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Impacts of Interval Computing on Stock Market Variability Forecasting

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Abstract This study uses the interval computing approach to forecast the annual and quarterly variability of the stock market. We find that the forecasting accuracy is significantly higher than the OLS lower and upper bound forecasting. The strength of the interval computing comes from its data processing. It uses lower and upper bound information simultaneously, no variability information is lost in parameter estimation. The quarterly interval (variability) forecasts suggest that the interval computing method outperforms the OLS lower and upper bound forecasting in both stable and volatile periods.

Keywords Interval forecast · Interval computing · The OLS lower and upper bound forecasting · Accuracy ratio

JEL Classification C53 · C82

1 Introduction

Interval forecasting has been the subject of active research over the past two decades as the traditional point forecasting fails to enhance accuracy. Despite numerous statistical methods, such as Bayesian, Bootstrapping, Box-Jenkins, GARCH, and Holt-Winters methods, used in generating interval forecasts, essentially, those interval forecasts are confidence-based interval forecasts, the combination of point forecasts and some

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variance of forecasts. That is, some arbitrarily determined percent of variance of forecasts is added to and subtracted from the point forecasts to form upper and lower forecast bounds. Given the fact that confidence-based interval forecasts are based on point data and point forecasts, poor forecast quality is a major problem for confidence-based interval forecasts, like for point forecasts (Chatfield 1993, 2001). According to Gardner (1988) and Granger (1996), the confidence-based interval forecasts are too narrow to have realistic coverage, because the uncertainty related to model selection and parameter estimation is not accounted for (De Gooijer and Hyndman 2006). Armstrong (2001) emphases that it is important to incorporate the uncertainty associated with the explanatory variables in interval forecasting. He and Hu (2007) find that indeed, the use of interval data for both dependent and explanatory variables can substantially increase accuracy of interval forecasts. When random variables are measured in intervals, more information (variability) is furnished for analysis, compared with point data. Therefore, the lower and upper bound forecasts based on the interval data and the rolling OLS method significantly outperform confidence-based interval forecasts which are generated from point data and point forecasts (He and Hu 2007). Given the fact that the OLS method uses the lower and upper bound information separately in parameter estimation and generating out-of-sample forecasts, it is inevitably to lose some volatility information in the estimating process.

In order to use the full variability information, the parameter estimating must deal with the lower and upper bounds simultaneously. In the mathematical and computing fields, a new method, interval computing, has been both theoretically and computationally developed since the mid 1960s (Moore 1966, 1979). In interval computing, both operands and computational results are intervals. By taking interval data (both lower and upper bounds) into considerations, it is reasonable to expect more reliable and accurate forecasts with interval arithmetic. The interval computing method has been effectively applied to many fields. Examples include, but not limited to, chemical engineering (Gau and Stadtherr 2000; Hua et al. 1996), reliable non-linear global optimization (Kearfott 1996; Hu et al. 2002; Kearfott and Hongthong 2005), data mining, decision making and game theory (Korvin et al. 2002). A detailed discussion of interval computing is contained in Sect. 3.1.

The interval computing creates a new forecasting format, variability forecasting, and may enhance forecasting accuracy to a higher level. The major purpose of this study is twofold. First, we employ the interval computing method to predict variability of the stock market. Then, the interval forecasts are compared with those based on the OLS rolling method reported in He and Hu (2007), in order to assess forecasting quality provided by the interval computing.

The remainder of this paper is organized as follows. Section 2 describes the estimation model and data, Sect. 3 discusses interval computing procedures, Sect. 4 presents empirical results, and Sect. 5 contains concluding comments.

2 Model and Data

This study uses the same interval data and models as He and Hu (2007). According to Chen et al. (1986), changes in the stock market (SP_t) are linearly determined by the following five macroeconomic factors: growth rate variations of seasonally adjusted Industrial Production Index (*IP*), changes in expected inflation (DEI_t) and unexpected inflation (UI_t), default risk premiums (DEF_t), and unexpected changes in interest rates ($TERM_t$):

$$SP_t = a_t + I_t(IP_t) + U_t(UI_t) + D_t(DEI_t) + F_t(DEF_t) + T_t(TERM_t) + e_t \quad (1)$$

Similar to Fama and French (1997) out-of-sample forecasting, the rolling coefficient estimates from Eq. 1 are multiplied by next period's explanatory variables to get out-of-sample forecasts for the stock market:

$$SP_t = a_{t-1} + I_{t-1}(IP_t) + U_{t-1}(UI_t) + D_{t-1}(DEI_t) + F_{t-1}(DEF_t) \\ + T_{t-1}(TERM_t). \quad (2)$$

The primary data set covers a period of January 1930-December 2004 and includes the following basic monthly series:

- *IP*: the growth rate of seasonally adjusted Industrial Production Index at the beginning of the month. The index is compiled by the Federal Reserve System. This study uses the one-month lead term of IP in the monthly data.
- *LONG*: the monthly returns on long-term U.S. government bonds (Stock, Bonds, Bills, and Inflation 2005 Yearbook, Ibbotson Associates).
- *CORP*: the monthly returns on long-term corporate bonds (Stock, Bonds, Bills, and Inflation 2005 Yearbook, Ibbotson Associates).
- *SHORT*: the monthly returns on one-month U.S. Treasury Bills (Stock, Bonds, Bills, and Inflation 2005 Yearbook, Ibbotson Associates). According to Fama and French (1993), SHORT_{t-1} is the proxy for "the general level of expected returns on bonds."
- *CPI*: the growth rate of the U.S. Consumer Price Index (Bureau of Labor Statistics).
- SP: the growth rate of Standard and Poor's 500 Stock Price Index.

For IP, CPI, and SP, the monthly growth rate is the first difference of natural logarithms in months of t and $t-1$; the annual growth rate is the first difference of natural logarithms in Decembers of t and $t-1$. The annual returns for LONG, CORP, and SHORT are compounded monthly returns of January through December. The following additional series are derived from the above basic series:

$DEF_t = CORP_t - LONG_t$. It represents the default risk premium (Fama and French 1993).
$TERM_t = LONG_t - SHORT_{t-1}$. It measures unexpected changes in interest rates (Fama and French 1993).
UI_t: unexpected inflation. It is proxied by the residuals from the following regression model (Fama 1981):
$CPI_t = \alpha_{t-1} + \beta SHORT_{t-1} + \eta_t$
EI_{t-1}: expected inflation at the end of month $t-1$. It is the difference of CPI_t and UI_t.
DEI_t: the change in expected inflation. It measures the difference of EI_t and EI_{t-1}.

The annual and quarterly interval data consist of the minimum and maximum monthly numbers in a year or a quarter. The OLS rolling method applies the lower and upper bound data to Eqs. 1, 2 to generate lower and upper bound forecasts; while the interval computing method directly uses the interval data and the two equations to produce interval forecasts.

3 Interval Algorithm and Estimation Accuracy

3.1 An Interval Least Squares Algorithm

An interval least squares algorithm has been developed and implemented in C++ in this study to estimate Eq. 1. Based on the least squares principle, the interval arithmetic (Moore 1979) is used to construct an interval valued linear system of equations. In this process, the product of two intervals is used mostly. In interval arithmetic, the product of two intervals is defined as [a, b] * [c, d] = [min {ac, ad, bc, bd}, max {ac, ad, bc, bd}].

In order to estimate the coefficients, it is necessary to solve an interval linear systems of equations $\mathbf{M}a = \mathbf{v}$, where $\mathbf{M}$ is a 6 by 6 interval matrix; a is the vector of the coefficients to be determined; and $\mathbf{v}$ is an interval vector. It is assumed that the coefficients are scalars initially. By taking M_{mid}, the mid-point matrix of **M,** and v_{mid}, the midpoint vector of **v**, a classic linear system of equations about a is constructed. The numerical estimates of the coefficients are obtained by using Gaussian elimination with scaled partial pivoting. This initial approach has the intuition of matching the center of two interval vectors. In order to better reflect time-varying relationships between the stock market and other macroeconomic variables, a rolling estimation period of ten consecutive years of annual data or 20 quarters of quarterly data is used to establish the interval linear system of equations for annual or quarterly forecasting. The rolling coefficient estimates in intervals obtained from Eq. 1 can be used to forecast changes of the stock market by estimating the out-of-sample forecasting model (Eq. 2).

However, the midpoint approach has not taken the widths into considerations yet. Therefore, it is essential to adjust the width of the forecasted interval. Consistent with the rolling estimation period, the width is adjusted by a rolling scalar constant which is equivalent to the average width of the S&P index intervals in the previous ten years or 20 quarters.

Hu and He (2007) illustrated step-by-step interval computing procedures with examples of in-sample (1939–2004) and out-of-sample (1940–2004) annual stock market interval forecasts. Today, software tools and applications for interval computing are available in mainstream programming languages such as C, C++, Fortran, Java, Lisp, as well as in computational algebra systems, such as Maple, MATLAB, and Mathematica.

3.2 Assessment of Forecasting Accuracy

The accuracy ratio defined in He and Hu (2007) and Hu and He (2007) is the primary measure of forecasting quality in this study. The accuracy ratio represents the commonly covered range by a forecasted and actual intervals divided by the maximum distance reached by the two intervals. If the actual interval is used as the denominator, the ratio is misleading. Consider the following example in He and Hu (2007): the predicted interval is [1, 5] and the actual interval is [1, 3]. The accuracy ratio should be 50% (2/4). If the actual interval is used as the denominator, the value of the ratio becomes 100%. Obviously, it is misleading. We use SP_{est} to represent the forecasted SP interval and define the *accuracy ratio* as: $\mathrm{w}(\mathrm{SP} \cap \mathrm{SP}_{\mathrm{est}})/\mathrm{w}(\mathrm{SP} \cup \mathrm{SP}_{\mathrm{est}})$, where w is the width function of an interval. The accuracy ratio is zero when $SP \cap SP_{\mathrm{est}}$ is empty.

The accuracy ratio is determined by the distance covered by a predicted and actual intervals as well as the maximum range stretched by the two intervals. Therefore, the shape of coverage is another interesting issue to analyze. We use the following five definitions of interval forecasts to categorize different shapes of coverage.

1. Over-forecast. It is defined as a forecasted interval that covers the whole actual interval, that is, both the lower and upper bounds of the forecasted interval exceed the actual interval. For example, the interval [2, 5] is an over-forecast for the interval [3, 4].
2. Under-forecast. A predicted interval that resides inside of the actual interval is an under-forecast. For example, the predicted interval [2, 4] is an under-forecast, if the actual interval is [1, 5].
3. Lower-forecast. It is a forecasted interval that covers a part of the actual interval and the lower bound of the forecasted interval exceeds the lower bound of the actual interval. For instance, the forecasted interval [0, 3] is called a lower-forecast, if the actual interval remains [1, 5].
4. Upper-forecast. It is opposite to the lower-forecast. The upper bound of a forecasted interval [3, 6] exceeds the upper bound of the actual interval [1, 5].
5. Zero-forecast. It is a forecasted interval with an accuracy ratio of zero. That is, the forecasted interval does not touch the actual interval, such as [1, 2] vs. [4, 5].

The average accuracy ratio and frequency in each category can provide us with detailed information about the strength and weakness of each interval forecasting method.

4 Empirical Results

4.1 Overall Accuracy of Interval Forecasts

Both annual and quarterly data indicate far wider intervals for the S&P stock index than other variables (Table 1). Two inflation measures, changes in expected inflation and unexpected inflation, have the narrowest intervals. Summary statistics in Table 1 suggest that quarterly interval data display larger unit variation measured by coefficient of variation, compared to the annual intervals.

The annual and quarterly interval data are used to generate out-of-sample variability forecasts for the stock market. We employ the interval computing approach to process interval inputs and produce interval forecasts based on Eqs. 1, 2. The results are compared with the OLS interval forecasts which are proven to be superior to point

Table 1 Summary statistics for interval data (in percent)

	SP	IP	UI	DEI	DEF	TERM
Panel A: Annual intervals (1930–2004)						
Upper bound mean	6.61	2.36	0.66	0.08	1.82	3.44
Standard deviation	5.20	2.41	0.86	0.06	1.22	2.70
Coefficient of variation	0.79	1.02	1.30	0.75	0.67	0.78
Lower bound mean	−6.30	−1.80	−0.51	−0.08	−1.66	−3.14
Standard Deviation	5.05	2.18	0.40	0.08	1.21	2.19
Coefficient of variation	−0.80	−1.21	−0.78	−1.00	−0.73	−0.70
Panel B: Quarterly intervals (Q1 1930–4 2004)						
Upper bound mean	3.28	1.12	0.29	0.04	0.94	1.76
Standard deviation	3.91	1.83	0.61	0.05	1.04	2.12
Coefficient of variation	1.19	1.63	2.10	1.25	1.11	1.20
Lower bound mean	−2.46	−0.56	−0.23	−0.04	−0.84	−1.43
Standard deviation	4.32	1.81	0.40	0.06	1.04	1.98
Coefficient of variation	−1.76	−3.23	−1.74	−1.50	−1.24	−1.38

SP = Growth rate in S&P stock index
IP = Growth rate in industrial production index
UI = Unexpected inflation
DEI = Changes in expected inflation
DEF = Default risk premium
TERM = Unexpected changes in interest rates
Coefficient of variation is in decimal numbers

forecast-based confidence interval forecasts by He and Hu (2007). Since the exact same inputs and models are used to make forecasts, differences between the two types of interval forecasts are simply consequences of different methods used in processing interval information. Accuracy ratios for 10-year rolling annual interval forecasts (64%) clearly suggest that the interval computing method is superior to the OLS lower and upper bound estimation (52%). The difference is significant at the one percent level (Table 2). The interval computing method can not only greatly enhance the accuracy of variability forecasting, but also increase the stability of annual rolling forecasting. The standard deviation of accuracy ratios is 16% for the interval computing, while 22% for the OLS. The difference is significant at the five percent level, as suggested by the F-statistic in Table 2. The higher accuracy of the interval computing is also evidenced by the significantly lower mean of forecast errors, 5.17% vs. 6.84%. The quarterly interval forecasts basically verify the annual findings. However, accuracy ratios for both methods (44% vs. 34%) are lower than annual forecasts. The result may reflect the fact that the quarterly interval variables have higher unit variation than annual ones.

A fundamental difference between the two methods lies in information processing. The interval computing method uses both lower and upper bound information simultaneously in estimating Eq. 1 and forming forecasts; while the OLS method separately uses lower and upper bound information in the process. The separation of lower and upper bounds in information processing deprives much variability information. This

Table 2 Out-of-sample forecasts based on interval computing and OLS lower and upper bound estimates

	Interval computing	OLS interval[a]
Panel A: Annual forecasts (1940–2004) with a 10-year rolling period		
Forecasting accuracy (FA)	0.64188	0.52449
Std. dev. of FA	0.16149	0.22040
Equality tests for the two pairs		*Test result*
T-statistic		3.46[b]
F-statistic		1.86[c]
Forecast error mean (IE)	0.05166	0.06843
Std. dev. of IE	0.03224	0.03727
Equality tests for the two pairs		*Test result*
T-statistic		−2.74[b]
F-statistic		1.34
Panel B: Quarterly forecasts (Q1,1935–Q4,2004) with a 20-quarter rolling period		
Forecasting accuracy (FA)	0.44063	0.34196
Std. dev. of FA	0.22301	0.25879
Equality tests for the two pairs		*Test result*
T-statistic		4.83[b]
F-statistic		1.35[c]
Forecast error mean (IE)	0.04659	0.05483
Std. dev. of IE	0.03470	0.03832
Equality tests for the two pairs		*Test result*
T-statistic		−2.67[b]
F-statistic		1.22[d]

Forecast error = Sum of absolute values of the differences between the forecasted and actual lower and upper bounds

FA = The range covered by both the forecasted and actual intervals divided by the maximum range stretched by the two intervals

[a] There are four annual and 14 quarterly OLS interval forecasts are mis-specified, that is, predicted upper bounds are smaller than predicted lower bounds. However, both FA and Forecast error for OLS interval are based on corrected mis-specified predicted bounds. We modified the computing procedure, so FA and Forecast error mean (IE) for OLS interval are slightly different from those reported in He and Hu (2007)

The T-statistic tests the null hypothesis of equality of means for accuracy ratios without the assumption of equal population variance

The F-statistic tests the null hypothesis of equality of variances. The 1%, 5%, and 10% significant levels are represented by b, c, and d, respectively

may be the reason why the OLS interval forecasts are less accurate than those generated by the interval computing. The separation of interval information also determines that the OLS lower and upper bound forecasts are independent of each other. As a result, the OLS interval forecasts can be mis-specified, that is, the predicted upper bound is lower than the predicted lower bound in an interval forecast. The mis-specified OLS interval forecasts are found in this study: four annual forecasts and 14 quarterly forecasts (Table 2). To correct this misspecification, we consider the lower predicted value as the lower bound and the higher value as the upper bound. The calculation of accuracy ratio and forecast error is based on corrected forecasts.

4.2 Accuracy of Different Categories of Forecasts

In order to further examine the effectiveness of interval forecasting, we classify all interval forecasts into five different categories. Table 3 provides detailed information about each category for annual forecasts. Forecasts from the interval computing are more likely to exceed the actual lower and upper bounds of S&P intervals. The frequency of this over-forecast (Over) is 25 out of a total of 65. Interval computing forecasts do not have a single zero accuracy ratio, that is, all interval computing forecasts touch the actual intervals. This is a major reason why the interval computing method enjoys a higher accurate level. Frequencies for other three categories are similar. Contrast to the interval computing approach, the OLS lower and upper bound prediction method produces 18 over-forecasts and four non-touch forecasts. The fre-

Table 3 Annual accuracy ratios and accumulative accuracy ratios in different categories

	Over	Under	Lower	Upper	Zero
Frequency in					
Interval computing	25	14	13	13	0
OLS interval	18	13	16	14	4
Average accuracy ratio in					
Interval computing	0.5800	0.6818	0.6917	0.6680	0
OLS interval	0.5421	0.6041	0.6073	0.4831	0
Standard deviation of accuracy ratios in					
Interval computing	0.1757	0.1837	0.1382	0.0918	0
OLS interval	0.1856	0.1952	0.1598	0.1667	0
Equality tests for Interval computing versus OLS interval					
T-statistic	0.68	1.06	1.52	3.60^{a}	
F-statistic	1.12	1.13	1.34	3.29^{b}	
Mean of accumulative accuracy ratios in					
Interval computing	7.5472	5.2618	4.7684	4.7018	0
OLS interval	5.3411	4.1034	5.2178	3.5683	0
Standard deviation of accumulative accuracy ratios					
Interval computing	4.4199	2.7374	2.7524	2.6227	0
OLS interval	2.9981	2.3422	2.9901	1.9170	0
Equality tests for Interval computing vs. OLS interval					
T-statistic	1.95^{c}	1.18	−0.42	1.27	
F-statistic	2.17^{b}	1.37	1.18	1.87	

Over = a forecasted interval that covers the whole actual interval
Under = a forecasted interval that resides inside of the actual interval
Lower = a forecasted interval that exceeds the lower bound of the actual interval
Upper = a forecasted interval that exceeds the upper bound of the actual interval
Zero = a forecasted interval that does not touch the actual interval
The T-statistic tests the null hypothesis of equality of means for accuracy ratios without the assumption of equal population variance
The F-statistic tests the null hypothesis of equality of variances. The 1%, 5%, and 10% significant levels are represented by a, b, and c, respectively

quencies of under-forecasts (forecasted intervals residing inside of actual intervals), upper-forecasts (forecasted intervals only exceeding upper bounds of actual intervals), and lower-forecasts (forecasted intervals only exceeding lower bounds of actual intervals) are similar for both methods.

The mean of accuracy ratios in each category is greater for the interval computing method, while standard deviation of accuracy ratios in each category is smaller for the computing, compared with the OLS lower and upper bound forecasts. However, only differences in the category of upper-forecasts, 66.8% vs. 48.3% for accuracy ratio and 9.18% vs. 16.67% for standard deviation, are statistically significant (Table 3).

Accumulative accuracy ratios, a combined measure of frequency and average ratio, in each category may provide a better picture for forecasting quality. Results for the category of over-forecasts are, once again, in favor of the interval computing, in terms of a significantly higher mean of accumulative accuracy ratios and lower standard deviation (Table 3). Results for other categories are not significantly different. Overall, annual interval forecasts suggest that the important strength of the interval computing approach is able to generate more and better over-forecasts, compared with the OLS interval forecasts.

When the higher frequency data, the quarterly interval data, are used, the accuracy of interval forecasts tends to decrease. The interval computing generates 12 non-touch (zero accuracy) quarterly forecasts, while there are as many as 36 for the OLS interval forecasting (Table 4). Furthermore, for both interval forecasting methods, the average accuracy ratios for other four categories are lower than that for annual forecasts.

However, the interval computing method still generates better quarterly forecasts. The significant T-statistics indicate that the under- and lower-forecasts developed by the interval computing enjoy the higher accuracy than those produced by the OLS method. When we take category frequencies into consideration, the interval computing outperforms the OLS method in two categories: over- and lower-forecasts, as suggested by the means of accumulative accuracy ratios, 16.63 vs. 7.32 and 19.58 vs. 16.29 (Table 4). The differences are significant at the one percent and five percent levels, respectively. The result that over-forecasts produced by the interval computing method have higher accumulative accuracy ratios than the OLS over-forecasts is in line with the annual results.

4.3 Volatility and Accuracy of Forecasts

Essentially, the interval forecasting is a variability forecasting. Can interval forecasting do a better job in a more volatile period? Results of this study provide a clear answer to the question. Interval forecasts obtained with both interval computing and the OLS methods have much higher accuracy ratios in 17 years with most volatile stock prices than with least volatile stock prices. The accuracy ratios are 64.56% vs. 54.52% for the interval computing method and 59.50% vs. 40.96% for the OLS method (Table 5). Both differences are statistically significant. However, the accuracy ratio for the interval computing in the volatile period is not significantly higher than that for the OLS method, although the difference of accuracy in the stable period is significant in favor of the interval computing.

Table 4 Quarterly accuracy ratios and accumulative accuracy ratios in different categories

	Over	Under	Lower	Upper	Zero
Frequency in					
Interval computing	75	36	88	69	12
OLS interval	39	47	85	73	36
Average accuracy ratio in					
Interval computing	0.4619	0.5560	0.4502	0.4217	0
OLS interval	0.4139	0.4107	0.3822	0.3811	0
Standard deviation of accuracy ratios in					
Interval computing	0.2169	0.1391	0.2216	0.1944	0
OLS interval	0.2294	0.2713	0.2212	0.2446	0
Equality tests for Interval computing vs. OLS interval					
T-statistic	1.08	3.17[a]	2.02[b]	1.10	
F-statistic	1.12	3.81[a]	1.00	1.58[c]	
Mean of accumulative accuracy ratios in					
Interval computing	16.629	10.322	19.576	14.967	0
OLS interval	7.3234	9.8570	16.291	14.627	0
Standard deviation of accumulative accuracy ratios in					
Interval computing	10.038	5.9144	11.347	8.9615	0
OLS interval	4.6564	6.0774	9.8934	8.4060	0
Equality tests for Interval computing vs. OLS interval					
T-statistic	6.75[a]	0.35	2.03[b]	0.23	
F-statistic	4.65[a]	1.06	1.32	1.14	

Over = a forecasted interval that covers the whole actual interval
Under = a forecasted interval that resides inside of the actual interval
Lower = a forecasted interval that exceeds the lower bound of the actual interval
Upper = a forecasted interval that exceeds the upper bound of the actual interval
Zero = a forecasted interval that does not touch the actual interval
The T-statistic tests the null hypothesis of equality of means for accuracy ratios without the assumption of equal population variance
The F-statistic tests the null hypothesis of equality of variances. The 1%, 5%, and 10% significant levels are represented by a, b, and c, respectively

Frequencies in different categories of forecasts suggest that over-forecasts are more likely to appear in periods with stable stock prices, while more under-forecasts are formed in volatile price situations. We find that in the stable period, the interval computing method produces 16 over-forecasts and zero under-forecasts, compared to one over-forecast and 10 under-forecasts in the volatile period. The OLS interval forecasts show the similar results, but in a less obvious manner.

Compared with the annual interval data, the quarterly data are more unstable, therefore, can better reveal the relationship between volatility and accuracy of interval forecasting. The accuracy of quarterly interval forecasts significantly increases in 70 most volatile quarters (45.19% for the interval computing and 35.06% for the OLS method) from 70 least volatile quarters (28.83% and 20.26%, respectively) (Table 6). Furthermore, in both periods, accuracy ratios are significantly higher (at the one percent level) for forecasts based on the interval computing than that from the OLS method. Finally, similar to the annual interval forecasts, the frequency of over-forecasts is the highest and the frequency of under-forecasts is the lowest for both methods in the stable period; while the opposite is true for the volatile period. This is because of that the interval computing method adjusted the width of the forecasting interval to the average width of SP intervals in the estimation window.

5 Concluding Comments

Results of this study indicate that the interval computing method can generate interval forecasts with a significant higher average accuracy ratio, compared with interval

Table 5 Annual forecasting quality in different periods

	Interval computing	OLS interval			
Panel A: 17 least volatile years (25% of the sample)					
Forecasting accuracy (FA)	0.54523	0.40959			
Std. dev. of FA	0.18018	0.22927			
Equality tests for the two pairs		*Test result*			
T-statistic		1.92[c]			
F-statistic		1.62			
	Over	Under	Lower	Upper	Zero
Frequency in					
Interval computing	16	0	1	0	0
OLS interval	10	0	3	2	2
	Interval computing	OLS interval			
Panel B: 17 most volatile years (25% of the sample)					
Forecasting accuracy (FA)	0.64559	0.59501			
Std. dev. of FA	0.16823	0.12254			
Equality tests for the two pairs		*Test result*			
T-statistic		1.00			
F-statistic		1.88			
	Over	Under	Lower	Upper	Zero
Frequency in					
Interval computing	1	10	1	5	0
OLS interval	1	7	2	7	0
	FA in stable period	FA in volatile period			
Panel C: The stable period vs. the volatile period					
Interval computing	0.54523	0.64559			
Equality tests for the two pairs		*Test result*			
T-statistic		−1.68[c]			
F-statistic		1.15			

Table 5 continued

	FA in stable period	FA in volatile period
OLS Interval	0.40959	0.59501
Equality tests for the two pairs		*Test result*
T-statistic		-2.94^{a}
F-statistic		3.50^{b}

Forecast error = Sum of absolute values of the differences between the forecasted and actual lower and upper bounds
FA = The range covered by both the forecasted and actual intervals divided by the maximum range stretched by the two intervals
The T-statistic tests the null hypothesis of equality of means without the assumption of equal population variance
The F-statistic tests the null hypothesis of equality of variances. The 1%, 5%, and 10% significant levels are represented by a, b, and c, respectively
Over = A forecasted interval that covers the whole actual interval
Under = A forecasted interval that resides inside of the actual interval
Lower = A forecasted interval that exceeds the lower bound of the actual interval
Upper = A forecasted interval that exceeds the upper bound of the actual interval
Zero = A forecasted interval that does not touch the actual interval

Table 6 Quarterly forecasting quality in different periods

	Interval computing	OLS interval			
Panel A: 70 least volatile quarters (25% of the sample)					
Forecasting accuracy (FA)	0.28829	0.20263			
Std. dev. of FA	0.17881	0.20610			
Equality tests for the two pairs		*Test result*			
T-statistic		2.63^{a}			
F-statistic		1.33			
	Over	Under	Lower	Upper	Zero
Frequency in					
Interval computing	45	0	15	7	3
OLS interval	24	2	15	11	18
	Interval computing	OLS interval			
Panel B: 70 most volatile quarters (25% of the sample)					
Forecasting accuracy (FA)	0.45190	0.35060			
Std. dev. of FA	0.19391	0.24468			
Equality tests for the two pairs		*Test result*			
T-statistic		2.71^{a}			
F-statistic		1.59^{c}			
	Over	Under	Lower	Upper	Zero
Frequency in					
Interval computing	0	29	20	18	3
OLS interval	1	28	20	15	6

Table 6 continued

	FA in stable period	FA in volatile period
Panel C: The stable period vs. the volatile period		
Interval computing	0.28829	0.45190
Equality tests for the two pairs		*Test result*
T-statistic		−5.19[a]
F-statistic		1.18
OLS Interval	0.20263	0.35060
Equality tests for the two pairs		*Test result*
T-statistic		−3.87[a]
F-statistic		1.41

Forecast error = Sum of absolute values of the differences between the forecasted and actual lower and upper bounds

FA = The range covered by both the forecasted and actual intervals divided by the maximum range stretched by the two intervals

The T-statistic tests the null hypothesis of equality of means without the assumption of equal population variance

The F-statistic tests the null hypothesis of equality of variances. The 1%, 5%, and 10% significant levels are represented by a, b, and c, respectively

Over = A forecasted interval that covers the whole actual interval

Under = A forecasted interval that resides inside of the actual interval

Lower = A forecasted interval that exceeds the lower bound of the actual interval

Upper = A forecasted interval that exceeds the upper bound of the actual interval

Zero = A forecasted interval that does not touch the actual interval

forecasts derived from the OLS lower and upper bound estimation. The strength of the interval computing comes from its data processing. It uses lower and upper bound information simultaneously, no variability information is lost in parameter estimation.

The annual results for different categories of forecasts suggest that the interval computing approach is able to generate more and better over-forecasts, compared with the OLS interval forecasts. When quarterly data are used, the accuracy of interval forecasting reduces. This may be the result of higher unit variation contained in the quarterly information. Nevertheless, the interval computing still outperforms the OLS method in two categories: over- and lower-forecasts, as suggested by the higher means of accumulated accuracy ratios.

Interval forecasting essentially is a variability forecasting. Our results suggest that interval forecasts generated by either method have higher accuracy ratios in the volatile period than in the stable period. Quarterly forecasts based on the interval computing have significantly higher accuracy than the OLS interval forecasts in both volatile and stable periods. However, annual forecasts from the interval computing outperform the OLS interval forecasts only in the stable period.

Results on different periods suggest that the frequency of over-forecasts is the highest and the frequency of under-forecasts is the lowest for both methods in the stable period; while the opposite is true for the volatile period.

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