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Monetary Policy Under Time-Varying Uncertainty Aversion

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Abstract In this paper we develop a framework to analyze the optimal policy of an inflation-targeting monetary authority that is not fully confident about its model and the degree of mistrust changes over time as the structure of the economy changes. These changes can include structural breaks as well as price, output or real exchange shocks. We use robust control to denote the degree of uncertainty aversion of the policy maker and a Markov chain to capture the time-varying nature of the uncertainty aversion. We find that in general a more aggressive interest rate policy is the optimal response to: (i) more uncertainty aversion and (ii) higher likelihood that the uncertainty aversion may appear in the future. Moreover, we find that the policy maker's welfare decreases when there is an increase in uncertainty aversion. However, the transition probabilities in the Markov-chain have ambiguous effects on the policy maker expected losses.

Keywords Model uncertainty · Robustness · Markov regime-switching · Monetary policy

JEL Classification C61 · E61

The opinions in this paper correspond to the authors and do not necessarily reflect the point of view of Banamex or Citigroup.

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1 Introduction

Uncertainty aversion is not necessarily time invariant. Policy makers can become more or less pessimistic according to events that they witness or experience. For example, in 2004 a surge in the global demand for commodities increased their international price, prompting a policy tightening by many central banks in the face of inflationary pressures throughout the year. This policy response to price shocks was due to several factors: (1) the direct impact of higher commodity prices on inflation, (2) the uncertainty about the evolution of commodity prices in the future, (3) the possibility of second round effects of the aforementioned shocks on the process of price formation and (4) the possibility of undesirable effects on inflation derived from the combination of continuing increases in commodity prices and the recovery experienced by the global economy.[1] Consequently, in the light of an increase in commodities international prices, those central banks seemed more uncertainty-averse when they tightened monetary policy mainly in response to precautionary motives.

In this paper we develop a model to study the problem of a monetary authority under a similar environment. In particular, we analyze the optimal policy response of an inflation-targeting monetary authority as the assumed structure of the economy changes which affects the level of confidence on its estimated model of the economy. The monetary authority may become distrustful of its model when a certain event or structural break occurs rather than remaining in the same state of trust all the time. That is, the estimated model of the economy works well in the absence of any significant change in the structure of the economy but the monetary authority mistrusts the model after certain events or structural breaks take place. These events include, for example, sharp and persistent shocks in prices, output or real exchange rates, whereas structural breaks may occur in the aftermath of a financial crisis. We assume that the monetary authority cannot assign unique probabilities to the alternative models (i.e. Knightian model uncertainty).

We model the time-varying degree of mistrust by considering two regimes: pessimistic and optimistic. In the latter, as the name indicates the monetary authority is optimistic about the accuracy of its own model while in the former it is pessimistic. We do not consider a jump diffusion process to account for rare events as in Liu et al. (2002); instead we use robust control to model uncertainty aversion. That is, the monetary authority's degree of mistrust about its own model in the pessimistic regime is modeled by introducing robust control as in Hansen and Sargent (2007). Hence, the monetary authority plays a fictitious game with an evil nature which distorts the model only in the pessimistic regime. I addition, we use a Markov chain to represent the possibility that the policy maker may become more pessimistic after these events or structural breaks take place and that the pessimism might subdue after some time. Therefore, the Markov chain captures the time-varying degree of uncertainty aversion about the policy maker's model. The transition probabilities of the Markov chain process can also be interpreted as bounds that limit the damage that evil nature can inflict on hitting targets. This bound, known as the free parameter in robust control,

[1] The possibility of persistent effects of the shocks observed in 2004 was highlighted in the Summary of the Quarterly Inflation Report October–December 2004 published by Banco de Mexico in January 2005.

measures the uncertainty aversion (or pessimism) of the policy maker and can take on two values. In the pessimistic regime the bound is between a lower bound and infinity (the policy maker is pessimistic about his model), while in the optimistic regime the bound approaches infinity (the policy maker is confident in his model). Notice that by limiting the damage of the evil nature, the transition probabilities also reflect the degree of mistrust in the model along with the free parameter. Markov chains have been the subject of recent interest in optimal control problems. For example, Zampolli (2006) combines optimal control and Markov regime-switching, finding more cautious optimal monetary policies in the presence of abrupt changes in one multiplicative parameter. Blake and Zampolli (2004) extend those results to find the optimal time-consistent monetary policy for models with forward-looking variables.

In general terms, our model captures an important feature of policy making under uncertainty, namely that the degree of uncertainty aversion of policy makers (and arguably human beings) can change according to the events they have recently witnessed or experienced. The framework developed in this study is applied to the small open economy model of Ball (1999) for its relative simplicity and parsimony. To simplify the exposition and follow the initial motivation of the paper, we let the evil nature distort only the Phillips curve by making price shocks more persistent. However, the model can be extended to include distortions on output and the real exchange. We analyze the optimal interest rate response and the monetary authority's welfare change to variations in the degree of uncertainty aversion and the transition probabilities of the Markov chain. We find four main results.

First, an increase (decrease) in the mistrust of the monetary authority about the accuracy of its own model in the pessimistic regime produces an aggressive (cautionary) response of the interest rate regardless of the current regime. In the optimistic regime, the possibility that in the next period the monetary authority can become more uncertainty averse produces an aggressive response of the interest rate. In the pessimistic regime this result implies that higher mistrust about the model produces a more aggressive response of the interest rate to reduce the inflation rate and the output gap.

Second, the transition probabilities of the Markov chain have different types of impact on the nature of the interest rate response. In general, in the optimistic regime, an increase in the probability that the monetary authority may become pessimistic in the future produces a more aggressive response of the interest rate. Alternatively, in the pessimistic regime an increase in the probability that the monetary authority's pessimism may decrease in the future produces a more cautionary response mainly for relatively large expected durations of the optimistic regime.

Third, the expected monetary authority's welfare decreases (increases) when there is an increase (decrease) in the degree of pessimism.

Finally, the transition probabilities in the Markov chain have different effects on welfare. Depending on the specific combinations of the transition probabilities increases in these probabilities can increase, decrease or not affect the expected policy maker's welfare. We identify these cases for a given level of time-varying uncertainty.

In general, our results show that when the pessimistic regime is more severe or more likely, the interest rate response is more aggressive. This suggests that considering the possibility that the monetary authority's degree of uncertainty aversion may increase in the future (even if the monetary authority is currently optimistic) produces a more aggressive interest rate response. In our model we have a cautionary approach (characterized by the use of robust control in the pessimistic regime) that is translated into an aggressive interest rate policy in the optimistic regime. This is consistent with most of the previous literature that has shown that monetary policy should be tightened under uncertainty about the persistence of inflation.[2] For example, Angeloni et al. (2003) explore the impact of nominal and real persistence on the transmission process of various shocks of the euro area. They find that it is better for policy makers to assume a relatively high degree of inflation persistence because the costs of making a mistake when there is less actual persistence are not as high as making the opposite mistake. Walsh (2004a) finds that a failure to reoptimize the Taylor rule coefficient carries very little cost when a shock is not very persistent, but a large cost for a very persistent disturbance. Coenen (2004) compares different models with different degrees of inflation persistence and finds that the policy maker should act under the assumption that inflation is characterized by a high degree of persistence. However, a relatively recent stream in the literature has shown that this cautionary approach is not always optimal. In particular, Walsh (2005a,b) conclude that when the monetary authority considers a social loss function, the optimal policy rule is to assume a low-persistence. Amano (2006) confirms the previous results by accounting for inflation persistence in the welfare-theoretic stabilization objective. In our case, the optimal cautionary interest rate policy we find takes place only for a small subset of cases when the monetary authority is already pessimistic about its model and the probability of becoming more optimistic increases.

The aggressive nature of the robust policies has been previously explored in the literature. Sargent (1999) and Walsh (2004a) both show that the use of robust control tends to produce aggressive responses of the policy maker. In particular, Sargent (1999) also uses Ball (1999) model and finds an aggressive interest rate response of the policy maker to higher model uncertainty. Thus, the general results of our paper follow the branch of the literature that advocates for an aggressive monetary policy in the presence of uncertainty about inflation persistence. However, we also show that a cautionary response can be optimal but only for a small set of cases of an already pessimistic monetary authority that may become less pessimistic. Hence, we extend previous findings to the case of time-variant uncertainty aversion and show a small subset of new results under robust control. That is, unlike the standard application of Hansen–Sargent robust control, we consider a time-variant bound on the evil nature's distortions. This setup has the advantage of producing an optimal solution that lies somewhere between the optimal policies of the certainty equivalence case and those derived from a purely robust control framework. In other words, the way we set up the control problem might be less suboptimal in the long run than assuming that the certainty equivalence principle always applies or that the model uncertainty aversion never ends.

In our model, the source of misspecification only affects the inflation equation through the incorporation of a disturbance. Walsh (2004b) finds that an optimal instrument rule and robust control lead to exactly the same implicit instrument rule, when

[2] Walsh (2004a) provides a thorough review on uncertainty and monetary policy.

there is uncertainty only about the disturbance processes. However, Walsh (2004b) also mentions that the two approaches predict different macroeconomic behavior. In the robust control framework, expectations are formed differently in the sense that they incorporate the behavior of an "evil" nature.

The framework developed in our paper is used for normative purposes. This implies that in the decision-making process the monetary authority anticipates the possibility of switching to a different degree of pessimism using the transition probability of the Markov chain. In a normative context it is initially possible to just consider a higher degree of uncertainty aversion instead of switching to a regime where the monetary authority is more pessimistic, making the specification of transition probabilities somewhat redundant. However, this setup is conceptually flawed because it assumes that, at the moment of optimization, the monetary authority believes that the degree of uncertainty aversion will never change in the future. This clearly contradicts the true belief of the monetary authority that uncertainty aversion could change. In practical terms this alternative setup will also yield different results from ours. In our setup the optimal solution includes a combination of the pessimistic and optimistic regimes, whereas the alternative formulation includes only one regime. That is, it is not the same to optimize every period considering the possibility of regime switching as to optimize every period, assuming the same regime but changing the degree of uncertainty aversion at each optimization.

The remainder of this paper is organized as follows. Section 2 presents the monetary authority basic problem given by the Ball (1999) model. Section 3 discusses the case of a monetary authority with a time-varying uncertainty aversion, which is modeled by combining robust control and Markov-Switching. We do not assume the reader is familiar with robust control and, consequently, provide a detailed explanation. Section 4 shows the optimal solution to the problem. Section 5 describes the procedure to find a reasonable level of robustness in the pessimistic regime. Section 6 defines and analyzes the optimal policy response and the welfare losses of the monetary authority to different degrees and probabilities of uncertainty aversion. Finally, Sect. 7 presents the conclusions.

2 The Basic Problem of the Monetary Authority

In this section we present the small open economy model of Ball (1999). We use this model for its simplicity in capturing the major monetary policy effects and concerns. Moreover, as stated by Ball (1999), the model follows the spirit of larger and more complicated macroeconomics models used by many central banks. The model consists of the following three equations[3]:

$$y_{t+1} = \alpha y_t - \beta (i_t - \pi_t) - \chi a_t + \eta_{t+1} \tag{1}$$

$$\pi_{t+1} = \delta \pi_t + \gamma y_t - f (a_t - a_{t-1}) + \varepsilon_{t+1} \tag{2}$$

$$a_t = \varphi (i_t - \pi_t) + \vartheta_t \tag{3}$$

[3] These equations attempt to capture empirical relationship and they do not necessarily come from "microfoundations".

where y_t is the output gap, i_t is the short term interest rate controlled by the monetary authority, π_t is the inflation rate and a_t is the real exchange rate (higher a_t is an appreciation of the domestic currency). In addition, η, ε and ϑ are white noise shocks whose variances are σ_η^2, σ_ε^2 and σ_ϑ^2, respectively. All parameters are positive.

Equation 1 is the open economy IS curve representing the relationship between monetary policy and output. In Eq. 1, the output gap depends on a demand shock (η_{t+1}), the lags of the real interest rate ($i_t - \pi_t$) and the real exchange rate, and the output gap lag. This equation captures the notion that lower real interest and exchange rates increase output. The lag on the output gap implies that there is persistence in output (or alternatively that the output is serially correlated). That is, there are output components that take more than one period to adjust.

Equation 2 is the open economy Phillips curve. In this equation, the inflation rate depends on an inflation shock and the lags of the inflation rate, output gap and real exchange rate change. Ball (1999) derives this equation from separate demand equations for domestic and foreign goods. This equation expresses the idea that an appreciation of the real exchange rate leads to lower import prices and inflation rate. Similarly, an increase in the output gap represents a higher domestic demand leading to higher inflation rates.

Equation 3 is a reduced-form equation that relates the real exchange rate to the real interest rate. This equation represents the notion that higher real interest rates make domestic assets more attractive, appreciating the real exchange rate. Moreover, Eq. 3 implies that interest rate policies can also be expressed as real exchange rate policies.

We assume that the monetary authority minimizes the following quadratic loss function:

$$\min_{i_t} \quad E_0 \sum_{t=0}^{\infty} \phi^t \left(\pi_t^2 + \lambda_y y_t^2 + \lambda_i i_t^2 \right) \tag{4}$$

where E_0 is the expectations operator at time zero, ϕ is the discount factor ($0 < \phi \leq 1$), λ_i is the penalization weight for variations in the interest rate and λ_y is the penalization weight for deviations of output from its potential. The loss function in Eq. 4 implicitly assumes zero as the inflation and output gap target. However, when inflation is expressed as a percentage deviation from the trend, then the optimal policy tracks the trend inflation rate. Thus, Eq. 4 represents an inflation targeting monetary authority that is also concerned about keeping the economy at its potential output level (by attempting to make the output gap as close to zero as possible) and stabilizing the interest rate. The importance of output and interest rate stabilization relative to the inflation target is given by λ_y and λ_i, respectively. Inflation targeting while attempting to keep the economy at its potential output is a common concern of central banks. Zampolli (2006) provides a summary of the main reasons offered in the literature for including interest rate stabilization in the loss function of the monetary authority: reducing the maturity mismatch risk to the financial sector, exerting greater influence on long-term bonds rates, lower transaction fictions, and reducing the frequency that a policy rule would produce negative nominal interest rates.

Hence, in this basic problem the monetary authority finds the optimal interest rate i_t^* that minimizes the loss function (Eq. 4) subject to the dynamics of the economy given by Eqs. 1–3.

3 Time-Varying Uncertainty Aversion

In this section we extend the basic model of Ball (1999), presented in the previous section, to include a time-varying uncertainty aversion of the policy maker as the structure of the economy changes. We proceed by first defining the two regimes. Second, we introduce model uncertainty in one of the regimes and use robust control to deal with this type of uncertainty. Third, we use Markov switching to model the time-varying uncertainty aversion of the policy maker. Finally, in the last subsection we set up the new optimal monetary authority's problem.

3.1 Pessimistic and Optimistic Regimes

We now consider an economy that experiences different events or structural breaks that may represent changes in the structure of the economy which can alter the degree of mistrusts of the monetary authority. As mentioned before these events or structural breaks could be financial crises, persistent and sharp shocks to prices, output or the exchange rate. In order to keep the model tractable we consider two regimes of the economy denoted by $r_{t+1} \in \{1, 2\}$.[4] Thus, $r_{t+1}$is the regime in period *t+1* which can take on two values:

$$r_{t+1} = \begin{cases} 1 & \text{if persistent and sharp shocks or structural breaks} \\ 2 & \text{if no significant events or structural breaks} \end{cases}$$

That is, in regime 2 ($r_{t+1} = 2$), there are no significant or persistent shocks or events which indicate to the monetary authority that the estimated model, given by Eqs. 1–3, works well in this regime. However, in regime 1 ($r_{t+1} = 1$), sharp and persistent shocks or structural breaks are observed and the estimated model may not be accurate since the structure of the economy could be changing. Consequently, the monetary authority mistrusts its own model in this regime. The monetary authority believes that in regime 1 the estimated model may not fully represent the agents' reactions, especially after a period of relative stability.[5] In this paper, we consider Knightian model uncertainty where the policy maker is unable to assign unique probabilities to the alternative models in regime 1. Hence, regime 1 is the pessimistic regime because the monetary authority mistrusts its model. Regime 2 is the optimistic regime since the policy maker believes that his model is a good approximation of the economy.

[4] The model we present can be easily extended to the case of N regimes, i.e. $r_{t+1} \in \{1, \ldots, N\}$. However, we decide to use two regimes to simplify the exposition.

[5] If the policy maker believes that his model is still accurate despite the events witnessed then there is no model uncertainty and the basic model presented in Sect. 2 is sufficient.

3.2 Model Uncertainty and Robust Control in Regime 1

The monetary authority deals with Knigthian model uncertainty in regime 1 by using robust control. We follow Hansen and Sargent (2007) treatment of robust control which is based on the Gilboa and Schmeidler (1989) minmax approach. The purpose of the policy maker under robust control is to find a policy rule that works reasonably well even if his model does not coincide with a true unknown model, as opposed to a policy rule that is optimal if it does but possibly disastrous if it does not. Thus, the policy maker defines a set of likely models around his original model in regime 1. In this set three important models are located: the true unknown model, the original policy maker's estimated model and the worst-case model. Following Gilboa and Schmeidler (1989), under robust control the policy maker adopts the policy rule dictated by the worst-case model. The policy maker does not truly believe that the worst case will take place (otherwise there will be no uncertainty), it only uses the worst case to obtain a policy rule that works relatively well under model uncertainty.

Robust control can also be interpreted as a zero-sum two-player game between the policy maker and an "evil" nature. In this fictitious game the evil nature introduces distortions to hurt the monetary authority. These distortions can take the place of more persistence shocks to inflation, output or the real exchange rate. To simplify the exposition and to follow with the motivation of the commodity price increases of the introduction, we consider an evil nature that introduces a distortion ω_{t+1} in the inflation equation and the monetary authority responds using the interest rate.[6] The evil nature uses this distortion to damage the policy maker by making cost-push shocks persistent. In principle, this distortion will be non-zero in regime 1 to affect the monetary authority and zero in regime 2 because there is no model uncertainty in this regime. Therefore, Eq. 2 is modified to include this possibility as follows:

$$\pi_{t+1} = \delta\pi_t + \gamma y_t - f\,(a_t - a_{t-1}) + \varepsilon_{t+1} + \omega_{t+1} \tag{5}$$

where the evil nature distortion ω_{t+1} is a new control variable included in the optimal robust control problem. The values of ω_{t+1} will depend on the next period regime, and ultimately on the history of the state variables as follows:

$$\omega_{t+1} = \omega\,(r_t, x_t, x_{t-1}, \ldots) \tag{6}$$

Equation 6 implies that the model in regime 1 is misspecified in unknown ways. These distortions include a wide range of misspecified dynamics such as wrong parameters, autocorrelated errors and non-linearities. However, the distortion ω_{t+1} introduced by the evil nature needs to be bounded or it will produce infinite damage to the policy maker in regime 1. Thus, the bound on the distortion is given by the following equation:

6 The model can be extended to the case where nature distorts output, prices and the real exchange rate in which case a vector of distortions is added to the model.

$$\sum_{t=0}^{\infty} \phi^t \omega_{t+1}\omega_{t+1} \leq \mu \tag{7}$$

where the parameter μ represents the robustness of the model or the degree of uncertainty aversion of the monetary authority in regime 1 and its value needs to come from outside the model. Higher values of μ allow higher values of the distortions ω_{t+1} producing a policy rule for a larger set of models and a more severe worst-case. Thus, when the monetary authority becomes more pessimistic about the accuracy of its model in regime 1 (more uncertainty averse) the value of μ increases. Similarly, lower values of μ represent a less pessimistic or less uncertainty averse monetary authority.

Equation 7 will represent another constraint in the new optimization problem in regime 1, and its Lagrange multiplier will be given by the positive parameter θ known as the "free" parameter of robust control. The role of μ is now taken by θ, where $\theta \in \left(\underline{\theta}, \infty\right)$. Since there is a bijective negative function from μ to θ, an increase in θ represents a decrease in the monetary authority's degree of uncertainty aversion (less pessimistic policy maker) in regime 1. When $\theta \rightarrow \infty$ the uncertainty aversion to model misspecification in regime 1 completely dissipates and the monetary authority is fully confident that his model represents the true state of the economy. Similarly, when θ decreases $\left(\theta \rightarrow \underline{\theta}\right)$ the monetary authority is more pessimistic about the accuracy of its model in regime 1. The lower bound of the free parameter, $\underline{\theta}$, represents the highest degree of uncertainty aversion for which is possible to obtain a robust policy.[7] Thus, in our model, we consider regime 2 as the state where the monetary authority is optimistic about the model $\theta \rightarrow \infty$ and denote the free parameter as θ_2. Similarly, regime 1 is defined as the state where the monetary authority is pessimistic about the model, $\underline{\theta} \leq \theta < \infty$ and denote the free parameter as θ_1. The actual choice of θ_1 and θ_2 is discussed in Sect. 5. Thus, regime 2 ($r_{t+1} = 2$) is the state where the monetary authority is fully confident about the model, $\theta_2 \rightarrow \infty$, the distortion is $\omega_{t+1} = 0$ and Eq. 5 becomes Eq. 2. Regime 1 ($r_{t+1} = 1$) is the state where the monetary authority is pessimistic about the model, $\underline{\theta} \leq \theta_1 < \infty$, the distortion is $\omega_{t+1} \neq 0$ which implies that Eq. 5 contains persistent price shocks as opposed to those in Eq. 2.

3.3 Time-Varying Uncertainty Aversion and Markov Chain

In this paper we argue that the desire for robustness under model uncertainty is not necessarily a time-invariant feature of economic behavior. In general, it is reasonable to suggest that human beings become more or less pessimistic depending on the events they have recently witnessed or experienced. For example, a person might drive more cautiously after observing or suffering a car accident just to return to the previous driving habits once the memory of the event has faded. An example in terms of our model is that if the policy maker observes a significant increase in commodity prices he might become more cautious about second-round effects of commodity price inflation, especially when a long period of relative price stability makes him believe that

7 Gonzalez and Rodriguez (2004) show in a small analytical model that there is no robustness for values $0 < \theta < \underline{\theta}$, where nature is actually benevolent.

his model (more specifically Eqs. 1–3) might misrepresent agent's reactions.[8] This implies that the policy maker's uncertainty aversion (or degree of pessimism about the model) θ can change as the assumed structure of the economy changes. That is, θ can alternate between θ_1 and θ_2 depending on the current state of the economy and possibly after some shocks or structural breaks are observed.

We use a Markov chain to model the time-varying nature of the policy maker's uncertainty aversion. In other words, the possibility of alternating between θ_1 and θ_2 is captured by using a Markov chain. We believe this is an appropriate choice, since as the experience of the Mexican central bank suggests, changes in the degree of pessimism about the model are not predicted with total certainty. In addition, a Markov chain allows the approximation of a more general non-linear process about the nature of time-varying uncertainty. Moreover, as the example above suggests, uncertainty aversion may be related to the current state of the economy. Thus, the regime r_{t+1} follows a first order Markov chain process with the following transition matrix[9]:

$$P = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix} \tag{8}$$

where $p = \Pr\{r_{t+1} = 2|r_t = 1\}$ is the probability that the economy alternates from the pessimistic regime 1 to the optimistic regime 2. On the other hand, $q = \Pr\{r_{t+1} = 1|r_t = 2\}$ is the probability to alternate from the optimistic regime 2 to the pessimistic regime 1. The transition probabilities *(p, q)* are assumed to be time-invariant and exogenous. These probabilities represent the uncertainty about the type of regime in the next period after shocks or structural breaks have been observed by the monetary authority. We assume that at time t the information set of the policy maker is the following:

$$I_t = \left\{x_t, y_t, a_{t-1}, r_t, P, \phi, \alpha, \beta, \chi, \lambda_y, \lambda_i, \delta, \gamma, f, \theta_t, \sigma_\varepsilon^2, \sigma_\eta^2, \sigma_\vartheta^2\right\} \tag{9}$$

The previous information set assumes that the regime of the economy r_{t+1} is revealed only at the end of period t, after the policy action has been decided. That is, when the policy maker chooses the policy rule, r_t is known but r_{t+1} is still uncertain. Hence, the uncertainty is about where the system will be at time $t+1, t+2$ and so forth.[10] Notice that since in regime 1 the policy maker faces Knightian model uncertainty, the regime shift is unstructured—i.e. there is no change in a particular parameter of the model when there is regime-switching. Finally, to simplify the exposition, the parameters of the model are time-invariant with exception of θ_{r_t} which as previously discussed represents time-varying degree of uncertainty aversion.

[8] This was actually one of the main concerns expressed by the Mexican central bank after the commodity price increase of 2004.

[9] Without loss of generality we use state and regime interchangeably.

[10] This is different to Zampolli (2006) assumption where the policy maker is uncertain about the current and future state of the economy. However, incorporating uncertainty about the current regime of the economy does not change our main results or the solution method.

An important aspect of our model is that the transition probabilities in the Markov chain also represent additional bounds on the evil nature. That is, the ability of the evil nature to inflict damage on the policy maker is constrained by the severity of the worst-case shock in regime 1 (θ_1) and by the transition probabilities (p, q). If the policy maker is in regime 2, the ability of the evil nature to affect the policy maker is constrained by the probability to transit to regime 1 (q) and by the size of θ_1. Similarly, if the policy maker is in regime 1, the evil nature is constrained by the expected duration of this regimen given by p^{-1} and by the severity of θ_1.

3.4 The New Optimal Control Problem of the Monetary Authority

In this subsection we set up the new optimization problem of the policy maker. We can summarize our model in general terms as an economy with an inflation targeting monetary authority, which experiences different shocks or structural breaks that can change the structure of the economy, specifically the Phillips curve equation. The monetary authority's model works well in the optimistic regime, but the monetary authority mistrusts its model in the pessimistic regime where the structure of the economy is suspected to have changed. Thus, the monetary authority is aware that its own degree of pessimism can change according to the observed events and introduces robust control only in the pessimistic regime to deal with model uncertainty. The probability of a changing degree of model mistrust is captured by using a Markov chain process between the optimistic and pessimistic regimes. In addition, the policy maker faces a set of constraints about the dynamic evolution of the economy.

We follow Zampolli (2006) set up of optimal control with regime shifts. However, our model differs in that we introduce robust control in regime 1. This implies that in the optimal control problem the policy maker attempts to minimize the loss function J using i_t whereas the evil nature tries to maximize it by using ω_{t+1}. Incorporating Eq. 7 and using θ_{r_t} as its Lagrange multiplier, the new optimal problem of the monetary authority becomes the following min-max:

$$v(y_t, \pi_t, r_t) = \min_{i} \max_{\omega_{t+1}} \ J = \Bigg\{ \pi_t^2 + \lambda_y y_t^2 + \lambda_i i_t^2 - \theta_{r_t} \omega_{t+1}^2 + \beta \sum_{r_{t+'1}=1}^{2} P_{r_t r_{t+1}} E_t \left[v(\pi_{t+1}, y_{t+1}, r_{t+1}) \right] \Bigg\} \quad \mathrm{r}_t = 1, 2 \tag{10}$$

subject to Eqs. 1, 3, 5 and 7.
where $r_t = 1, 2$ represents the current regime which follows a Markov process given by Eq. 8 and $P_{r_t, r_{t+1}}$ represents the transition probability between regime r_t and r_{t+1}. From Eq. 8 this implies that $P_{1,2} = p, \ P_{1,1} = 1 - p, \ P_{2,1} = q$ and $P_{2,2} = 1 - q$. Hence, $\theta_{r_{t+1}}$ contained in $v(\pi_{t+1}, y_{t+1}, r_{t+1})$ of Eq. 10 represents whether $\underline{\theta} \le \theta_1 < \infty$ or $\theta_2 \to \infty$ in the next period. This feature allows us to model the time-varying uncertainty aversion about the original policy maker's model given by Eqs. 1–3. The solution to the new policy maker's problem will produce a policy rule for the interest rate that

introduces the possibility that the policy maker's degree of pessimism about his own model may change as the structure of the economy changes.

The next step is to transform the new monetary authority's problem in matrix form which allows us to solve it as linear quadratic control problem. We first define the control and state vectors in period t and regime r_t as follows:

$$u_t(r_t) = \begin{pmatrix} i_t \\ \omega_{t+1} \end{pmatrix}, x_t(r_t) = \begin{pmatrix} y_t \\ \pi_t \\ a_{t-1} \end{pmatrix} \text{ where } r_t = 1, 2.$$

Therefore, for each period the problem generates two sets of controls and states that correspond to regime 1 and 2. Substituting the value of a_t into Eqs. 1 and 2 and setting $\varsigma \equiv \beta + \chi\varphi$, $\psi \equiv \delta + f\varphi$, and $\tau \equiv f\varphi$, we obtain the following matrices of parameters:

$$A = \begin{bmatrix} \alpha & \varsigma & 0 \\ \gamma & \psi & f \\ 0 & -\varphi & 0 \end{bmatrix}, B = \begin{bmatrix} -\varsigma & 0 \\ -\tau & 1 \\ \varphi & 0 \end{bmatrix}, Q = \begin{bmatrix} \lambda_y & 0 & 0 \\ 0 & \lambda_\pi & 0 \\ 0 & 0 & \lambda_i \end{bmatrix}, R_t = \begin{bmatrix} 0 & 0 \\ 0 & -\theta_{r_t} \end{bmatrix} \; \mathrm{r}_t = 1, 2$$

Since the error ϑ now enters directly in Eqs. 1 and 2, the new additive errors for these equations are defined as η_y and ε_π with standard deviations of σ_y and σ_π, respectively. Thus, the new additive error vector is denoted by ξ_{t+1} and its corresponding 3×3 variance-covariance matrix by Ω.

The next step is to substitute all the newly defined matrices into the new monetary authority's problem given by Eqs. 10, 5, 7, 3 and 1 to obtain a linear quadratic problem. Since the Riccati equations of a linear quadratic problem with Markov switching emerges from the first-order conditions alone and the first-order conditions for extremizing a quadratic criterion function match those of an ordinary (non-robust) linear quadratic problem with Markov switching and an extra set of controls (see Hansen and Sargent 2007), then the new monetary authority problem with varying uncertainty aversion consists of finding the controls $\{u_t(r_t)\}_{t=0}^{\infty}$ in order to extremize the loss function J.[11]

$$\begin{aligned} v(x_t, r_t) = \underset{x_t, u_t}{ext} \; J = x_t^{'} Q x_t + u_t^{'} R_t u_t \\ + \phi \sum_{r+1=1}^{2} P_{r_t, r_{t+1}} E_t\Big[v(x_{t+1}, r_{t+1})\Big] \quad r_t = 1, 2 \end{aligned} \tag{11}$$

Subject to the state-space representation of the model

$$x_{t+1} = Ax_t(r_t) + Bu_t(r_t) + \xi_{t+1} \qquad \mathrm{r}_t = 1, 2. \tag{12}$$

[11] In this case extremizing refers to minimize the loss function J by using the interest rate and maximize it by using ω_{t+1}.

The continuation value of the dynamic problem at t is $v(\cdot)$ and it is a function of the state variables and the current regime (r_t). Thus, the problem generates a value of $v(\cdot)$for each regime.

4 Optimal Solution with Unstructured Regime Shifts

In this section, we show the solution to the inflation targeting monetary authority problem with time-varying uncertainty aversion given by the system of Eqs. 11–12. Solving this optimal control problem with unstructured regime shifts is equivalent to finding a contingent policy rule $u_t(r_t)$. The solution is given by the following feedback rule:

$$u(x_t, r_t) = -F_{r_t} x_t \quad \text{where } F_{r_t} = \begin{bmatrix} Fi_{r_t} \\ F\omega_{r_t} \end{bmatrix} \quad \mathrm{r}_t = 1, 2. \tag{13}$$

where $Fi_{r_t}$is a (1×3) matrix that shows the optimal rule of the interest rate to the state variables (y_t, π_t, a_{t-1}). Similarly, $Fw_{r_t}$is a (1×3) matrix that contains the optimal rule of the evil nature. The term r_t denotes that for each policy rule the model produce two sets of values, one for each regime. Following Hansen and Sargent (2007), Kendrick (2002) and Zampolli (2006); substituting the matrices and vectors in our problem and after extensive matrix algebra we find the feedback matrices:

$$\begin{aligned} F_{r_t} = &\left(\sum_{r_{t+1}=1}^{2} P_{r_t, r_{t+1}} \left[\phi B^{'} V_{r_{t+1}} B + R^{'}_{r_{t+1}} \right] \right)^{-1} \\ &\times \left(\phi \sum_{r_{t+1}=1}^{2} P_{r_t, r_{t+1}} B^{'} V_{r_{t+1}} A \right) \quad r_t = 1, 2 \end{aligned} \tag{14}$$

where the matrices V_{r+1} in Eq. 14 are (3×3) positive semi-definite and represent the set of Ricatti equations. Similarly, we obtain the solution to the Ricatti Equations:

$$\begin{aligned} V_{r_t} = &\, Q + \phi \sum_{r_{t+1}=1}^{2} P_{r_t, r_{t+1}} A' V_{r_{t+1}} A - \phi^2 \left(\sum_{r_{t+1}=1}^{2} P_{r_t, r_{t+1}} A' V_{r_{t+1}} B \right) \\ &\times \left(\sum_{r_{t+1}=1}^{2} P_{r_t, r_{t+1}} \left[\phi B^{'} V_{r_{t+1}} B + R^{'}_{r_{t+1}} \right] \right)^{-1} \\ &\times \left(\sum_{r_{t+1}=1}^{2} P_{r_t, r_{t+1}} B' V_{r_{t+1}} A \right) \quad r_t = 1, 2 \end{aligned} \tag{15}$$

Equation 13 and 14 show that the solution in Eq. 15 produces one Ricatti matrix for each regime (V_1, V_2). In addition, the Ricatti equations for each regime are interrelated. That is, $V_2$is part of the solution of V_1 and vice versa. Therefore, we iterate jointly on V_1 and V_2 until convergence is achieved for each of them.[12] Notice that the solution to the Ricatti equation for each regime $\left(V_{r_t}\right)$ places a weight on the Ricatti equation of the next period regime $\left(V_{r_{t+1}}\right)$ equal to the probability of transiting to that regime $\left(P_{r_t,r_{t+1}}\right)$.

An important difference between the Ricatti equations obtained here and those in Zampolli (2006) is the time-varying nature of the matrix $R_{r_{t+1}}$ in Eq. 15 of our model which in our case contains the uncertainty aversion parameter θ.

Since the problem is linear quadratic the value function of the monetary authority's losses takes the following form:

$$v(x_t, r_t) = x_t^{'} V_{r_t} x_t + d_{r_t} \qquad \mathrm{r}_t = 1, 2. \tag{16}$$

where d_{r_t} is a scalar that can take different values on each regime. Equation 16 implies that the losses to the policy maker are conditional of being in regime r_t. The scalar d_{r_t} is given by the following equation:

$$d = (I_2 - \phi P)^{-1} \phi P \left\{tr(V_{r_t}\Omega)\right\} \tag{17}$$

where d is a (2×1) vector that contains the values of d_{r_t} for each regime, I_2 is a (2×2) identity matrix and tr is the trace operator.[13] Given that in the Ball (1999) model the optimal steady state values of the state variables are zero, the steady state losses to the policy maker in each regime are given by the vector d. That is, the first element of the vector d is the value of the loss conditional of being in regime 1 and the second element is the value of the loss conditional of being in regime 2.

The expected duration of regime 1 and 2 are given by p^{-1} and q^{-1}, respectively. Therefore, we compute the expected losses as follows:

$$\overline{v}(x_t) = p^{-1} v(x_t, r_t = 1) \ + q^{-1} v(x_t, r_t = 2) \tag{18}$$

where $p^{-1}v(x_t, r_t = 1)$ is the expected duration of regime 1 times the losses in regime 1 and $q^{-1}v(x_t, r_t = 2)$ is the expected duration of regime 2 times the losses regime 2.

5 Selection of the Robust Control Free Parameter in the Pessimistic Regime 1

As explained in Sect. 3.2, the monetary authority's degree of pessimism about its model is given by the free parameter θ. Under robust control the value of θ needs to come from outside the model. The selection of the free parameter is important because values of θ_{r_t} close to $\underline{\theta}$ represent a monetary authority extremely pessimistic about its model in regime r_t. Large values of θ_1 indicate that the monetary authority

[12] The matlab implementation of the problem can be obtained from the authors upon request.

[13] In the general case where there are N number of regimes the d vector has N rows and the identity matrix is $N \times N$.

Table 1 Parameter values

Economy						Loss	
α	0.72	φ	0.2	$\sigma^2_{y,\pi}$	−0.00013	λ_y	1
ς	0.564	f	0.2	$\sigma^2_{y,\vartheta}$	−0.00013	λ_π	1
γ	0.49	σ^2_y	0.000041	$\sigma^2_{\vartheta,\pi}$	0.00003	λ_i	0
τ	0.4	σ^2_π	0.000083			ϕ	0.96
ψ	1.4	σ^2_ϑ	0.00063				

is completely confident about the model in regime 1. Therefore, reasonable values of θ_1 prevent the policy maker from appearing catastrophist instead of cautious.

We follow Hansen and Sargent (2007) and use detection error probability theory to choose these reasonable values of θ_1 and θ_2. However, we extend their procedure to include the transition probabilities in the Markov chain and simultaneously choose θ_1 and θ_2. Hence, we consider three models of the economy: the original, the distorted and the time-varying model. The original model discussed in Sect. 2 has no model uncertainty and it is represented by Eqs. 1–3. The distorted model includes the evil nature distortion ω_{t+1}, the free parameter θ and is given by Eqs. 1, 3 and 5. The time-varying model represents the possibility of alternating between regimes where there is model uncertainty in one regime and no uncertainty in the other regime. Thus, the time-varying model can be written as follows:

$$\text{time-varying model} = (1-q)(\text{original model}) + q\ (\text{distorted model})$$

The objective is to find values of θ_1 and θ_2 for which it is statistically difficult to distinguish between the original model and the time-varying model. This allows us to rule out values of θ_1 and θ_2 that imply extremely pessimistic cases.

In particular, the procedure consists of obtaining two types of probabilities: (i) the probability of choosing the original model when the data were generated by the time-varying model and (ii) the probability of choosing the time-varying model when the data were generated by the original model. The average of these two probabilities is the probability of making an error in the detection of the model—i.e. the detection error probability denoted by κ. Note that if there is no robustness ($\theta \to \infty \Rightarrow \omega \to 0 \Rightarrow$ original model = distorted model) the original and the time-varying model are the same and the detection error probability is equal to 0.5 ($\kappa = 0.5$). On the other hand, when the level of robustness is infinite $\left(\theta \to \underline{\theta} \Rightarrow \omega \to \infty\right)$ the detection error probability is equal to zero ($\kappa = 0$). Hansen and Sargent (2007) recommend the use of θ associated with detection error probabilities between 0.1 and 0.2 since they correspond to commonly used confidence intervals of 95% and 90%, respectively.

We parameterize the model using the values in Zampolli (2006) which correspond to the UK economy. However, these values are similar to those obtained for the U.S. and the Euro area (see Orphanides and Wieland 2000; Sargent 1999) and are shown in Table 1.

We use the parameters in Table 1 to find the combinations of θ_1, θ_2, p and q that produce the recommended values of κ=0.1 and 0.2. We perform 1,000 simulations using a time horizon of 150 periods ($T = 150$). Each simulation represents a random draw of the additive noise. The detailed procedure is shown in Appendix A.
An important advantage of our procedure is that allows the transition probabilities as well as θ_1 and θ_2 to determine κ. Therefore, for a given level of κ we may have different values of θ_1 and $\theta_2$depending on the values of (p, q).

6 Optimal Policy Response Under Time-Varying Uncertainty

In this section, we analyze numerically the response of the interest rate and the policy maker's losses to changes in the degree of model uncertainty in regime 1 and for different values of the transition probabilities (p, q). From Eq. 13, we can write the optimal interest rate policy rule as follows:

$$i_t = fi_{y,r_t} y_t + fi_{\pi,r_t} \pi_t + fi_{a,r_t} a_{t-1} \tag{19}$$

where fi_{y,r_t}, fi_{π,r_t} and fi_{a,r_t} are the response of the interest rate to the output gap, inflation and the lagged exchanged rate. We substitute the parameter values for the UK shown in Table 1 into problem (11)–(12) and obtain the optimal policy responses and welfare losses. Since the worst-case shock is only used to obtain a robust interest rate policy rule and to keep the exposition succinct, we present the response of the worst-case shock in Appendix B.

6.1 Optimal Interest Rate Response

In our model the degree of time-varying uncertainty aversion of the monetary authority (denoted byκ) is composed by the probability that its degree of pessimism will change in the future given by (p, q) and by the degree of uncertainty aversion in regime 1 given by θ_1. We define an aggressive (cautionary) response when a change in a parameter produces a higher (lower) value of the feedback coefficient of the interest rate. Aggressive (cautionary) response implies a stronger (weaker) use of the interest rate that in the Ball (1999) model depreciates (appreciates) the exchange rate, reduces (increases) inflation and the output gap.

The effects of changes only in p or q (without offsetting changes in θ_1 and θ_2 to keep κ constant) are relatively straight forward. The interest rate response will be aggressive (cautionary) for changes that increase (decrease) the time-varying model uncertainty. Thus, increases only in p will produce a higher κ and generate a cautionary response in both regimes. In the optimistic regime 2, a higher p represents a lower expected duration of the pessimistic regime 1. In the pessimistic regime 1, it represents a higher probability to transit to the optimistic regime. Increases in q generate a lower κ and produce an aggressive response of the interest rate in both regimes. In the pessimistic regime, a higher q means a lower expected duration of the optimistic regime, whereas in the optimistic regime implies a higher probability of the pessimistic regime.

Table 2 Optimal response of the interest rate coefficient (Fi)

			fi_y	fi_π	fi_a	fi_y	fi_π	fi_a
q	p		$\kappa = 0.1$			$\kappa = 0.2$		
0	0	$r = 2$	1.152	1.89	0.178	1.152	1.891	0.178
0.1	0.75	$r = 1$	1.289	2.19	0.238	1.239	2.08	0.216
		$r = 2$	1.272	2.15	0.23	1.229	2.056	0.211
0.25	0.75	$r = 1$	1.304	2.22	0.243	1.247	2.094	0.219
		$r = 2$	1.304	2.22	0.243	1.247	2.094	0.219
0.5	0.75	$r = 1$	1.346	2.29	0.259	1.268	2.133	0.227
		$r = 2$	1.395	2.42	0.283	1.294	2.2	0.24
0.75	0.75	$r = 1$	1.478	2.53	0.306	1.334	2.252	0.25
		$r = 2$	1.753	3.22	0.444	1.454	2.563	0.313
1	0.75	$r = 1$	1.775	3.05	0.409	1.813	3.12	0.424
		$r = 2$	3.956	8.33	1.465	4.188	8.842	1.568
0.1	0.5	$r = 1$	1.277	2.17	0.234	1.231	2.066	0.213
		$r = 2$	1.228	2.05	0.21	1.201	1.995	0.199
0.25	0.5	$r = 1$	1.285	2.18	0.237	1.236	2.074	0.215
		$r = 2$	1.251	2.1	0.22	1.215	2.024	0.205
0.5	0.5	$r = 1$	1.307	2.22	0.245	1.247	2.094	0.219
		$r = 2$	1.307	2.22	0.245	1.247	2.094	0.219
0.75	0.5	$r = 1$	1.369	2.34	0.267	1.277	2.148	0.23
		$r = 2$	1.483	2.62	0.324	1.333	2.293	0.259
1	0.5	$r = 1$	1.649	2.82	0.364	1.667	2.853	0.371
		$r = 2$	3.408	7.12	1.224	3.496	7.315	1.263
0.1	0.25	$r = 1$	1.282	2.18	0.235	1.233	2.07	0.214
		$r = 2$	1.19	1.97	0.194	1.177	1.942	0.188
0.25	0.25	$r = 1$	1.285	2.18	0.236	1.235	2.073	0.215
		$r = 2$	1.208	2.01	0.201	1.188	1.964	0.193
0.5	0.25	$r = 1$	1.293	2.2	0.239	1.238	2.079	0.216
		$r = 2$	1.242	2.07	0.215	1.208	2.006	0.201
0.75	0.25	$r = 1$	1.309	2.23	0.245	1.246	2.094	0.219
		$r = 2$	1.309	2.23	0.245	1.246	2.094	0.219
1	0.25	$r = 1$	1.522	2.6	0.32	1.475	2.505	0.301
		$r = 2$	2.973	6.17	1.033	2.762	5.697	0.939

Since changes in p and q affect κ, we concentrate the analysis to the effect of (p,q) for a constant value of κ by adjusting θ_1 and θ_2 (as outlined in Sect. 5).

Table 2 presents the optimal policy feedback coefficients of the interest rate for κ=0.1 and 0.2 and selected values of p and q. The first two columns show different values of p and q while the third column indicates the regime. Columns 4-6 display the response of the interest rate with respect to each state variable for a robustness level of $\kappa = 0.1$. Similarly, columns 7-9 and 10-12 show the response of the interest rate for $\kappa = 0.2$. Higher values of κ represent lower model uncertainty.

Table 2 shows that the response of the interest rate is positive with respect to all state variables for all combinations of p, q, κ and regimes, i.e. the interest rate is always positively correlated with the output gap, inflation and the lagged exchange rate. The positive sign of the response coefficients is the standard solution to the Ball model. Hence, we concentrate our analysis in the change of the responses rather than their sign. The first row in Table 2 is the response when $p = 0$ and $q = 0$. That is, it represents the response when the monetary authority feels optimistic that its model is an accurate representation of the true unknown model and it will not transit to a regime where the model is only an approximation. In this case the policy maker is always in regime 2. The smallest response takes place precisely when $p = q = 0$. This implies that the introduction of the possibility that the policy maker may mistrust his own model in the future produces an aggressive response of the interest rate even when he is optimistic about the accuracy of the model. Moreover, comparing the interest rate response to the same state variable but for different values of κ, we find that higher degrees of model uncertainty produce more aggressive responses of the interest rate with respect to all control variables in both regimes. In regime 1, the policy maker faces the Knigthian model uncertainty about his model which implies that higher values of θ_1 produce higher worst-case shocks of cost-push inflation. Under robust control the monetary authority follows the worst-case policy rule. Therefore, in regime 1 the policy maker behaves as if the worst case of cost-push shocks on inflation (within the reasonable limits given by θ_1) were going to take place and reacts aggressively to counteract the effect of the worst-case shock on inflation, output gap and the exchange rate. In regime 2, the policy maker cannot predict with certainty if his degree of uncertainty aversion will increase in the future. However, the possibility of this increase on the policy maker's uncertainty aversion produces an aggressive response of the interest rate even when the policy maker is optimistic about his model in the current regime. The monetary authority in regime 2 does not wait until it arrives to regime 1 to have an aggressive response because it takes the interest rate one period to affect the output gap and the inflation rate.

This aggressive response of the policy maker to higher model uncertainty under robust control has been documented before by Gonzalez and Rodriguez (2004), Walsh (2004a) and Sargent (1999) among others and it contrasts with the traditional cautionary response shown in Brainard (1967). The difference is that in the latter the policy maker faces Bayesian multiplicative parameter uncertainty in a linear quadratic problem. This makes the policy maker to be more concerned about the variance of the state variables (output gap and inflation) rather than their actual level and consequently higher parameter uncertainty triggers a cautionary response.[14]

In order to complement our analysis, we also use the contour graphs of the responses shown in Fig. 1. The graphs allows us to observe the responses for all the values of p and q which are not feasible in Table 2 but at the cost of restricting the analysis to one value of κ. We use $\kappa = 10\%$ in Fig. 1 but the results for $\kappa = 20\%$ follow the same pattern.

In general terms we find an aggressive or insensitive response of the interest rate to changes in p or q in both regimes for a given level of time-varying uncertainty aversion. The only exception is found for a relatively small region of regime 1 where the response is cautionary. Next, we explain the interest rate response to changes in p and q for each regime and then compare across regimes.

[14] The cautionary response will always be the case as long as the uncertain multiplicative parameter belongs to the monetary policy transmission channel (see Söderström 2002).

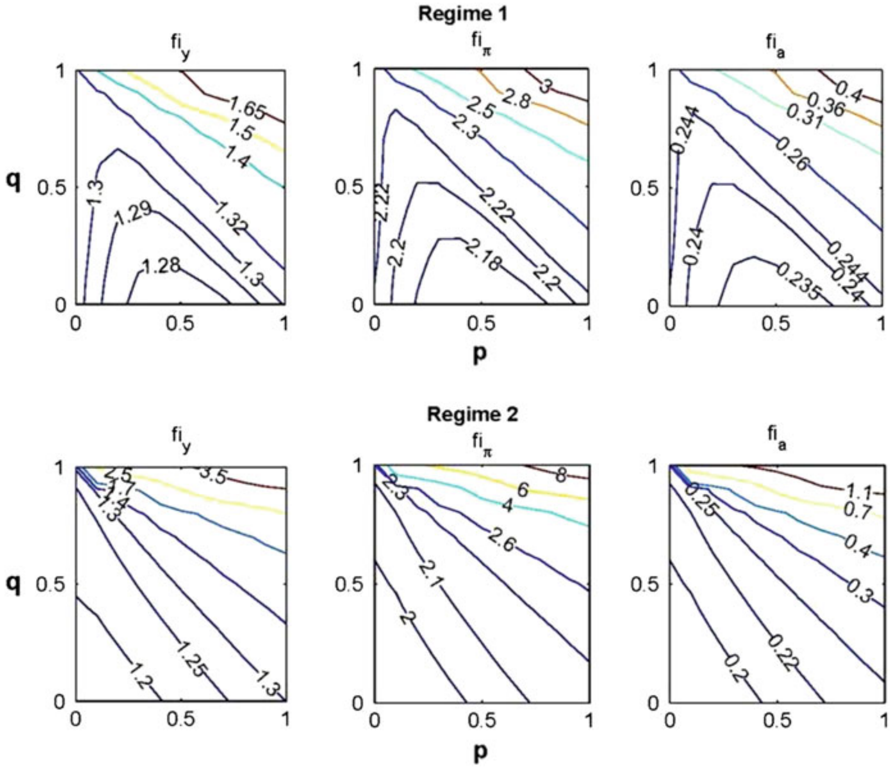

Fig. 1 Response of the interest rate for $\kappa = 0.1$

6.1.1 Response in the Optimistic Regime 2

The interest rate response in the optimistic regime 2 is aggressive to changes in either p or q for a given value of κ. In the case of higher q there are two underlying effects. As mentioned above a higher q produces an aggressive response. However, to keep the level of time-varying model uncertainty (κ) constant, the degree of pessimism in regime 1 decreases (i.e. higher θ_1 or lower ω) which tends to reduce the response. Figure 1 shows that the increase in q dominates the impact of a lower ω. Thus, for a given degree of time-varying model uncertainty, the monetary authority prepares for the higher possibility of becoming pessimistic by making an aggressive use of the interest rate in the optimistic regime because of the lagged effect that the interest rate has on inflation and the output gap.

Regarding p in regime 2, the interpretation is similar. Higher p reduces the response since it decreases the expected duration of regime 1. To keep constant the time-varying model uncertainty given by κ, the degree of pessimism in regime 1 increases which tends to increase the response. The net result of the two effects in Fig. 1 indicates that when the policy maker is optimistic a decrease in the expected duration of the pessimistic regime produces an aggressive response of the interest rate for a given degree of time-varying model uncertainty.

6.1.2 Response in the Pessimistic Regime 1

Regarding the pessimistic regime 1, we find an aggressive response to changes in q for a given level of κ. In the pessimistic regime 1, a higher q implies a lower expected duration of the optimistic regime 2 which increases the response. To keep κ constant the degree of pessimism about the model in regime 1 decreases (higher ω) which tends to reduce the response. Thus, the aggressive response to higher q in regime 1 in Fig. 1 indicates that when the policy maker is pessimistic about his model, a lower expected duration of the optimistic regime produces an aggressive response of the interest rate for a given level of time-varying model uncertainty.

The effect of p in regime 1 can be divided into two different sections. These sections can be better observed by tracing a straight line from $p = 0.5$ to $q = 1$. In the left part of this imaginary line, we find that increases in the probability of transiting to the optimistic regime reduce the response for a given level of κ. Higher values of p by themselves generate a cautionary response but to keep κ constant the pessimism in regime 1 has to increase producing an aggressive response. Thus, our results indicate that for this left section the effect of higher p dominates the impact of higher ω. In the right part of this auxiliary line, we find that there is an aggressive response of the interest rate to higher values of p. Roughly speaking, when policy maker is pessimistic about his model and the probability of transiting to the optimistic regime is small, increases in this probability produce a cautionary response mainly for large expected durations of the optimistic regime for a given level of time-varying model uncertainty. Moreover, the response is aggressive when the expected duration or the probability of the optimist regime is large.

6.1.3 Responses Across Regimes

A comparison between responses show that the feedback coefficients are greater in the pessimistic regime than in the optimistic regime when $q < (1-p)$ or alternatively when $p < (1-q)$. This implies that when probability of transiting to the other regime is low compared to the expected duration of that other regime, the response will be more aggressive if the policy maker is in the pessimistic regime 1.[15] In the opposite case when the when $q > (1 - p)$ or alternatively when $p > (1 - q)$, feedback coefficients are greater in regime 2 than in regime 1. This implies that when probability of transiting to the other regime is high compared to the expected duration of that other regime, the response will be more aggressive if the policy maker is in the optimistic regime 2. Finally, the feedback coefficients are the same in both regimes when $q = (1 - p)$ or alternatively when $p = (1 - q)$.

[15] Since p^{-1} is the expected duration of regime 1, high values $(1 - p)$ imply low values of p and consequently a larger expected duration of regime 1. Similarly, high values of values $(1 - q)$ imply low values of q and consequently a larger expected duration of regime 2.

Table 3 Expected policy maker losses

		$\kappa = 0.1$			$\kappa = 0.2$		
q	p	Total	$r = 1$	$r = 2$	Total	$r = 1$	$r = 2$
0	0		0.07022	0.05302		0.06385	0.05302
0.1	0.75	0.75917	0.06700	0.06698	0.70129	0.06189	0.06188
0.25	0.75	0.37403	0.07013	0.07013	0.33993	0.06374	0.06374
0.5	0.75	0.25779	0.07737	0.07732	0.22628	0.06790	0.06787
0.75	0.75	0.27027	0.10158	0.10112	0.21335	0.08012	0.07990
1	0.75	0.67512	0.29210	0.28565	0.55490	0.23991	0.23502
0.1	0.5	0.75133	0.06275	0.06258	0.71010	0.05926	0.05916
0.25	0.5	0.39300	0.06554	0.06548	0.36575	0.06098	0.06095
0.5	0.5	0.28039	0.07010	0.07010	0.25494	0.06374	0.06374
0.75	0.5	0.26835	0.08055	0.08044	0.23196	0.06961	0.06955
1	0.5	0.63883	0.21426	0.21031	0.53280	0.17860	0.17559
0.1	0.25	0.84449	0.06088	0.06010	0.80705	0.05799	0.05751
0.25	0.25	0.50947	0.06387	0.06350	0.47824	0.05990	0.05966
0.5	0.25	0.40016	0.06672	0.06664	0.36994	0.06168	0.06162
0.75	0.25	0.37245	0.06983	0.06983	0.33982	0.06372	0.06372
1	0.25	0.67705	0.13574	0.13408	0.58896	0.11806	0.11672

6.2 Welfare Losses

In this subsection we analyze the effect on the monetary authority expected losses from changes in q, p and κ given by Eq. 18. Table 3 displays the monetary authority losses for different values of q, p and κ. Table 3 shows that higher degrees of uncertainty aversion (or pessimism) of the monetary authority increases the monetary authority's losses. That is, when the monetary authority believes that his model is farther away from the true unknown model, he assumes a more severe worst-case shock ($\omega$is higher) and consequently losses are larger. Alternatively, higher values of κ imply a more optimistic monetary authority and a lower worst-case shock of inflation which reduces the losses.[16]

We analyze the effect welfare losses by also using a contour graph for all values p, q and $\kappa = 0.1$.

While we are mostly interested in the total expected losses we also show the expected losses in each regime to explain the underlying effects on the losses. Figure 2 shows from left to right expected total, regime 1 and regime 2 losses for different combinations of p and q. Unsurprisingly, expected losses in regime 1 and 2 follow a similar as the interest rate responses shown in Fig. 1. Combining this information with Eq. 18, we expect changes in q to have opposite effects on the total expected

[16] As shown in Sect. 3.4 the worst-case shock affects the loss function directly through $\theta_1 \omega_{t+1} \omega_{t+1}$.

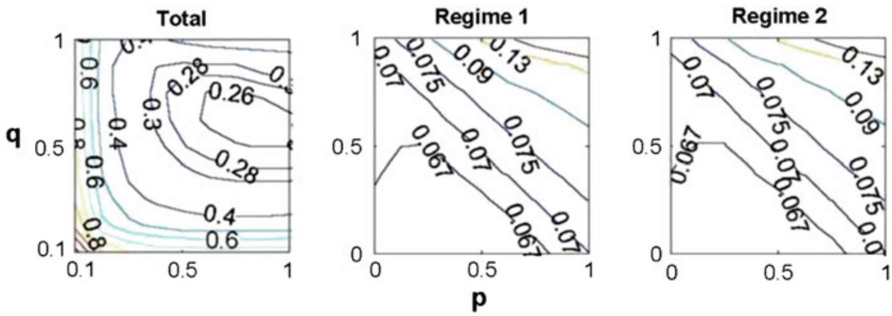

Fig. 2 Welfare losses for κ=0.1

losses. Higher q tends to increase losses in both regimes but it also tends to reduce duration of these higher losses of regime 2. Similarly, for most values of p, higher values of p tend to increase losses in both regimes but they also reduce the expected duration of the higher losses in regime 1.

The leftmost graph in Fig. 2, shows the final results on the total expected losses of these opposite effects of p and q. Higher values of p decrease expected total losses when p is small (between 0.1 and 0.4) and when both $0.3 < q < 0.8$ and $0.4 < p < 0.7$. That is, when the expected duration of the pessimistic regime is large or when the optimistic and pessimistic regime have both a middle duration, an increase in the probability to transit to the optimistic regime makes the policy maker better off for a given degree of κ. For values $p > 0.3$ the response is mostly insensitive for the smallest and largest values of q ($0 < q < 0.3$ and $0.8 < q < 1$). In these areas the two opposite effects of higher p roughly offset each other. For middle to high values of q ($0.5 < p < 0.7$) and a large p ($p > 0.7$), higher values of p slightly increase the losses. In this area, the effect of higher losses in both regimes as a result of larger p is greater than the reduction in the expected duration of the pessimistic regime since this regime has a low expected duration. In other words, when the duration of the pessimistic regime is low a further reduction is out weighted by the aggressive response of the interest rate making the monetary authority worse off.

Regarding the net effect of q on the total expected losses, we find that when $0.6 < q < 1$ and $0.3 < p < 1$, higher values of q increase the losses. This implies that when the duration of the optimistic regime is relatively small and the duration of the pessimistic regime is not large then increases in the duration of the optimistic regime makes the policy maker worse off for a given degree of model uncertainty. The monetary authority's total expected welfare increases for higher values of q when $0.3 < p < 1$ and $0 < q < 0.6$. Hence, increases in the expected duration of the optimistic regime make the policy maker better off when the expected duration of the optimistic regime is relatively long and the expected duration of the pessimistic regime is not large. Finally, expected total losses are mostly insensitive to changes in q when $p < 0.3$, implying that when the expected duration of the pessimistic regime is large then changes in the expected duration of the optimistic regime have a negligible effect on the monetary authority's expected welfare.

7 Conclusions

Casual empiricism suggests that human beings can become more or less pessimistic depending on the events that they have just witnessed or experienced. The commodity price shocks observed in 2004 is our initial motivation for this paper. As prices increased, monetary authorities worried about the persistence of these price shocks, and, as a result, tightened monetary policy. Moreover, these shocks affected the confidence of the monetary authority about the accuracy of its own model to accurately reflect agents' reactions. In this paper, we develop a framework to analyze the problem of an inflation-targeting monetary authority in a similar environment. We consider a monetary authority whose degree of uncertainty aversion can vary with time as the structure of the economy changes which is reflected in different events that can range from structural breaks (for example in the aftermath of a financial crisis) to price, output or real exchange shocks. To keep the model tractable we consider two regimes. In the optimistic regimes the events observed do not affect the structure of the economy and the monetary authority is confident that its model works well. In the pessimistic regime the events experienced by the economy produces a monetary authority that mistrusts its model, as it is reasonable to believe that the estimated model may misrepresent the agents' reactions, particularly after a period of relative stability. We model the fear of model misspecification in the pessimistic regime by using robust control. The possibility of time-varying uncertainty aversion is captured by a Markov chain between the pessimistic and optimistic regimes. We apply this framework to the open economy model of Ball (1999).

In general we find that the interest rate response is more aggressive when: (i) the degree of mistrust about the model of the monetary authority in the pessimistic regime increases and (ii) the likelihood of future uncertainty aversion increases. These results follow the previous research findings in regard to monetary policy under model uncertainty being more aggressive. Our results confirm this aggressive approach to monetary policy in the presence of time-varying uncertainty aversion about the policy maker's model. However, we also find a cautionary response for a small set of cases when the policy maker already mistrust its model and believes the duration of this pessimism is short.

Appendix A

In this appendix we describe the steps to computer the detection error probability in our model, the Matlab implementation of this procedure can be obtained from the authors upon request.

We define the vector of parameters for the original model: $\widetilde{B} = \begin{bmatrix} -\kappa \\ -\tau \\ \varphi \end{bmatrix}$

The solution to the original model is then given by the following equation:

$$i_t = -\widetilde{F}_t \widetilde{x}_t \tag{A-1}$$

where the tilde above a matrix or vector denotes the value in the original model. The optimal value of the state is then given by the following:

$$\widetilde{x}_{t+1} = \left(A - \widetilde{B}\widetilde{F}_t\right)\widetilde{x}_t + \widetilde{\xi}_{t+1} \tag{A-2}$$

Combining Eqs. 12 and 13 in the text we obtain the following expression for the states in the time-varying model which are then given by the following:

$$x_{t+1} = (A - BF_t)\, x_t(r_t) + \xi_{t+1} \qquad r_t = 1, 2. \tag{A-3}$$

Substituting $\widetilde{x}$ in the original model and the x in state 1 because that is the state when the evil nature introduces the distortion we obtain the following expression:

$$\widehat{x}_{t+1} = (1 - q)\,\widetilde{x}_t + q x_{t+1, r_t = 1} + \widehat{\xi}_{t+1} \tag{A-4}$$

where the hat above a matrix or vector denotes the value in the time-varying model.

Next, we follow the approach introduced by Hansen and Sargent (2007). Thus, the log likelihood ratio under the original model is the following:

$$\text{Log-likelihood ratio original} = \frac{1}{T}\sum_{t=0}^{T-1}\{0.5\widetilde{\omega}_{t+1}\widetilde{\omega}_{t+1} - \widetilde{\omega}_{t+1}\xi_{t+1}\} \tag{A-5}$$

where the $\widetilde{\omega}$ is the mean of the worst-case shock under the original model that is, the mean over -$F\omega\widetilde{x}$. We ran 1,000 Monte Carlo simulations where each of them chooses a random value of ξ for $T = 150$ and count the instances where the log-likelihood is negative.

The log-likelihood ratio under the time-varying model is the following

$$\text{Log-likelihood ratio time-varying} = \frac{1}{T}\sum_{t=0}^{T-1}\left\{0.5\widehat{\omega}_{t+1}\widehat{\omega}_{t+1} + \widehat{\omega}_{t+1}\widehat{\xi}_{t+1}\right\} \tag{A-6}$$

where the $\widehat{\omega}$ is the mean of the worst-case shock under the time-varying model that is, the mean over -$F\omega\widehat{x}$. We ran 1,000 simulations where each of them chooses a random value of $\widehat{\xi}$ for $T = 150$ and count the instances where the log-likelihood is negative.

The error detection probability is then given by the following equation:

$$\kappa = 0.5[\text{prob(original)} + \text{prob(time-varying)}] \tag{A-7}$$

where the *prob(original)* and *prob(time-varying)* are the proportion of instances in which the likelihood ratio in (A-5) and (A-6), respectively, was negative.

This procedure is undertaken for each set of parameters including θ_1, θ_2 and (p, q) until we find κ=0.1, 0.2. In particular, we start by setting (p, q) and assigning a value for θ_1 much lower than θ_2. Then, we change the values of θ_1 and θ_2 while trying to keep the difference between them as large as possible until the desired κ is found.

Appendix B

In this Appendix we show the table with the values of the worst-case shock response. From Eq. 13 in the text, we can write the optimal worst-case shock response as follows:

$$\omega_{t+1} = f\omega_{y,r_{t-1}} y_t + f\omega_{\pi,r_{t-1}} \pi_t + f\omega_{a,r_{t-1}} a_{t-1} \tag{B-1}$$

Table B-1 Optimal response of the worst-case shock coefficient (Fw)

			fi_y	fi_π	fi_a	fi_y	fi_π	fi_a
q	p		$\kappa = 0.1$			$\kappa = 0.2$		
0.1	0.75	$r = 1$	0.0790	0.1971	0.0394	0.0507	0.1282	0.0256
		$r = 2$	0.0636	0.1588	0.0318	0.0413	0.1045	0.0209
0.25	0.75	$r = 1$	0.0795	0.1971	0.0394	0.0508	0.1279	0.0256
		$r = 2$	0.0795	0.1971	0.0394	0.0508	0.1279	0.0256
0.5	0.75	$r = 1$	0.0813	0.1979	0.0396	0.0515	0.1282	0.0256
		$r = 2$	0.1322	0.3232	0.0646	0.0814	0.2031	0.0406
0.75	0.75	$r = 1$	0.0871	0.2027	0.0405	0.0540	0.1301	0.0260
		$r = 2$	0.3561	0.8481	0.1696	0.1956	0.4791	0.0958
1	0.75	$r = 1$	0.1010	0.2134	0.0427	0.0659	0.1404	0.0281
		$r = 2$	2.7958	6.4585	1.2917	2.1950	5.1257	1.0251
0.1	0.5	$r = 1$	0.0785	0.1970	0.0394	0.0505	0.1282	0.0256
		$r = 2$	0.0391	0.0989	0.0198	0.0261	0.0667	0.0133
0.25	0.5	$r = 1$	0.0787	0.1967	0.0393	0.0505	0.1280	0.0256
		$r = 2$	0.0487	0.1223	0.0245	0.0321	0.0815	0.0163
0.5	0.5	$r = 1$	0.0794	0.1967	0.0393	0.0508	0.1278	0.0256
		$r = 2$	0.0794	0.1967	0.0393	0.0508	0.1278	0.0256
0.75	0.5	$r = 1$	0.0816	0.1974	0.0395	0.0518	0.1283	0.0257
		$r = 2$	0.1931	0.4706	0.0941	0.1153	0.2870	0.0574
1	0.5	$r = 1$	0.0985	0.2128	0.0426	0.0633	0.1383	0.0277
		$r = 2$	2.2463	5.2471	1.0494	1.7741	4.1984	0.8397
0.1	0.25	$r = 1$	0.0787	0.1970	0.0394	0.0505	0.1282	0.0256
		$r = 2$	0.0178	0.0456	0.0091	0.0123	0.0316	0.0063
0.25	0.25	$r = 1$	0.0786	0.1967	0.0393	0.0506	0.1283	0.0257
		$r = 2$	0.0221	0.0561	0.0112	0.0151	0.0386	0.0077
0.5	0.25	$r = 1$	0.0785	0.1957	0.0391	0.0504	0.1274	0.0255
		$r = 2$	0.0353	0.0883	0.0177	0.0235	0.0595	0.0119
0.75	0.25	$r = 1$	0.0782	0.1939	0.0388	0.0507	0.1276	0.0255
		$r = 2$	0.0782	0.1939	0.0388	0.0507	0.1276	0.0255
1	0.25	$r = 1$	0.0910	0.2055	0.0411	0.0588	0.1348	0.0270
		$r = 2$	1.6162	3.8573	0.7715	1.3282	3.2157	0.6432

where $f\omega_y$, $f\omega_{\pi,t_t}$ and $f\omega_a$ are the responses of the worst-case shock to the output gap, inflation and the lagged exchange rate. Table B-1 shows the optimal response of the evil nature for κ=0.1 and κ= 0.2.

The worst-case shock responses follow a similar pattern as the interest rate response since the evil nature and the policy maker are playing a zero-sum game. Hence, the monetary authority uses the interest rate to attempt to offset the actions of the evil nature. The latter tries to hit the policy maker as much as possible and it is bounded by the degree of uncertainty and the transition probabilities. Therefore, the same rationale exposed in the text for the interest rate applies for the worst-case shock of the evil nature.

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