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FORMATION OF THERMAL STRUCTURES WITH BLOWUP DURING SOLAR FLARES

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A mathematical model is proposed describing the initial phase of solar flare heating in the corona. The model is based on the nonlinear heat equation with a sign-alternating volume source, which is derived from the energy equation of the electron component of stationary plasma. Flares are assumed to arise as a result of sausage-type instabilities of a magnetic tube and creation of collapsing magnetic traps. A source function is chosen and the model parameters are fitted. Thermal structures arising under supercritical perturbations in a homogeneous temperature background are calculated and their specific features are examined. With nonclassical heat conduction, flares produce structures for which the half-width of the energy release region shrinks over time. The effect of "emission measure reduction" observed in the early phase of the flare is associated with the decrease of the flare filling factor due to the decrease of the structure half-width.

Keywords: phenomenological mathematical model, solar corona midroflares, heating function, blowup, thermal structures, nonclassical heat conduction.

Introduction

Solar flares in the corona involve explosive release of energy in the form of radiation, heat and accelerated particle streams, and ejection of mass. The impulse phase of solar flares lasts from a few seconds to several minutes, while the quantity of energy released in this time may reach billions of megatons in TNT equivalents. Understanding the flare mechanism is one of the key problems in the physics of the Sun. Earthbound and satellite observations of solar flares ensuring high temporal, spatial, and spectral resolution make it possible to assemble an overall picture, but the energy release mechanism remains unclear.

The key to understanding this phenomenon should clearly be sought in the structure and dynamics of the solar magnetic field, whose energy may exceed many fold the thermal plasma energy. Solar flares as a rule occur near the neutral line of the sunspot magnetic field, separating between regions of north and south polarity. The most commonly discussed flare mechanism is magnetic reconnection in a neutral current sheath, which is assumed to arise in the corona above the active region with magnetic sunspots [1]. During a flare, the primary field energy is converted into thermal energy and the plasma temperature rapidly rises. Magnetic reconnection and microflares are often also considered as mechanisms heating the "stationary" solar corona to temperature of 2 MK [3].

Heating models describing heat propagation in a magnetic tube require solution of a system of hydrodynamic equations. The problem of nonstationary heating of the chromosphere in the impulse phase is solved by numerical calculation of transient processes for the hydrodynamic response. The general picture described by this model is consistent with observations, but its interpretation involves certain difficulties, in particular, the theoretical calculations fail to produce the "emission measure reduction" effect observed in the early phase of the flare [4].

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Numerical solutions of complex systems of equations may miss particular important effects. The nature of flare plasma heating largely depends on source nonlinearity. Yet numerical solutions of gas-dynamic problems usually specify a boundary condition in the form of a rectangular impulse or an impulse with linear buildup and decay of accelerated heat and particle fluxes.

It is shown in [4, 5] that during a flare the temperature increases with blowup (hyperbolically). In the present article, we model the explosive growth of energy in a stationary electron plasma during a flare on the basis of a nonlinear heat equation with a volume heat source. This model is chosen because under certain conditions it describes the formation of localizes nonstationary structures that evolve with blowup. Localized nonstationary structures adequately describe short-lived microflares with characteristic space and time scales. They arise in the presence of supercritical small-scale background temperature perturbations. The half-width of the region of high-intensity energy release contracts, which reduces the filling factor of the emitting volume and is manifested as the emission measure reduction effect observed during flares. Nonlinear functions and parameters have been selected that reflect the dependence of the volume source and the heat conductivity on temperature for the solar corona plasma.

The Dynamics of Combustion Processes on Zero Background. Three Typs of Blowup Regimes

It was discovered in the 1970s that in a medium with nonlinear heat conductivity and a volume heat source combustion processes evolve with blowup. Under certain conditions, heat localization is observed, leading to stratification of the continuum into distinct structure — regions of high-intensity combustion. The temperature inside these regions may be several orders of magnitude higher than the background temperature. The localized structures have a characteristic size – the fundamental length, and exist for a finite time.

To study the unusual properties of blowup regimes, we model them by a quasilinear heat equation with heat conductivity and a volume heat source that are power functions of the temperature $T(r,t)$:

$$\frac{\partial T}{\partial t} = \operatorname{div}(\chi_0 T^{\sigma} \operatorname{grad} T) + q_0 T^{\beta}, \qquad t > 0, \qquad r \in D_1, \tag{1}$$

where t is time, r is the space coordinate, χ_0, $q_0 > 0$, $\sigma > 0$, $\beta > 1$ are parameters. With appropriately chosen parameter values, Eq. (1) describes thermonuclear combustion of a plasma with electron or radiation heat conduction.

The combustion is initiated by specifying an initial temperature distribution in some spatial region $D_2 \subseteq D_1$:

$$T(r,0) = T_0(r) \le M < \infty. \tag{2}$$

Once begun, the combustion propagates over the zero background. Usual boundary conditions of matching with the zero background are specified on the combustion wavefront.

The properties of the solutions of this problem for Eq. (1) with various parameter values have been studied in detail. Many results have been presented in [6], and original papers on blowup regimes have been collected in [7].

The nonlinear equation (1) for given parameter values has infinitely many solutions dependent on the initial distribution (2). However, only some of these solutions determine the evolution of the entire system. These are the self-similar solutions, which play the role of attractors for all other solutions of the Cauchy problem with arbitrary initial values. The self-similar solutions are determined by the parameters of the nonlinear medium and are independent of the initial distribution (2). They are accordingly called eigenfunctions (EFs) of the nonlinear medium.

In the radially symmetrical case, the self-similar solutions have the form

$$T(r,t) = g(t)\Theta(\xi), \quad \xi = \frac{r}{\psi(t)}, \tag{3}$$

where ξ is the self-similar variable, $\Theta(\xi)$ a self-similar solution, and $g(t)$ and $\psi(t)$ are time functions. The self-similar equation for the function $\Theta(\xi)$ and the form of the functions $g(t)$ and $\psi(t)$ can be obtained by substituting expression (3) in Eq. (1) [6],

$$g(t) = \left(1-\frac{t}{\tau}\right)^m, \quad \psi(t) = \left(1-\frac{t}{\tau}\right)^n, \quad m = -\frac{1}{\beta-1}, \quad n = \frac{\beta-\sigma-1}{2(\beta-1)}. \tag{4}$$

The self-similar equation has the form

$$\frac{\chi_0}{(\sigma+1)}\Delta\Theta^{\sigma+1} = -\frac{m}{\tau}\Theta + \frac{n}{\tau}\xi\frac{\partial\Theta}{\partial\xi} - q_0\Theta^{\beta}, \tag{5}$$

where τ is an arbitrary parameter of the generalized separation of variables (3), which is interpreted as the blowup time for $\tau > 0$. Indeed, it follows from (3) and (4) that for $\tau > 0$ the self-similar solutions have a finite lifetime $t = \tau$ and evolve with blowup: $g(t) \to \infty$ as $t \to \tau$.

It has been shown that the self-similar equation with appropriate boundary conditions has solutions for every $\tau > 0$. These solutions are called the eigenfunctions (EF) of the self-similar problem. For various τ, the EFs are obtained from one another by a similarity transformation: they define the same self-similar solution (3) at different times.

Depending on the values of the parameters σ and β, we distinguish three types of blowup regimes — HS, S, and LS — defined by the self-similar solutions (3).

1) ***HS-Regime.*** For $1 < \beta < \sigma + 1$, there exists a unique radially symmetric EF with compact support that has a single maximum at the center of symmetry. This function describes a spatially propagating combustion wave. During the blowup time τ the combustion encompasses the entire space.

2) ***S-Regime.*** For $\beta = \sigma + 1$, the self-similar problem also has a unique EF with a single maximum at the symmetry center. The corresponding self-similar solution describes a nonstationary structure localized in a region of certain size — the fundamental length L_T. Indeed, in this case, as it follows from (3) and (4), the front and all other profile points of the self-similar solution (3) do not propagate in space (because $\xi = r$). Inside the localization region the temperature increases with blowup, whereas outside this region the temperature remains zero; the combustion region half-width does not change. In the one-dimensional case, we have the analytical solution [6]

$$T(r,t) = T_0 g(t)\Theta_S(r), \quad 0 \le r \le 0.5 * L_T,$$

$$\Theta_S(r) = \cos^{2/\sigma}\left(\frac{\pi r}{L_T}\right), \quad L_T = \frac{2\pi}{\sigma}\sqrt{\frac{\chi_0}{q_0}(\sigma+1)}\,. \tag{6}$$

In the radially symmetrical case, the eigenfunction $\Theta_S(r)$ has the same form (6), but with a different fundamental length. The fundamental length increases with the dimension of the space. An approximate solution for the spherically symmetrical and cylindrically symmetrical cases has been obtained in [8].

3) ***LS-Regime.*** For $\beta > \sigma + 1$, all the profile points of the self-similar solution (3) move toward the center (because $r(t) = \xi\psi(t)$ decreases with time) and grow with blowup. As a result, the effective width of the combustion region shrinks and for $t = \tau$ the temperature takes an infinite value only at one point — the center of symmetry. In this case, the self-similar problem does not have a solution in a bounded spatial region and the front is at infinity. It has been shown for initial perturbations with compact support (2) that the combustion is strictly localized and the self-similar solution adequately describes combustion inside the localization region. A bound of the fundamental length in the LS-regime has been obtained for the one-dimensional case. This bound depends on the energy of the initial perturbation W_0 $\left(W_0 = \max T_0(r) \times \|\mathrm{supp}\, T_0\|\right)$ [6]:

$$L_{(LS)}{}^{(\beta-\sigma-3)} = \frac{1}{d^2} W_0{}^{(\beta-\sigma-1)}, \quad d = \pi\sqrt{\frac{\chi_0}{q_0}\frac{2(\beta+\sigma+1)}{\sigma(\beta-1)}}\,, \tag{7}$$

Comparisons theorems have been applied to show that the bound does not exceed the fundamental length in the S-regime for the smaller value $\beta = \sigma + 1$.

Localization and appearance of nonstationary dissipative structures are thus observed for $\beta \geq \sigma + 1$ in S- and LS-regimes.

Further studies have shown that in the LS-regime there exists a finite, strictly defined spectrum of EFs, the number of which depends on the parameters σ and β [7, 8, 9]. The EFs differ from one another by architecture, symmetry, location of maxima, shape and size of localization regions. The spectrum always contains the first EF $\Theta_1(\xi)$ — a simple structure with a single maximum at the center. There exist radially symmetrical EFs and complex-architecture structures with multiple maxima [9].

EFs have different degrees of stability and different attraction regions. On the whole we can assert that every solution initiated by initial perturbation (2) tends to reach a self-similar regime and evolve in accordance with some EF. A structurally stable self-similar solution with a wide attraction region is a simple structure with a single maximum. This structure determines the characteristic size of the structures forming in the nonlinear medium and the fundamental length of the localization region (7) [8, 9].

Dynamics of Blowup Regimes on a Homogeneous Nonzero Background

Virtually all theoretical results concerning the properties of blowup regimes have been obtained for the case when combustion starts in an absolutely cold medium at zero temperature. Inside the combustion region, the temperature rises by many orders of magnitudes, and the background temperature may indeed be assumed equal to zero. This approximation, however, is not applicable to modeling solar flares in the corona, as they occur on a high temperature background.

We will investigate the specific features of combustion dynamics evolving with blowup on a homogeneous nonzero background. We modify the volume heat source so that Eq. (1) admits the existence of a stable homogeneous stationary solution. Consider the nonlinear heat equation with a source in the form

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}(\chi_0 T^\sigma \frac{\partial T}{\partial x}) + Q(T), \tag{8}$$

$$Q(T) = q_0(T^{\beta_1} - T_0)(T^{\beta_2} - T_1), \quad T_0 < T_1, \quad 0 < \sigma < 1, \quad \beta_1 + \beta_2 > 1.$$

Equation (8) has two stationary solutions: $\tilde{T}_0 = T_0^{1/\beta_1}$, $\tilde{T}_1 = T_1^{1/\beta_2}$. It is easy to show that in the corresponding ODE system

$$\frac{dT}{dt} = Q(T), \quad T(t=0) = T^0 \tag{9}$$

the smaller stationary solution $\tilde{T}_0$ is stable and the greater stationary solution $\tilde{T}_1$ is unstable. If the initial temperature is $T^0 > \tilde{T}_1$, then the system evolves with blowup, reaching infinite temperature in a finite time. If $T^0 < \tilde{T}_1$, then the ODE solution goes to the stable stationary solution $\tilde{T}_0$. The unstable stationary solution $\tilde{T}_1$ thus acts as a threshold.

Now consider the nonlinear heat equation (8). Linear stability analysis shows that small perturbations of the stationary solution $\tilde{T}_0$ die away, whereas small perturbations of the stationary solution $\tilde{T}_1$ grow, so that as in the ODE system the stationary solution $\tilde{T}_0$ is stable and $\tilde{T}_1$ is unstable. Are there supercritical perturbations of the stationary solution $\tilde{T}_0$ that lead to a combustion "flare" with blowup? Do we observe heat localization and do nonstationary structures appear?

Intuitively it is clear that the evolution of combustion with blowup at high temperatures is driven by the principal nonlinear term $T^{\beta_1}T^{\beta_2}$ in the source:

$$Q(T) = q_0(T^{\beta_1} - T_0)(T^{\beta_2} - T_1) = q_0(T^{\beta_1}T^{\beta_2} - T_1T^{\beta_1} - T_0T^{\beta_2} + T_0T_1).$$

The other terms play a progressively diminishing role as the temperature increases, and the solutions of Eq. (8) go to the solutions of Eq. (1) with $\beta_1 + \beta_2 = \beta$. In the developed stage of the process, HS-regime is observed for $1 < \beta_1 + \beta_2 < \sigma + 1$, S-regime for $\beta_1 + \beta_2 = \sigma + 1$, and LS-regime for $\beta_1 + \beta_2 > \sigma + 1$. It is also reasonable to assume that some small perturbations of the homogeneous stable stationary solution decay, while other supercritical perturbations increase with blowup. For $\beta_1 + \beta_2 \geq \sigma + 1$ we observe heat localization on the background and formation of nonstationary structures, whereas for $1 < \beta_1 + \beta_2 < \sigma + 1$ the combustion propagates over the background.

Equation (8) has been investigated in [10] for the case of the S-regime with $\sigma = 1$, $\beta_1 = \beta_2 = 1$. It has been proved that that there indeed exist supercritical perturbations of the stationary solution for which the combustion process evolves with blowup, and it has been shown numerically that there is heat localization on a nonzero background. Furthermore, self-similar procedures have shown that the temperature distributions approaches a self-similar solution (6), which is determined by Eq. (1). Some calculations have been carried out also for LS- and HS-regimes.

In the present study, we have carried out a detailed numerical analysis of the Cauchy problem for Eq. (7) with various initial perturbations of the stationary solution $\tilde{T}_0$. Our conjectures have been confirmed. Indeed,

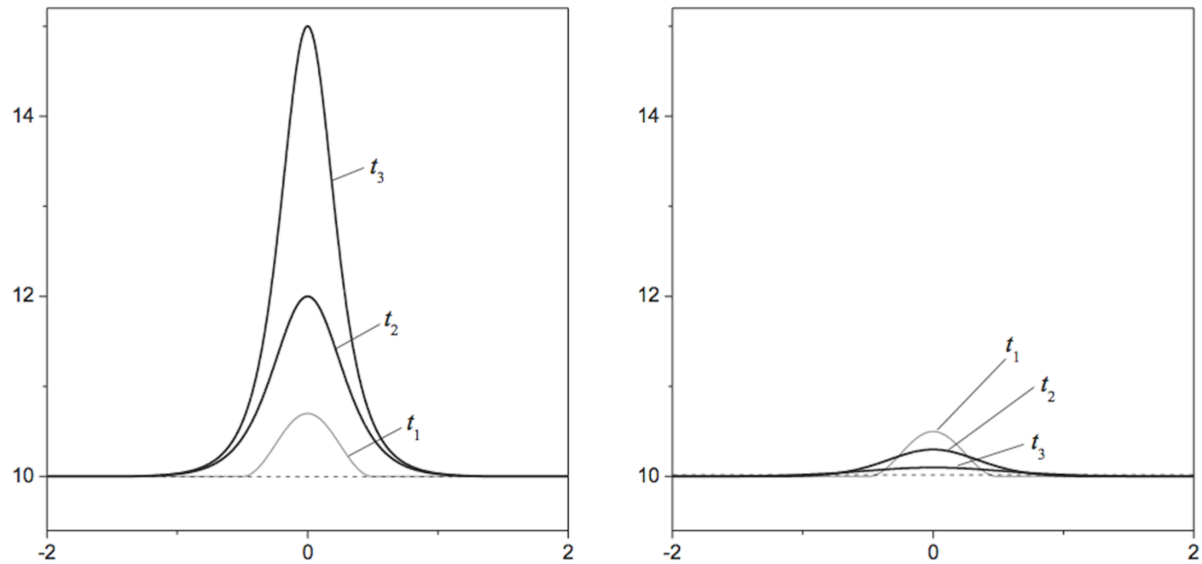

Fig. 1. Growth (a) and decay (b) of initial background perturbation as a function of its magnitude, $\chi_0 = 0.05$, $\sigma = 1$, $\beta_1 = 1$, $\beta_2 = 2$, LS-regime.

depending on parameter values, all three combustion regimes with blowup exist and they arise in the presence of supercritical background perturbations. Below we present some characteristic calculations for various values of the parameters σ, β_1, β_2 with $\tilde{T}_0 = 10^{1/\beta_1}$, $\tilde{T}_1 = 105^{1/\beta_2}$, $q_0 = 1$.

1) Figure 1 shows temperature distributions at various time instants $(0 < t_1 < t_2 < t_3)$. We see that if the initial perturbation is below some threshold, then it relaxes to the background (Fig. 1b); if, on the other hand, it exceeds the threshold, we observe growth with blowup (Fig. 1a). Note that the perturbation decays in the classical regime, when the perturbation temperature asymptotically goes to the background temperature T_0 in a theoretically infinite time.

2) Figure 2 shows the temperature distributions at various time instants $(0 < t_1 < t_2 < t_3 < t_4)$ with combustion evolving in HS (Fig. 2a), LS (Fig. 2b), and S (Fig. 2d) regimes with blowup. The following parameter values have been used:

$$\text{S-regime} \quad (\beta_1 + \beta_2 = \sigma + 1): \quad \chi_0 = 0.06, \quad \sigma = 2, \quad \beta_1 = 1, \quad \beta_2 = 2.$$

$$\text{HS-regime} \quad (\beta_1 + \beta_2 < \sigma + 1): \quad \chi_0 = 0.06, \quad \sigma = 2.6, \quad \beta_1 = 1, \quad \beta_2 = 2.$$

$$\text{LS-regime} \quad (\beta_1 + \beta_2 > \sigma + 1): \quad \chi_0 = 0.005, \quad \sigma = 1, \quad \beta_1 = 1, \quad \beta_2 = 2.$$

In all cases combustion is initiated by specifying some compact-support positive supercritical perturbation $\tilde{T}_0$ in the region D_1 (the temperature $T^0(r)$ at $t = 0$ exceeded $\tilde{T}_0$ in D_1). We see that for the HS-regime (Fig. 2a) the combustion region expands and there is no localization. In LS- and S-regimes, localization of combustion is observed and nonstationary structures of contracting (Fig. 2b) or constant (Fig. 2d) half-width appear on the nonzero background.

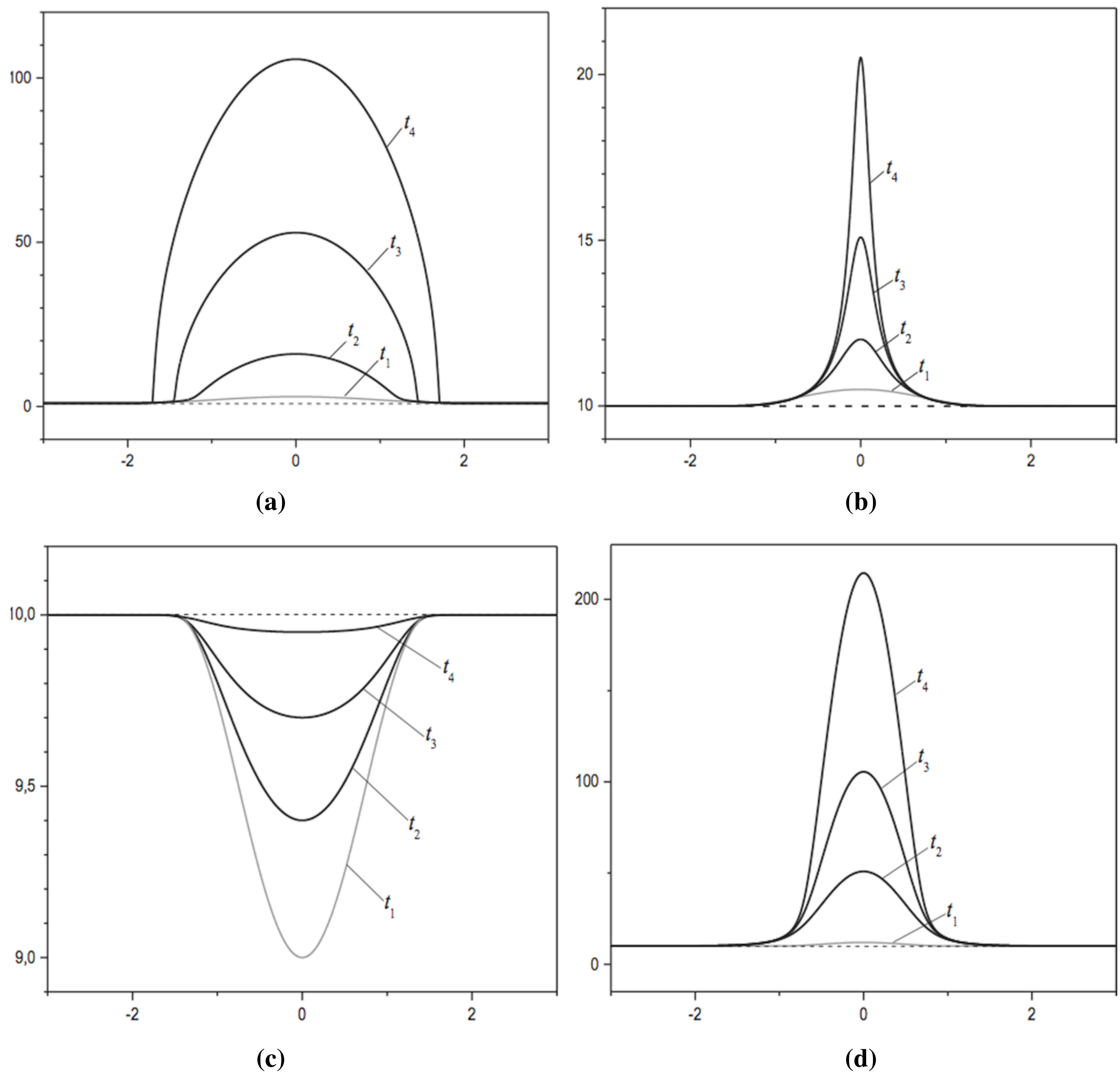

Fig. 2. Evolution of a negative perturbation in HS (a), LS (b), and S (d) regimes with blowup and its relaxation to the background (c).

The calculations show that a negative initial perturbations of the stationary solution $\tilde{T}_0$ $(T^0(r) < \tilde{T}_0$, $r \in D_1)$ decays for all parameter values. Figure 2c shows the relaxation of a negative initial perturbation to the background for parameter values corresponding to the LS-regime. The approach to the stationary solution occurs in the ordinary classical regime, because linear terms play increasing role near the background, producing exponential decay.

3) The existence for the same parameter values of compact-support initial perturbations that may lead both to a blowup regime and to relaxation produces interesting localized structures that evolve with blowup and have a finite lifetime. Figure 3 shows some types of structures that may arise in the given problem. The parameters correspond to the LS-regime $(\beta_1 + \beta_2 > \sigma + 1)$: $\sigma = 0.5$, $\beta_1 = 1$, $\beta_2 = 2$. The values of the other constants are $\tilde{T}_0 = 10$, $\tilde{T}_1 = \sqrt{105}$, $q_0 = 1$, $\chi_0 = 0.05$ or 0.2.

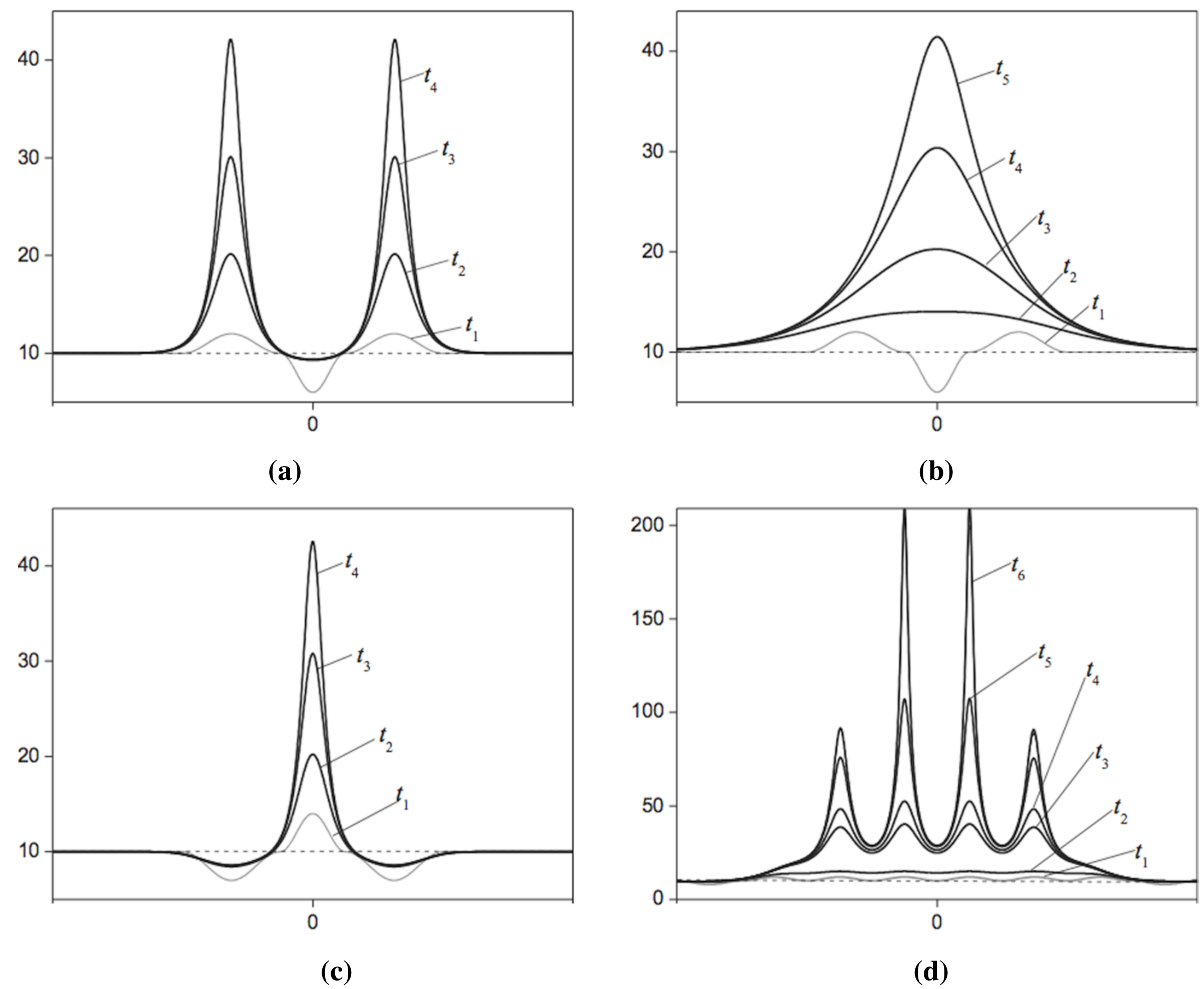

Fig. 3. Some types of structures arising in the LS-regime.

If the initial perturbation contains sections of type 1, where the temperature drops below the background, and sections of type 2, where it rises above the critical level on the background, then on sections of type 2 combustion is triggered with blowup and localized structures form. On sections of type 1, relaxation to the background is observed. Since in the ordinary regime decay is slow, whereas a blowup regime evolves on the adjoining sections, the temperature in type 1 regions hardly rises during the blowup time, when the temperature in the localization region rises by several order of magnitude. As a result, an illusion creates the impression of structures that contain sections in which the temperature drops below the background and does not change until the blowup. Examples of such structures with one and two maxima are shown in Figures 3a and 3c. Structures have been constructed with multiple maxima and minima that drop below the background. Such structure arise if type 2 sections exceed the fundamental length L_{LS} .

4) If the region with the supercritical perturbation is smaller than L_{LS} , then the initial perturbation spreads out in the initial stage of the process and stops only after some time, when the temperature starts increasing vigorously with blowup. In this case, we may observe a structure with a single maximum (Fig. 3b) or a complex structure with multiple maxima (Fig. 3d). We see that equal initial background perturbations in Figs. 3a and 3b result in different dynamics, since the diffusion coefficient in case b is 4 times greater than in case a, and the fundamental length is twice as large (see (7)).

Phenomenological Model of the Initial Flare Heating Phase

1) Blowup Regime. In the study of flares, we have to consider the question of the heating regime, which can be determined from time series data. We usually determine a trend by selecting the best available approximation: linear, power, logarithmic, or exponential (in standard PC software packages). Approximation with a hyperbola is seldom considered, possibly due to its behavior near the asymptote, where it increases without bound on a finite time interval. And yet, it is this hyperbolic behavior that has special interest, because it reflects the nonlinear character of the source and, in particular, corresponds to a regime with blowup.

A differential method has been proposed in [11] to resolve this question. It uses time series data to evaluate the function

$$H(t) = \frac{T_t'}{T}. \tag{10}$$

If $H(t) = \text{const}$, then the process is clearly exponential. If $|H(t)|$ increases, then the process can be conventionally called accelerating; if it decreases, then the process can be called slowing. Temperature profiles obtained from the analysis of X-ray photographs of flares have shown that the lifetime of a flare can be divided into several intervals with different non-exponential heating and cooling regimes [4]. The behavior of both temperature and emission measure was studied in [5], so that the energy balance equation could be applied to investigate the properties of the heating source and some features of flare gas dynamics. It was established that in the early stage the flare goes through accelerated heating with blowup, which is accompanied by an increase of $H(t)$. Once $H(t)$ has reached its maximum value, the heating slows down, i.e., $H(t)$ begins decreasing. In particular, for the flare of 5 July 2009, the first 150 seconds involved accelerated heating with blowup from 0.7 keV to 1.1 keV. Further heating occurred in a slowing-down mode, with $H(t)$ decreasing. This phase was probably governed by radiation cooling and energy losses in hydrodynamic processes. This can be taken as a proof of the existence of blowup in the flare, which is a necessary condition for heat localization.

This conclusion is also important for resolving the question of the flare mechanism. To observe accelerated heating, the power-function volume source $Q(T) \sim T^{\beta}$ should have $\beta > 1$. Subsequent suppression of the blowup regime and transition to slowed heating is determined by the increase of radiation cooling due to the "evaporation" of the chromosphere and increase of the emission measure [5]. Such a heating source with $\beta \approx 3/2$ arises when charged particles are accelerated in a magnetic trap formed by convergent sausage-type instabilities of a magnetic tube [12]. The existence of sausage-type instabilities in flare tubes is suggested by observations of small-scale high-temperature structures [13]. It is assumed that the flare scenario constitutes transformation of the kinetic energy of a perturbation (sausage-type instability) propagating from the photosphere through an arcade of magnetic tubes.

2) Emission Measure and Filling Factor. An important effect was discovered in the flare of 5 July 2009. X-ray observations show that the heating was initially accompanied by a two-stage decrease of the emission measure *EM*,

$$EM = \int n^2 dV, \tag{11}$$

n is concentration, V is volume, by approximately 30% in 80 sec, followed by recovery to the initial level. There are intervals where $EM \sim T^{\gamma}$: in the first interval with $0 < t < 80$ sec, $\gamma = -0.64$; in the second inter-

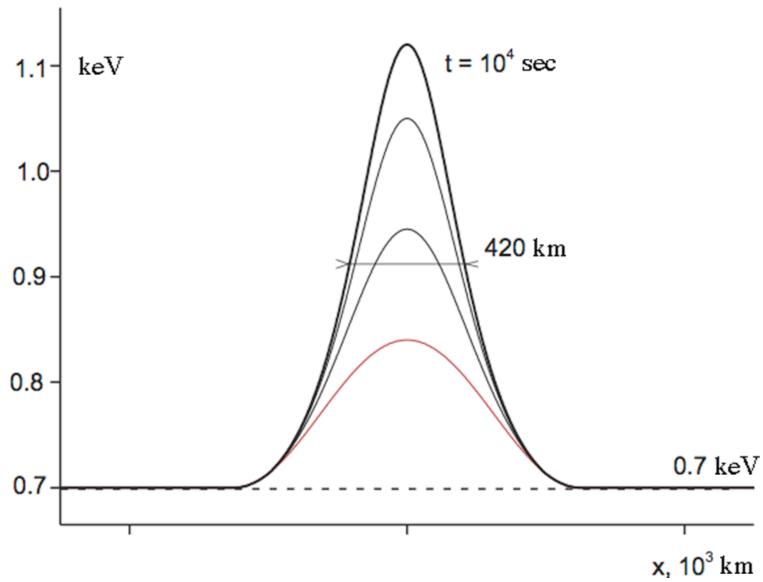

Fig. 4. Structure formation.

val with $80 < t < 150$ sec, $\gamma = 0.6$. The observed effect, in all probability, is not connected with a real decrease in n or V. While the total flare volume and the number of particles are conserved, nonhomogeneous turbulent heating enhances small-scale perturbations reducing their half-width. This reduces the filling factor and is manifested as an emission measure reduction effect. A reduction of the effective emitting volume of the hot region occurs in LS- and S-regimes if the initial perturbation exceeds the fundamental length. Then combustion proceeds with half-width reduction even in the S-regime, until it has shrunk to the size determined by the parameters and the self-similar solution. Hence it follows that if the source has $\beta = 3/2$, the power exponent of thermal conductivity should be less than $\beta \le \beta - 1$, i.e., $\sigma \le 0.5$. This markedly differs from the classical thermal conductivity determined by thermal electrons, where $\sigma = 5/2$ [12]. If we assume weak diffusion with $\sigma \le 0.5$, then we inevitably reach the conclusion of anomalous thermal conductivity of the flare plasma.

3) The Model. The energy equation for the electron component of stationary flare plasma in a magnetic tube can be reduced to a nonlinear heat equation (8). Let us chose the parameters describing solar flares.

The maximum source power exponent is $\beta = 3/2$, and thus $\beta_1 + \beta_2 = 1.5$. Since flare observations suggest decrease of the half-width of spatial structures over time, we take $\sigma \le 0.5$.

Observations further suggest that local flare brightness increases occur on a temperature background of about $T_0 \approx 7 \times 10^6\,\text{K} = 0.7$ keV on spatial scales of the order of several hundreds of kilometers. Local temperature increase up to about 1.1 keV is observed in a finite time. Hence we find for the coefficients $q_0 \approx 7 \times 10^{-4}\,\text{sec}^{-1}$, $\chi_0 \approx 10^3\,\text{cm}^2$. The parameters determining the background and the threshold level are respectively $T_0 = 10^{1/\beta_1}$, $T_1 = 22^{1/\beta_2}$. Figure 4 presents the calculation results for the following parameter values: $\sigma = 0.5$, $\beta_1 = 0.75$, $\beta_2 = 0.75$, $T_1 = 1$, $T_2 = 1.05$.

For comparison with experimental data, the calculations are presented in real space-time scales and for real temperatures. The choice of parameters corresponds to an S-regime with blowup, when the heating region is spatially localized. Using formula (6) to obtain a bound on the fundamental length, we obtain $L_S \approx 150$ m. The characteristic size L_T of the observed flare structures is hundreds of km, i.e., three orders of magnitude

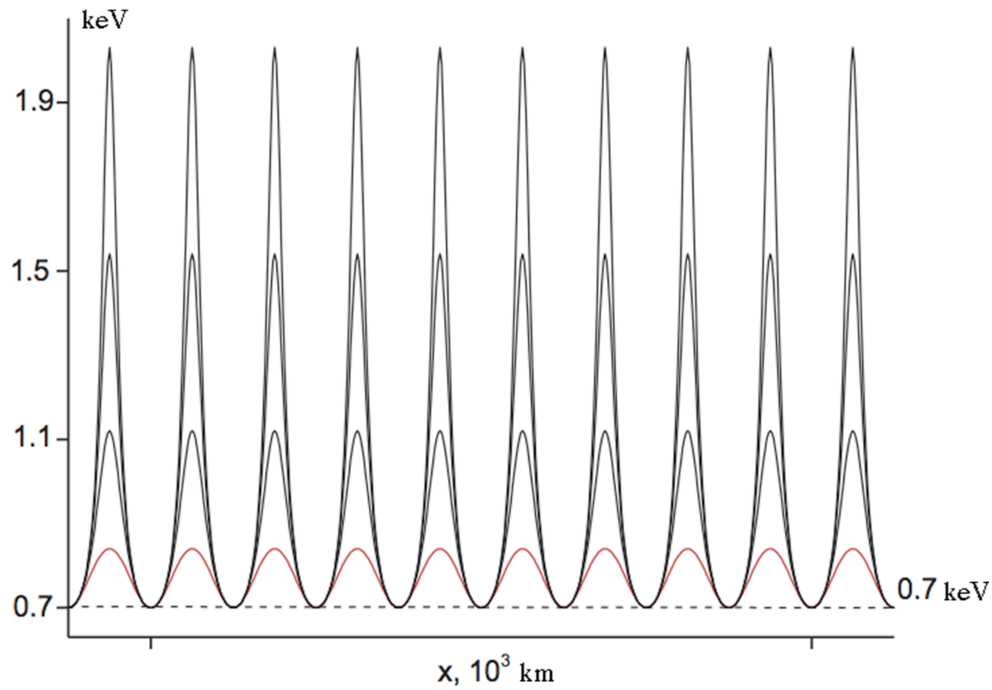

Fig. 5. Periodic structures describing the evolution of temperature perturbations in a flare region.

greater than L_S. This implies that if the initial supercritical perturbation arises in a region of size L_T, then the half-width of the forming structure shrinks over time until it reaches the fundamental length L_S. After that it remains constant, and the solution progressively approaches the self-similar solution (6). Seeing that $L_S << L_T$, this may occur only at the late stage of the process once the amplitude has increased by several orders of magnitude. In our problem, heating from 0.7 keV to 1.1 keV corresponds only to the initial stage of the blowup regime and the half-width of the combustion region contracts, which is manifested as emission measure reduction.

Let us examine the evolution of several flares that appear simultaneously on the background. Since background perturbations are of characteristic size L_T greater than the fundamental length L_S, the combustion region does not spread in the initial stage of the process, as in the converse case (see Fig. 3d); then on each section of the medium where supercritical energy is released we observe development of a single structure with a contracting half-width. In other words, the growth of initial perturbations occurs in the form of autonomously evolving and mutually independent structures. Figure 5 shows the calculated evolution of a periodic initial background perturbation.

Note that the calculations are performed by using MATLAB package over fine meshes (400 – 1800 nodes in x), the time step t is decreased in accordance with the growth in the blowup regime. In order to control reliability of the computed solutions, we use thickening the grid in x.

CONCLUSION

A mathematical model is proposed describing the initial phase of flare heating in the solar corona. The model is based on a nonlinear heat equation with a sign-alternating volume source. This equation is obtained from the energy equation of the electron component of a stationary plasma as the number of variables is reduced due to the empirical relationship of the concentration and temperature $n \sim T^{\gamma}$ in intervals with $\gamma \approx \text{const}$.

It is assumed that the flare arises as a result of sausage-type instabilities in a magnetic tube and formation of collapsing magnetic traps. The convergence of sausage-type instabilities accelerates the charged particles (Fermi mechanism) and creates a heating source with temperature dependence $\sim T^{3/2}$.

A function describing the source has been selected and parameters for a model with nonclassical thermal conductivity have been chosen.

We have performed calculations and investigated the formation of thermal structure that arise in the presence of supercritical perturbations on a homogeneous temperature background.

We have demonstrated the formation of structures during the flare in which the half-width of the energy release region contracts over time. The "emission measure reduction" effect observed at the early phase of the flare is associated with the decrease of the flare filling factor due to half-width reduction of the structures.

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