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An Economic Model of Oil Exploration and Extraction

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Abstract In this paper we present empirical facts on oil exploitation and a model that can replicate some of these facts. In particular, we show that the time path of the oil price, on the one hand, and the extraction rate, on the other hand, seem to follow a U-shaped and an inverted U-shaped relationship, respectively, which is confirmed by simple non-parametric estimations. Next, we present a theoretical model where a monopolistic resource owner maximizes inter-temporal profits from exploiting a non-renewable resource where the price of the resource depends on the extraction rate and on cumulated past extraction. The resource is finite and only a part of the resource is known while the rest has not yet been discovered. The analysis of that model demonstrates that the extraction rate and the price of the resource show the empirically observed pattern if the stock of the initially known resource is small.

Keywords Non-renewable resources · Optimal control · Non-parametric estimation · Oil production

JEL Classification Q30 · C61

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1 Introduction

Already more than 70 years ago Hotelling (1931) analyzed the problem of how a non-renewable resource should be optimally exploited. This is indeed an important issue from an economic point of view because non-renewable resources, such as oil, gas or ore, are important factors of production and exhaustible. Even if recycling can be used and allows to reuse a certain part of the resources, this does not change the fact that the major part of non-renewable resources is lost in the production process. With a limited amount of resources available, this implies that most of the resources used today are not available in the future.

Although Hotelling presented his model in the 30's of the last century, it was only in the 70's of the last century that his contribution received the attention it deserved. This is not too surprising since it was only in the early 70's that people became aware of the limited availability of natural resources, in particular of oil. Thus, the Club of Rome argued that the process of economic growth cannot continue given a finite amount of natural resources (cf. Meadows 1972) and the two oil crises in the 1970's lead to a drastic rise of the price for that resources. Therefore, economists spent great efforts in finding the optimal rate at which a non-renewable resource should be exploited (see e.g.Dasgupta and Heal 1979 or Conrad and Clark 1987).

The basic model presented by Hotelling assumes that the market for the exhaustible resource is perfectly competitive. A representative supplier of the resource solves an inter-temporal optimization problem where the problem is to find the optimal rate of extraction given the price trajectory. The solution to that problem, which is equivalent to the social optimum, shows that the price of the resource grows at the interest rate that is used to discount profits. This rule is the so-called Hotelling's rule which is at the heart of the economics of non-renewable resources. While the price of the resource grows, the extraction rate monotonically declines over time.

The basic model presented by Hotelling can be varied in several directions. One obvious extension is to assume that the extraction of resources incurs costs. In this case, the basics of the Hotelling rule dose not change. Hence, in optimum the net price, i.e. price minus marginal cost of extraction, rises at the interest rate. If costs are taken into account, the time path of costs can be crucial as to the price path of the resource. If, for example, marginal costs decline over time due to technical progress, it may well be that the price of the resource first declines before it rises again (see e.g. Khanna 2003). A fact, that seems to hold for some resources such as oil for example, as the next section will show. Another possibility to reconcile theory with first declining prices that rise at a later stage is to assume that exploratory efforts build up a stock of reserves that is used to satisfy demand for that resource, as suggested by Pindyck (1978). Then, a U-shaped price path results if the initial reserves are sufficiently small to induce falling prices in the early periods and reserves must rise, requiring exploration to exceed production over an interval of time. Another model that predicts a U-shaped price path is the one by Liu and Sutinen (1982) where benefits of consuming the resource rise with an increase in the extracted resource stock and exploration costs increase with an increase in cumulative exploration. In that model the price path may display a U-shaped form, too, given some technical conditions.

An important variation of the basic Hotelling model is to assume that the supplier of the resource can control the price at least to a certain degree. Thus, the 70's of the last century demonstrated that the assumption of perfect competition does not necessarily hold for the oil market. Hence, another variation of the Hotelling model consists in assuming that the market structure is given by a monopoly where the monopolist again maximizes inter-temporal profits. The result to this optimization problem gives the modified Hotelling's rule, stating that the net marginal revenue must rise at a rate equal to the interest rate. In addition, it can be shown that, with a monopolistic market structure, the resource is exhausted at a later point in time compared to the case of perfect competition. This is due to the monopolist offering a lower amount of the resource at a higher price compared to suppliers under perfect competition.

With the rising growth rates of some Asian economies in the last few years, in particular China and India, the demand for some resources has drastically increased, particularly for oil. Therefore, the question of how the price of oil and the production of that resource have evolved has again become a matter of interest both for policy makers as well as for economists. In this paper we intend to contribute to that line of research. Thus, we present some facts as concerns the evolution of the oil price and as concerns the extraction of deposits and we present a theoretical model that is able to replicate some of these facts.

The rest of the paper is organized as follows. The next section presents facts on the oil price and on oil exploration and a simple non-parametric estimation of the evolution of the oil price and of U.S. oil production. Section 3 presents a theoretical model that can replicate some of the empirical facts. In contrast to Pindyck (1978) and Liu and Sutinen (1982) we consider a monopolistic resource owner who knows only a certain part of the total stock of resource but discovers at each point of time new stocks that add to the known stock of the resource. Section 4, finally, concludes.

2 Some Facts on the Oil Price and Oil Exploration

In this section we summarize some important facts about the production of oil over the last century and about the oil price. The data are taken from BP (2006) and from the Energy Information Administration (2007). We should like to point out that we focus on the long-run evolution of the oil price and oil extraction. Hence, aspects such as speculation and surprisingly strong demand perspectives that are important in the short-run are likely to play a minor role. Further, the strong demand from newly industrializing countries may bring us into a regime where scarcity of oil will become more and more important as regards the determination of its price (cf. Hamilton 2008). The latter holds even if technical progress reduces dependence on natural resources to a certain degree, as pointed out by Krautkraemer (2005).

Figure 1 shows the price for oil over the last 140 years. The figure clearly demonstrates that both the nominal and the real price first declined before they began to rise. In order to get an idea about the data generating process behind these data, as a function of time, we estimate the relationship between the oil price as dependent variable and time as the independent variable. We take the data in levels, i.e. we do not use first differences of the oil price, and we estimate the relationship in a non-parametrical way.

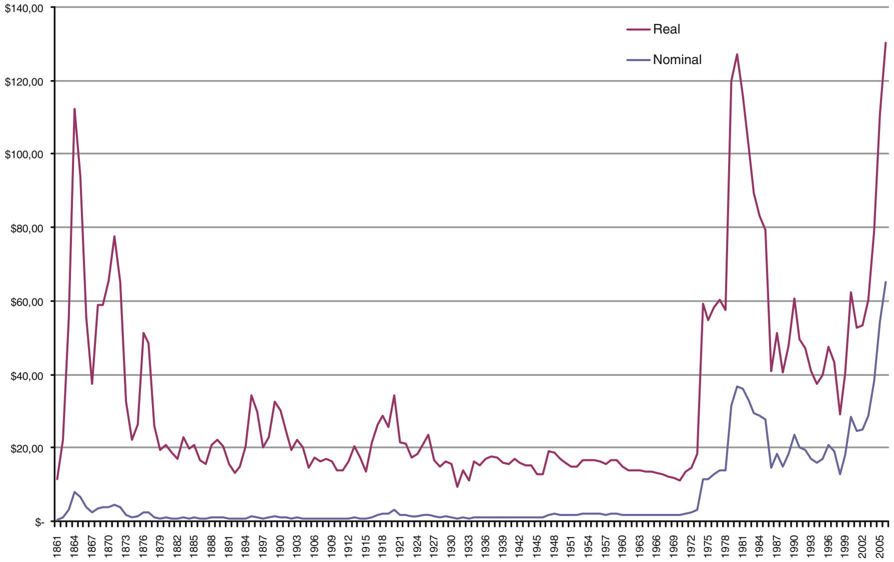

Fig. 1 Nominal and real oil price 1861–2005.

Performing a simple p-spline estimation confirms the conjecture of a first decreasing and then increasing time path of the real oil price. The detailed result of the estimation is given in Table 1 in appendix A.[1] Figure 2 shows the estimated function with the smoothing parameter set equal to 0.01.[2] The dotted lines give the 95% significance interval and the function is such that its average value is equal to zero.

Since oil reserves are finite oil production will decline sooner or later. The Peak-Oil theory, developed by Hubbert (1956), states that oil production first rises and then declines, implying that oil production typically follows a bell-shaped curve. Current estimates of the peak in oil production posit that this point has already been reached in most oil producing countries (see e.g.Schindler and Zittel 2008).

This does not mean that in some regions oil production cannot remain at the current high level or even be increased, but most countries definitely reached their maximal oil production already long ago. In the US, for example, the peak in oil production was reached in 1971, as shown in Fig. 3, where production first rises and then declines.

Performing p-spline estimation gives the estimated curve as a function of time.[3] Figure 4 shows that the increase in oil production up to 1971 is followed by a decline, except for the first half of the 1980's where oil production temporarily rose.

[1] All estimations were done with the package mgcv, version 1.3–23, in R, version 2.5.0, that can be downloaded from http://www.r-project.org/. For a short introduction into p-spline estimation see Greiner (2009) and a more thorough treatment can be found in Ruppert et al. (2003).

[2] Selecting the smoothing parameter data driven by resorting to the generalized cross validation criterion would give a value of 8.7×10^{-5} and a more wiggly function without altering the initial decrease and final increase of the estimated function.

[3] For this estimation, the smoothing parameter is selected by applying the GCV criterion. The estimation output is in Table 2 in appendix A.

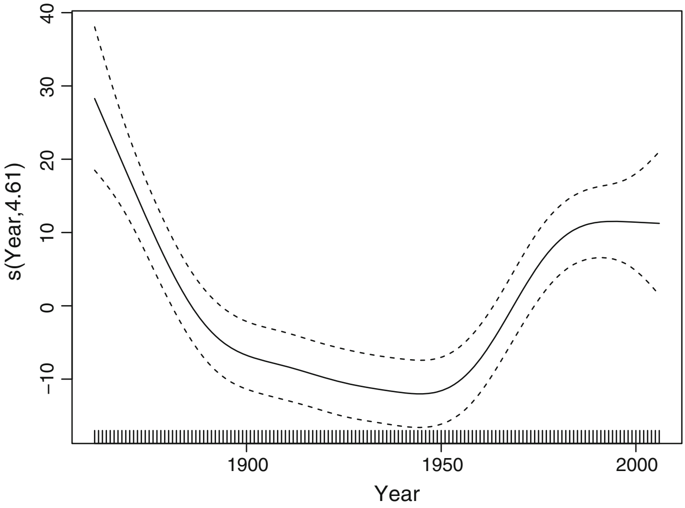

Fig. 2 Estimation of the (real) oil price as a function of time.

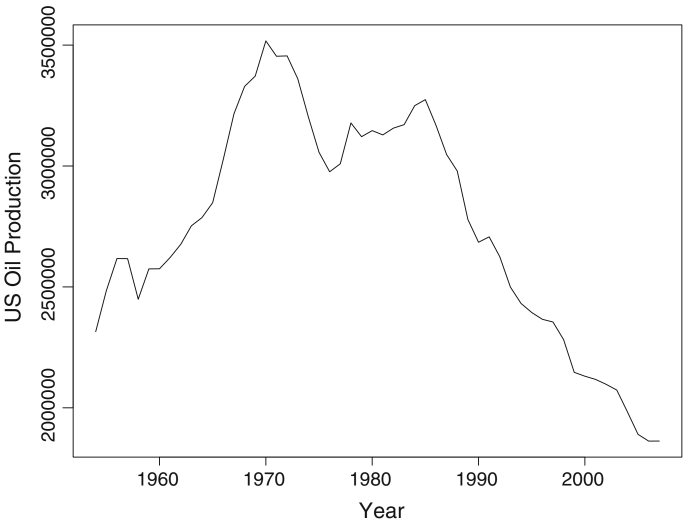

Fig. 3 Annual US oil production from 1954–2007 (in Thousand Barrels).

3 The Model

The next issue is whether we can replicate the above mentioned facts by an economic model. To do so we present a model where we assume that the total stock of the resource consists of a certain part that is known and of a certain part that is unknown up to time t but can be discovered and, then, becomes known.

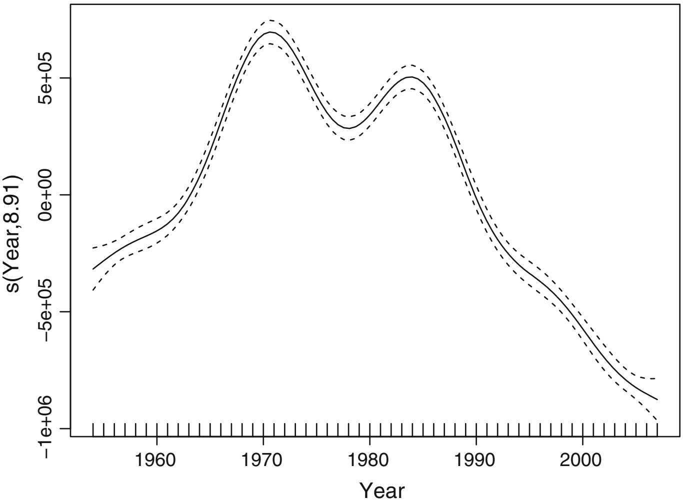

Fig. 4 Estimation of the extraction rate as a function of time.

3.1 The Analytical Model

Assume that the goal is to optimally exploit a non-renewable resource $x(t)$, where t is the time argument. At time t, a certain part of the resource has already been discovered and is known, with $x^k(t)$ denoting the stock of the resource which is known. The rest of the total amount of the resource has not yet been discovered and we denote by $x^n(t)$ the stock of the resource which is not known. Thus, we have $x(t) = x^k(t) + x^n(t)$.

At each point in time a certain part of the hidden resource is discovered with a certain rate $f \geq 0$. We assume that the rate at which the resource is discovered is a function which positively depends on the amount of the resource not yet discovered,[4] i.e. $f = f(x^n)$, with $f'(x^n) > 0$. This is certainly reasonable because the larger the amount of the resource not yet discovered, the easier it is to find new deposits. The more the resource is exploited the more difficult it becomes to detect new deposits.

The stock of the known resource is exploited and declines at the rate u which gives the amount of exploitation at each point in time. At the same time, it also rises because at each point in time a certain part of the resource is discovered and raises the stock of the known resource. The differential equation describing the time path of the known resource is written as,

$$\dot{x}^k = -u + f(x^n) = -u + f(x - x^k), \tag{1}$$

where we have used $x^n = x - x^k$.

It should be noted that the stock of the unknown resource declines at the rate at which new deposits are discovered. Thus, the evolution of the stock of the unknown resource is given by,

[4] In the following we delete the time argument if no ambiguity arises.

$$\dot{x}^n = -f(x^n). \tag{2}$$

The total stock of the resource evolves according to

$$\dot{x} = \dot{x}^k + \dot{x}^n = -u. \tag{3}$$

Thus, in the long-run, i.e. for $t \to \infty, u = 0$ must hold because the resource is non-renewable and because the stock is finite.

Using Eq. (3) we can rewrite (1) as follows,

$$\dot{x}^k = -u + f\left(x_o - \int_{-\infty}^{t} u(\nu)d\nu - x^k\right) = -u + f\left(x_o - y - x^k\right), \tag{4}$$

with y cumulated past extraction and x_o the initially totally available resource that must exceed past extraction plus the stock of the known resource, i.e.

$$y(t) = \int_{-\infty}^{t} u(\nu)d\nu, \;\; x_o \geq y + x^k.$$

As to the optimization problem we assume that a monopolistic firm wants to maximize the discounted stream of profits resulting from exploiting the stock of the known resource. The demand curve is given by $p(u, y) > 0$ and is a strictly negative function of the amount of resources supplied at each point in time, u, and positively depends on cumulated past exploitation, y. With respect to u the demand curve is characterized by the usual assumptions, $p_u(\cdot) < 0$ and $p_{uu}(\cdot) < -2p_{uu}(\cdot)/u$. The latter assumption implies that the marginal revenue is a declining function of u. In addition, we posit that the price remains finite, $p(0, y) < \infty$.

As regards the derivative of the demand function with respect to cumulated past extraction, y, we posit $p_y(\cdot) \geq 0$. If the inequality sign is strict the price for the resource is the higher the more of the resource has already been extracted. This assumption is reasonable because, given the finiteness of the resource, the price will be higher when less of the resource is left. If the level of remaining reserves is declining the economic agents will expect declining supply in the future and thus will expect the price to be rising. Through the future markets oil will be bought in order to trade barrels of oil when the oil is in short supply leading to higher demand and a higher price.

The firm incurs costs resulting from exploiting the resource and we posit that it is the more expensive to exploit the resource the smaller the actual stock of the resource is. This holds because at first those deposits are exploited which are less costly. When more and more of the resource has been exploited those deposits have to be exploited which are less accessible and the exploitation of which is associated with higher costs. Thus, it becomes both more difficult to find new deposits as well as more expensive to extract the resource when a large fraction of the resource has already been discovered and exploited.

Costs of extracting the resource are given by $u\,C(x_o - y)$, with $0 \geq C_{(x_o-y)} > -\infty$. This implies that marginal costs are positive, as usual, and that costs are the smaller, the less the resource has been exploited. The latter assumption is intuitively plausible because extraction of resources starts with those deposits which can be exploited less costly. It should be noted that all of the known resource will be extracted if demand for $u = 0$ exceeds extraction costs, i.e. $p(0, y) \geq C(x_z^k) \geq C(x_o - y)$, with $x_z^k \geq 0$ that value of the known resource below which no new deposits are discovered. If this does not hold the resource may not be completely extracted because extraction costs become too high.

Denoting by $r > 0$ the constant discount rate, the optimization problem is formulated as

$$\max_u \int_0^\infty e^{-rt}\,(p(u, y) - C(x_o - y))\,u\,dt, \tag{5}$$

subject to

$$\dot{x}^k = -u + f\left(x_o - y - x^k\right), \qquad x^k(0) > 0 \tag{6}$$

$$\dot{y} = u, \qquad y(0) \geq 0 \tag{7}$$

$$\lim_{t\to\infty} x^k \geq 0 \tag{8}$$

To get insight into the optimal solution, we formulate the current-value Hamiltonian $H(\cdot)$ which is written as

$$H = (p(u, y) - C(x_o - y))u + \lambda_1(f(x_o - y - x^k) - u) + \lambda_2\,u \tag{9}$$

with λ_1 and λ_2 denoting costate variables or shadow prices of x^k and y.

Assuming that the demand for the resource is sufficiently high such that an interior solution exists, the necessary optimality conditions are obtained as

$$\frac{\partial H}{\partial u} = \lambda_2 - \lambda_1 + up_u(\cdot) + p(\cdot) - C(\cdot) = 0 \tag{10}$$

$$\dot{\lambda}_1 = r\lambda_1 + \lambda_1 f'(\cdot) \tag{11}$$

$$\dot{\lambda}_2 = r\lambda_2 + \lambda_1 f'(\cdot) - u\,C_{(x_o-y)}(\cdot) - u\,p_y(\cdot) \tag{12}$$

Equation (11) shows that the shadow price of the resource grows at a rate larger than r since $f'(\cdot) > 0$ holds. For example, if $f(\cdot)$ is linear, λ_1 will grow at the rate $r+\xi$, with $\xi > 0$ the slope of the function $f(\cdot)$. Hence, comparing our model to the outcome of the standard optimal control problem, without extraction costs, of a monopolistic resource owner demonstrates that the shadow price in our model rises faster. The reason is that in our model a certain part of the unknown resource is discovered at each point in time which makes the shadow price of the known resource increase faster. It should be mentioned that the shadow price of the resource in our model also grows at a higher rate than in the standard model with perfect competition where the shadow price increases at the rate r at which the market price of the resource grows, too. The latter rule is the well-known Hotelling's rule which characterizes the optimal extraction of a non-renewable resource. Thus, Hotelling's rule does not hold in our model implying that the extraction of the resource is not optimal.

In the following we further specify the function underlying our model. As concerns the demand function, we assume that it is given by

$$p(u, y) = \left(\frac{1}{\gamma + \eta u - \mu y}\right)^{\alpha}, \ \alpha > 0, \ \gamma > 0, \ \eta > 0, \ \mu \geq 0 \tag{13}$$

and the cost function is

$$C(x_o - y) = (\phi/2)\,(x_o - y)^{-2}, \ \phi > 0 \tag{14}$$

The rate at which new deposits of the resource are discovered is linear in its argument,

$$f(x_o - y - x^k) = \xi\,(x_o - y - x^k - x_z^k), \ \xi > 0, \ x_z^k \geq 0 \tag{15}$$

For x_z^k sufficiently large so that $p(0, y) \geq C(x_z^k) \geq C(x_o - y)$ holds,[5] the known resource is completely exploited such that $(x^k)^{\star} = 0$. The steady state value for y, then, is given by $y^{\star} = x_o - x_z^k$ and $u^{\star} = 0$.

In order to gain insight into the transitional dynamics of our model we resort to numerical simulations in the next section.

3.2 Numerical Results

Next we use the dynamic programming method by Grüne (1997) and applied by Grüne and Semmler (2004) to study the dynamics of the model with oil extraction. The dynamic programming method can explore the local and global dynamics by using a coarse grid for a larger region and then employing grid refinement for smaller regions. Since it does not use first or second order Taylor approximations to solve for the local dynamics, dynamic programming can provide one with the truly global dynamics in a larger region of the state space.[6] The algorithm is explained in detail in Grüne and Semmler (2004).

Using this algorithm we solve the model of Sect. 3.1 where we set the following parameter values: $\alpha = 2$, $\xi = 0.5$, $r = 0.03$, $\gamma = 0.05$, $\phi = 4$, $\eta = 4$ and $\mu = 0.05$. Moreover, we set $x_o = 6$ and $x_z^k = 3$.

Figure 5 shows the two optimal trajectories in the state space of known, x^k, and already exploited, y, resources. The two trajectories correspond to different initial conditions for the known oil resource, $x^k(0)$. The lower trajectories represents the

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[5] This holds for $x_z^k \geq \gamma^{\alpha/2}\sqrt{\phi/2}$. With (14) the inequality must be strict for $C_{(x_o - y)} > -\infty$.

[6] For details on the analysis of why the dynamic programming algorithm is globally significantly more accurate than algorithms using the second order approximations see Becker et al. (2007).

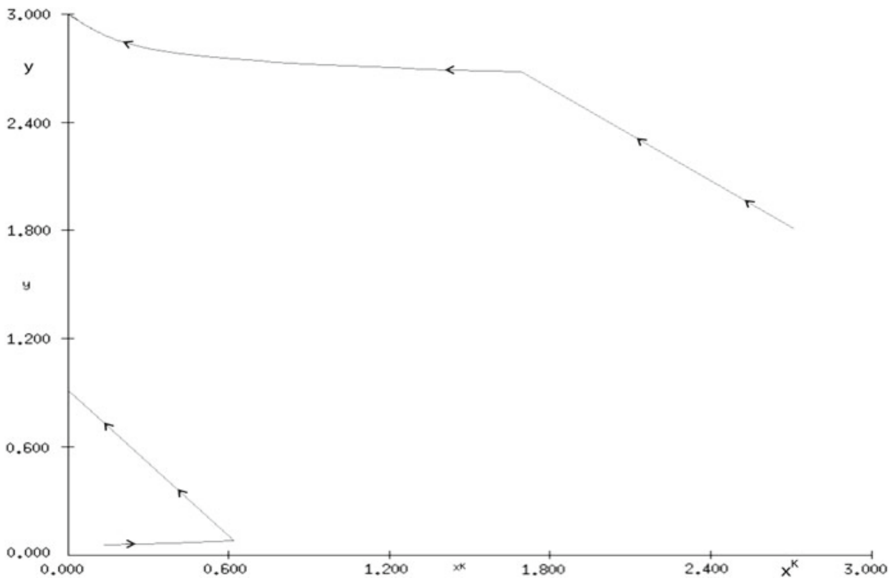

Fig. 5 Optimal trajectories of x^k and y for different initial conditions.

optimal extraction path for mostly unknown resources, whereas the upper trajectory shows the path of the oil resource when most of the oil resource is known.

The implied time path for the optimal extraction rate of the upper trajectory depicted in Fig. 5 implies a monotonically declining time path of the exploitation rate u and a monotonically increasing price for oil p.

The more interesting case is the path of the optimal extraction rate u and the corresponding price p when the initial stock of the known resource is small. Then, the optimal extraction rate is hump-shaped, first increasing then decreasing, and the price movement due to the eventually exhausted resource first falls and then rapidly rises, as shown in Figs. 6 and 7.

The observation that in the case of the upper trajectory of Fig. 5 the price will monotonically increase is a very plausible scenario since the total stock of the resource is overwhelmingly known and does not have to be discovered. Thus, the function $f(\cdot)$ in Eq. (1) is not positive or only slightly positive, and the known resource does not rise. Therefore the extraction rate is not likely to rise, but rather to fall. In the case of the lower trajectory the oil resource is not known. It has to be discovered and its discovery adds to the total oil resources which increases first and then decreases. Hereby the optimal extraction rate will rise first and then fall. It is the latter effect that produces the U-shaped price movement as we can in fact also observe in the real data.

4 Conclusions

In this paper we have presented facts on oil exploitation as well as a theoretical model that can replicate some of these facts. The model we presented consisted of a monopolistic owner of the resource who knows only a certain part of the total stock of the

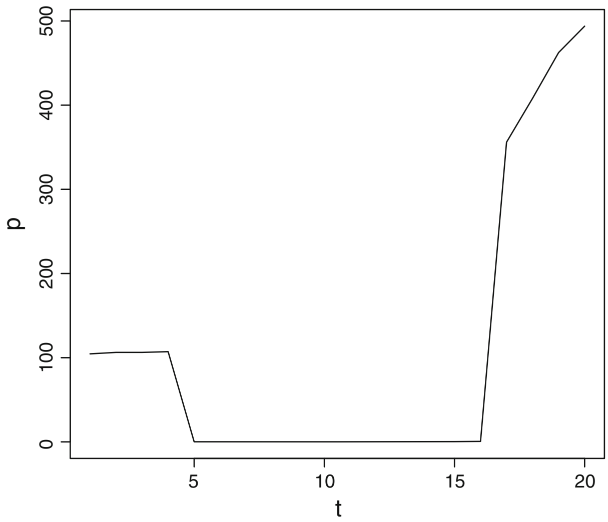

Fig. 6 U-shaped price movement.

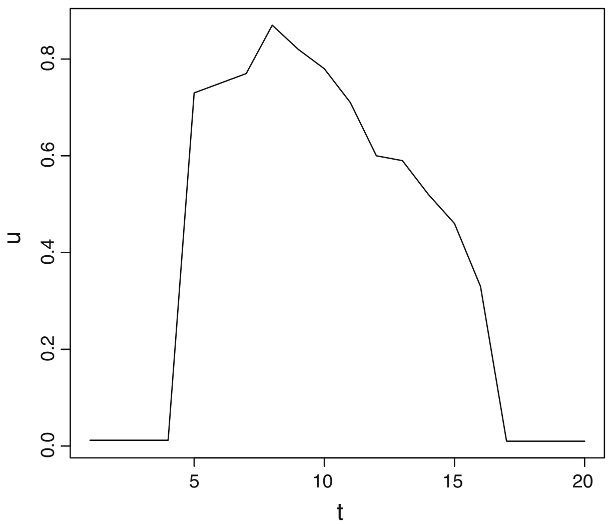

Fig. 7 Optimal extraction rate.

resource and who discovers new reserves at a certain rate. Assuming that the price of the resource depends on the current extraction rate and on cumulated extraction, we could show that the optimal oil extraction rate may follow an inverted U-shaped time path. The price of the resource in that case first declines and, then, rises again. A prerequisite for that outcome is that the initial stock of the known resource is small. If the stock of resources initially known is large, the extraction rate and the price of the resource follow monotonically declining and rising time paths, respectively.

Appendix A: Estimation Results

See Tables 1 and 2.

Table 1 Estimation results producing Fig. 2

	Parametric coefficients:		
	Estimate	Stand. error (t-stat)	P-value
(Intercept)	26.101	1.205 (21.65)	$< 2.2 \times 10^{-16}$
	Approximate significance of smooth terms:		
	edf	F	P-value
s(Year)	4.607	13.21	3.82×10^{-15}
	R^2(adj) = 0.385	n = 146	

Table 2 Estimation results producing Fig. 4

	Parametric coefficients:		
	Estimate	Stand. error (t-stat)	P-value
(Intercept)	$2.7 \cdot 10^6$	8704 (315.3)	$< 2 \times 10^{-16}$
	Approximate significance of smooth terms:		
	edf	F	P-value
s(Year)	8.908	305.9	2×10^{-16}
	R^2(adj) = 0.981	n = 54	

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