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OPTIMIZATION OF ECONOMIC INDICATORS IN OPEN-PIT MINING

N. L. Grigorenko, D. V. Kamzolkin, and D. G. Pivovarchuk

We investigate the application of the Real Options approach to the optimization of open-pit mining. The Real Options approach introduces investment as an additional control parameter for profit maximization. In the context of applying the Real Options approach to open-pit mining optimization, we consider a model with two-stage investments. Open-pit mining requires both extracting and processing capacities. These capacities in turn require investments, which are divided into two parts: investments to create the initial capacities and investments to increase existing capacities in the process of mining. The initial and augmented capacities as well as the capacity augmentation time are control parameters that can be chosen with the objective of increasing the mining profits. In this article, we assume that the market price of the mineral is a random process described by a stochastic differential equation. A control strategy is a rule that at every time instant, making use of the available information, determines the mining rate, establishes if additional investments are required at the given time, and if yes, calculates the investment amount. The problem involves the construction of an optimal mining control strategy that maximizes the mean discounted profit from the open-pit mine.

1. The Model and Statement of the Problem

We consider a model of excavation by layer in an open-pit mine. The total mine volume to be excavated is a. The quantity of ore excavated at the current time t (in the current year) is given by the function $u(t)$. The excavated ore is divided into two parts — ore for processing and stored ore. The two parts are determined by the mineral concentration threshold $g(t)$ in the ore. We assume that the threshold concentration is chosen so as to ensure full utilization of processing capacities.

We can choose the value of the function $u(t)$ at each time t. The range of possible $u(t)$ is constrained by the maximum excavation capacity Q and the maximum ore processing capacity P. These two parameters will be called respectively the mining and the processing capacity. Thus, $u(t) \in [P, Q]$. Creation of these capacities requires initial investments $IC(P, Q)$.

The duration of the open-pit excavation is not fixed: it is determined by the chosen control function $u(t)$. Let $T(u(\cdot))$ be the time when the open-pit mine had been fully excavated. The current state of the excavation process is characterized by the variable $x(t)$, which describes the ore volume excavated by the given time t. The excavated ore volume can be linked with the excavation rate $u(t)$ by the following differential equation:

$$\dot{x} = u(t), \qquad x(0) = 0, \qquad x(T) = a. \tag{1}$$

We assume that the mining and processing capacities can be changed once only. Let the initial capacities be $P_0,\ Q_0$ and at some time $\tilde{t}$ the capacities are changed to $P_1,\ Q_1$. Then the excavation rate $u(t)$ may take the following values:

$$\begin{cases} P_0 \le u(t) \le Q_0, & t \in [0, \tilde{t}\,), \\ P_1 \le u(t) \le Q_1, & t \in [\tilde{t}, T]. \end{cases}$$

Lomonosov Moscow State University, Faculty of Computation Mathematics and Cybernetics, Moscow, Russia.

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At each time $t,$ a useful mineral is extracted from the ore and it is sold in the market at the going price $s(t)$.

We have the excavation rate function $u(t)$ that ensures completion of open-pit excavation in time $T(u(\cdot))$; the market price of the mineral $s(t)$ is also known for all $t \in [0, T(u(\cdot))]$. The mining profit allowing for the change in capacity at time $\tilde{t}$ is calculated from the formula

$$
\begin{aligned}
NPV(u(\cdot) \mid s(\cdot), P_0, Q_0, P_1, Q_1, \tilde{t}\,) \;=\; & \int\limits_0^{\tilde{t}} e^{-\delta t}\left[-mu(t) - pP_0 - s(t)\frac{\alpha P_0^2}{2u(t)} + s(t)\alpha P_0\right]dt \\
& + \int\limits_{\tilde{t}}^{T(u(\cdot))} e^{-\delta t}\left[-mu(t) - pP_1 - s(t)\frac{\alpha P_1^2}{2u(t)} + s(t)\alpha P_1\right]dt \\
& - \; IC(P_0, Q_0) \; - \; e^{-\delta \tilde{t}}\Big(IC(P_1, Q_1) - IC(P_0, Q_0)\Big).
\end{aligned}
$$

The last two terms in the formula for NPV describe the amount invested in creating the initial capacities (P_0, Q_0) and the discounted amount invested in increasing the capacities at time $\tilde{t}$. The investment amount is calculated from the formula

$$IC(P, Q) = MC(Q) + PC(P),$$

where $MC(Q)$ is the investment required to attain the mining capacity Q and $PC(P)$ is the investment required to attain the processing capacity P.

The model of the excavation process includes the following variables and parameters:

t — the current time,

$x(t)$ — the ore mass excavated by time t,

α — the maximum concentration of the mineral [%/100],

a — total ore mass,

m — excavation cost [money units/mass unit],

p — processing cost [money units/mass unit],

c — market price of the mineral [money units/mass unit],

δ — discounting factor [(%/100)/year],

$u(t)$ — excavation rate [money units/mass unit].

Let us set the value for the initial capacities (P_0, Q_0). We assume that there is a certain set of alternative capacities to which the process can switch at time $\tilde{t}$. These alternative capacities are denoted by

$$\{(P_l, Q_l)\}, \quad l \in \{1, \ldots, L\}.$$

To formalize capacity adjustment, consider an additional time function $l = l(t)$. This function may take the values $\{0, 1, \ldots, L\}$, which index the corresponding capacities (P_l, Q_l). Initially at time $t = 0$ the function $l(t)$

takes the value 0, so that the initial capacity is (P_0, Q_0). We assume that this function switches not more than once to a value from the set $\{1, \dots, L\}$. This means that we can switch not more than once to a new capacity (P_l, Q_l).

In this way, we have eliminated the explicit inclusion of the time $\tilde{t}$ in the expression for NPV, but at the same time the capacity parameters (P, Q) became dependent on time, i.e., $(P_{l(t)}, Q_{l(t)})$. The time $\tilde{t}$ is determined by the switching point of the function $l(t)$. We denote this fact by $\tilde{t}(l)$. The value to which the function $l(t)$ switches will be denoted by $\tilde{l}(l)$.

Using the new notation, we rewrite the expression for NPV as

$$NPV(u(\cdot), l(\cdot) \mid s(\cdot), P_0, Q_0) = \int_0^{T(u(\cdot))} e^{-\delta t}\left[-mu(t) - pP_{l(t)} - s(t)\frac{\alpha P_{l(t)}^2}{2u(t)} + s(t)\alpha P_{l(t)}\right]dt$$

$$- \; IC(P_0, Q_0) \; - \; e^{-\delta \tilde{t}(l)}\Big(IC(P_{\tilde{l}(l)}, Q_{\tilde{l}(l)}) - IC(P_0, Q_0)\Big), \qquad (2)$$

$$P_{l(t)} \le u(t) \le Q_{l(t)}, \quad t \in [0, T(u(\cdot))].$$

The market price of the mineral $s(t)$ is assumed to be a stochastic process described by the Ito stochastic differential equation [2, 3]

$$ds = k(\mu - \ln s)s\,dt + \sigma s\,dxi, \qquad (3)$$

where $k,\ \mu,\ \sigma$ are given constant, $\xi(t)$ is the Brownian motion satisfying the conditions

$$\xi(0) = 0, \qquad E\xi(t) = 0, \qquad E\xi^2(t) = t.$$

The symbol $E\xi$ denotes the expectation of the random variable ξ.

We obtain that the price of the mineral $s(t)$ on $[0, T]$ is a stochastic function. Because of the price uncertainty, it is important to make clear assumptions about information availability. At each instant during open-pit mining we know the exact state of the excavation process $x(t)$ and the exact price of the extracted mineral $s(t)$. This available information is used to construct the control strategy for the ore excavation rate and for the switching time to a new capacity.

The excavation rate function thus has the form $u(t, x(t), s(t))$, i.e., it depends on the available information about the current state of the project. The capacity switching time and the index of the new capacity similarly depend on the available information and are determined by the function $l(t, x(t), s(t))$. We assume that the function $l(t, x(t), s(t))$ can have at most one switching point on every admissible trajectory $(x(t), s(t))$. In this way, we construct an adaptive control strategy that determines the control given the observed state $x(t)$ and the observed price $s(t)$.

A priori we do not know which price scenario will be realized in the market. We only have the stochastic equation that describes the set of possible price development scenarios and characterizes the probability of each scenario. Thus the adaptive strategy for the excavation function $u(t, x(t), s(t))$ and the capacity switching function $l(t, x(t), s(t))$ can be characterized by the mean profit (averaged over the set of possible scenarios). This mean is given by the formula

$$\overline{NPV}(u(\cdot), l(\cdot) \mid s_0, P_0, Q_0) = E_{s(\cdot)}\big[NPV(u(\cdot), l(\cdot) \mid s(\cdot), P_0, Q_0)\big],$$

where s_0 is the given initial price of the mineral.

The optimization problem for the excavation of an open-pit mine with switching of mining and processing capacities is posed as a maximization problem on the set of admissible adaptive strategies

$$(u(t, x(t), s(t)), l(t, x(t), s(t))) \; : \quad \overline{NPV}(s_0) = \max_{P_0, Q_0, u(\cdot), l(\cdot)} E_{s(\cdot)} \big[NPV(u(\cdot), l(\cdot) \mid s(\cdot), P_0, Q_0)\big].$$

2. General Statement of the Problem

Consider a discrete controlled process describing ore excavation,

$$x_{i+1} = f_i(x_i, u_i), \quad i = 0, \dots, N.$$

The index i corresponds to discrete time intervals (each one year long). The corresponding times will be denoted t_i, with $t_0 = 0$. The variable x_i is the quantity of ore excavated by time t_i, the control parameter u_i is the excavation rate on the interval $[t_i, t_{i+1})$. The initial and the final states of the process are specified by the equalities

$$x_0 = 0, \qquad x_n = a.$$

We do not know in what step the final condition is satisfied: this depends on the rate of excavation. We denote this step by n_u.

The set of values that the variable u_i may take in each step of the process is denoted by U. This set is characterized by mining and processing capacities. The creation of these capacities requires initial investment, which we denote by $IC(U)$.

We assume that the capacity may be changed once. This means that we know the initial capacity and at a certain step of the excavation process we can switch to a greater capacity. The capacities to which the process can switch are defined by the collection of sets

$$U^0 \subset U^1 \subset \ldots \subset U^L.$$

The sets are ordered by increasing capacity.

Initially at time t_0 we choose the capacity U^{l_0}, $l_0 \in \{0, \dots, L\}$. Then at some time $\tilde{t}$ we can change the capacity to $U^{\tilde{l}}$, $\tilde{l} \in \{0, \dots, L\}$, where $\tilde{l} > l_0$.

Thus, two control parameters are chosen in each step i of the process: the excavation rate u_i and the capacity index l_i. At most one switching point is allowed in the sequence of indexes $l_0, l_1, \dots, l_N$.

The amount invested in the creation and expansion of capacity is determined by the formula

$$IC = IC(U^{l_0}) + e^{-\delta \tilde{t}}(IC(U^{\tilde{l}}) - IC(U^{l_0})).$$

The mineral mined on the interval $[t_i, t_{i+1})$ is sold in the market at the current price c_i. The profit from the sale is determined by the function $g_i(c_i, u_i)$.

The total discounted profit from the sale of the mineral in the market is given by the formula

$$J(u, c) = \sum_{i=0}^{n_u(c)} e^{-\delta t_i} g_i(c_i, u_i).$$

The market price at each time is a random variable. Consider a sequence of prices at discrete times corresponding to the excavation of the open-pit mine:

$$c = (c_0, c_1, \dots, c_{N-1}), \quad c_i \in C.$$

We assume that this sequence of random variables has the Markov property with known transition probabilities

$$p_{i+1}(c_{i+1} \mid c_i).$$

The set of prices C is finite.

The probability that a particular price behavior scenario

$$\bar{c} = (\bar{c}_0, \bar{c}_1, \ldots, \bar{c}_{N-1})$$

is realized is given by the formula

$$P\{c = \bar{c}\} = p_0(\bar{c}_0) p_1(\bar{c}_1 \mid \bar{c}_0) \ldots p_{N-1}(\bar{c}_{N-1} \mid \bar{c}_{N-2}).$$

A programmed control of the excavation process

$$(u_0, u_1, \ldots, u_N), \qquad (l_0, l_1, \ldots, l_N),$$

is called admissible if $u_i \in U^{l_i}$, $i = 0, \ldots, N-1$, the sequence $(l_0, l_1, \ldots, l_N)$ has at most one switching point, and the corresponding trajectory $x = (x_0, \ldots, x_N)$ satisfies the terminal condition $x_{n_u(c)} = a$ for some $n_u(c) \in \{0, \ldots, N-1\}$.

Consider the class of adaptive controls. An admissible adaptive control is the set of functions

$$u(\cdot) = \big(u_0(x_0, c_0), u_1(x_1, c_1), \ldots, u_{N-1}(x_{N-1}, c_{N-1})\big),$$

$$l(\cdot) = \big(l_0(x_0, c_0), l_1(x_0, c_0), \ldots, l_{N-1}(x_{N-2}, c_{N-2})\big),$$

that on each trajectory $\{x_i\}_{i=0,\ldots,N}$, $\{c_i\}_{i=0,\ldots,N-1}$ induce a feasible programmed control

$$(u_0, \ldots, u_{N-1}), \quad (l_0, \ldots, l_{N-1}).$$

The mean profit from open-pit mining for a given adaptive control strategy $u(\cdot)$, $l(\cdot)$ is defined as the mean profit over the set of possible price scenarios

$$\bar{J}(u(\cdot), l(\cdot) | c_0) = E_c \left[\sum_{i=0}^{n_u(c)} e^{-\delta t_i} g_i(c_i, u_i(x_i, c_i)) \; - IC(U^{l_0}) - e^{-\delta \tilde{t}(l)} (IC(U^{\tilde{l}(l)}) - IC(U^{l_0})) \right].$$

The problem is to find an optimal initial capacity $l_0(x_0, c_0)$ and an optimal adaptive strategy $u(\cdot)$, $l(\cdot)$ that maximize the mean profit:

$$\bar{J}(u(\cdot), l(\cdot) \mid c_0) \to \max_{u(\cdot), l(\cdot)} . \tag{4}$$

3. The Algorithm

The problem is solved by a dynamic programming algorithm.

The first step of the algorithm is to evaluate the price function for constant capacities. For each possible capacity U^l, $l \in \{0, \ldots, L\}$, we construct a price function $V_i^l(x_i, c_i)$.

The price function $V_i^l(x_i, c_i)$ is evaluated as follows:

(i) For $k = N$, let

$$V_N^l(a, c^N) = 0, \quad c^N \in C,$$

$$G_N^l = \{a\};$$

(ii) For each $k \in \{0, \dots, N-1\}$, let

$$V_k^l(a, c^k) = 0, \quad c^k \in C;$$

(iii) For each $k \in \{1, \dots, N\}$, each $x^{k-1} \in G_{k-1}^l$ such that $x^{k-1} \neq a$, and each $c^{k-1} \in C$, let

$$V_{k-1}^l(x^{k-1}, c^{k-1}) = \max_{v_{k-1} \in U_{k-1}^l(x^{k-1})} \Big[e^{-\delta t_{k-1}} g_{k-1}(c^{k-1}, v_{k-1}) + \sum_{c_k \in C} p_k(c_k | c^{k-1}) V_k^l(f_{k-1}(x^{k-1}, v_{k-1}), c_k) \Big],$$

where

$$G_{k-1}^l = \{x^{k-1} \mid \exists v_{k-1} \in U^l \colon f_{k-1}(x^{k-1}, v_{k-1}) \in G_k^l\},$$

$$U_{k-1}^l(x^{k-1}) = \{v_{k-1} \in U^l \mid f_{k-1}(x^{k-1}, v_{k-1}) \in G_k^l\}.$$

Note that superscripts denote the currently observed value of the variable.

The value $V_i^l(x_i, c_i)$ is the optimal mean profit (for a constant capacity U^l) that can be earned from continuing open-pit mining if the current step i produced a quantity x_i of ore and the current price of the mineral is c_i.

In the second step of the algorithm, we compute the optimal value with at most one change of capacity. The price function corresponding to this problem is $W_i^l(x_i, c_i)$. The superscript l indicates that we construct a price function for the case when the initial capacity is from the set U^l.

The price function

$$W_i^l(x_i, c_i), \quad l \in \{0, \dots, L\}, \tag{5}$$

is defined by the following relationships:

(i) For $k = N$,

$$W_N^l(a, c^N) = 0, \quad c^N \in C;$$

(ii) For each $k \in \{0, \dots, N-1\}$,

$$W_k^l(a, c^k) = 0, \quad c^k \in C;$$

(iii) For each $k \in \{1, \dots, N\}$, each $x^{k-1} \in G^l_{k-1}$ such that $x^{k-1} \neq a$, and each $c^{k-1} \in C$,

$$W^l_{k-1}(x^{k-1}, c^{k-1}) = \max_{v_{k-1} \in U^l_{k-1}(x^{k-1})} \Bigg[e^{-\delta t_{k-1}} g_{k-1}(c^{k-1}, v_{k-1}) + \max \left\{ \begin{array}{l} \sum_{c_k \in C} p_k(c_k | c^{k-1}) W^l_k(f_{k-1}(x^{k-1}, v_{k-1}), c_k); \\ \max_{q \in \{l+1, \dots, L\}} \Big\{ -e^{-\delta t_{k-1}}(IC(U^q) - IC(U^l)) \\ \qquad + \sum_{c_k \in C} p_k(c_k | c^{k-1}) V^q_k(f_{k-1}(x^{k-1}, v_{k-1}), c_k) \Big\}; \end{array} \right\} \Bigg].$$

Once the functions $W^l_i(x_i, c_i),\ l \in \{0, \dots, L\}$ have been constructed, we find the optimal mean profit for the case when a one-time capacity expansion is allowed. This is calculated from the maximization problem

$$\bar{J}^*(c^0) = \max_{l \in \{0, \dots, L\}} \{W^l_0(0, c^0) - IC(U^l)\}. \tag{6}$$

The capacity $l^*_0(0, c^0)$ that maximizes the above expression determines the optimal initial capacity.

Let us consider how we can use the price function $W^{l^*_0}_i(x_i, c_i)$ to construct an optimal adaptive control strategy. We denote the optimal strategy by

$$u^*(\cdot) = \big(u^*_0(x_0, c_0), u^*_1(x_1, c_1), \dots, u^*_{N-1}(x_{N-1}, c_{N-1})\big),$$

$$l^*(\cdot) = \big(l^*_0(x_0, c_0), l^*_1(x_0, c_0), \dots, l^*_{N-1}(x_{N-2}, c_{N-2})\big).$$

Initially, according to the initial condition we have $x_0 = 0$. Let the initial price of the mineral be $c_0 = c^0$. The optimal initial capacity l^*_0 corresponding to the initial price c^0 is determined as the maximizer of (6). The optimal excavation rate u^*_0 and the next capacity index l^*_1 are obtained from the equality

$$W^{l^*_0}_0(0, c^0) = e^{-\delta t_0} g_{k-1}(c^0, u^*_0) + \max \left\{ \begin{array}{l} \sum_{c_1 \in C} p_1(c_1 | c^0) W^{l^*_0}_k(f_0(0, u^*_0), c_0); \\ -e^{-\delta t_0}(IC(U^{l^*_1}) - IC(U^{l^*_0})) \\ \qquad + \sum_{c_1 \in C} p_1(c_1 | c^0) V^{l^*_1}_0(f_0(0, u^*_0), c_0); \end{array} \right\}.$$

Having found the optimal value u^*_0, we calculate the optimal volume of excavated ore

$$x^*_1 = f_0(0, u^*_0).$$

Take the time t_{k-1}. Assume that the excavated ore volume is x^*_{k-1}, the current observed price is c^{k-1}, and the current capacity is l^*_{k-1}. Find the optimal excavation rate u^*_{k-1} and the next capacity index l^*_k. Two cases are possible. The first case: the capacity has already been switched, i.e., $l_{k-1}^* \neq l_0^*$. Then $l_k^* = l_{k-1}^*$ and u_{k-1}^* is determined from the equality

$$V_{k-1}^{l_{k-1}^*}(x_{k-1}^*, c^{k-1}) = e^{-\delta t_{k-1}} g_{k-1}(c^{k-1}, u_{k-1}^*) + \sum_{c_k \in C} p_k(c_k|c^{k-1}) V_k^{l_{k-1}^*}(f_{k-1}(x_{k-1}^*, u_{k-1}^*), c_k).$$

The second case: the capacity has not been switched yet, i.e., $l_{k-1}^* = l_0^*$. Then the optimal excavation rate u_{k-1}^* and the next capacity index l_k^* are determined from the equality

$$\begin{aligned} W_{k-1}^{l_0^*}(x_{k-1}^*, c^{k-1}) &= e^{-\delta t_{k-1}} g_{k-1}(c^{k-1}, u_{k-1}^*) \\ &+ \max \left\{ \begin{array}{l} \displaystyle\sum_{c_k \in C} p_k(c_k|c^{k-1}) W_k^{l_0^*}(f_{k-1}(x_{k-1}^*, u_{k-1}^*), c_k); \\ -e^{-\delta t_{k-1}}(IC(U^{l_k^*}) - IC(U^{l_0^*})) \\ \qquad + \displaystyle\sum_{c_k \in C} p_k(c_k|c^{k-1}) V_k^{l_k^*}(f_{k-1}(x_{k-1}^*, u_{k-1}^*), c_k); \end{array} \right\}. \end{aligned}$$

The optimal excavated volume in step k is evaluated from

$$x_k^* = f_{k-1}(x_{k-1}^*, u_{k-1}^*).$$

4. Calculation Results

In this section, we use concrete values for the model parameters corresponding to some open-pit mine and apply the proposed approach to find an optimal control excavation strategy.

We take the following values for the model parameters:

maximum concentration of the mineral	$\alpha = 0.015$
total ore mass	$a = 600$ [million tons]
excavation cost	$m = 1$ [per ton of ore]
processing cost	$p = 5$ [per ton of ore]
discount factor	$\delta = 0.1\%/100$ [per annum]
range of excavation capacities	$Q_{\max} \in [9, 60]$ [million tons per year]
range of processing capacities	$P_{max} \in [4, 35]$ [million tons per year]
price range	$c \in [500, 1500]$ [$ per ton of mineral]
$MC(Q)$	$1 \cdot 10^8 \cdot (Q/6 \cdot 10^{-7})^{0.8}$
$PC(P)$	$5 \cdot 10^8 \cdot (P/2 \cdot 10^{-7})^{0.7}$

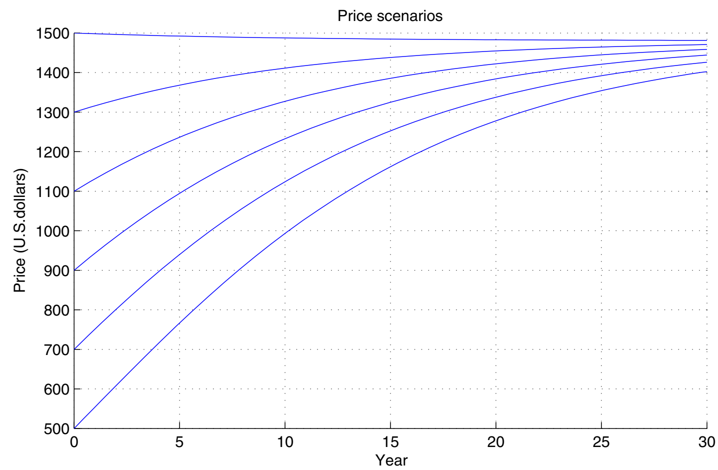

Fig. 1. Price scenarios

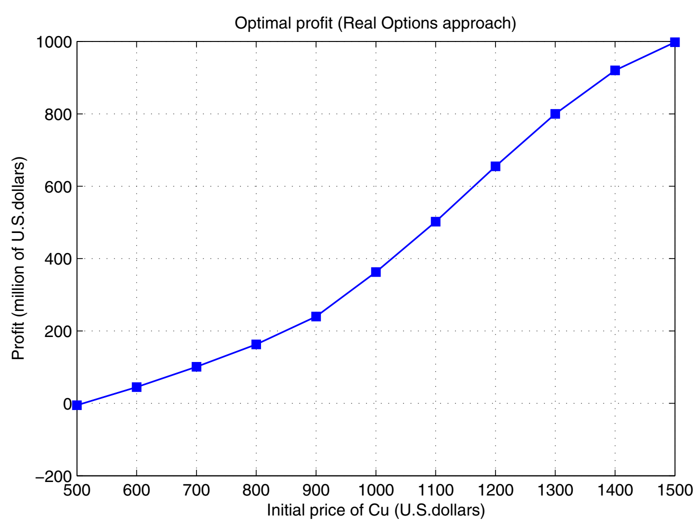

Fig. 2. Optimal profit

Price equation parameters:

$$\mu = 7.3, \qquad \sigma = 0.04, \qquad k = 0.1.$$

We assume that the price of the mineral ranges from 500 to 1500 in the course of mining. The parameters chosen for the stochastic equation indicate that the most probable scenario is when the price rises from its initial value to the maximum value 1500. Figure 1 illustrates the most probable price scenarios with various initial prices.

The mining and processing capacities are not given. We only have the range of their possible values. Optimal initial and expanded capacities are to be determined.

The first step of our algorithm evaluates the price function (5) and uses it to compute the optimal mean profit from open-pit mining $\bar{J}^*(c^0)$ by formula (6), for various values of the initial price c^0. Figure 2 presents the calculation results. As expected, the profit is higher the higher the initial price of the mineral, because the most probable scenario is that the prices increase over time.

Alongside with the evaluation of the optimal mean profit, we evaluate the optimal initial capacities and the optimal additional investment strategy given the observed price. The optimal initial capacities depend on the initial price of the mineral (see Figs. 3 and 4). The graphs in Figs. 3 and 4 lead to the following conclusion: the higher the initial price of the mineral, the larger are the required initial capacities.

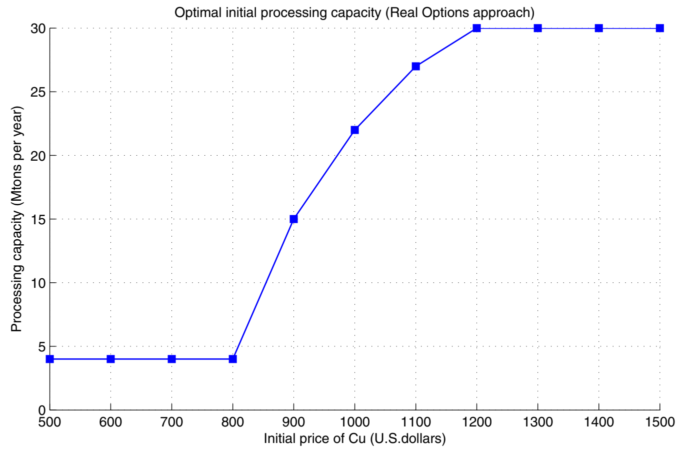

Fig. 3. Optimal initial processing capacities

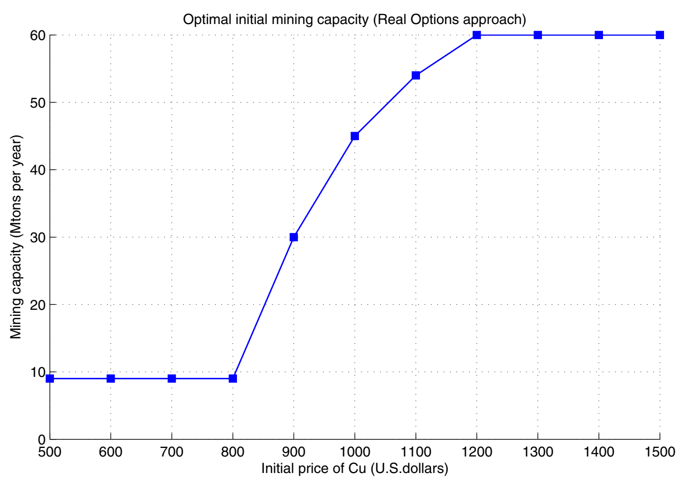

Fig. 4. Optimal initial mining capacity

Figure 5 illustrates the optimal strategy for additional investments in capacity expansion. Note that the capacity expansion strategy depends on the initial capacity. Figure 5 illustrates the strategy with initial processing capacity 4 (Mtons per year). The capacity expansion strategy depends on the quantity of ore excavated up to the given time and the currently observed price. The figure shows at what observed state x and what price c the capacity should be increased: as long as the trajectory $(x(t), c(t))$ is in the white region, the initial capacity should be kept; when the trajectory reaches the colored region, the strategy is to switch to the capacity corresponding to the color that the trajectory has hit. We assume that the strategy can be increased only once. The graph shows that the capacity should be increased when the price reaches the threshold 1100, and the increased capacity is determined by the quantity of ore mined up to the given time. Figure 6 shows the optimal capacity expansion strategy when the initial processing capacity is 16 (Mtons per year).

The next step of our analysis is to evaluate the optimal capacities to which the strategy should switch so as to maximize the profit. This step also includes evaluation of the optimal capacity switching time. Here it is important to remember that we are constructing an adaptive strategy, i.e., decisions are made using observations of the current state of the mining process and the current price. Therefore, in general, we cannot pinpoint a unique optimal switching time or a unique capacity. The observation-based optimal adaptive strategy is constructed by the described scheme. As an example, we consider the most probable price scenarios from Fig. 1.

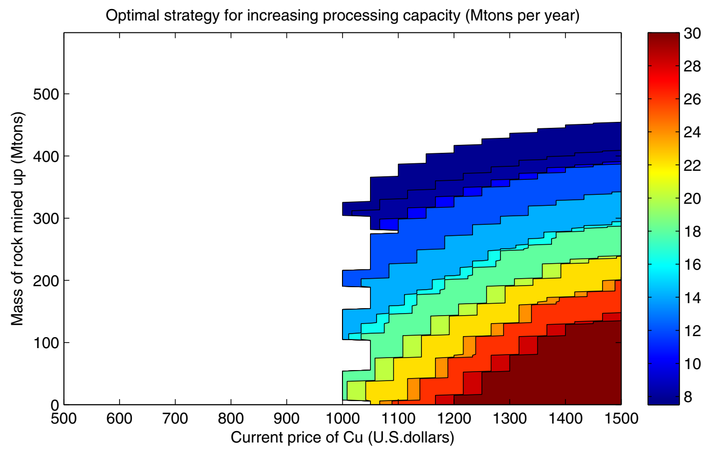

Fig. 5. Optimal strategy for increasing processing capacity. Initial capacity 4 Mtons.

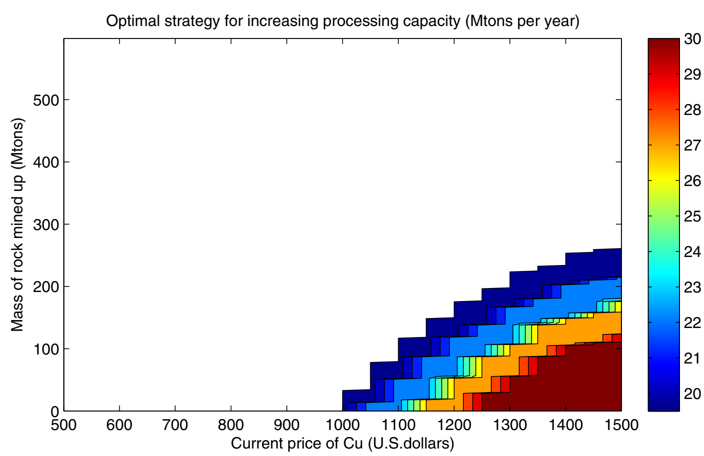

Fig. 6. Optimal strategy for increasing processing capacity. Initial capacity 16 Mtons.

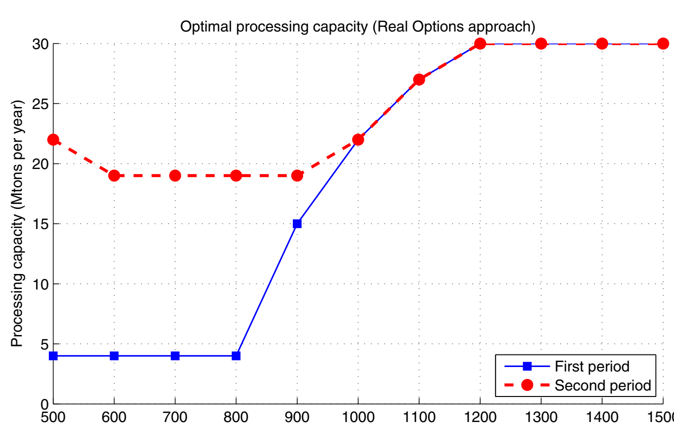

Fig. 7. Optimal processing capacities

The second-period line in Figs. 7, 8 plots the optimal increase of capacity as a function of the initial price. In particular, we see that if the initial price is greater than 1000, the most profitable strategy is to start with the optimal strategy without any switching during the mining process. Figure 9 shows the optimal time for capacity switching. From these graphs we draw the following conclusion: if the initial price is low and subsequent price

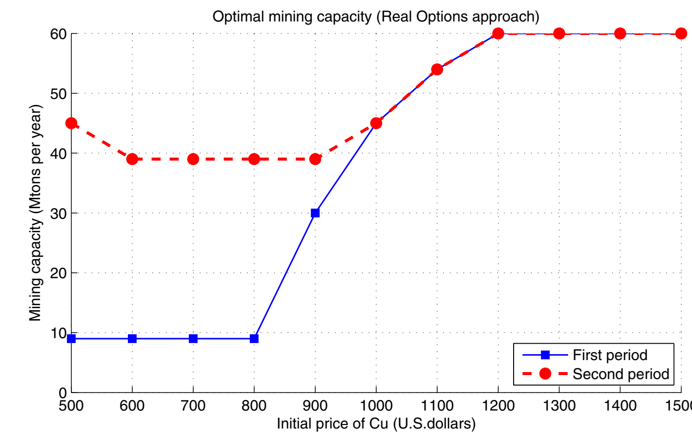

Fig. 8. Optimal mining capacities

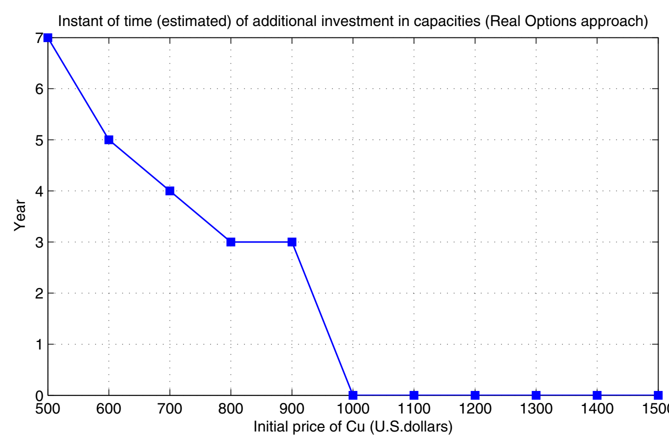

Fig. 9. Switching time (estimate) for additional investment in capacity

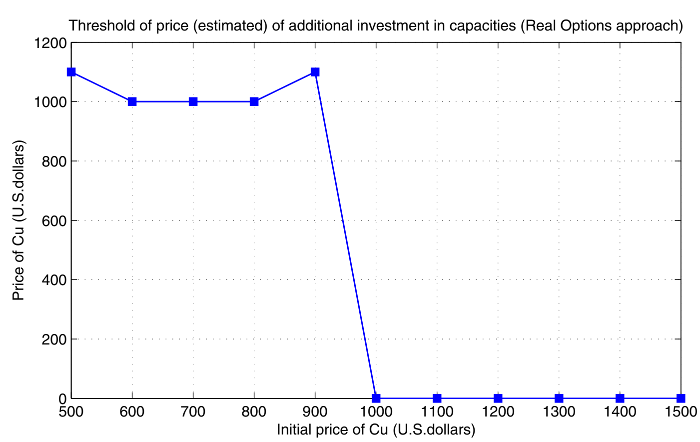

Fig. 10. Price threshold (estimate) for additional investment in capacity

increase is expected, it is better to invest initially in small capacities and to increase them subsequently when the prices have risen.

We stress again that the capacity switching time and the magnitude of capacity increase are shown in our graphs for the most probable scenarios. Fig. 10 plots the current price when the decision to invest in more capacity is made. We see that if the additional price encourages a capacity increase, it is best to make the switch when the price has reached 1000–1100. These values provide a heuristic price threshold. When the price reaches this threshold, the capacity should be increased.

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