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ORIGINAL PAPER

A new modeling approach investigating the diffusion speed of mobile telecommunication services in EU-15

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Abstract The objective of this paper is to investigate the impact of the time-delay effect on the diffusion of mobile telecommunication services in EU. It has been proved from several studies that the time-delay between the awareness and the adoption phase of mobile services-potential users determines the speed of the mobile telecommunication service diffusion and can be used effectively for ranking or cluster purposes in cases when the diffusion of a new product in different countries is studied. The proposed modeling approach originates from the well-known logistic model where it is assumed that the ordinary contagion process does not take place instantly but after some certain amount of time. A proper modification of the proposed model described by a time lag ordinary differential equation can be solved analytically and its properties for several parameters' combination are investigated. Moreover, a new diffusion speed index is proposed and the correlation between the time-delay index and the proposed diffusion speed index is examined. Finally the model is applied to real data concerning the mobile services diffusion in 15 counties of EU from 1990 to 2002. Based on the estimated parameters of the model produced for each country a ranking and a clustering of the EU countries based on their derived diffusion speed and time-delay indexes are provided.

Keywords Time-delay model · Diffusion speed · Innovation diffusion modeling · Technology marketing-management · Modeling telecommunication services

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1 Introduction

Today, forecasting technology in economic activity is no more avoidable than in forecasting weather in daily life. In fact, voluminous literature has explored different growth-curve models in forecasting the diffusion process of new technologies. Early contributions to this subject are attributed to scientists who noted the analogy between the epidemic process and the social adoption process (Griliches, 1957; Mansfield, 1961; Bass, 1969; Fisher & Pry, 1971; Blackman, 1972; Sharif & Kabir, 1976; Sharif and Ramanathan, 1984). They came to a general agreement that the proportion of adopters rises at an accelerating rate during the early stages of the diffusion process and then at a declining rate until the population of potential adopters has been exhausted. Later, economists and technologists joined the field to predict the marketability of new products, trajectory of technology process, penetration rate of the advanced manufacturing technologies (Skiadas, 1985, 1986, 1987; Kumar & Kumar, 1992; Mead & Islam, 1998). Initiated with a simple logistic function, various curves have been empirically derived to investigate the patterns of technological growth process. These curves differ from one another in terms of number of parameters, the point of inflection, the symmetric or non-symmetric shape of their shape, etc. A more general approach for the selection of growth functions as the logistic and other functions proposed is based on the Lagrangian theory (Skiadas, 1995). According to this approach some popular innovation diffusion models as the logistic, exponential, and Gompertz arise as the simplest case from more general forms.

The attempt to improve the fitting and forecasting performance of the proposed models gave rise to deterministic, stochastic, and even chaotic approaches. By introducing a stochastic term in the adoption-diffusion equations an improvement of deterministic models was done. The stochastic modeling approach is very useful to define the upper and lower bounds of an adoption-diffusion trend. Interesting results where obtained in the case of technology diffusion processes, as is that of the electricity consumption (Giovanis & Skiadas, 1999).

The time-delay effect is more difficult to model. Time-delay equations lead to complicated and even chaotic solutions (Skiadas, 2005). However, some simplified forms of time-delay adoption-diffusion equations can be of particular interest. By using these equation forms important characteristics of the diffusion process as the time lag and the speed of diffusion are defined along with the diffusion trend (Skiadas, Rompogiannakis, Apostolou, & Dimoticals, 2005).

This paper is using an earlier qualitative approach (Poznanski, 1983) and a quantitative study (Skiadas, 1986) originating from the logistic innovation diffusion model which incorporates the time-delay between the awareness and the adoption phase during the classical contagion process between the adopters and the potential adopters of a new technology. It has been proven that the time-delay that affects the performance of a new technology launching and speed and it can be used for comparison purposes, in order to study the innovation diffusion among groups of potential adopters with different characteristics. The proposed model can be solved analytically and presents very attractive properties well documented in the field of innovation diffusion representation. Additionally, an expression on the relationship between the time-delay parameter and the diffusion speed is presented. The time-delay innovation diffusion model is applied to the data of mobile telephony services diffusion in EU-15, in order to determine the existence of penetration patterns in relation to the time delay between the awareness and the adoption phase of the potential adopters. Finally, the outcomes are used for a ranking of the investigated countries.

2 A model expressing the time-delay of adoption-diffusion process

The diffusion of an innovation in a stable and homogeneous system with no external influence is traditionally expected to follow a symmetric S-shaped pattern represented by the well-known logistic curve (Griliches, 1957). More specifically, let X_t denote the number of agents that have adopted the new technology in time t. Let X^* denote the total number of potential adopters. Then the following ordinary differential equation expresses the dynamics of the innovation diffusion process through the contagion process between the adopters and the potential adopters:

$$\frac{\mathrm{d}X_t}{\mathrm{d}t} = \frac{b}{X^*} X_t \left(X^* - X_t\right), \tag{1}$$

which implies that b represents the growth rate of the numbers of adopters relative to the proportion of agents who have not yet adopted the innovation. The innovation's penetration level follows an S-shaped pattern with maximum diffusion speed reached when half of the total number of potential adopters has adopted the new technology.

This traditional approach in defining the innovation diffusion process assumes that the process takes place in a stable and homogeneous system in which the innovation spreads without any affection of the system's structure. In such cases, the diffusion follows a symmetric pattern similar to those provided by (1). The symmetry is also retained in the presence of external influences (e.g., promotional activities), which are not acting directly to the system's structure. However, many studies have proven that the presence of symmetry is not the general rule in innovation diffusion process (Mahajan, Muller, & Bass, 1990; Skiadas, 1985, 1986, 1987). In the majority of new technology penetration patterns the asymmetry is caused by several factors such as cultural status, economic conditions, demographics (population density, urbanization, and educational level), governmental policy, technology utility, technology familiarity, etc. (Bakalis, Abeln, & Enid, 1997). The incorporation of such a critical aspect of the diffusion process into the process representation efforts not only provides more flexible models but can also lead to the revelation of several interesting properties of the innovation diffusion process.

Equation 1 assumes an immediate interaction between the adopters and the potential adopters of a new product leading to a symmetric diffusion pattern.

However, this assumption is not always true since there is always a time-delay between the time of interaction occurrence and the adoption time. Thus, the potential adopters $(X^* - X_t)$ at time t interact with the adopters $X_{(t-T)}$ at time $(t - T)$. Taking into account the above consideration, the original logistic model takes the following form:

$$\frac{\mathrm{d}X_t}{\mathrm{d}t} = \frac{b}{X^*} X_{(t-T)} \left(X^* - X_t\right), \tag{2}$$

where T is the mean value of all time-delays occurring between the adopters and the potential adopters of the technology under investigation. Equation 2 cannot be easily handled and therefore an appropriate transformation is needed to order to have an approximate solution. By applying the Taylor series expansion to the expression $X_{(t-T)}$ we have:

$$X_{(t-T)} = X_t - T \cdot \frac{\mathrm{d}X_t}{\mathrm{d}t} + \frac{T^2}{2} \cdot \frac{\mathrm{d}^2 X_t}{\mathrm{d}t^2} - \frac{T^3}{3!} \cdot \frac{\mathrm{d}^3 X_t}{\mathrm{d}t^3} + \cdots \tag{3a}$$

Provided that the parameter T is not to large compared to the total time interval, the two first terms of the right hand side of Eq. 3a could be retained. Then Eq. 3a can be written as:

$$X_{(t-T)} = X_t - T \cdot \frac{\mathrm{d}X_t}{\mathrm{d}t}. \tag{3b}$$

Introducing Eq. 3b into Eq. 2 the following delay ordinary differential equation results:

$$\frac{\mathrm{d}X_t}{\mathrm{d}t} = \frac{b}{X^*} \cdot \left[X_t - T \cdot \frac{\mathrm{d}X_t}{\mathrm{d}t}\right] \cdot \left(X^* - X_t\right). \tag{4a}$$

The appropriate rearrangements in Eq. 4a yield:

$$\frac{\mathrm{d}X_t}{\mathrm{d}t} = \frac{b}{1 + b \cdot T} \cdot \frac{X_t \cdot (X^* - X_t)}{X^* - \frac{b \cdot T}{1 + b \cdot T} \cdot X_t}. \tag{4b}$$

Setting

$$b^* = \frac{b}{1 + b \cdot T} \tag{5a}$$

and then

$$b^* \cdot T = 1 - \sigma. \tag{5b}$$

Equation 4b takes the form:

$$\frac{\mathrm{d}X_t}{\mathrm{d}t} = b^* \cdot \frac{X_t \cdot (X^* - X_t)}{X^* - (1 - \sigma) \cdot X_t}. \tag{6}$$

Equation 6, is a special case of a family of generalized innovation diffusion models proposed by (Skiadas, 1985, 1986) aiming to represent the innovation diffusion process. When $\sigma = 1$ then Eq. 6 results in the above-described logistic model, whereas when $\sigma = 0$ it results in the exponential model. The solution of ordinary differential equation (6) has given by Skiadas (1985) and has the following form:

$$\ln (X_t) - \sigma \cdot \ln \left(X^* - X_t\right) = \ln (X_0) - \sigma \cdot \ln \left(X^* - X_0\right) + b \cdot t, \tag{7}$$

where X_0 represents the number of adopters at time 0.

The inflection point of the above model is given by Skiadas (1985) and has the following form:

$$X_{\text{inf}} = X^* \cdot \frac{1 - \sqrt{\sigma}}{1 - \sigma}. \tag{8}$$

The inflection point is considered as a measure of asymmetry in every technology diffusion case. Equation 8 reveals that the proposed model is very flexible since the inflection point takes values from 0 to X^* depending on the values of parameter σ. When $\sigma = 1$, $X_{\text{inf}} = X^*/2$ which is the inflection point of the logistic model.

3 Pattern identification in mobile telephony diffusion in EU-15

3.1 Model identification results

Mobile telecommunications has recently developed into a popular field of innovation diffusion studies. In fact, researchers have conducted studies on a national level (Wright, Upritchard, & Lewis, 1997; Frank, 2004), a multi-national level (Gruber & Verboven, 2001; Gruber, 2001), and on a worldwide level (Dekimpe, Parker, & SarVary, 1996). These, multi-national or cross-country studies examine the reasons and dynamics behind the differences in the adoption or diffusion processes of a set of countries. The present approach is trying to identify the existence of standardized patterns in mobile telephony diffusion in EU-15 due to the different time-delay effects between adopters and potential adopters during the contagion process.

The available data express the penetration level of mobile telephony in EU-15 from 1990 until 2002 and has been taken from OECD communication outlook (2000, 2001, 2002). The proposed model is applied to the available data by using an appropriate non-linear regression algorithm (Skiadas, 1987). The results for the 15 countries under investigation are summarized in Table 1.

As it can be seen, the model identification performance is very good since it explains for every country more than 99% of the process variance. The parameter σ is statistically significant for every country showing that the assertion of the existence of time-delay between the awareness and adoption phases is true. Based on the outcomes, the time-delay varies from 0.33 to 1.79 years. Figure 1 shows the time-delay parameters for each country under investigation. Among the countries with the smaller time-delay parameter are Portugal, France, and Greece, while the countries with the bigger time-delay parameter

Table 1 Parameter estimates, MSE, and % variance explained for the diffusion of mobile telephony diffusion in EU-15 (standard errors in parentheses)

Country	X_0	b^*	X^*	σ	$V(e_t)$	MSE	Variance explained (%)	T	X_{inf} (%)
Austria	0.059	0.701	82.459	0.230	1.648	1.141	99.89	1.10	56
	(0.025)	(0.050)	(1.026)	(0.073)					
Belgium	0.067	0.620	78.790	0.146	0.110	0.076	99.99	1.38	57
	(0.007)	(0.010)	(0.354)	(0.015)					
Denmark	1.878	0.357	95.520	0.416	2.987	2.068	99.72	1.64	58
	(0.451)	(0.041)	(15.076)	(0.340)					
Finland	1.674	0.447	88.994	0.655	2.267	1.569	99.82	0.77	49
	(0.390)	(0.046)	(4.198)	(0.253)					
France	0.016	0.819	67.193	0.699	0.315	0.218	99.96	0.37	37
	(0.006)	(0.049)	(1.246)	(0.135)					
Germany	0.048	0.652	69.997	0.048	3.938	2.726	99.60	1.46	57
	(0.023)	(0.100)	(1.404)	(0.015)					
Greece	0.229	0.768	89.000	0.617	0.477	0.220	99.97	0.50	50
	(0.055)	(0.044)	(2.242)	(0.134)					
Netherlands	0.046	0.693	74.422	0.137	1.756	1.216	99.85	1.24	54
	(0.023)	(0.056)	(0.964)	(0.063)					
Ireland	0.132	0.589	76.599	0.141	0.531	0.368	99.96	1.46	56
	(0.028)	(0.025)	(0.595)	(0.040)					
Italy	0.221	0.579	94.533	0.364	0.356	0.246	99.98	1.10	60
	(0.033)	(0.019)	(0.998)	(0.051)					
Luxembourg	0.223	0.534	99.556	0.101	4.891	3.386	99.74	1.68	77
	(0.085)	(0.041)	(2.257)	(0.049)					
Portugal	0.030	0.801	85.127	0.738	0.613	0.425	99.95	0.33	46
	(0.012)	(0.053)	(1.656)	(0.151)					
Spain	0.027	0.750	83.563	0.473	2.236	1.548	99.82	0.70	50
	(0.010)	(0.083)	(2.943)	(0.200)					
Sweden	3.150	0.330	99.392	0.447	1.881	1.302	99.84	1.68	60
	(0.460)	(0.029)	(9.747)	(0.214)					
UK	0.204	0.538	81.853	0.039	2.358	8.556	99.04	1.79	69
	(0.091)	(0.101)	(2.486)	(0.011)					

are UK, Luxembourg, Germany, and Denmark, Sweden. It's obvious that a catching-up process is present in the diffusion of mobile telecommunications (Gruber & Verboven, 2001) since the countries with high-technology level or countries which belong to the originators of the mobile technology present a bigger time-delay parameter than other countries which develop the industry later on. Finally, three countries, Finland, Greece, and Spain present almost symmetric diffusion pattern (inflection point $\approx$ 50% of the saturation level), while all the others not.

3.2 Time-delay effect and speed of diffusion

It is interesting to examine the relationship between the time-delay effect and the speed of the diffusion process. A frequently utilized measure for the speed is

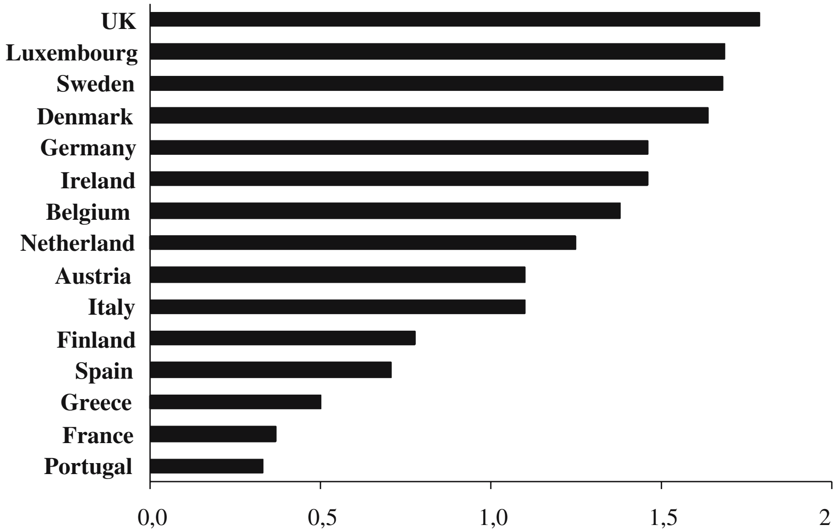

Fig. 1 Time-delay parameters for EU-15

the reciprocal of characteristic duration, a measure expressing the time required to grow from 10 to 90% of the estimated saturation level. Solving Eq. 7 for $1/t$ yields the Speed of Diffusion (SPD):

$$\mathrm{SPD} = \frac{1}{t} = b^* \cdot \left[\ln\left(\frac{X_t}{X_0}\right) - \sigma \cdot \ln\left(\frac{X^* - X_t}{X^* - X_0}\right) \right]^{-1}, \tag{9}$$

where, for each country, X_0 represents the 10% of the saturation level and X_t represents the 90% of the saturation level. Figure 2 shows the results concerning the speed of mobile telephony penetration for each country under investigation.

Figure 3 shows the mobile telephony speed of EU-15 countries w.r.t. their estimated time-delay effect.

From this cross comparison study it is obvious that the speed of diffusion tends to increase as the time-delay decreases. To confirm this visual observation a linear regression model was used relating the speed of diffusion with the time-delay and the square of the time-delay. The last independent variable is used in order to confirm that this relationship is not linear but rather quadratic. The results of the linear regression are given in Table 2.

From Table 2 it can be seen that the model fitting performance is very good ($R^2 = 0.97$) and the established assumptions are well supported since the sign of the time-delay is negative and the parameter of T^2 is statistically significant.

However, the form of the regression curve is better estimated by an inverse function or by a negative exponential curve. This last case is illustrated with a solid line in Fig. 3. The regression equation applied is of the form:

$$\mathrm{TL} = \alpha + \beta\, \mathrm{e}^{-\lambda(\mathrm{SPD})},$$

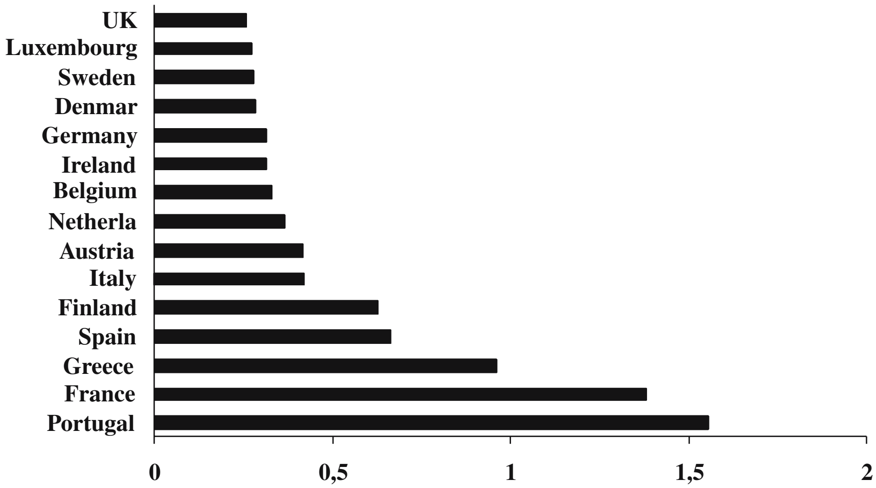

Fig. 2 The speed of mobile telephony penetration for EU-15

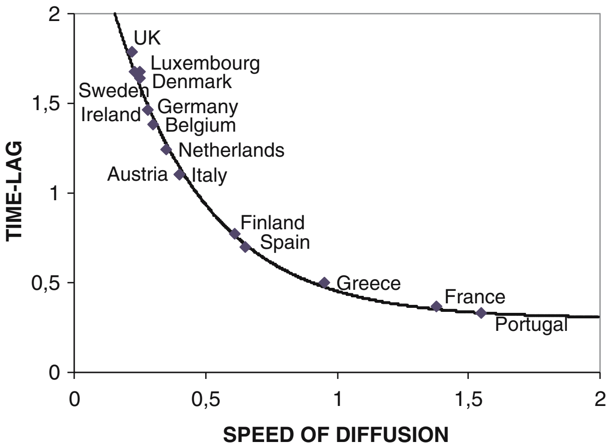

Fig. 3 Time-delay and speed of mobile telephony diffusion for EU-15

where TL is the time-lag, SPD is the speed of diffusion and α, β, λ are parameters of the regression equation. For the regression curve shown in Fig. 3 the following values for the parameters are obtained: $\alpha = 0.3$, $\beta = 2.646$, $\lambda = 2.85$. The negative exponential curve applied gives very good fitting explaining quite satisfactory the relation between time-lag and speed of diffusion.

4 Conclusions

This paper proposed a new modeling approach for the investigation of diffusion of mobile telecommunications services in EU-15. It was found that the proposed model which incorporates the notion of the time-delay between the

Table 2 Linear regression results for the relationship between speed and time-delay

	df	SS	MS	F	Significant F	
ANOVA						
Regression	2.000	2.336	1.168	168.96	0.000	
Residual	12.000	0.083	0.007			
Total	14.000	2.419				
	Coefficients	Standard error	t Stat	P-value	Lower 95%	Upper 95%
Intercept	2.151	0.114	18.788	0.000	1.901	2.400
T	−2.537	0.249	−10.189	0.000	−3.080	−1.995
T^2	0.851	0.117	7.301	0.000	0.597	1.106
Multiple R	0.983					
R^2	0.966					
Adjusted R^2	0.960					
Standard Error	0.083					
Observations	15.000					

awareness and the adoption phases of a new product plays an important role in studies of new product penetration in different groups of potential agents. The model was applied to the data of mobile telecommunication in EU-15 and the time-delay effect was used for the ranking of the countries under investigation with respect to their ability to adopt and diffuse the new technology. Furthermore, a new speed index was developed aimed to measure the speed of innovation diffusion. The relationship between the speed of diffusion and the time-delay effect was studied revealing that they are related in an inverse mode, i.e., as the time-delay effect of diffusion increases the speed of diffusion decreases in an inverse mode.

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