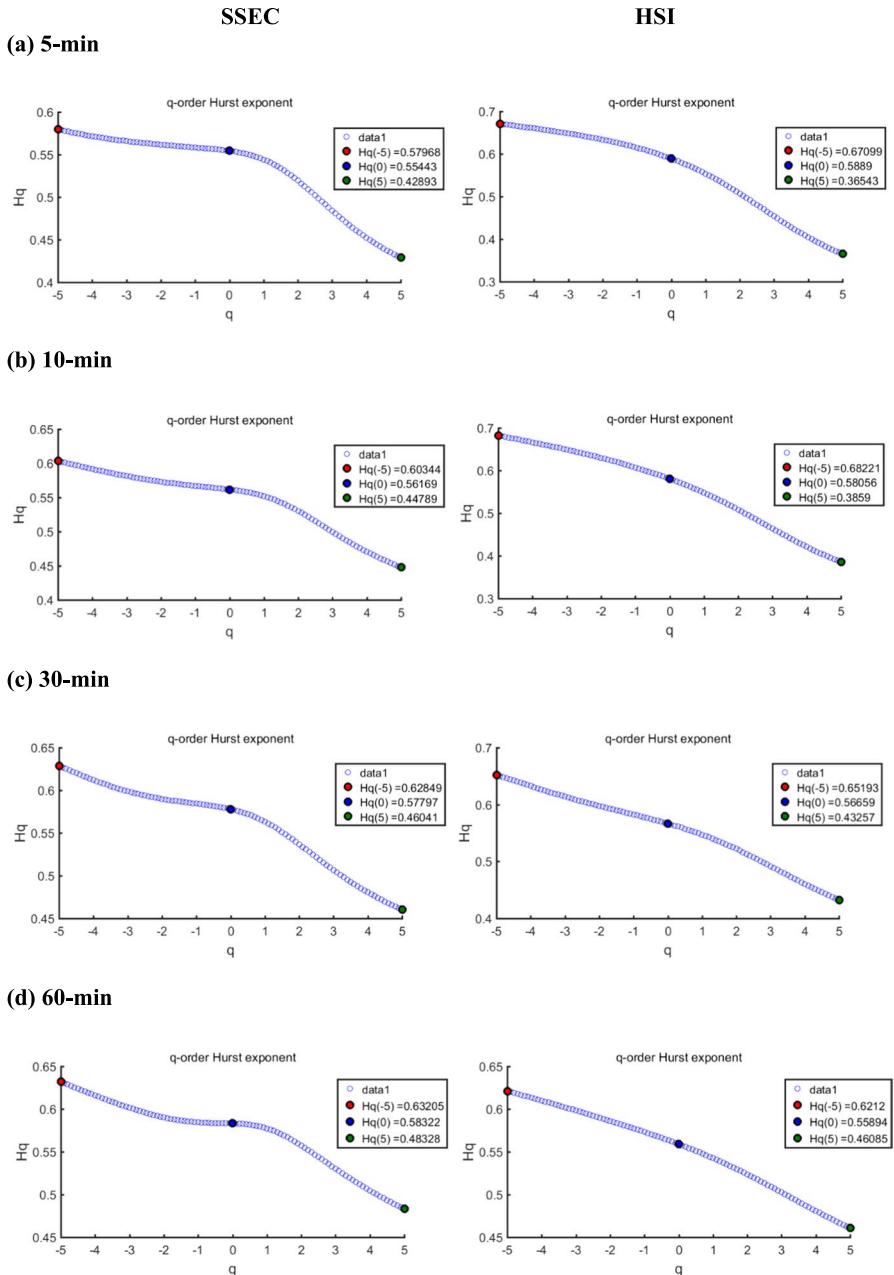**(b) 10-min**

(c) 30-min
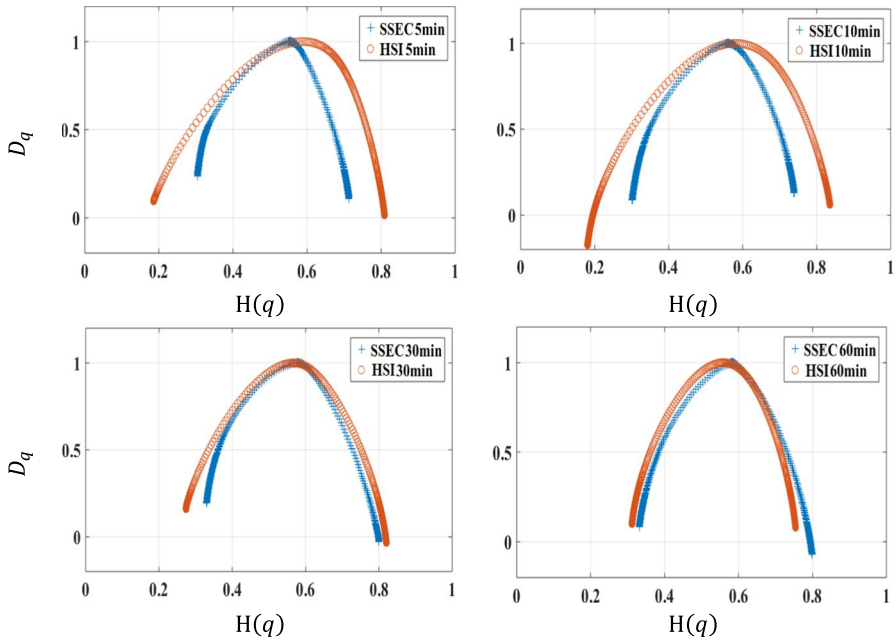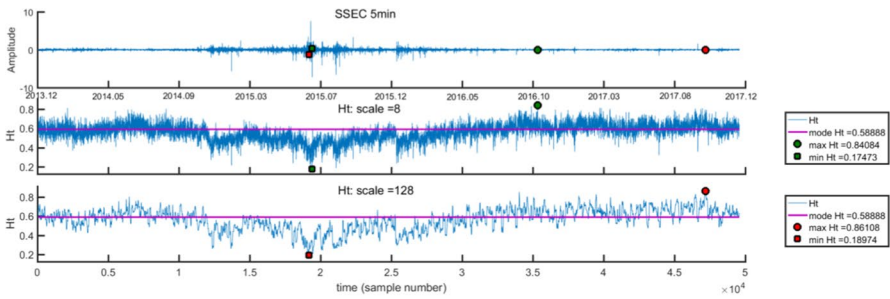

(d) 60-min

Fig. 4 The slope of H(q) in different time scales

Fig. 5 Multifractal spectrum of different time scales

results indicate that financial investors with a log horizon should pay attention to the asymmetric long-range correlation.

Figure 10 displays the excess asymmetry in multifractality for SSEC and HSI, for 5-min, 10-min, 30-min, and 60-min frequencies. The excess asymmetry is the difference between the Hurst exponent at upward and downward trends. If $\Delta H(q)$ is different (equal) to zero, it indicates evidence of (symmetric) asymmetric multifractality. The higher the $\Delta H(q)$, the more asymmetric the market is. If $H^+(q) > H^-(q)$, this indicates a higher cross-correlation exponent when the time series has a positive trend than when it has a negative trend. As we can see, there is a strong excess asymmetry in multifractality for all markets regardless of the frequencies. For the HSI market, we observe a negative and a downward trend of excess asymmetry in multifractality. These results show that the multifractality is much stronger in downward stock price movements. For the SSEC market, the excess asymmetry in multifractality oscillates between negative and positive values, regardless of the four frequencies. This result indicates the validity of our A-MF-DFA method. Both markets exhibit different behavior in terms of excess asymmetry in multifractality.

Figure 11 plots the overall, upward, and downward generalized Hurst exponent $(H(q)$, $H^+(q)$, and $H^-(q))$ values in the SSEC and HSI return dynamics for different q values ranging from −10 to +10. We find that the Hurst exponent values decrease with scale increases for overall, downward, and upward trends. This result is in line with the findings of Cao et al. (2013) where they find that values of the generalized Hurst exponents for Chinese stock markets (Shanghai stock exchange composite index and the Shenzhen component index) both decrease with increases in q,
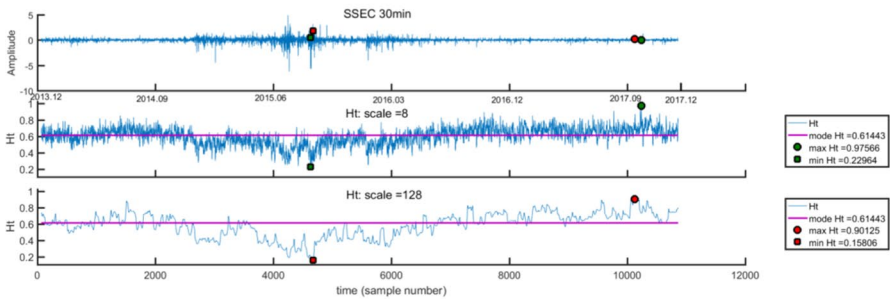
(a) 5-min SSEC

(b) 10-min SSEC
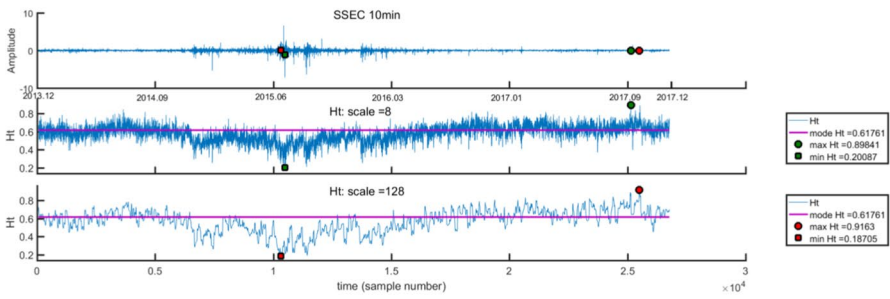
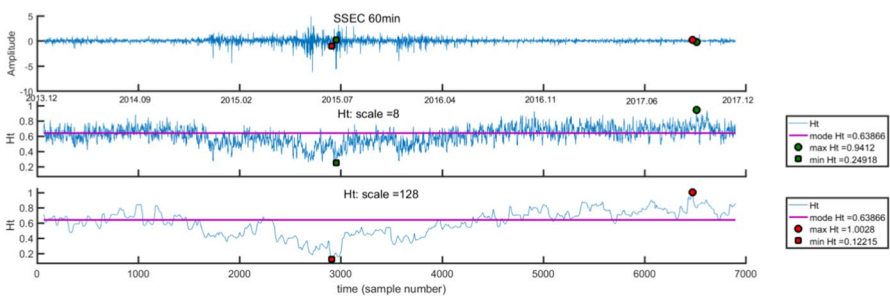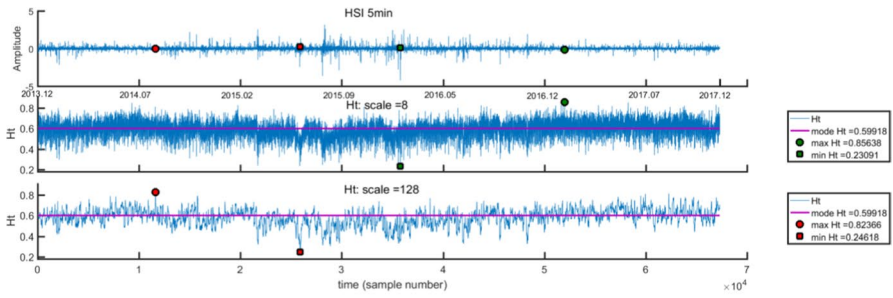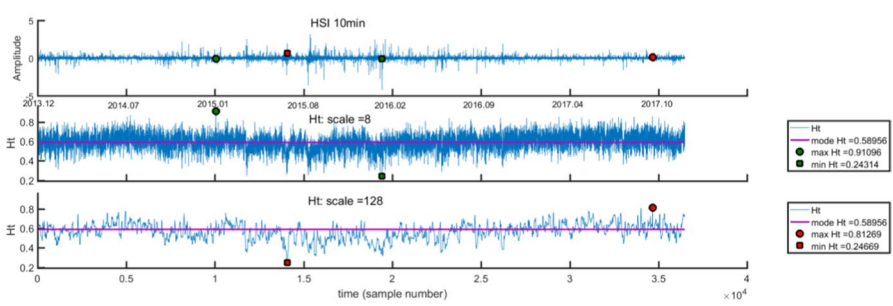**(c) 30-min SSEC**

(d) 60-min SSEC
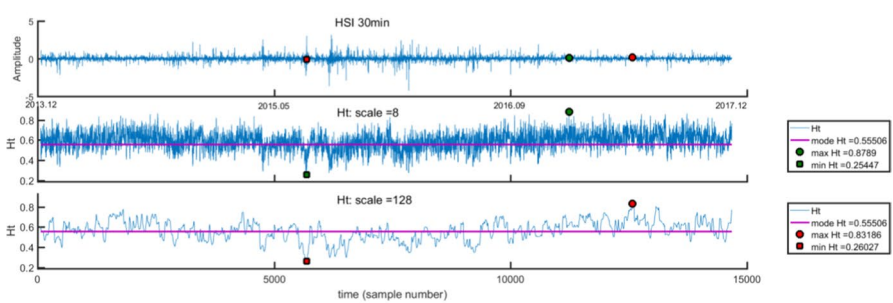

Fig. 6 Time-varying dynamics of Hurst exponents for SSEC

(a) 5-min HSI

(b) 10-min HSI

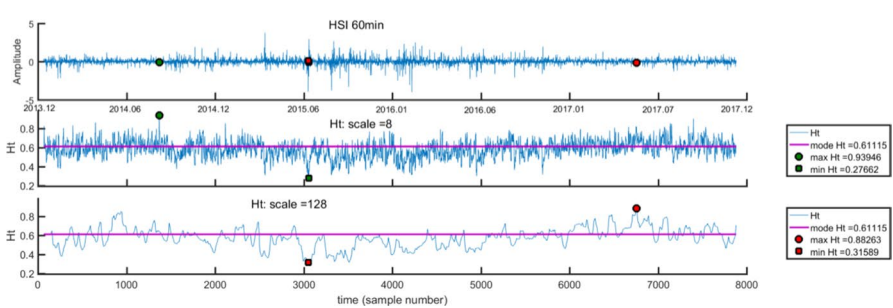**(c) 30-min HSI**

(d) 60-min HSI

Fig. 7 Time-varying dynamics of Hurst exponents for HSI

Table 2 Generalized Hurst exponent for different time scale intraday returns

| q	SSEC				HIS			
	5 min	10 min	30 min	60 min	5 min	10 min	30 min	60 min
− 20	0.66876	0.69530	0.74931	0.74494	0.75991	0.78849	0.76871	0.70801
− 19	0.66642	0.69302	0.74662	0.74213	0.75733	0.78602	0.76600	0.70561
− 18	0.66384	0.69050	0.74362	0.73902	0.75449	0.78329	0.76301	0.70297
− 17	0.66097	0.68771	0.74027	0.73557	0.75135	0.78024	0.75969	0.70007
− 16	0.65777	0.68460	0.73650	0.73173	0.74786	0.77681	0.75599	0.69685
− 15	0.65418	0.68112	0.73224	0.72741	0.74398	0.77295	0.75184	0.69327
− 14	0.65013	0.67719	0.72737	0.72255	0.73963	0.76857	0.74717	0.68928
− 13	0.64552	0.67272	0.72179	0.71704	0.73475	0.76356	0.74186	0.68479
− 12	0.64028	0.66762	0.71533	0.71076	0.72926	0.75778	0.73580	0.67973
− 11	0.63427	0.66175	0.70780	0.70358	0.72309	0.75109	0.72880	0.67399
− 10	0.62736	0.65494	0.69896	0.69531	0.71617	0.74328	0.72065	0.66748
− 9	0.61944	0.64703	0.68853	0.68576	0.70847	0.73416	0.71107	0.66008
− 8	0.61042	0.63784	0.67624	0.67472	0.70001	0.72350	0.69971	0.65171
− 7	0.60036	0.62731	0.66191	0.66204	0.69086	0.71119	0.68619	0.64237
− 6	0.58972	0.61564	0.64570	0.64769	0.68116	0.69731	0.67021	0.63212
− 5	0.57967	0.60344	0.62849	0.63205	0.67099	0.68220	0.65192	0.62119
− 4	0.57162	0.59170	0.61212	0.61605	0.66016	0.66619	0.63251	0.60984
− 3	0.56587	0.58142	0.59886	0.60138	0.64806	0.64905	0.61399	0.59819
− 2	0.56169	0.57317	0.58999	0.59019	0.63346	0.62985	0.59754	0.58610
− 1	0.55833	0.56697	0.58441	0.58440	0.61455	0.60741	0.58245	0.57320
0	0.55442	0.56169	0.57797	0.58322	0.58889	0.58055	0.56659	0.55893
1	0.54436	0.55209	0.56326	0.57730	0.55358	0.54794	0.54714	0.54261
2	0.51946	0.53026	0.53674	0.55744	0.50715	0.50836	0.52196	0.52370
3	0.48425	0.49960	0.50665	0.53036	0.45417	0.46393	0.49179	0.50261
4	0.45247	0.47080	0.48075	0.50477	0.40474	0.42135	0.46059	0.48099
5	0.42892	0.44788	0.46041	0.48327	0.36543	0.38590	0.43257	0.46085
6	0.41202	0.43017	0.44455	0.46563	0.33631	0.35822	0.40959	0.44342
7	0.39949	0.41623	0.43198	0.45111	0.31487	0.33673	0.39147	0.42888
8	0.38976	0.40498	0.42179	0.43904	0.29875	0.31975	0.37726	0.41684
9	0.38192	0.39566	0.41337	0.42894	0.28627	0.30600	0.36598	0.40684
10	0.37541	0.38781	0.40629	0.42039	0.27633	0.29464	0.35689	0.39845
11	0.36987	0.38110	0.40026	0.41310	0.26822	0.28509	0.34944	0.39133
12	0.36510	0.37528	0.39506	0.40684	0.26147	0.27694	0.34321	0.38523
13	0.36092	0.37019	0.39052	0.40141	0.25577	0.26991	0.33794	0.37995
14	0.35724	0.36569	0.38654	0.39667	0.25087	0.26379	0.33342	0.37534
15	0.35396	0.36171	0.38300	0.39250	0.24663	0.25841	0.32950	0.37128
16	0.35103	0.35814	0.37985	0.38881	0.24291	0.25365	0.32605	0.36769
17	0.34839	0.35494	0.37703	0.38552	0.23962	0.24942	0.32301	0.36448
18	0.34600	0.35204	0.37448	0.38258	0.23669	0.24563	0.32029	0.36160
19	0.34382	0.34942	0.37216	0.37992	0.23406	0.24222	0.31785	0.35900
20	0.34183	0.34703	0.37006	0.37752	0.23169	0.23914	0.31565	0.35665
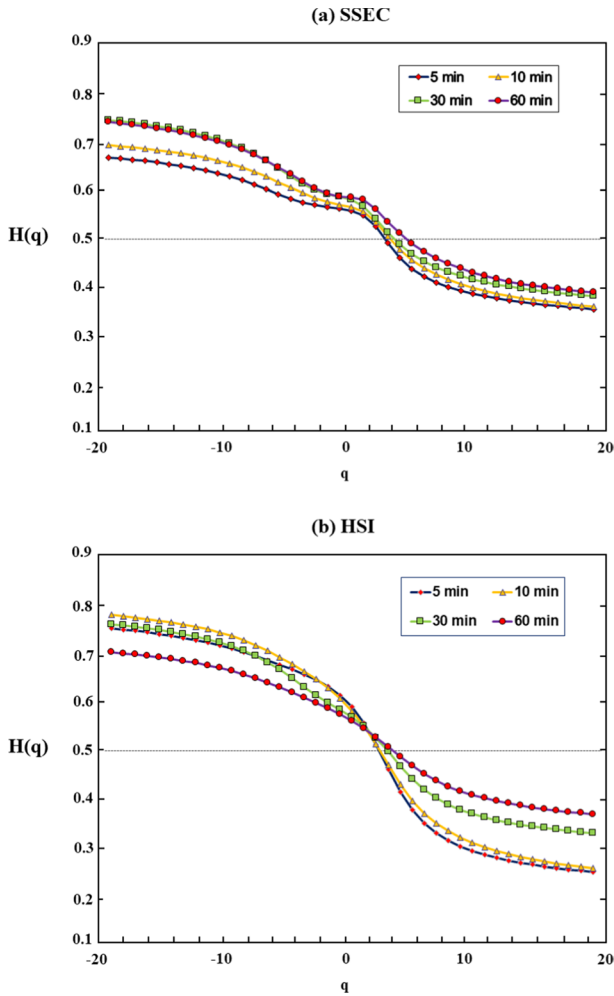
Fig. 8 Generalized Hurst exponent for different time scale intraday returns

suggesting gradually weakened correlations for both down and uptrends. In addition, we find that the upward Hurst exponent values for SSEC markets are superior to downward Hurst exponent values for all scales. In contrast, the reverse is true for the HSI market. We note that the deviation between upward and downward Hurst exponents is more significant for negative scales than positive scales. We note that the gap between the uptrend and downtrend is important for negative scales (small fluctuations) for HSI and for positive scales (large fluctuations) for SSEC. This result implies that the multifractality and its asymmetry are more apparent in the HSI market for small fluctuations and for large fluctuations in the SSEC market. This suggests that equity investors are more interested in the market with large multifractality. This supports evidence of long-range temporal correlations, suggesting higher predictability patters and abnormal profits.
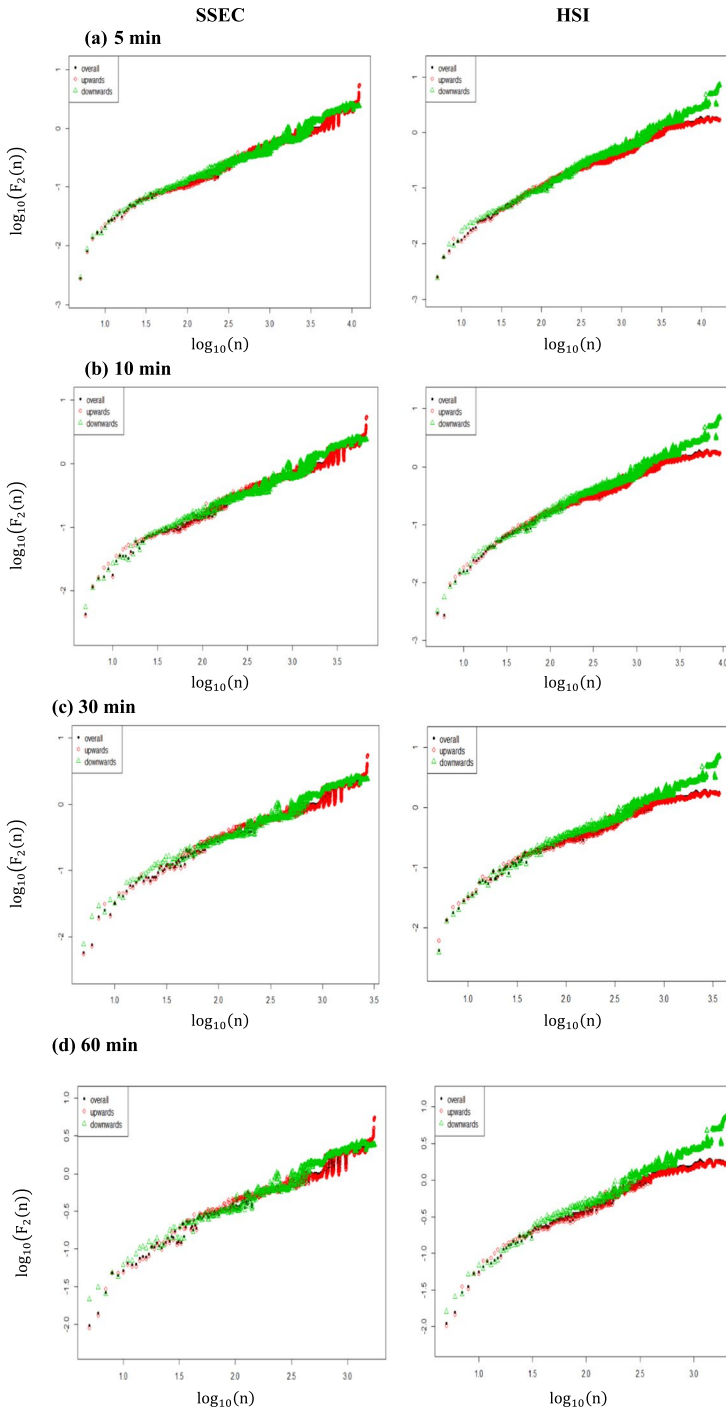
Fig. 9 Asymmetric MF-DFA functions $F_2(n)$ vs the time scale (n). *Note* This figure represents the plot of $log_{10}\big(F_2(n)\big)$ vs. $log_{10}(n)$ for each intraday returns

Figure 12 shows the multifractal spectrum for overall, downward, and upward trends. We find evidence of asymmetric multifractality for the two markets and for the four frequencies. The multifractal spectrum of different time scales is different for upward and downward trends. Taking the 5-min frequency as an example, we observe that the downward multifractality of Hong Kong stock markets has a much larger width than the upward multifractality for all frequencies. The reverse is true for the Chinese stock market. As a result, the Hong Kong stock market shows strong evidence of asymmetric behaviors between upwards and downwards markets. This finding indicates that the Hong Kong stock market is more sensitive to downwards market conditions rather than upwards market conditions.

Following Wang et al. (2009), we quantify the level of inefficiency by utilizing the market deficiency measure (*MDM*) as follows:

$$MDM = \frac{1}{2}(|h(-5) - 0.5| + |h(5) - 0.5|) = \frac{1}{2}\Delta h. \tag{13}$$

It worth noting that a cryptocurrency market is efficient if all fluctuations, including small ($q = -5$) and large ($q = +5$), follow a random walk process. The *MDM* value will therefore be zero for an efficient market and high for an inefficient market.

We quantify the *MDM* and report the measurement results of market efficiency in Fig. 13. The graphical evidence shows that both the SSEC and HSI markets are highly inefficient under a downward market trend rather than an upward trend. For overall and upward trends, we observe that the HIS is less inefficient than the SSEC market for all frequencies. On the other hand, the inefficiency increases with time factor for both markets but is more pronounced for the SSEC market. This result demonstrates that markets become more inefficient as time increases. This result also justifies the appropriateness of using the A-MF-DFA method, relative to the symmetric MF-DFA method. Thus, ignoring the asymmetry in multifractality mis-allocates the resources. The changes in efficiency during different market statuses (bear and bull) enhance our understanding of market behavior and optimize asset allocation and the management of portfolio risk. In sum, the SSEC market is more inefficient than HSI markets. The high inefficiency in the Chinese stock exchange market may be attributed to China's stock market crash in 2015, when the government implements a rescue program that includes a massive government stock purchases from 1000 firms. The market responsiveness to macroeconomic policies and global financial events, as well as the changes in the market environment, affect individual investor sentiment during bear and bull market scenarios, which explains the difference in the efficiency level of both Chinese and Hong Kong stock exchange markets.

It is worth noting that the last few years have been marketed by fierce competition in international stock markets, including the Hong Kong and mainland China markets. The Chinese government has launched a series of reforms and policies to enhance the development and liquidity of their stock markets. The Shanghai-Hong Kong Connect Program (SHCP) implemented on April 10, 2014 is one of the most important programs allowing foreign investors that have stock accounts in Hong Kong to buy shares of some connected firms in the Chinese stock market. Moreover,

n, which varies from 5 to $N/4$ (where N is the number of observations in the time series). The y-axis represents the difference between $log_{10}(F_2^+(n))$ and $log_{10}(F_2^-(n))$

China's stock market is not fully liberalized, which means that local (domestic) investors cannot easily invest overseas.

To test the validity of our results, we augment our study with some important robustness tests. We first test the null hypothesis that the Hurst exponent under scale 2 equates 0.5 in the value of the random walk hypothesis (H(q = 2) = 0.5) against the alternative hypothesis that (H(q = 2) ≠ 0.5). The results of this test are reported in Table 3. On the other hand, we test the equality of the slope under downwards and upwards trends using two mean equality tests (e.g., Satterthwaite & Welch, 1946; Anova of Welch, 1951) and two variance equality tests (e.g., Brown & Forsythe, 1974; Levene, 1960). The results are presented in Table 4. Looking at these two tables, we can draw two important conclusions. First, the Hong Kong and Chinese stock markets are inefficient for overall, upward, and downward trends. Second, the downward inefficiency is statistically and significantly different to upward inefficiency (Table 3).

3.3 The Determinants of Time-Varying Hurst Exponent: The Role of BTC

To determine the driver factors in the time varying of Hurst exponents (H_t),[5] we estimate a QRA model[6] specified as follows:

$$Q_\tau(H_t) = c(\tau) + \beta_{1\tau} rBTC\_Price_t + \beta_{2\tau} \Delta BTC\_Vol_t + \beta_{3\tau} Log(BTC_{Cap}) + \xi_t,$$

(11)

where $c(\tau)$ denotes a constant term and Q_τ indicates the τth quantile in the explained variables of Hurst exponents (*H*). Moreover, the $\beta_{k\tau}$ parameter monitors the impact at the τth quantile of the dependent variable using three explanatory variables, namely BTC returns (*rBTC_Price*), changes in BTC volumes (ΔBTC_Vol), and log BTC USD trading capitalizations (Log(*BTC_Cap*)). Among all cryptocurrencies, BTC is the most important crypto asset. In terms of market capitalization ($1 trillion). BTC values experience a first upside pattern from 2008 until 2017 followed by a crash by about 65% in the first quarter of 2018. The second BTC price crash occurred in September 2018. The BTC price is soaring in early 2021 after a first decrease in March 2020 (price increases by 700%). The impact of BTC prices on stock market returns and volatility attracts a special attention. The existing empirical literatures shows spillovers between BTC and stock markets (Ahmed, 2021; Jiang et al., 2022; Singh, 2021; Uzonwanne, 2021; Zhang et al., 2021). Goodell and Goutte (2021) shows that COVID-19 intensifies

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[5] The descriptive statistics of the Hurst exponent show evidence of non-normality and nonlinearity. The results are available upon request.

[6] For more information on QRA, see Koenker (2005), Koenker and Bassett (1982), Koenker and D'Orey (1987), and Koenker and Hallock (2001).
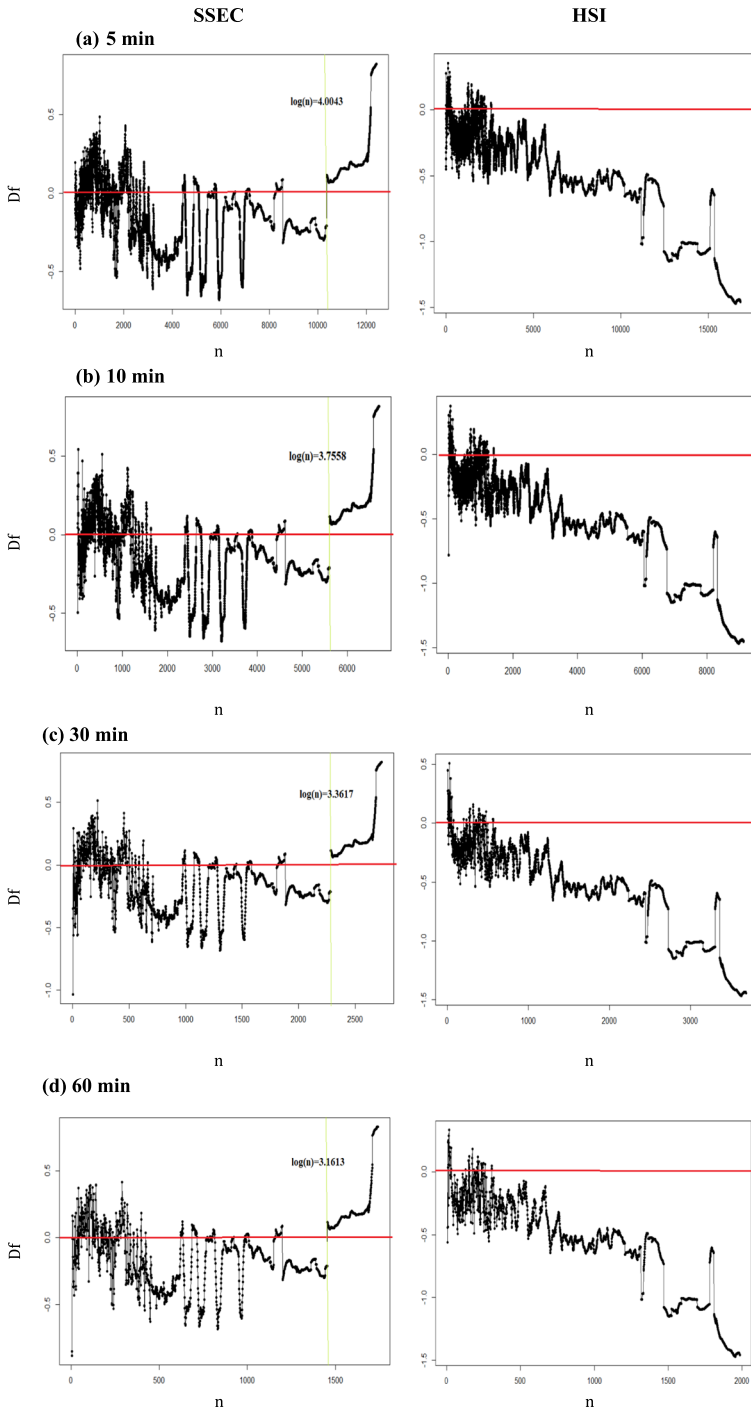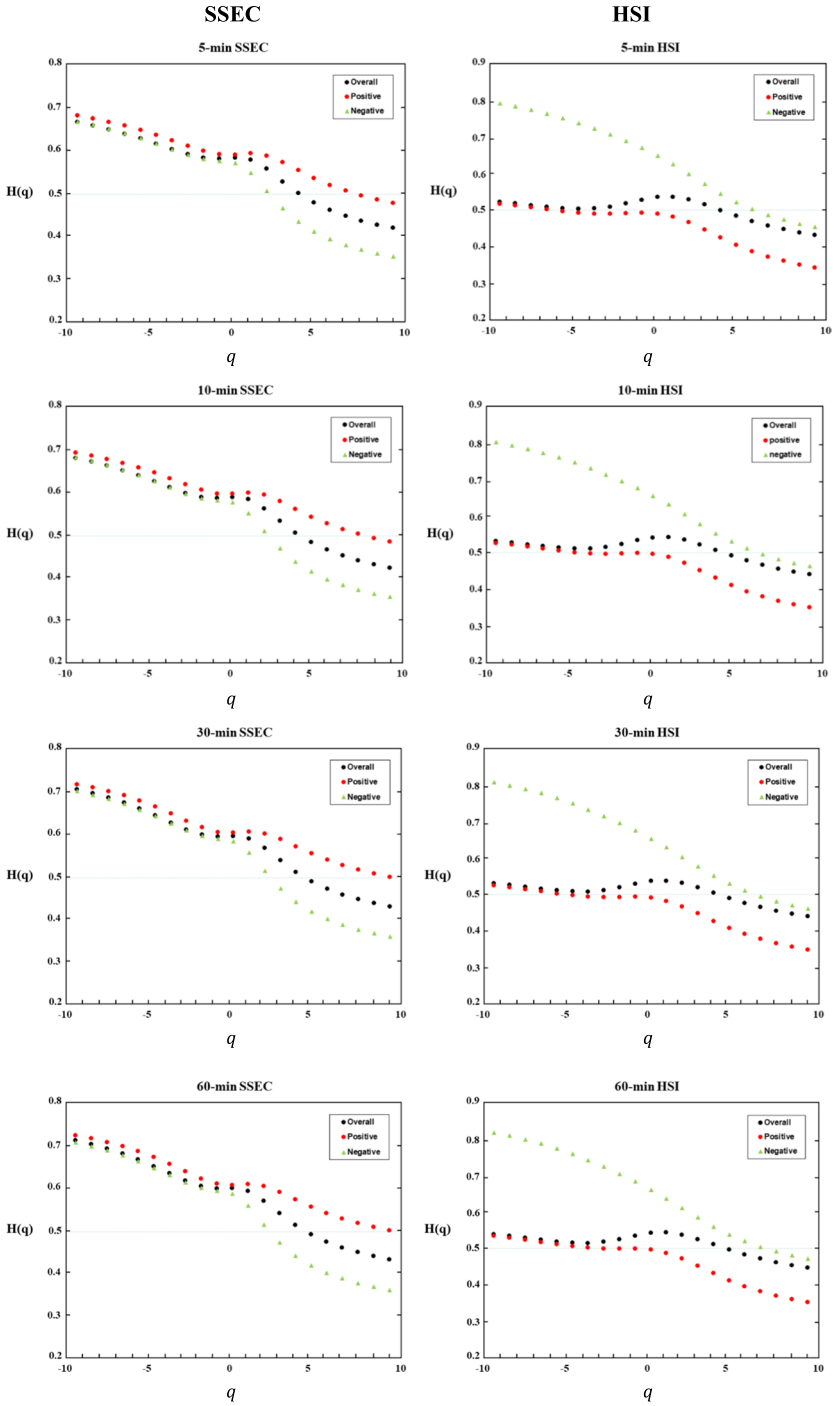
SSEC   HSI

(a) 5 min

(b) 10 min

(c) 30 min

(d) 60 min

Fig. 11 Plots of Hurst exponents for SSEC and HSI stock markets. *Notes* This figure shows the overall, ▶ upward, and downward generalized Hurst exponent ($H(q)$, $H^+(q)$, and $H^-(q)$) values in the SSEC and HSI return dynamics for different q values ranging from -10 to $+10$. (i) If $0 < H < 0.5$, then the intraday returns $x(t)$ is not persistent; (ii) if $0.5 < H < 1$, $x(t)$ exhibits persistence; (iii) if $H = 0.5$, then $x(t)$ follows a random series

the co-movements between cryptocurrencies and equity indexes. Salisu et al. (2019) find that BTC is a good predictor instrument of stock markets. QRA examines the conditional dependence of specific quantiles of the efficiency degree with respect to the conditioning control variables. It enables us to examine the nonlinear relationships between Hurst level and the control variables when the level of efficiency ranges from the lowest (lower quantiles or q = 0.05) to highest (upper quantile or q = 0.95). The QRA offers valuable information on the impacts of BTC returns, changes in BTC volumes, and log BTC USD trading capitalizations on the degree of efficiency under different market efficiency circumstances. This method explores whether the effects of control variables on varying level of efficiency are symmetric or asymmetric. The 5-min data for BTC are obtained from the Bitfinex Exchange. The data for the other frequencies are not available, which is why we carried out the regression using 5-min data.

Table 5 reports the estimates of QRA between the Hurst exponents of SSEC and HSI markets and BTC price returns, changes in BTC volumes, and BTC USD trading capitalizations across seven quantiles. The results indicate that BTC price returns have no impact on the efficiency in both Chinese and Hong Kong markets for all quantiles. BTC volumes negatively and significantly influence the efficiency level of Chinese stock markets during lowest, normal, and highest quantiles. This result indicates that the decrease in BTC volumes amplifies the inefficiency for different levels of Hurst exponent. For Hong Kong stock markets, the BTC volume negatively influences the efficiency during periods in which the Hurst exponent value is low (q = 0.05 and q = 0.1 or lowest inefficiency). The BTC USD trading capitalizations contribute significantly to the efficiency level of Chinese stock markets under different quantiles. For Hong Kong, we find asymmetric relationships between BTC USD trading capitalizations and efficiency level. Figure A1 plots the changes in the quantile regression coefficients of efficiency level conditioning by three driver variables.

For robustness, we carry out the nonparametric Wald robustness test (see Koenker & Bassett, 1982 for more information on this test) to check for the stability of the slopes over quantiles. This test checks all slope heterogeneity across any two quantiles. More specifically, we test the null of homogeneity of slopes for two quantiles against the alternative hypothesis of heterogeneity of slopes. Table 6 reports, as an example, the results of the Wald tests across the quantiles q = 0.05 vs. q = 0.5 and q = 0.05 vs. q = 0.95. The remaining results are available upon request. This table rejects the null of parameter homogeneity across quantiles for BTC USD trading capitalizations, indicating that the estimated coefficients are time varying, implying that the relationship between trading capitalization and efficiency level varies significantly across quantiles.
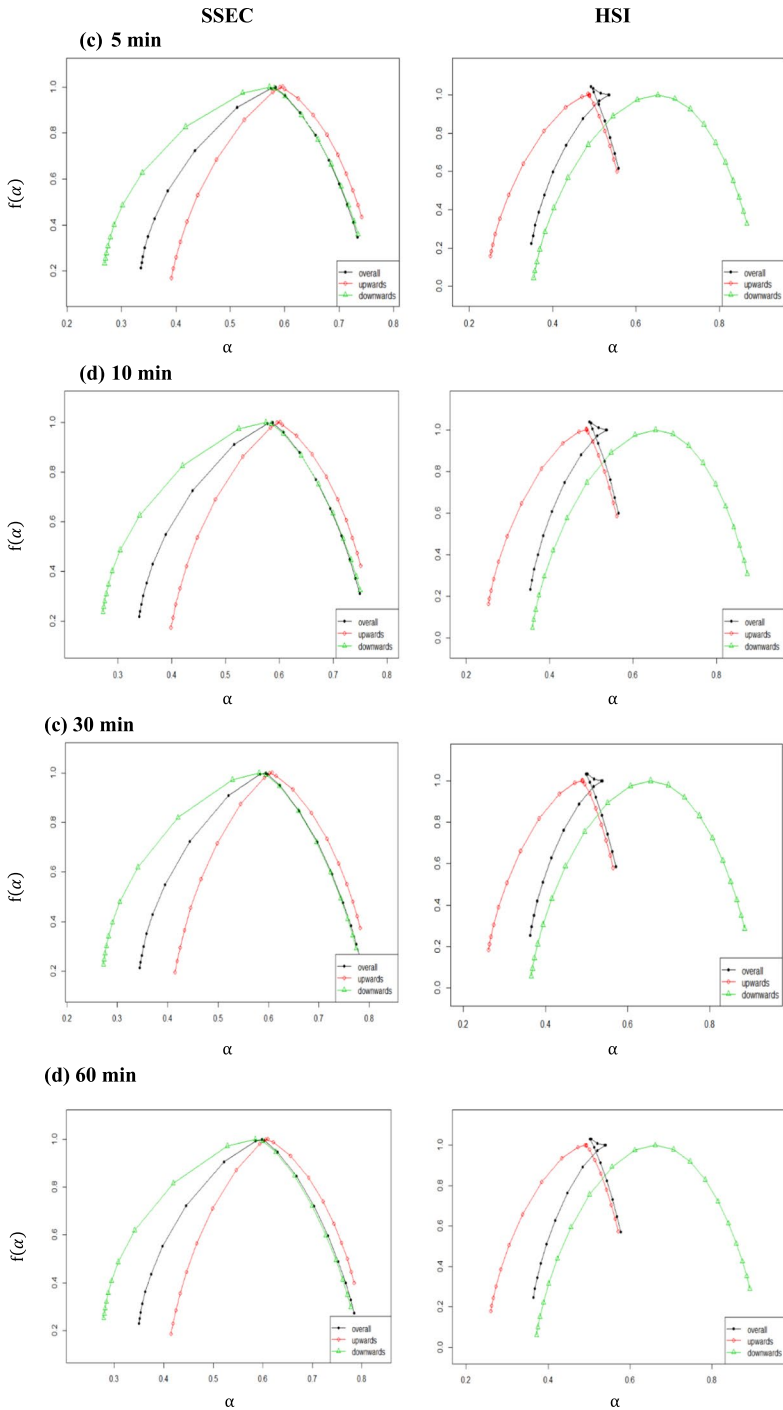
SSEC

HSI

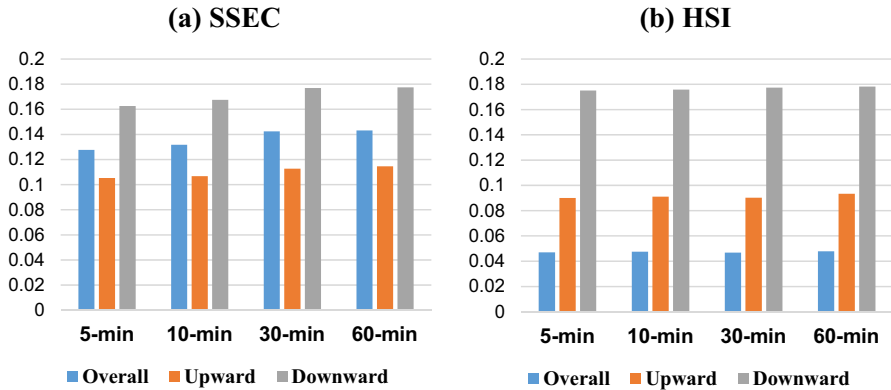**Fig. 12** Asymmetric multifractal spectrum. *Note* The multifractal spectra $f(\alpha)$ versus α where q ranges from -5 to 5

Fig. 13 Measurement of market efficiency using MDM

Table 3 t-test results of the null hypothesis for H(q = 2) = 0.5

	SSEC			HSI		
	Overall	Upward	Downward	Overall	Upward	Downward
5-min	2.8894	6.0088	0.9127	-0.5458	-3.8272	5.0716
	[0.009]	[0.000]	[0.372]	[0.591]	[0.001]	[0.000]
10-min	3.1242	6.3664	1.0925	-0.1521	-3.6226	5.1915
	[0.005]	[0.000]	[0.287]	[0.880]	[0.002]	[0.000]
30-min	3.4688	6.9395	1.3648	0.3828	-3.4602	5.3428
	[0.002]	[0.000]	[0.187]	[0.705]	[0.002]	[0.000]
60-min	3.6250	7.0160	1.4747	0.8284	-3.1751	5.5441
	[0.002]	[0.000]	[0.155]	[0.417]	[0.004]	[0.000]

The generalized Hurst exponent in the case of $q = 2$, i.e., H_q, is identical to the standard Hurst exponent, which can be used to test the long-memory property of a time series. [***] denotes the rejection of the null hypotheses at the 1% significance level

4 Conclusions

Upward and downward movements of stock prices have important implications for asset allocations and risk management. Equity investors follow price patterns and formulate investment strategies given market trends. On the other hand, market efficiency, long memory, and multifractality issues are not only influenced by the time factor but also by market trends. This study sheds light on the asymmetry in multifractality, long-range memory, and evolving efficiency of Chinese and Hong Kong stock markets. We use the A-MF-DFA method and generalized Hurst exponent for intraday data using four different frequencies (5-min, 10-min, 30-min, and 60-min). We also examine the drivers (BTC prices, BTC volume and BTC USD trading capitalizations) of time-varying efficiency across quantiles.

Using the MF-DFA method, our results provide evidence of crossover time scale for both markets and for all frequencies, indicating that variation in the trajectory

Table 4 Robustness tests for heterogeneity of slopes

	Equality mean tests		Equality variance tests		
	Satterthwaite-Welch	Anova	Bartlett	Levene	Brown-Forsythe
SSEC					
5-min	88.071	5796.9	21.845	4.6788	2.7548
	[0.000]	[0.000]	[0.000]	[0.003]	[0.042]
10-min	261.82	1797.7	22.701	5.9933	3.0887
	[0.000]	[0.000]	[0.000]	[0.000]	[0.027]
30-min	275.20	1044.1	18.772	5.8374	3.4063
	[0.000]	[0.000]	[0.000]	[0.001]	[0.018]
60-min	261.06	819.94	12.533	3.0876	2.295
	[0.000]	[0.000]	[0.013]	[0.027]	[0.078]
HIS					
5-min	97.065	108.41	34.177	4.5588	2.9463
	[0.000]	[0.000]	[0.000]	[0.004]	[0.033]
10-min	86.227	8081.6	41.183	7.0486	4.5261
	[0.000]	[0.000]	[0.000]	[0.003]	[0.005]
30-min	63.569	5433.9	44.442	10.896	9.070
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
60-min	125.01	294.21	10.552	3.7384	3.3557
	[0.000]	[0.000]	[0.032]	[0.012]	[0.019]

This table presents the mean equality tests (Satterth-Welch and Anova statistics) and the variance equality tests (Bartlett, Levene, and Brown-Forsythe) for the Hurst exponent coefficients of MF-DFA vs. A-MF-DFA models

of the local slope. These results provide supporting evidence for the multifractality hypothesis. Moreover, the multifractality of Hong Kong stock markets has a very large width compared to the Chinese stock markets for all frequencies, suggesting that the multifractality in Hong Kong stock price dynamics is much higher than in the Chinese stock prices. For both markets, the Hurst exponent is time varying along the sample period regardless of the frequencies. More interestingly, the values of the Hurst exponent increase with time factor. Both markets are inefficient according to the values of the Hurst exponent. In addition, the inefficiency is higher under small fluctuations ($q < 0$) than large fluctuations ($q > 0$). Using the A-MF-DFA method, the results reveal significant distinctions between uptrend and downtrend values throughout various time scales. The extent of the symmetry is significant at higher time scales. This result provides evidence against the symmetric multifractality assumption. The Hurst exponent values decrease with scale increases for overall, downward, and upward trends. In addition, we find that the upward Hurst exponent values for SSEC market are superior to downward Hurst exponent values for all scales. In contrast, the reverse is true for the HSI market. We note that the deviation between upward and downward Hurst exponents is more significant for negative scales than for positive scales. Using the MDM measure, we show that the Chinese and Hong Kong stock markets are highly inefficient under downward

Table 5 Quantile regression estimates for the Hurst exponents

	Q(0.05)	Q(0.10)	Q(0.25)	Q(0.50)	Q(0.75)	Q(0.90)	Q(0.95)
Panel A: SSEC_5min							
c	0.2977*** (0.0064)	03429*** (0.0044)	0.4125*** (0.0024)	0.5027*** (0.0019)	0.5769*** (0.0017)	0.6349*** (0.0018)	0.6653*** (0.0023)
$\beta_1(rBTC\_Price)$	0.0181 (0.0041)	−0.0080 (0.0058)	−0.0106 (0.0203)	−0.0009 (0.0013)	−0.0016 (0.0133)	−0.0034 (0.0161)	−0.0024 (0.0222)
$\beta_2(\Delta BTC\_Vol)$	−0.0072*** (0.0006)	−0.0064*** (0.0004)	−0.0056*** (0.0003)	−0.0039*** (0.0002)	−0.0025 (0.0002)	−0.0034*** (0.0161)	−0.0016*** (0.0003)
$\beta_3(Log(BTC\_Cap))$	0.0326*** (0.0015)	0.0299*** (0.0010)	0.0026*** (0.0005)	0.0177*** (0.0004)	0.0117*** (0.0003)	0.0075*** (0.0002)	0.0061*** (0.0005)
PseudoR²	0.0326	0.0327	0.0340	0.0177	0.0144	0.0071	0.0042
Panel B: HSI_5min							
c	0.3806*** (0.1140)	0.4347*** (0.0089)	0.5198*** (0.0058)	0.5785*** (0.0053)	0.6517*** (0.0050)	0.7058*** (0.0073)	0.7389*** (0.0097)
$\beta_1(rBTC\_Price)$	−0.0852 (0.0850)	−0.0174 (0.0514)	0.0257 (0.0433)	−0.0186 (0.0342)	−0.0226 (0.0422)	0.0778 (0.0441)	−0.0227 (0.1161)
$\beta_2(\Delta BTC\_Vol)$	−0.0041*** (0.0009)	−0.0023*** (0.0009)	−0.0004 (0.0006)	−0.0004 (0.0005)	0.0005 (0.2368)	0.0011 (0.0007)	−0.0010 (0.0009)
$\beta_3(Log(BTC\_Cap))$	0.0131*** (0.0024)	0.0081*** (0.0020)	−0.0011 (0.0014)	−0.0004 (0.0012)	−0.0035 (0.0011)	−0.0048*** (0.0017)	−0.0049** (0.0024)
PseudoR²	0.0091	0.0032	0.0009	0.0007	0.0012	0.0021	0.0016

Standard errors are in parentheses

, and * , * indicate significance at the 5% and 1% levels, respectively

Table 6 Wald tests for the equality of slopes (0.05 against 0.5 and 0.95 quantiles)

	Against the 0.5 quantile		Against the 0.95 quantile	
	Test-statistic	p-value	Test-statistic	p-value
Panel A: SSEC_5min				
$\beta_1(rBTC\_Price)$	-0.0105	0.6206	0.0174	0.7112
$\beta_2(\Delta BTC\_Vol)$	-0.0004	0.2131	-0.0011	0.1110
$\beta_3(Log(BTC\_Cap))$	0.0022^{***}	0.0004	0.0034^{**}	0.0359
Panel B: HSI_5min				
$\beta_1(rBTC\_Price)$	0.0403	0.4467	-0.0251	0.8592
$\beta_2(\Delta BTC\_Vol)$	0.0006	0.4667	-0.0022	0.1388
$\beta_3(Log(BTC\_Cap))$	-0.0033^{*}	0.0673	0.0072^{**}	0.0477

This table presents the estimated results of the Wald test for equality of slopes (0.05 against each of 0.5 and 0.95 quantiles)

The asterisks *, ** and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively

market status compared to upward trends. In addition, the Hong Kong market is less inefficient than the Chinese market for all frequencies. The Chinese and Hong Kong markets are more inefficient under 60-min data than 5-min data. The robustness tests show that the coefficient of the Hurst exponents is statistically different for overall, upward, and downward trends. The downward inefficiency is statistically and significantly different to upward inefficiency.

Finally, we find that BTC price returns have no impact of the efficiency of both Chinese and Hong Kong markets for all quantiles. BTC volumes negatively and significantly influence the efficiency level of the Chinese stock market during the lowest, normal, and highest quantiles. For the Hong Kong stock market, the BTC volume negatively influences the efficiency when Hurst exponent values are low. The BTC USD trading capitalizations contribute significantly to the efficiency level of the Chinese stock market under different quantiles. The study finds asymmetric relationships between BTC USD trading capitalizations and the efficiency level for the Hong Kong stock market.

Our results have important implications for equity investors. First, investors should account for asymmetric multifractality to better understand the price dynamics, and to predict volatility and crashes (Grech & Pamula, 2008; Wei & Wang, 2008). Second, investors in Chinese and Hong Kong stock markets can exploit abnormal returns more during downward market conditions compared to upward trends. The evidence of dependencies implies some capacity for the predictability of stock prices. Third, investors should keep a close eye on the BTC market to better understand the price dynamics and efficiency of the Chinese and Hong Kong stock markets and optimize the investment strategies. Our findings also assist policymakers and regulators in implementing policy actions and measures, mainly during downward trends, to ensure transparency and stability and boost the confidence of retail and institutional investors. These measures protect the investment environment in equity shares during turmoil periods and promote the sustainability and development of the stock markets for financial stability.

Our paper can be extended by analyzing the effects of investors' happiness on the time-varying efficiency of stock markets.
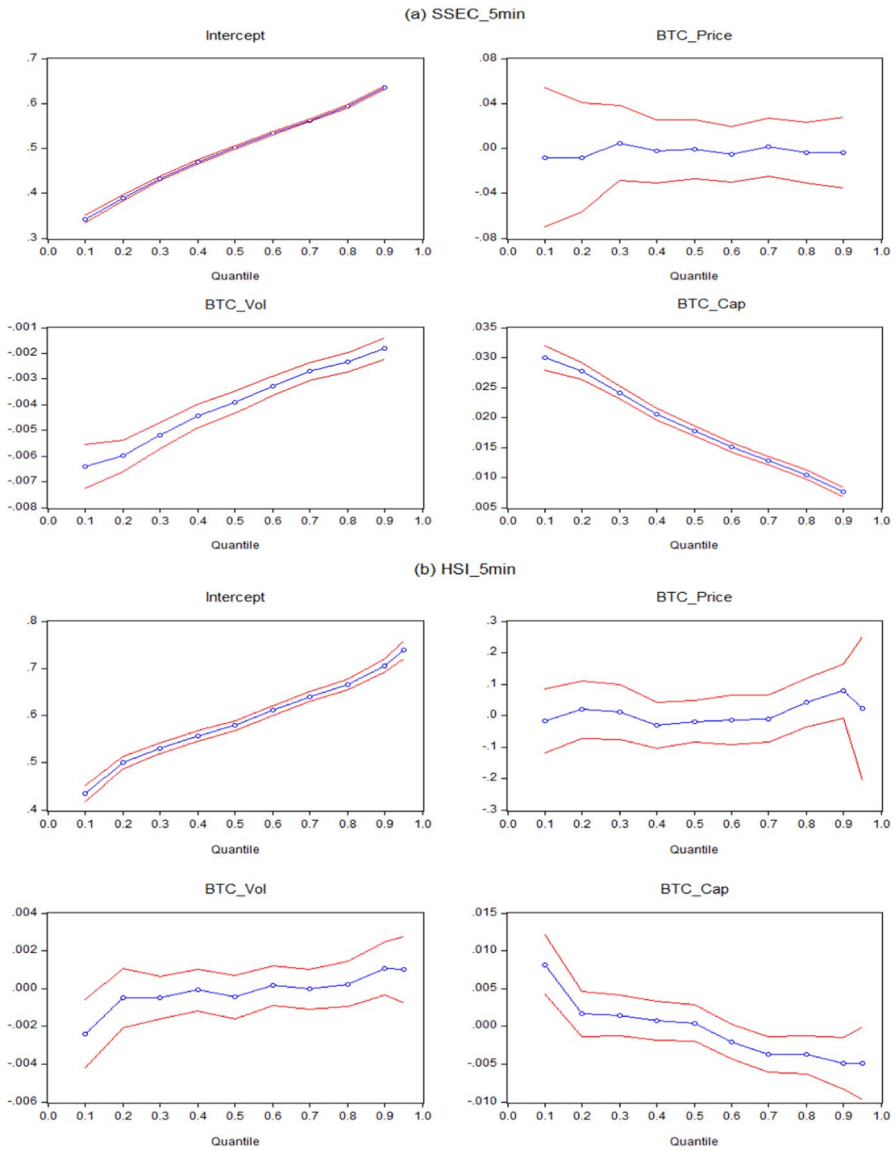
Appendix

See Fig. 14

Fig. 14 Changes in the quantile regression coefficients

Declarations

Conflict of interest The authors certify that there is no conflict of interest.

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