



Analysis of China's Regional Economic Environmental Performance: A Non-radial Multi-objective DEA Approach

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Abstract

One of the hot topics is how to achieve more accurate results of economic and environmental efficiency evaluation in China. Previous data envelopment analysis (DEA) literature on environmental performance measurement often follow the concept of non-radial efficiency measure for calculating the performance on resources and economic-environmental factors respectively. This paper proposes a non-radial and multi-objective generalized DEA model for economic-environmental efficiency evaluation. The results illustrate that this model can not only analyze the relationship between DEA efficiency and Pareto optimality of the multi-objective programming problem defined on the production possibility set, but also obtain the performance improvement direction by using the projection of decision making units. Finally, a case on measuring the economic-environmental performance of Chinese provincial regions is employed to indicate that the proposed model can be helpful to promote the accuracy of economic-environmental efficiency evaluation.

Keywords Data envelopment analysis (DEA) · Undesirable outputs · Non-radial · Multi-objective

1 Introduction

Over the past decades, China's economy has experienced rapid growth through the reform and opening-up policy (Ding and Li 2014). According to the China National Bureau of Statistics, the gross domestic product (GDP) reached exceed 80 trillion Yuan, and contributes more than 30% of global economic growth in 2017. In addition, Chinese GDP has exceeded the USA in the light of purchase power parity, and

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will surpass the scale of the United States in the light of market rates of exchange by 2030. However, the rapid economic development has brought challenges of imbalance and insufficiency development problems, such as regional disparities, resource shortage, and environmental pollution (Zhang et al. 2016; Yang et al. 2017). To address these issues, economic environmental performance analysis has been widely studied by governments and academics.

Data envelopment analysis (DEA) has been widely applied for evaluating economic environmental efficiency since it was firstly proposed by Charnes et al. (1978). It is a well-known non-parametric method to evaluate the performance of a set of decision making units with multiple inputs and multiple outputs. Based on different empirical axioms and corresponding to different characteristics of the production possibility set and production frontiers, many DEA models, such as CCR model (Charnes et al. 1978), namely the BCC model (Banker et al. 1984), the FG model (Färe and Grosskopf 1985) and the ST model (Seiford and Thrall 1990), are developed and applied in various areas, such as educational institutions (Sagarra et al. 2014; Thanassoulis et al. 2017), hospitals (Chowdhury and Zelenyuk 2016; Toloo and Jalili 2016), financial industries (Aggelopoulos and Georgopoulos 2017; Zhou et al. 2018). Considering the multiple inputs and outputs of economic regions in China, the DEA approach is selected as our tool for performance evaluation in this paper.

In literature, DEA models have been widely applied to economic environmental performance evaluation problems. A direct approach for measuring environmental performance originates from the idea of incorporating undesirable outputs with productive efficiency measurement pioneered by Färe et al. (1989). Zhou et al. (2008) apply a DEA method to analyze the Chinese industrial eco-efficiency under the assumption of variant returns to scale (VRS). Chu et al. (2016) focus on the eco-efficiency analysis of Chinese provincial-level regions, regarding each region as a two-stage network structure. Fei et al. (2016) integrate the goal of maximizing the desirable outputs and that of disposing the undesirable outputs to evaluate the performance of industrial systems for Chinese administrative regions. Masuda (2016) measures the eco-efficiency of wheat production in Japan at a regional level by using a combined methodology of DEA and life cycle assessment. Zhu et al. (2016) propose a SBM-DEA model based on natural resource input orientation to evaluate the efficiency of natural resource utilization for 26 provincial regions in mainland China from 2005 to 2012. Sueyoshi et al. (2017) define social sustainability as the simultaneous achievement of economic prosperity and environmental protection, and evaluate the degree of social sustainability across provinces in China. Beltrán-Estève and Picazo-Tadeo (2017) assess environmental performance in the European Union (EU) using Luenberger productivity indicators, directional distance functions and Data Envelopment Analysis techniques. Song et al. (2017) evaluate China's provincial environmental efficiency by using Ray slack-based model. Other relative researches on DEA based economic environmental performance evaluation can be seen in Mardani et al. (2016) which provide a quite comprehensive literature review.

The above economic environmental analysis approaches mainly develop radial measures to deal with performance evaluation problems. However, there are some shortages in using radial efficiency measures. For example, it often leads to the case

where a lot of DMUs have the same efficiency score of 1 and hence difficulty in ranking the environmental performance of these DMUs only based on their efficiency scores. Non-radial DEA models seem to be more efficient in measuring environmental performance, as they have a higher discriminating power in evaluating the efficiencies of DMUs. In addition, if more information, for example, the preference of decision makers, is available, radial DEA models are not easy to incorporate the information by assigning different weights to different undesirable outputs.

There are also some non-radial DEA models measuring economic efficiency have been developed in environmental performance measurement, for example, Meng et al. (2013), Sueyoshi and Wang (2014), Huang et al. (2014) and Krivonozhko et al. (2014). However, rarely study considers their applicability in comprehensively considering environmental and economic performance measurement. It is therefore worthwhile to extend the traditional DEA models into the case comprehensively considering inputs, desirable outputs and undesirable outputs. Due to the increasing depletion of non-renewable resources, the resources needed by social economic development are more and more difficult to obtain. While protecting the environment, we also pay more attention to the resources invested and try to maintain economic growth. Hence, this paper aims to introduce a non-radial and multi-objective generalized DEA model measuring pure environmental performance and productive efficiency.

In this paper, we develop a non-radial and multi-objective generalized DEA model to evaluate the economic environmental performance of 30 Chinese provincial regions in 2016. The contribution of this paper is in four aspects. First, the proposed DEA model outlines some particular DEA models including (CCR) multi-objective DEA model, (BCC) multi-objective DEA model, (FG) multi-objective DEA model, and (ST) multi-objective DEA model. Second, we analyze the relationship between DEA efficiency and Pareto Optimality. Third, we define the definition on the projection of DMUs, and obtain adjustable volumes of inputs and outputs by using the projection of DMUs. Finally, a empirical study of measuring economic-environmental performance of Chinese provincial regions, which indicates that this new model promotes the accuracy of economic and environmental efficiency evaluation.

The rest of the paper is organized as follows. The methodology is presented in Sect. 2. Section 3 analyzes the empirical study and discusses the results. Conclusions and suggestions for future research are given in Sect. 4.

2 Non-radial and Multi-objective Generalized DEA Model

2.1 Production Technology

When desirable outputs and undesirable outputs are jointly produced, we first study the concept of production technology. Assume that $x = (x_1, x_2, \dots, x_m)$, $y = (y_1, y_2, \dots, y_s)$ and $z = (z_1, z_2, \dots, z_k)$ denote the vectors of inputs, desirable outputs and undesirable outputs respectively. namely Production possibility set is defined as $T = \{(x, y, z): x \text{ can produce } (y, z)\}$

The production technology T has been well-defined conceptually, but it cannot be directly used to the environmental DEA technology. When undesirable outputs are considered, Färe et al. (2004) introduced the production possibility set exhibiting constant returns to scale by the piecewise linear combination of the observed data. Now suppose undesirable outputs can be also changed the same as input and desirable output and we extend the production possibility set. There are n DMUs and for $DMU_j(j=1,2,\dots,n)$ the observed input, desirable output and undesirable output vectors are respectively $x_i=(x_{1j}, x_{2j}, \dots, x_{mj})$, $y_r=(y_{1j}, y_{2j}, \dots, y_{sj})$ and $z_t=(z_{1j}, z_{2j}, \dots, z_{kj})$, T can be concretely formulated as follows:

$$T = \left\{ (x, y, z) : \begin{aligned} &\sum_{j=1}^n \lambda_j x_{ij} \leq x_i, \quad i = 1, \dots, m; \\ &\sum_{j=1}^n \lambda_j y_{rj} \geq y_r, \quad r = 1, \dots, s; \\ &\sum_{j=1}^n \lambda_j z_{tj} \leq z_t, \quad t = 1, \dots, k; \lambda_j \geq 0, \quad j = 1, \dots, n \end{aligned} \right\} \quad (1)$$

where λ_j are intensity variables. Multi-objective programming is composed of T :

$$\min F(X, Y, Z) \text{ s.t. } (X, Y, Z) \in T \quad (2)$$

where $F(X, Y, Z) = (X, -Y, Z)^T$.

2.2 Multi-objective Performance Measure

There are many radial DEA-based models for measuring environmental performance in the process of measuring environmental efficiency. However, these models have some limitations. One of limitations is that they adjust all inputs, desirable outputs or undesirable outputs by the same proportion to the efficient targets. In addition, their discriminating power is so weak that many DMUs cannot be directly compared and ranked. Out of realistic or economic considerations, however, decision makers or government officials may prefer different efficient targets. Therefore, it is meaningful and practical to extend radial DEA model to non-radial one measuring environmental performance.

In the framework of DEA environmental performance evaluation, non-radial DEA models have been well developed in the past. Despite the abundance of non-radial DEA models, rarely of them comprehensively consider inputs, desirable outputs and undesirable outputs simultaneously. In the real production process, people always expect to put in the less and get more desirable outputs and fewer undesirable outputs. The analytic structure of environmental efficiency evaluation considering undesirable outputs model is described in Fig. 1. In this figure We can see desirable and undesirable outputs will increase with the increase of inputs during production. If we blindly focus on economic growth, then this will bring increasing pollution that can be hard for us to take. Moreover, raw materials are limited and then the

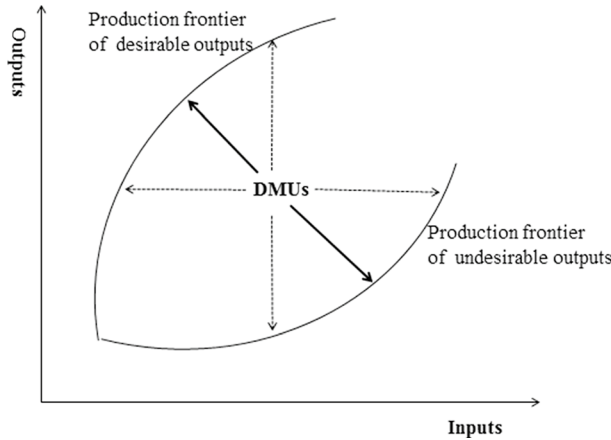


Fig. 1 The structural relationship of inputs, desirable outputs and undesirable outputs

increase in inputs is limited. If we are obsessed with controlling pollution by reducing undesirable outputs, then it would reduce the desirable outputs and hold back economic development. So we must seek a balance point between them to seek the efficiency optimization.

For a given DMU_{j_0} under evaluation ($0 \leq j_0 \leq n$), then based on the idea, we introduce a new non-radial and multi-objective DEA model for measuring environmental performance as follows:

$$\begin{aligned}
 \min a &= (a_1, a_2, \dots, a_m); \max b = (b_1, b_2, \dots, b_s); \min c = (c_1, c_2, \dots, c_k) \\
 \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij} \leq a_i x_{ij_0}, \quad i = 1, 2, \dots, m \\
 & \sum_{j=1}^n \lambda_j y_{rj} \geq b_r y_{rj_0}, \quad r = 1, 2, \dots, s \\
 & \sum_{j=1}^n \lambda_j z_{tj} \leq c_t z_{tj_0}, \quad t = 1, 2, \dots, k \\
 & a_i \leq 1, \quad b_r \geq 1, \quad c_t \leq 1, \forall i, r, t \\
 & \lambda_j \geq 0, \quad j = 1, 2, \dots, n
 \end{aligned} \tag{3}$$

where a_i ($i=1, 2, \dots, m$) indicates the efficiency value of i th input; b_r ($r=1, 2, \dots, s$) indicates the efficiency value of r th desirable output; c_t ($t=1, 2, \dots, k$) indicates the efficiency value of t th undesirable output.

There is very obvious economic significance of model (3). If the optimum value $a_i^* < 1$, $b_r^* > 1$ and $c_t^* < 1$, there exists a decision making unit by which we can get more desirable outputs and fewer undesirable outputs, but its inputs are not greater than decision making unit evaluated. Thus the decision making units DMU_{j_0} evaluated is not DEA efficient. Therefore, only if $a_i = 1$ ($i=1, 2, \dots, m$), $b_r = 1$ ($r=1, 2, \dots, s$) and $c_t = 1$ ($t=1, 2, \dots, k$), DMU_{j_0} is DEA efficient.

However, the multi-objective programming (3) is not solved, and its DEA efficiency is difficult for us to discuss. In addition, the duality of model (3) is hard to study. To this end, we extend the generalized DEA (GDEA) model given in Zhu (2014) into a Non-radial generalized tri-DEA model measuring economic-environmental performance.

Let $w_i^1 (i = 1, 2, \dots, m)$ indicates the degree of i th production input and $w_r^2 (r = 1, 2, \dots, s)$ indicates the importance of r th desirable output; $w_t^3 (t = 1, 2, \dots, k)$ is normalized user-specified weights for adjusting the t th pollutant which reflects the desirability degree of decision makers in adjusting the current level of this pollutant. The model is proposed as follows.

$$\begin{aligned}
 \min h_0 &= \sum_{i=1}^m w_i^1 a_i - \sum_{r=1}^s w_r^2 b_r + \sum_{t=1}^k w_t^3 c_t \\
 \text{s.t.} \quad &\sum_{j=1}^n \lambda_j x_{ij} \leq a_i x_{ij_0}, \quad i = 1, 2, \dots, m \\
 &\sum_{j=1}^n \lambda_j y_{rj} \geq b_r y_{rj_0}, \quad r = 1, 2, \dots, s \\
 &\sum_{j=1}^n \lambda_j z_{tj} \leq c_t z_{tj_0}, \quad t = 1, 2, \dots, k \\
 &\delta_1 \left(\sum_{j=1}^n \lambda_j + \delta_2 (-1)^{\delta_3} \lambda_{n+1} \right) = \delta_1 \\
 &a_i \leq 1, b_r \geq 1, c_t \leq 1, \forall i, r, t \\
 &\lambda_j \geq 0, j = 1, 2, \dots, n, \lambda_{n+1} \geq 0
 \end{aligned} \tag{4}$$

where h_0 indicates efficiency value of DMU_{j_0} comprehensively considering inputs, desirable outputs and undesirable outputs.

In addition, $\sum_{i=1}^m w_i^1 + \sum_{r=1}^s w_r^2 + \sum_{t=1}^k w_t^3 = 1, 0 \leq w_i^1 \leq 1, 0 \leq w_r^2 \leq 1$ and $0 \leq w_t^3 \leq 1$. The greater $w_i^1 (i = 1, 2, \dots, m)$, the more decision makers' input; The greater $w_r^2 (r = 1, 2, \dots, s)$ is, the more the desirable output decision makers want to get; The greater $w_t^3 (t = 1, 2, \dots, k)$ is, the more to priority reduce the emissions of pollutants decision makers take. Moreover, not all the variables in real-life word are discretionary (or controllable), while the discretion of variables should be controlled by set the value of weights (Tsai et al. 2011). For example, we can set $w_1^1=0$ if the first input variables is non-discretionary (or uncontrollable). The decision makers try various devices to reduce the inputs and undesirable outputs and to increase desirable outputs as much as possible. If the optimum value $a_i^* = 1 (i = 1, 2, \dots, m)$, $b_r^* = 1 (r = 1, 2, \dots, s)$ and $c_t^* = 1 (t = 1, 2, \dots, k)$, DMU_{j_0} is DEA efficient. Otherwise, DMU_{j_0} is not DEA efficient.

For convenience of study DEA efficiency, Pareto efficient solution and the projection of decision making units, slack variables $S_i^- (i = 1, 2, \dots, m)$, $S_r^+ (r = 1, 2, \dots, s)$ and $S_t^- (t = 1, 2, \dots, k)$ are introduced into model (4), The model (5) is formulated as follows:

$$\begin{aligned}
 \min h_0 &= \sum_{i=1}^m w_i^1 a_i - \sum_{r=1}^s w_r^2 b_r + \sum_{t=1}^k w_t^3 c_t \\
 \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij} + S_i^- = a_i x_{ij_0}, \quad i = 1, 2, \dots, m \\
 & \sum_{j=1}^n \lambda_j y_{rj} - S_r^+ = b_r y_{rj_0}, \quad r = 1, 2, \dots, s \\
 & \sum_{j=1}^n \lambda_j z_{tj} + S_t^- = c_t z_{tj_0}, \quad t = 1, 2, \dots, k \\
 & \delta_1 \left(\sum_{j=1}^n \lambda_j + \delta_2 (-1)^{\delta_3} \lambda_{n+1} \right) = \delta_1 \\
 & a_i \leq 1, b_r \geq 1, c_t \leq 1, \forall i, r, t \\
 & \lambda_j \geq 0, j = 1, 2, \dots, n, \lambda_{n+1} \geq 0.
 \end{aligned} \tag{5}$$

The duality of model (5):

$$\begin{aligned}
 \max \quad & -\delta_1 \mu_0 - \beta_1 + \beta_2 - \beta_3 \\
 \text{s.t.} \quad & \omega^T X_j - \mu^T Y_j + \gamma^T Z_j + \delta_1 \mu_0 e^T \geq 0, \quad j = 1, 2, \dots, n \\
 & \omega^T X_0 + \beta_1 = e_m^T K^1 \\
 & \mu^T Y_0 - \beta_2 = e_s^T K^2 \\
 & \gamma^T Z_0 + \beta_3 = e_k^T K^3 \\
 & \delta_1 \delta_2 (-1)^{\delta_3} \mu_0 \geq 0 \\
 & \beta_1 \geq 0, \beta_2 \geq 0, \beta_3 \geq 0 \\
 & \omega^T \geq 0, \mu^T \geq 0, \gamma^T \geq 0
 \end{aligned} \tag{6}$$

where $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})^T$ is the input vector for $DMU_j, j=1, \dots, n, Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})^T$ is the desirable output vector for $DMU_j, j=1, \dots, n, Z_j = (z_{1j}, z_{2j}, \dots, z_{kj})^T$ is the undesirable output vector for $DMU_j, j=1, \dots, n; K^1 = (w_1^1, w_2^1, \dots, w_m^1), K^2 = (w_1^2, w_2^2, \dots, w_s^2), K^3 = (w_1^3, w_2^3, \dots, w_k^3); e = (1, 1, \dots, 1)^T \in E^n; \beta_1, \beta_2$ and β_3 are slack variables. δ_1, δ_2 and δ_3 are 0–1 binary parameters. Using Wei et al. (2008) technique, different values of parameters δ_1, δ_2 and δ_3 lead to the different generalized tri-DEA models measuring economic-environmental performance (where ‘*’ indicates either 0 or 1):

Case 1 When $(\delta_1, \delta_2, \delta_3) = (0, *, *)$, the generalized tri-DEA model is reduced to the (CCR) generalized tri-DEA model:

$$\begin{aligned}
\min h_0 &= \sum_{i=1}^m w_i^1 a_i - \sum_{r=1}^s w_r^2 b_r + \sum_{t=1}^k w_t^3 c_t \\
s.t. \quad & \sum_{j=1}^n \lambda_j x_{ij} \leq a_i x_{ij_0}, \quad i = 1, 2, \dots, m \\
& \sum_{j=1}^n \lambda_j y_{rj} \geq b_r y_{rj_0}, \quad r = 1, 2, \dots, s \\
& \sum_{j=1}^n \lambda_j z_{tj} \leq c_t z_{tj_0}, \quad t = 1, 2, \dots, k \\
& a_i \leq 1, b_r \geq 1, c_t \leq 1, \quad \forall i, r, t \\
& \lambda_j \geq 0, \quad j = 1, 2, \dots, n,
\end{aligned} \tag{7}$$

Case 2 When $(\delta_1, \delta_2, \delta_3) = (1, 0, *)$, the generalized tri-DEA model is reduced to the (BCC) generalized tri-DEA model:

$$\begin{aligned}
\min h_0 &= \sum_{i=1}^m w_i^1 a_i - \sum_{r=1}^s w_r^2 b_r + \sum_{t=1}^k w_t^3 c_t \\
s.t. \quad & \sum_{j=1}^n \lambda_j x_{ij} \leq a_i x_{ij_0}, \quad i = 1, 2, \dots, m \\
& \sum_{j=1}^n \lambda_j y_{rj} \geq b_r y_{rj_0}, \quad r = 1, 2, \dots, s \\
& \sum_{j=1}^n \lambda_j z_{tj} \leq c_t z_{tj_0}, \quad t = 1, 2, \dots, k \\
& a_i \leq 1, \quad b_r \geq 1, \quad c_t \leq 1, \quad \forall i, r, t \\
& \sum_{j=1}^n \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, 2, \dots, n,
\end{aligned} \tag{8}$$

Case 3 When $(\delta_1, \delta_2, \delta_3) = (1, 1, 0)$, the generalized tri-DEA model is reduced to the (FG) generalized tri-DEA model:

$$\begin{aligned}
\min h_0 &= \sum_{i=1}^m w_i^1 a_i - \sum_{r=1}^s w_r^2 b_r + \sum_{t=1}^k w_t^3 c_t \\
s.t. \quad & \sum_{j=1}^n \lambda_j x_{ij} \leq a_i x_{ij_0}, \quad i = 1, 2, \dots, m \\
& \sum_{j=1}^n \lambda_j y_{rj} \geq b_r y_{rj_0}, \quad r = 1, 2, \dots, s \\
& \sum_{j=1}^n \lambda_j z_{tj} \leq c_t z_{tj_0}, \quad t = 1, 2, \dots, k \\
& a_i \leq 1, \quad b_r \geq 1, \quad c_t \leq 1, \quad \forall i, r, t \\
& \sum_{j=1}^n \lambda_j \leq 1, \quad \lambda_j \geq 0, \quad j = 1, 2, \dots, n,
\end{aligned} \tag{9}$$

Case 4 When $(\delta_1, \delta_2, \delta_3) = (1, 1, 1)$, the generalized tri-DEA model is reduced to the (ST) generalized tri-DEA model:

$$\begin{aligned}
 \min h_0 &= \sum_{i=1}^m w_i^1 a_i - \sum_{r=1}^s w_r^2 b_r + \sum_{t=1}^k w_t^3 c_t \\
 \text{s.t.} \quad &\sum_{j=1}^n \lambda_j x_{ij} \leq a_i x_{ij_0}, \quad i = 1, 2, \dots, m \\
 &\sum_{j=1}^n \lambda_j y_{rj} \geq b_r y_{rj_0}, \quad r = 1, 2, \dots, s \\
 &\sum_{j=1}^n \lambda_j z_{tj} \leq c_t z_{tj_0}, \quad t = 1, 2, \dots, k \\
 &a_i \leq 1, \quad b_r \geq 1, \quad c_t \leq 1, \quad \forall i, r, t \\
 &\sum_{j=1}^n \lambda_j \geq 1, \quad \lambda_j \geq 0, \quad j = 1, 2, \dots, n.
 \end{aligned} \tag{10}$$

According to decision maker’s preferences, they may choose the relative importance of different input, desirable output and undesirable output categories and different DMUs (see Chen 2003).

Definition 1 Let $\omega^0, \mu^0, \gamma^0, \mu_0^0, \beta_1^0, \beta_2^0, \beta_3^0$ be the optimal solution of model (6). If $\omega^0 > 0, \mu^0 > 0, \gamma^0 > 0, \beta_1 = \beta_2 = \beta_3 = 0$ and the optimum $-\delta_1 \mu_0^0 = e_m^T K^1 - e_s^T K^2 + e_k^T K^3$, then DMU_{j_0} is called DEA efficiency.

Definition 2 Let $(\tilde{X}, \tilde{Y}, \tilde{Z}) \in T$. If there no existence of (X, Y, Z) , satisfying $F(X, Y, Z) < F(\tilde{X}, \tilde{Y}, \tilde{Z}), (X, Y, Z) \in T$, then $(\tilde{X}, \tilde{Y}, \tilde{Z})$ is called weak Pareto solution of multi-objective programming.

Definition 3 Let $(\tilde{X}, \tilde{Y}, \tilde{Z}) \in T$. If there no existence of (X, Y, Z) , satisfying $F(X, Y, Z) \leq F(\tilde{X}, \tilde{Y}, \tilde{Z}), (X, Y, Z) \in T$, then $(\tilde{X}, \tilde{Y}, \tilde{Z})$ is called Pareto solution of multi-objective programming.

Lemma 1 (weak dual theorem) Let $(\lambda_j, \lambda_{n+1}, a_i, b_r, c_t)$ be a feasible solution of model (4), $(\omega, \mu, \gamma, \mu_0, \beta_1, \beta_2, \beta_3)$ be a feasible solution of model (6), then

$$\sum_{i=1}^m w_i^1 a_i - \sum_{r=1}^s w_r^2 b_r + \sum_{t=1}^k w_t^3 c_t \geq -\delta_1 \mu_0 - \beta_1 + \beta_2 - \beta_3$$

Proof Let $(\lambda_j, \lambda_{n+1}, a_i, b_r, c_t)$ and $(\omega, \mu, \gamma, \mu_0, \beta_1, \beta_2, \beta_3)$ be feasible solutions of model (4) and (6), respectively. Since

$$\begin{aligned}
 \sum_{j=1}^n \lambda_j x_{ij} - a_i x_{ij_0} &\leq 0, \quad i = 1, 2, \dots, m, \quad - \sum_{j=1}^n \lambda_j y_{rj} + b_r y_{rj_0} \leq 0, \quad r = 1, 2, \dots, s, \\
 \sum_{j=1}^n \lambda_j z_{tj} - c_t z_{tj_0} &\leq 0, \quad t = 1, 2, \dots, k,
 \end{aligned}$$

and

$$\omega^T \geq 0, \mu^T \geq 0, \gamma^T \geq 0,$$

then

$$\omega^T \sum_{i=1}^m \left(\sum_{j=1}^n \lambda_j x_{ij} - a_i x_{ij_0} \right) + \mu^T \sum_{r=1}^s \left(- \sum_{j=1}^n \lambda_j y_{rj} + b_r y_{rj_0} \right) + \gamma^T \sum_{t=1}^k \left(\sum_{j=1}^n \lambda_j z_{tj} - c_t z_{tj_0} \right) \leq 0,$$

that is

$$\omega^T \sum_{i=1}^m \sum_{j=1}^n \lambda_j x_{ij} - \mu^T \sum_{r=1}^s \sum_{j=1}^n \lambda_j y_{rj} + \gamma^T \sum_{t=1}^k \sum_{j=1}^n \lambda_j z_{tj} \leq \omega^T \sum_{i=1}^m a_i x_{ij_0} - \mu^T \sum_{r=1}^s b_r y_{rj_0} + \gamma^T \sum_{t=1}^k c_t z_{tj_0}.$$

Note that

$$\omega^T X_j - \mu^T Y_j + \gamma^T Z_j + \delta_1 \mu_0 e^T \geq 0,$$

we have

$$\omega^T \sum_{j=1}^n \lambda_j X_j - \mu^T \sum_{j=1}^n \lambda_j Y_j + \gamma^T \sum_{j=1}^n \lambda_j Z_j \geq -\delta_1 \mu_0 e^T \sum_{j=1}^n \lambda_j.$$

Since $\lambda_{n+1} \geq 0$ and $\delta_1 \delta_2 (-1)^{\delta_3} \mu_0 \geq 0$, then $\lambda_{n+1} \delta_1 \delta_2 (-1)^{\delta_3} \mu_0 \geq 0$. From the above analysis, we have

$$\begin{aligned} & \sum_{i=1}^m w_i^1 a_i - \sum_{r=1}^s w_r^2 b_r + \sum_{t=1}^k w_t^3 c_t \\ &= \left(\sum_{i=1}^m a_i \omega^T X_0 - \sum_{r=1}^s b_r \mu^T Y_0 + \sum_{t=1}^k c_t \gamma^T Z_0 \right) \\ & \quad - \left(\beta_1 \sum_{i=1}^m a_i - \beta_2 \sum_{r=1}^s b_r + \beta_3 \sum_{t=1}^k c_t \right) \\ & \geq \omega^T \sum_{i=1}^m \sum_{j=1}^n \lambda_j x_{ij} - \mu^T \sum_{r=1}^s \sum_{j=1}^n \lambda_j y_{rj} + \gamma^T \sum_{t=1}^k \sum_{j=1}^n \lambda_j z_{tj} - \beta_1 + \beta_2 - \beta_3 \\ & \geq \omega^T \sum_{j=1}^n \lambda_j X_j - \mu^T \sum_{j=1}^n \lambda_j Y_j + \gamma^T \sum_{j=1}^n \lambda_j Z_j - \beta_1 + \beta_2 - \beta_3 \\ & \geq -\delta_1 \mu_0 e^T \sum_{j=1}^n \lambda_j - \beta_1 + \beta_2 - \beta_3 \\ & = -\delta_1 \mu_0 (1 - \delta_2 (-1)^{\delta_3} \lambda_{n+1}) - \beta_1 + \beta_2 - \beta_3 \\ & = -\delta_1 \mu_0 - \beta_1 + \beta_2 - \beta_3. \end{aligned}$$

From Lemma 1, it is not hard to have the following corollary. □

Corollary 1 *If $\omega^0, \mu^0, \gamma^0, \mu_0^0, \beta_1^0, \beta_2^0, \beta_3^0$ is a feasible solution of model (6) such that*

$$-\delta_1 \mu_0^0 - \beta_1^0 + \beta_2^0 - \beta_3^0 = e_m^T K^1 - e_s^T K^2 + e_k^T K^3,$$

then $\omega^0, \mu^0, \gamma^0, \mu_0^0, \beta_1^0, \beta_2^0, \beta_3^0$ is an optimal solution of model (6).

Theorem 1 *If (X_0, Y_0, Z_0) is Pareto solution of multi-objective programming, then the corresponding DMU_{j_0} is DEA efficient.*

Proof (Proof by contradiction) Assume DMU_{j_0} is not DEA efficient, by Lemma 1 and Corollary 1, then the optimum $-\delta_1 \mu_0^0 < e_m^T K^1 - e_s^T K^2 + e_k^T K^3$, $h_0 > e_m^T K^1 - e_s^T K^2 + e_k^T K^3$. It suggests that decision makers can make further efforts to reduce inputs or undesirable outputs and increase desirable outputs such that the optimum declines to $e_m^T K^1 - e_s^T K^2 + e_k^T K^3$. That is, the DMU_{j_0} relatively efficient.

Assume (X_*, Y_*, Z_*) are input–output volumes adjusted, $X_* < X_0, Y_* > Y_0, Z_* < Z_0$ and (X_*, Y_*, Z_*) are input–output volumes of DMU_{j_0} . Based on the above analysis, DMU_{j_0} known is DEA efficient and $a_i = 1$ ($i = 1, 2, \dots, m$), $b_r = 1$ ($r = 1, 2, \dots, s$) and $c_t = 1$ ($t = 1, 2, \dots, k$). Therefore, a conclusion can be drawn: (X_*, Y_*, Z_*) meets the following conditions:

$$\sum_{j=1}^n \lambda_j X_j \leq X_*, \sum_{j=1}^n \lambda_j Y_j \geq Y_*, \sum_{j=1}^n \lambda_j Z_j \leq Z_*, \lambda_j \geq 0, j = 1, 2, \dots, n,$$

Then $(X_*, Y_*, Z_*) \in T$. That is, there exists (X_*, Y_*, Z_*) satisfying $F(X_*, Y_*, Z_*) < F(X_0, Y_0, Z_0)$, $(X_*, Y_*, Z_*) \in T$. By Definition 2, (X_0, Y_0, Z_0) is not weak Pareto solution of multi-objective programming, nor is it Pareto solution. Obviously, it contradicts with the known condition of (X_0, Y_0, Z_0) being Pareto solution of multi-objective programming. Therefore, DMU_{j_0} is DEA efficient. □

Theorem 2 *If DMU_{j_0} is DEA efficient, then the corresponding (X_0, Y_0, Z_0) is Pareto solution of multi-objective programming.*

Proof (Reduction to absurdity) Assume (X_0, Y_0, Z_0) is not Pareto solution of multi-objective programming, then there exists (X, Y, Z) satisfying $F(X, Y, Z) < F(X_0, Y_0, Z_0)$, $(X, Y, Z) \in G$. By $(X, Y, Z) \in T$ and model (4), then $\lambda_j \geq 0$ ($j = 1, 2, \dots, n$), such that $\sum_{j=1}^n \lambda_j x_{ij} \leq a_i x_{ij_0}$, $i = 1, 2, \dots, m$, $\sum_{j=1}^n \lambda_j y_{rj} \geq b_r y_{rj_0}$, $r = 1, 2, \dots, s$, $\sum_{j=1}^n \lambda_j z_{tj} \leq c_t z_{tj_0}$, $t = 1, 2, \dots, k$. By model (5), we have $S_i^- = a_i x_{ij_0} - \sum_{j=1}^n \lambda_j x_{ij}$, $S_r^+ = -b_r y_{rj_0} - \sum_{j=1}^n \lambda_j y_{rj}$, $S_t^- = c_t z_{tj_0} - \sum_{j=1}^n \lambda_j z_{tj}$. Then $\lambda_j \geq 0$ ($j = 1, 2, \dots, n$), S_i^- ($i = 1, 2, \dots, m$), S_r^+ ($r = 1, 2, \dots, s$) and S_t^- ($t = 1, 2, \dots, k$) is a feasible solution of model (5) and $(S_i^-, S_r^+, S_t^-) \geq 0$. Therefore, DMU_{j_0} is DEA efficient. This contradicts with the known conditions. The assumption does not hold. □

2.3 Frontier Projection

In this section, we will focus on the projection of decision making units. If DMU_{j_0} is not efficient, DMU_{j_0} may be efficient through adjusting the inputs and outputs. The adjustable inputs and outputs are called the projections of the efficient production frontier. From the perspective of multi-objective programming, efficient production frontier is surface composed of Pareto solutions. DEA efficient production frontier is defined as follows:

Definition 4 If $\widehat{\omega} > 0, \widehat{\mu} > 0, \widehat{\gamma} > 0$ and hyperplane $L = \{(X, Y, Z) \mid \widehat{\omega}^T X - \widehat{\mu}^T Y + \widehat{\gamma}^T Z = 0\}$ satisfies $T \subset \{(X, Y, Z) \mid \widehat{\omega}^T X - \widehat{\mu}^T Y + \widehat{\gamma}^T Z \geq 0\}$ and $L \cap T \neq \emptyset$, then L is called the efficient surface of production possibility set T and $L \cap T$ is called the production frontier of production possibility set T .

Definition 5 Let $\lambda_j^0, \lambda_{n+1}^0, a_i^0, b_r^0, c_t^0, S_i^{-0}, S_r^{+0}, S_t^{-0}$ be an optimal solution of model (5) and $\hat{x}_{ij_0} = a_i^0 x_{ij_0} - S_i^{-0}, \hat{y}_{rj_0} = b_r^0 y_{rj_0} + S_r^{+0}, \hat{z}_{tj_0} = c_t^0 z_{tj_0} - S_t^{-0}, (\hat{X}_0, \hat{Y}_0, \hat{Z}_0)$ is called projection on the production frontier of production possibility set T .

Obviously, $\hat{x}_{ij_0} = a_i^0 x_{ij_0} - S_i^{-0} = \sum_{j=1}^n \lambda_j^0 x_{ij}, \hat{y}_{rj_0} = b_r^0 y_{rj_0} + S_r^{+0} = \sum_{j=1}^n \lambda_j^0 y_{rj}, \hat{z}_{tj_0} = c_t^0 z_{tj_0} - S_t^{-0} = \sum_{j=1}^n \lambda_j^0 z_{tj}.$

If DMU_{j_0} is weak DEA efficient, then $\hat{x}_{ij_0} = x_{ij_0} - S_i^{-0}, \hat{y}_{rj_0} = y_{rj_0} + S_r^{+0}$ and $\hat{z}_{tj_0} = z_{tj_0} - S_t^{-0}$; If DMU_{j_0} is DEA efficient, then $\hat{x}_{ij_0} = x_{ij_0}, \hat{y}_{rj_0} = y_{rj_0}$ and $\hat{z}_{tj_0} = z_{tj_0}$. Further, we can draw the conclusion that the corresponding projection $(\hat{X}_0, \hat{Y}_0, \hat{Z}_0)$ of DMU_{j_0} constitutes a new decision making unit which is DEA efficient. That is, the new decision making unit $(\hat{X}_0, \hat{Y}_0, \hat{Z}_0)$ locates in the production frontier of convex polyhedral cone $C(\hat{T})$, where, $\hat{T} = \{(X_1, Y_1, Z_1), (X_2, Y_2, Z_2), \dots, (X_n, Y_n, Z_n)\}$ is the corresponding reference set of input and output data. Convex cone $C(\hat{T}) = \left\{ \sum_{j=1}^n \lambda_j (X_j, Y_j, Z_j) \mid \lambda_j \geq 0, j = 1, 2, \dots, n \right\}$ generated by set \hat{T} is data envelopment of n points in reference set (Yu et al. 1996). Therefore, DEA is helpful to estimate the unknown economic production function. If DMU_{j_0} is not DEA efficient, then a_i^0, b_r^0, c_t^0 are not all 1 and $S_i^{-0}, S_r^{+0}, S_t^{-0}$ are not all 0.

According to the complementary slackness theorem of linear programming, the optimal solution vector of model (5) is greater than 0, $\omega^0 > 0, \mu^0 > 0, \gamma^0 > 0$.

Theorem 3 Let $(\hat{X}_0, \hat{Y}_0, \hat{Z}_0)$ be projection on the production frontier of DMU_{j_0} , then $(\hat{X}_0, \hat{Y}_0, \hat{Z}_0)$ is DEA efficient.

Proof By Definition 5, $\hat{x}_{ij_0} = a_i^0 x_{ij_0} - S_i^{-0}, \hat{y}_{rj_0} = b_r^0 y_{rj_0} + S_r^{+0}, \hat{z}_{tj_0} = c_t^0 z_{tj_0} - S_t^{-0}$. For the first condition of model (6), applying complementary slackness condition of Karush–Kuhn–Tucker theorem in mathematical programming, we have

$$\lambda_j^0 (\omega^{0T} X_j - \mu^{0T} Y_j + \gamma^{0T} Z_j) = 0, \quad j = 1, 2, \dots, n,$$

then

$$\omega^{0T} \hat{X}_0 - \mu^{0T} \hat{Y}_0 + \gamma^{0T} \hat{Z}_0 = 0.$$

For $\forall (X, Y, Z) \in G$, we have

$$\sum_{j=1}^n X_j \lambda_j \leq X, \quad \sum_{j=1}^n Y_j \lambda_j \geq Y, \quad \sum_{j=1}^n Z_j \lambda_j \leq Z, \quad \lambda_j \geq 0, \quad j = 1, 2, \dots, n.$$

Note that $\omega^{0T} X_j - \mu^{0T} Y_j + \gamma^{0T} Z_j \geq 0$, then

$$\begin{aligned} \omega^{0T} X - \mu^{0T} Y + \gamma^{0T} Z &\geq \omega^{0T} \left(\sum_{j=1}^n X_j \lambda_j \right) - \mu^{0T} \left(\sum_{j=1}^n Y_j \lambda_j \right) + \gamma^{0T} \left(\sum_{j=1}^n Z_j \lambda_j \right) \\ &= \sum_{j=1}^n (\omega^{0T} X_j - \mu^{0T} Y_j + \gamma^{0T} Z_j) \lambda_j \\ &= \omega^{0T} \hat{X}_0 - \mu^{0T} \hat{Y}_0 + \gamma^{0T} \hat{Z}_0 = 0. \end{aligned}$$

Therefore, $(\hat{X}_0, \hat{Y}_0, \hat{Z}_0)$ is a Pareto solution of multi-objective programming. Assume $(\hat{X}_0, \hat{Y}_0, \hat{Z}_0)$ is not a Pareto solution. By Definition 3, there exists $(X', Y', Z') \in G$ and $X' \leq \hat{X}_0, Y' \geq \hat{Y}_0, Z' \leq \hat{Z}_0$. Since $\omega^0 > 0, \mu^0 > 0, \gamma^0 > 0$, then $\omega^{0T} X' - \mu^{0T} Y' + \gamma^{0T} Z' < \omega^{0T} \hat{X}_0 - \mu^{0T} \hat{Y}_0 + \gamma^{0T} \hat{Z}_0$. It contradicts with the above results. Therefore, $(\hat{X}_0, \hat{Y}_0, \hat{Z}_0)$ is a Pareto solution of multi-objective programming. From Theorem 1, $(\hat{X}_0, \hat{Y}_0, \hat{Z}_0)$ is DEA efficient. \square

From Theorem 3, DMU_{j_0} is not DEA efficient, but its projection may be DEA efficient. The projection on relatively DEA efficient surface of DMU_{j_0} , in fact, points out the non-efficient reasons and provides a feasible scheme to improve the efficiency of DMU_{j_0} simultaneously. In the actual production process, therefore, people can take advantage of the projection to achieve the relative DEA efficiency. Expressions of projection can be rewritten as follows:

$$\begin{aligned} \hat{x}_{ij_0} - x_{ij_0} &= -((1 - a_i^0)x_{ij} + S_i^{-0}), & \hat{y}_{rj_0} - y_{rj_0} &= (b_r^0 - 1)y_{rj_0} + S_r^{+0}, \\ \hat{z}_{tj_0} - z_{tj_0} &= -((1 - c_t^0)z_{tj} + S_t^{-0}), \end{aligned}$$

then we can get the adjustable volume of input, desirable output and undesirable output. For i th input, the amount reduced is $((1 - a_i^0)x_{ij} + S_i^{-0})$, $i = 1, 2, \dots, m$; for r th desirable output, the amount increased is $(\alpha_r^0 - 1)y_{rj_0} + S_r^{+0}$, $r = 1, 2, \dots, s$; for t th undesirable output, the amount reduced is $((1 - c_t^0)z_{tj} + S_t^{-0})$, $t = 1, 2, \dots, k$.

3 Application

3.1 Variables and Data

In this section we conduct the application of economic-environmental performance of thirty Chinese provinces in 2016 to evaluate the non-radial and multi-objective generalized DEA approaches in assessing the impact of contextual variables on inputs, desirable outputs and undesirable outputs. A lot of researches have been done to measure economic-environmental efficiency of Chinese regions, such as Zhang and Chen (2017) and Song et al. (2018). Following their researches, population, capital stock and power consumption are viewed as three inputs to produce one desirable output gross domestic production (GDP). As byproducts, the two undesirable outputs are wastewater emissions and SO₂ emissions, respectively. The corresponding input–output measures are listed in the following Table 1.

The data on these variables were collected from the 2017 China Statistics Yearbook, the 2017 China Energy Database, and the 2017 China Environment Database. Due to data unavailability, we exclude the Taiwan, Hong Kong, Macao and Tibet provinces from the economic-environment performance analysis. The raw data is shown in Table 2.

3.2 Performance Evaluation

The non-radial and multi-objective generalized DEA model (7) (based on CRS assumption) is applied to calculate the economic-environmental efficiency score of thirty Chinese provinces in 2016. According to the model (7), the weight w_i^1 , w_r^2 and w_t^3 affect only the value of the objective function, having no influence on the constraint conditions. Following the research of Zhang and Choi (2013), we set the weight of input, desirable output and undesirable output as (1/3, 1/3, 1/3). Further, we set the weight of all the variable population (X_1), capital stock (X_2), power

Table 1 Variables of inputs and outputs

Type	Variable	Units
Inputs	Population	Millions persons
	Capital stock	Billions Yuan
	Power consumption	Billions KWH
Desirable output	GDP	Billions Yuan
Undesirable outputs	Wastewater emissions	Millions tons
	SO ₂ emissions	Thousands tons

Table 2 Raw data of inputs and outputs

Regions	Population (X_1)	Total capital stock (X_2)	Power consumption (X_3)	GDP (Y_1)	Wastewater emissions (Z_1)	SO ₂ emissions (Z_2)
Beijing	21.73	794.39	102.03	2566.91	1664.19	33.2
Tianjin	15.62	1277.94	80.79	1788.54	915.34	70.6
Hebei	74.70	3175.0	326.45	3207.05	2887.95	789.4
Shanxi	36.82	1419.8	179.72	1305.04	1392.91	686.4
Inner Mongolia	25.20	1508	260.50	1812.81	1046.96	625.7
Liaoning	43.78	669.22	203.74	2224.69	2282.02	507.7
Jilin	27.33	1392.32	66.76	1477.68	970.73	188.1
Heilongjiang	37.99	1064.83	89.66	1538.61	1383.35	338.2
Shanghai	24.20	675.59	148.60	2817.87	2207.59	74.2
Jiangsu	79.99	4966.32	545.90	7738.83	6166.24	570.1
Zhejiang	55.90	3027.61	387.32	4725.14	4308.57	268.4
Anhui	61.96	2703.34	179.50	2440.76	2406.66	281.6
Fujian	38.74	2323.74	196.86	2881.06	2370.16	189.3
Jiangxi	45.92	1969.42	118.25	1849.9	2210.92	276.9
Shandong	99.47	5332.29	539.08	6802.45	5075.91	1134.5
Henan	95.32	4041.51	298.92	4047.18	4020.55	413.6
Hubei	58.85	3001.17	176.31	3266.54	2747.87	285.6
Hunan	68.22	2835.33	149.57	3155.14	2987.57	346.8
Guangdong	109.99	3330.36	561.01	8085.49	9382.61	353.7
Guangxi	48.38	1823.68	135.97	1831.76	1931.86	201.1
Hainan	9.17	389.04	28.73	405.32	440.97	17
Chongqing	30.48	1604.81	92.49	1774.06	2020.61	288.3
Sichuan	82.62	2881.2	210.10	3293.45	3528.26	488.3
Guizhou	35.55	1320.4	124.18	1177.67	1007.20	647.1
Yunnan	47.71	1611.94	141.05	1478.84	1810.89	526.2
Shaanxi	38.13	2082.53	135.71	1939.96	1665.65	318
Gansu	26.10	966.4	106.52	720.04	663.25	272
Qinghai	5.93	352.81	63.75	257.25	272.75	113.7
Ningxia	6.75	379.42	88.69	316.86	339.49	236.9
Xinjiang	23.98	1028.75	231.65	964.97	939.07	480.7
Total	1376.53	59,949.16	59,69.81	77,891.87	71,048.10	11,023.3

consumption (X_3), GDP (Y), wastewater emissions (Y_1^b) and SO₂ emission (Y_2^b) as (1/9, 1/9, 1/9, 1/3, 1/6, 1/6). Then, our DEA approach should be similar with the non-radial DDF proposed in Zhang and Choi (2013). Table 3 shows the obtained efficiency scores of the thirty provinces in 2016.

It can be seen from Table 3 that the non-radial and multi-objective generalized DEA model has a higher discriminating power. There are significant difference between the efficiency scores for different kind of variables, in which the efficiency

Table 3 Efficiency scores of 30 Chinese provinces in 2016 based on all variables

Regions	Inputs			Desirable output	Undesirable outputs	
	a_1	a_2	a_3		b_1	c_1
Beijing	1	1	1	1	1	1
Tianjin	1	1	1	1	1	1
Hebei	0.505	0.434	0.542	1.389	1	0.073
Shanxi	0.494	0.468	0.475	1.646	1	0.040
Inner Mongolia	0.619	0.624	0.296	1	1	0.077
Liaoning	0.431	1	0.452	1	0.648	0.064
Jilin	0.458	0.328	0.880	1	0.987	0.102
Heilongjiang	0.475	0.620	0.946	1.387	1.000	0.082
Shanghai	1	1	1	1	1	1
Jiangsu	0.819	0.482	0.563	1	0.814	0.176
Zhejiang	0.716	0.483	0.485	1	0.711	0.228
Anhui	0.507	0.425	0.822	1.521	1	0.170
Fujian	0.630	0.384	0.582	1	0.788	0.197
Jiangxi	0.548	0.467	1	1.608	0.872	0.139
Shandong	0.666	0.454	0.577	1.151	1	0.089
Henan	0.551	0.475	0.825	1.532	1	0.194
Hubei	0.610	0.437	0.956	1.298	1	0.192
Hunan	0.467	0.411	1	1.193	0.817	0.140
Guangdong	0.622	0.751	0.573	1.000	0.559	0.296
Guangxi	0.521	0.506	0.871	1.627	1	0.192
Hainan	0.628	0.541	0.941	1.678	1	0.517
Chongqing	0.646	0.449	1	1.312	0.747	0.104
Sichuan	0.542	0.568	1	1.605	0.971	0.140
Guizhou	0.370	0.364	0.497	1.319	1	0.031
Yunnan	0.496	0.536	0.787	1.889	1	0.069
Shaanxi	0.570	0.382	0.752	1.324	1	0.104
Gansu	0.434	0.958	0.550	1.800	1	0.188
Qinghai	0.764	1	0.365	2.023	1	0.170
Ningxia	0.797	1	0.307	1.960	1	0.086
Xinjiang	0.511	0.436	0.249	1.501	1	0.039

scores of desirable outputs should be no smaller than 1 while that of inputs and undesirable outputs should be no larger than 1. In 30 regions, only Beijing, Tianjin and Shanghai are DEA efficient and the rest are not DEA efficient. In 2016, 24 regions have the same GDP performance index of “1”, and only Shanxi, Yunnan, Gansu, Qinghai, Ningxia and Xinjiang need to further improve GDP. For the input performance indexes, however, only 3 regions (Beijing, Tianjin and Shanghai) have always been equal to “1”, and the rest of the regions need to pay attention to the distribution of inputs, especially, the conservation of power consumption. From Table 3 we can also find that the majority of regions have smaller environmental

Table 4 Efficiency scores of 30 Chinese provinces in 2016 based on discretionary variables

Regions	Inputs	Desirable output	Undesirable outputs	
	a_3	b_1	c_1	c_2
Beijing	1	1	1	1
Tianjin	1	1	1	1
Hebei	0.704	1.640	1	0.215
Shanxi	0.598	1.905	1	0.109
Inner Mongolia	0.355	1.128	1	0.129
Liaoning	0.452	1	0.648	0.064
Jilin	0.880	1	0.987	0.102
Heilongjiang	0.682	1	0.721	0.059
Shanghai	1	1	1	1
Jiangsu	0.563	1	0.814	0.176
Zhejiang	0.678	1.397	0.994	0.318
Anhui	0.822	1.521	1	0.170
Fujian	0.582	1	0.788	0.197
Jiangxi	1	1.608	0.872	0.139
Shandong	0.577	1.151	1	0.089
Henan	0.825	1.532	1	0.194
Hubei	0.736	1	0.771	0.148
Hunan	0.838	1	0.685	0.118
Guangdong	0.573	1	0.559	0.296
Guangxi	0.871	1.627	1	0.192
Hainan	0.561	1	0.596	0.308
Chongqing	0.762	1	0.569	0.080
Sichuan	1	1.605	0.971	0.140
Guizhou	0.696	1.638	1	0.112
Yunnan	0.943	2.115	1	0.157
Shaanxi	0.752	1.324	1	0.104
Gansu	0.550	1.800	1	0.188
Qinghai	0.365	2.023	1	0.170
Ningxia	0.307	1.960	1	0.086
Xinjiang	0.322	1.770	1	0.114

performance indexes, only 10 regions (Beijing, Tianjin, Shanxi, Inner Mongolia, Shanghai, Yunnan, Gansu, Qinghai, Ningxia and Xinjiang) have wastewater emissions performance indexes of “1” and 3 regions (Beijing, Tianjin and Shanghai) have SO_2 emissions performance indexes of “1”.

However, the inputs in regional economic activities are different from that of microeconomic production, in which the inputs are not discretionary or need not to decrease. In this application, some of the input variables (population and capital stock) are not discretionary. It is unreasonable to reduce these inputs for attaining higher environmental efficiency. Then, we can set weight of all the variable

population (X_1), capital stock (X_2), power consumption (X_3), GDP (Y), wastewater emissions (Y_1^b) and SO_2 emission (Y_2^b) as (0, 0, 1/3, 1/3, 1/6, 1/6) for showing the different discretionary on variables. Then, we can calculate the efficiency score as shown in Table 4.

Table 4 shows the efficiency result with considering variables' discretionary in which power consumption (X_3) is the only disposable input. Then, we can obtain more reasonable results from Table 4. By comparison, we can find that the efficiency scores of inputs and outputs are different from that shown in Table 3 for some DMUs. Some of the DMUs need to decrease the power consumption and pollution emission while keep the present level of GDP, such as

Table 5 Adjustable volumes of discretionary inputs and outputs

Regions	Power consumption	GDP	Waste water emissions	SO_2 emissions
Beijing	0.00	0.00	0.00	0.00
Tianjin	0.00	0.00	0.00	0.00
Hebei	96.69	2052.06	0.00	619.99
Shanxi	72.18	1181.56	0.00	611.63
Inner Mongolia	168.09	232.91	0.00	544.95
Liaoning	111.72	0.00	802.37	475.22
Jilin	8.02	0.00	12.71	168.99
Heilongjiang	28.50	0.00	385.83	318.30
Shanghai	0.00	0.00	0.00	0.00
Jiangsu	238.30	0.00	1148.97	470.01
Zhejiang	124.85	1878.19	27.47	182.99
Anhui	31.95	1271.37	0.00	233.59
Fujian	82.34	0.00	502.30	152.04
Jiangxi	0.00	1125.08	282.17	238.42
Shandong	227.88	1026.83	0.00	1033.24
Henan	52.42	2154.27	0.00	333.39
Hubei	46.47	0.00	630.09	243.35
Hunan	24.16	0.00	942.02	305.99
Guangdong	239.63	0.00	4140.59	249.12
Guangxi	17.53	1148.02	0.00	162.56
Hainan	12.62	0.00	178.19	11.76
Chongqing	21.97	0.00	870.44	265.35
Sichuan	0.00	1992.33	101.36	419.93
Guizhou	37.80	751.94	0.00	574.75
Yunnan	8.10	1649.16	0.00	443.55
Shaanxi	33.59	629.20	0.00	284.77
Gansu	47.98	575.93	0.00	220.84
Qinghai	40.50	263.16	0.00	94.40
Ningxia	61.50	304.14	0.00	216.60
Xinjiang	157.05	743.49	0.00	425.84

Jiangsu, Fujian and Chongqing. Some DMUs such as Sichuan and Jiangxi need to use the given level of input to produce more desirable outputs and emit less pollution. While the other DMUs need to do some improvements from all the sides of input, desirable output and undesirable output.

3.3 Frontier Projection

On the basis of the expressions: $\hat{x}_{ij_0} - x_{ij_0} = -((1 - a_i^0)x_{ij} + S_i^{-0})$, $\hat{y}_{rj_0} - y_{rj_0} = (b_r^0 - 1)y_{rj_0} + S_r^{+0}$, $\hat{z}_{tj_0} - z_{tj_0} = -((1 - c_t^0)z_{tj} + S_t^{-0})$, we calculate the adjustable volumes of inputs and outputs. We should point out that the adjustable volumes of the input population and total capital are not listed in our calculation because they can not be adjusted through a short term of operation. The results are shown in Table 5.

From Table 5, we can find that only three regions, namely Beijing, Tianjin and Shanghai, should keep the present level of production while all the poor performance DMUs should find out the adjustable volumes for power consumption, GDP, waste water and SO₂ emission to become efficient. We can see that the GDP of Shanxi, Yunnan Gansu Qinghai Ningxia and Xinjiang should be increased. Adjustable volumes of inputs are relatively large and the deductible volumes of power consumption are more noteworthy. The projection is consistent with the corresponding results in Table 3. For the environmental performance indexes, most countries need to reduce the corresponding volumes of waste water emissions and SO₂ emissions exclusive of Beijing, Tianjin and Shanghai. Through the adjustments, we can improve the level of economic-environmental efficiency and achieve DEA efficiency.

Based on the efficiency results, we analysis the type of return to scale for all the DMUs and the benchmarks for all the inefficient DMUs. The results of return to scale measured by using the new proposed models are listed in Table 6. Only three efficient DMUs, namely Beijing, Tianjin and Shanghai are Constant Return to Scale (CRS). Nine provinces are Decrease Return to Scale (DRS), most of which locate at the west area of China such as Gansu, Qinghai, Ningxia and Xingjiang or at the northeast of China such as Heilongjiang, Jining and Liaoning. While all the rest inefficient DMUs are Increase Return to Scale (IRS). Moreover, we can find that the inefficient DMUs select different efficient DMUs as benchmark. Most of them set Beijing (17 DMUs) or Beijing and Tianjin (7 DMUs) as benchmarks, they need to study their mode on economic development. While only one DMU (Liaoning) use Shanghai as benchmark, which means that Shanghai's mode might not be suitable for most of the regions.

4 Conclusions

During the past three decades, China's economy has achieved significant development. However, the rapid economic growth is causing China to pay heavily due to the increasing environmental pollution. Therefore, it is worth to achieve more accurate results of economic environmental efficiency evaluation to provide some policy suggestions. During the past years, non-radial DEA models integrating with

Table 6 Return to scale and benchmarks

Regions	Return to Scale	Beijing	Tianjin	Shanghai
Beijing	CRS	✓		
Tianjin	CRS		✓	
Hebei	IRS	✓	✓	
Shanxi	IRS	✓	✓	
Inner Mongolia	IRS		✓	
Liaoning	DRS		✓	✓
Jilin	DRS	✓		
Heilongjiang	DRS	✓		
Shanghai	CRS			✓
Jiangsu	IRS	✓		
Zhejiang	IRS	✓		
Anhui	IRS	✓		
Fujian	IRS	✓		
Jiangxi	IRS	✓		
Shandong	IRS	✓		
Henan	IRS	✓		
Hubei	IRS	✓		
Hunan	IRS	✓		
Guangdong	IRS	✓		
Guangxi	IRS	✓		
Hainan	DRS	✓		
Chongqing	DRS	✓		
Sichuan	IRS	✓		
Guizhou	IRS	✓	✓	
Yunnan	IRS	✓	✓	
Shaanxi	IRS	✓		
Gansu	DRS		✓	
Qinghai	DRS	✓	✓	
Ningxia	DRS	✓	✓	
Xinjiang	DRS	✓	✓	

undesirable outputs were frequently used in environmental performance measurements because they have a higher discriminating power in environmental performance comparisons. In this paper, we extend previous studies and present a non-radial and multi-objective generalized DEA approach to measuring environmental performance, which synthetically takes account of decreasing inputs, increasing desirable outputs and decreasing undesirable outputs simultaneously in DEA models and consists of a non-radial DEA-based models for multilateral environmental performance comparisons. It thus allows the decision maker to have the kinds of preferable projections and allows that the projection would not be in Pareto-inefficient portions of the production frontier which may occur in the radial projection.

The proposed non-radial and multi-objective generalized DEA approach has been applied to for modeling economic-environmental performance of 30 Chinese regions in 2016. It is found that multi-objective generalized DEA model has a higher discriminating power. Only 3 regions are DEA efficient while the rest are not DEA efficient in which some of them are Decrease Return to Scale. This suggests that we need to pay more attention on the allocation and rational use of resources together with the control of environment pollution, and it's more important to increase the quality of economic development but not the scale. Moreover, most of the inefficient DMUs select Beijing and Tianjin as benchmarks to improve their performance that means it's more valuable and feasible to extend "Beijing mode" and "Tianjin mode" to poor performance regions in China rather than "Shanghai mode".

It should be point out that there exists some shortages of the multiple-objective although it is a feasible approach to calculate the environmental efficiency. One shortage is that the DMs are not able to assign any objective weights sometimes due to the limited knowledge and perception capability of human beings. Another shortage is that it may lead to nonlinear problem in multiple-objective programming which is not easy to obtain the global optimal solution. Therefore, we will aim to develop our approach with uncertain objective weights and study the solving algorithms for nonlinear programs in the next step.

Future study should also be considered in the following two ways. Firstly, generalize the non-radial and multi-objective DEA model for calculating the efficiency on resource allocation and pollution production Simultaneously, in which we should confront that how to allocate production resources and how to handle the loss caused by undesirable outputs. Besides, the approach proposed in this paper should also be extended into Malmquist index for calculating dynamic efficiency scores among difference years.

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