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ORIGINAL PAPER

A new method for the construction and optimization of quadrangular adaptive well pattern

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Abstract A proper well pattern will have considerable effects on the oil field production, with the ultimate recovery of hydrocarbon enhanced and the water production rate reduced. Comparing with triangular well pattern, quadrangular well pattern has more advantages in some cases and has broader application prospects. However, constructing an optimal quadrangular well pattern is more complicated and has not gain enough research. Facing this situation, a new method of constructing quadrangular adaptive well pattern is proposed in this paper. This quadrangular adaptive well pattern is generated using frontal Delaunay quad-mesh generation method. Boundaries, faults, and existing wells can be constrained and treated as control variables to determine the well spacing, a gradient-based algorithm coupled with reservoir numerical simulator is used to optimize the well pattern. Comparing to conventional regular well pattern, the new well pattern will adjust its shape according to the heterogeneity in different parts of the reservoir, achieving the optimal effect using fewest wells. Two different examples are applied to demonstrate the proposed methodology. The results show that the method proposed can be successfully applied to the construction and optimization of well pattern for large-scale reservoirs and improve the ultimate recovery significantly.

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1 Introduction

In petroleum industry, to maximize the water flooding effect and enhance the oil recovery, injection and production wells are usually arranged into certain geometry to form injection and production units, named well pattern. A good well pattern will effectively reduce oilfield production input, increase stable production period, and enhance oil recovery efficiency as a result. However, acquiring an optimal well pattern has certain difficulty because of the complexities of well pattern design. Numerous factors such as the location of boundaries and faults, the existing wells, as well as static and dynamic uncertainties, must be taken into consideration in the design and optimization of well pattern.

Many efforts have been devoted into the construction and optimization of well pattern. In 1974, Rosenwald and Green firstly used the mixed integer programming to find the optimal well pattern; since then, well pattern optimization models using variety of different algorithms appeared constantly. Bittencourt et al. [1] firstly applied genetic algorithm to well pattern optimization. They built a hybrid genetic algorithm system (HGA) which contains the genetic algorithm and polyhedron and taboo search. Guyaguler et al. [2] used artificial neural networks (ANN) together with genetic algorithm to optimize well pattern, form hybrid optimization framework, and greatly reduce the calculation time. Badru and Kabir [3] expanded Guyaguler's research by applying HGA into horizontal well optimization. Özdogan and Horne [4] put time parameter into well pattern optimization and proposed a quasi-history work flow. Ermolaev and Larionov [5] used ant colony algorithm combining with genetic algorithm to optimize the effective pattern well spacing and well pattern, which reduced the calculation time. Bangerth et al. [6] used simulated annealing and random perturbation for their optimization work. Zandvliet et al. [7] firstly proposed the irregular well pattern optimization method using adjoint gradient algorithm. Maschio et al. [8] combined genetic algorithm and quality map to reduce the combination number in the process of optimization. Onwunalu et al. [9] combined genetic algorithm with statistic agency and reduced more than 90 % of simulation runs. Litvak et al. [10] expanded Onwunalu's work by adding discrete and continuous variables. Wang et al. [11] combined particle swarm optimization (PSO) with retrospective optimization framework to achieve well pattern optimization. Ma et al. [12] used gradient-based stochastic method to optimize horizontal wellbore placement in unconventional gas reservoirs. Mauranda and Barrere [13] used kriging interpolation with quality map to determine the best position and number of injection wells. Chen et al. [14] applies an efficient optimization technique based on a multivariate adaptive regression spline (MARS) technique to determine the optimal design of well pattern. Humphries and Haynes [15] combined a stochastic global algorithm (particle swarm optimization) and mesh adaptive direct search to compare several simultaneous and sequential approaches to the joint placement and control problem. Ding et al. [16] studied a couple of optimization algorithm and proved that the standard particle swarm optimization algorithm (SPSO) is effective and the applicability of modified particle swarm optimization algorithm and quality map method (QM + MPSO) is promising.

In the oil field practice, well patterns are usually classified into four-, five-, seven-, and nine-spot well patterns as well as row well pattern based on the arrangement of injection wells and production wells. And all of the well patterns above can be derived from two basic types: triangular and quadrangular well patterns. In our recent paper [17], a method of construction and optimization of triangular well pattern has been developed. However, comparing with triangular well pattern, quadrangular well pattern has many advantages. For example, quadrangular well pattern can be easily transformed into different types of well pattern by simply changing some production wells into injection wells, or by inserting new wells (shown in Fig. 1), while these transformation will be difficult for a triangular well pattern. So, quadrangular well pattern owns more flexibility compared with triangular well pattern and it is in favor of well pattern adjustment for long-term production.

However, comparing with triangular well pattern, acquiring an optimal quadrangular well pattern is far more difficult because generating quadrangular meshes requires more complicated algorithm. So far, there is not much research on the quadrangular well pattern's construction and optimization, and research on the adaptability of quadrangular well pattern is even scarce. Facing this situation, a new method for the construction of quadrangular adaptive well pattern is proposed in this paper, which can construct optimal quadrangular well pattern according to the characteristic of the reservoir. The frontal Delaunay quad-mesh generation method is employed to build quadrangular meshes, then well pattern will be obtained with injection wells placed on the vertices and production wells inside of the quadrangular meshes.

Detailed explanation about well pattern construction will be shown in Section 2. After the well pattern is constructed, an optimization model which accounts for the profit of hydrocarbon and cost of drilling and producing is established to optimize the well pattern. The heterogeneity of the reservoir will be taken into consideration in the model to help determine the optimal well spacing. So, the optimal well pattern obtained will have varying well spacing which adapts to the heterogeneity of the reservoir, achieving optimal effect using fewest wells. Details about the optimization method will be shown in Section 3. This paper also uses two examples to testify the effect of the model in heterogeneous reservoir. Results are presented in Section 4.

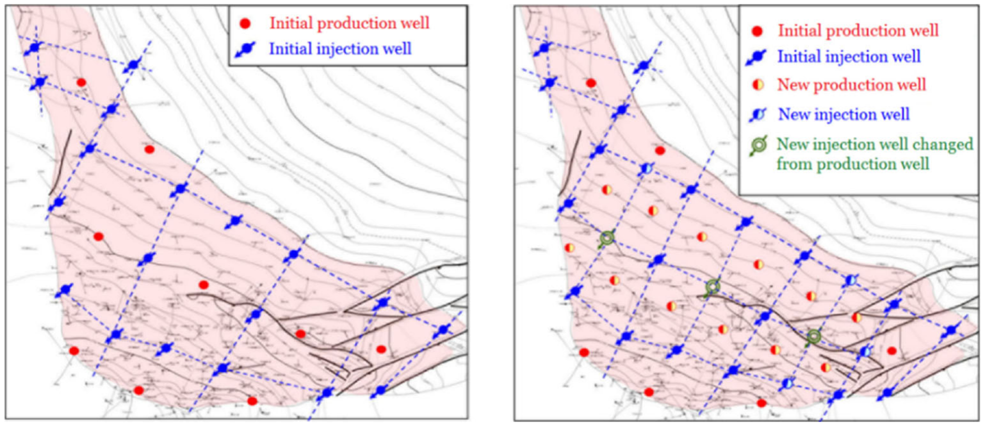
2 The construction of quadrangular adaptive well pattern

To construct quadrangular well pattern, a given domain which represents the reservoir should be firstly subdivided into a series of quadrangular meshes, then wells will be placed onto the vertexes of the meshes. Several factors should be taken into consideration to get the optimal well pattern arrangement.

2.1 A brief introduction to quadrangulation

In this research, a quad-mesh generation method derived from computer graphics is adopted. So far, there exist essentially two approaches to generate quad-meshes: direct method [18, 19] and indirect method [20, 21]. The indirect method firstly subdivides the given domain into several triangles then combines these triangles into quadrangles. Comparing with direct method, indirect methods have the advantage to rely on triangle meshing algorithms that are relatively simple to implement and that have some mathematical properties that allow faster building.

In 2006, Tchon and Camarero proposed an indirect method for generating quad-meshes [22]. In their approach, they started from equilateral triangles and applied vertex relocation that aims at aligning vertices on prescribed directions to form quadrangles (shown in Fig. 2). However, this method leads to a large amount of calculation and some



(a) Well pattern before adjustment (b) Well pattern after adjustment

Fig. 1 Example of quadrangular well pattern in real reservoir

vertexes become redundant and have to be removed during the relocation. In 2010, Remacle, Henrotte, and Baudouin improved the method by generating Delaunay right-angled triangles using L^∞-norm, namely: frontal Delaunay quad-mesh generation method [23]. By using this method, right-angled triangles are directly generated and the process of triangle transformation is omitted, so the computational efforts significantly decreased and no vertexes are relocated and removed. In our study, this method is adopted to generate quad-meshes.

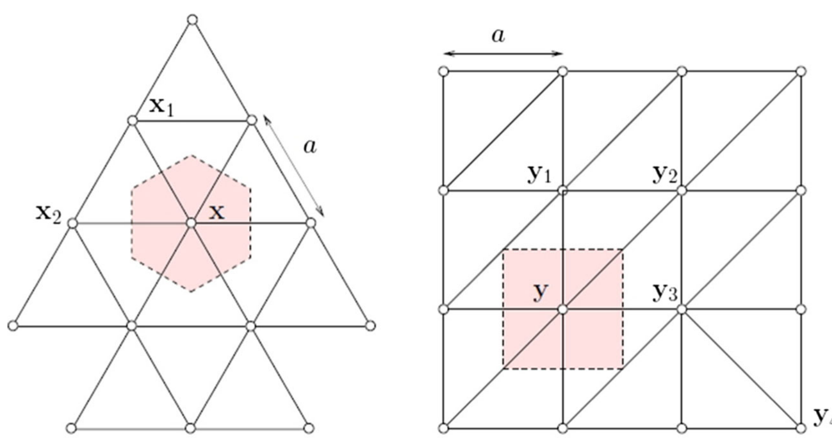
In the R^2 plane, the distance between two points $\mathbf{X}_1(x_1, y_1)$ and $\mathbf{X}_2(x_2, y_2)$ is usually based on the Euclidean norm (L^2-norm). Other distances can be defined based on other norms:

- The L^1-norm distance: $\|\mathbf{x}_2 - \mathbf{x}_1\|_1 = |x_2 - x_1| + |y_2 - y_1|$,
- The L^2-norm distance: $\|\mathbf{x}_2 - \mathbf{x}_1\|_2 = (|x_2 - x_1|^2 + |y_2 - y_1|^2)^{1/2}$,
- The L^p-norm distance: $\|\mathbf{x}_2 - \mathbf{x}_1\|_p = (|x_2 - x_1|^p + |y_2 - y_1|^p)^{1/p}$,
- The L^∞-norm distance: $\|\mathbf{x}_2 - \mathbf{x}_1\|_\infty = \lim_{p\to\infty} \|\mathbf{x}_2 - \mathbf{x}_1\|_p = \max(|x_2 - x_1|, |y_2 - y_1|)$,

Shown in Fig. 2, comparing with the equilateral triangle, the right-angled triangle is with different side lengths in L^2-norm, so it will not be generated by ordinary Delaunay triangulation method [24–28]. However, if distances between points are measured in the L^∞-norm, side lengths of the right-angled triangle in Fig. 2b will be equal: $\|\mathbf{y} - \mathbf{y}_1\|_\infty = \|\mathbf{y} - \mathbf{y}_2\|_\infty = a$. On this basis, the Delaunay triangulation method can be used to generate right-angled triangles in the L^∞-norm.

The "cross field" [29] is a parameter of each point $\mathbf{u}(x, y)$ in the domain, and it is adopted to control the orientation of the quadrangles. As shown in Fig. 3, in two dimensions, cross field of a point is controlled by the angle $\theta(\mathbf{u})$ that defines the orientation of a local frame at each point of the domain [23]. Quadrangles generated will be aligned with the cross field.

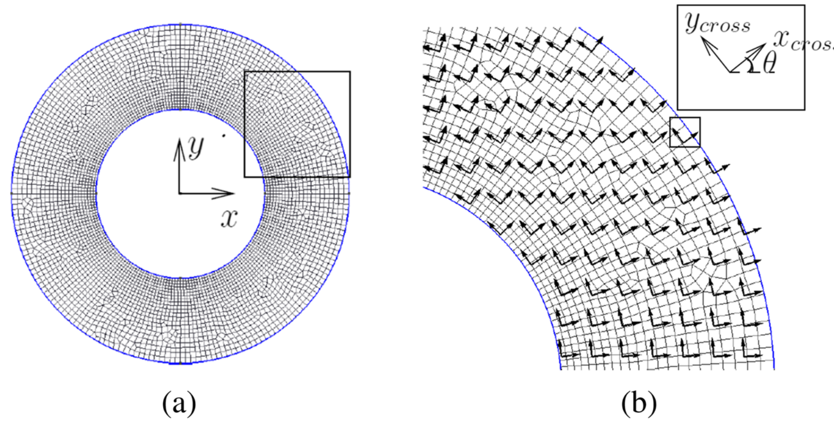
As Fig. 4 shows, constrained by the cross field and the L^∞-norm distance, the Delaunay right-angled triangles will



(a) (b)

Fig. 2 Equilateral triangles (**a**) and right-angled triangles (**b**)

Fig. 3 Meshes (**a**) and cross fields (**b**) in an annular domain

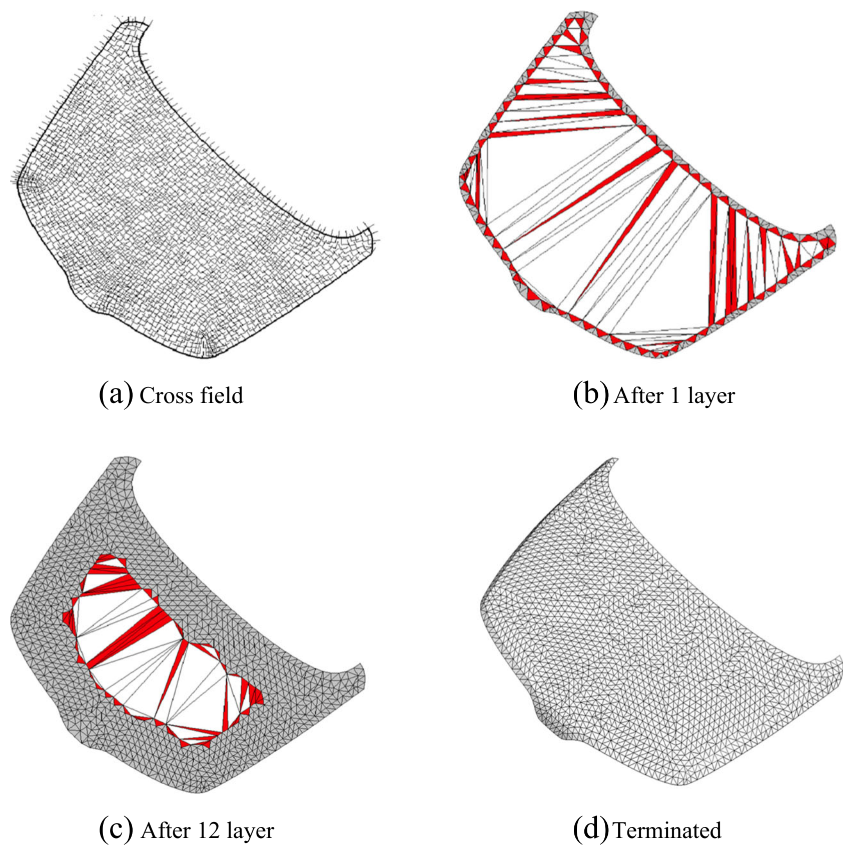


be generated paralleling to the adjacent boundary layer by layer [23]. Because of the complex boundary conditions, it is impossible to make sure all the generated triangles to be right-angled triangles, so some little error is allowed. After all, the domain is divided by the triangles, two adjacent right-angled triangles which share the same bevel edge will be combined together and the shared bevel edge will be removed. The whole procedure proceeds as follows, and the schematic figure of it is shown visually in Fig. 5:

1) Set initial points of the domain (Fig. 5a).
2) Connect adjacent initial points into a loop to define the domain to be quadrangulated, and the lines connecting the points are defined as initial boundaries (Fig. 5b).
3) Subdivide the initial boundaries into several short segments, and a series of new points are inserted among these segments (Fig. 5c).
4) Subdivide the domain into Delaunay right-angled triangles with these boundary points as vertexes (Fig. 5d).
5) Combine two adjacent triangles into one quadrangle and delete the diagonal of each quadrangle (Fig. 5e).

According to the procedure above, a set of quadrangular meshes are generated. As is mentioned above, because of the boundaries, it is impossible to make all the squares

Fig. 4 The process of frontal Delaunay right-angled triangle generation (*white triangles are waiting triangles, red triangles are active triangles and grey triangles are resolved triangles*)



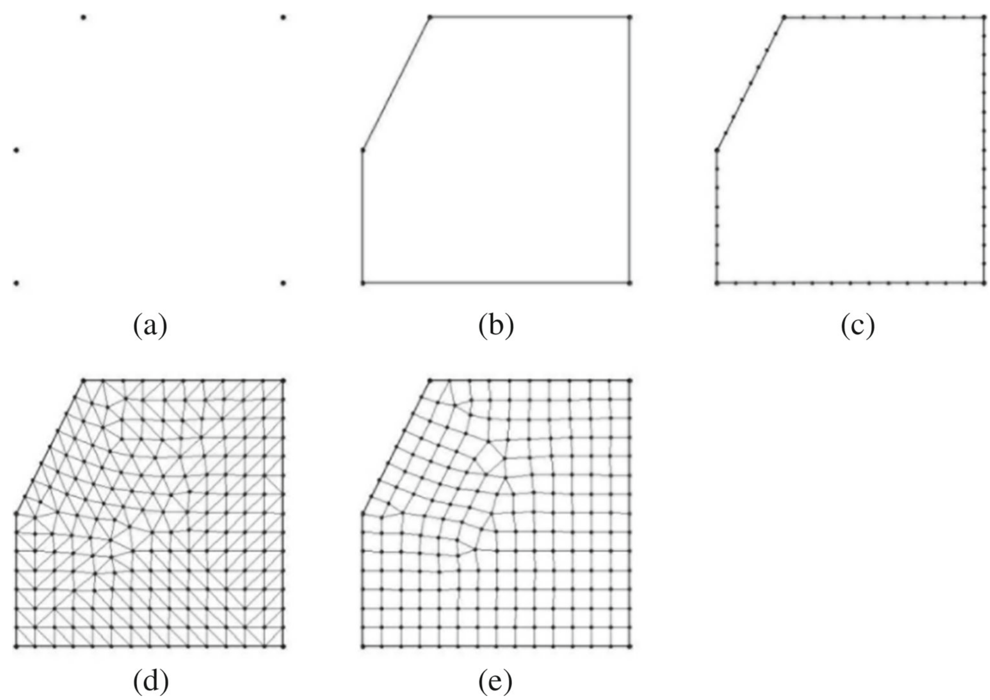


Fig. 5 Example of frontal Delaunay quad generation

parallel to each other. So, some individual non-right angled triangles should be kept to fill the gap.

2.2 Construction of adaptive quadrangular well pattern

When the frontal Delaunay quadrangulation method is used to construct well pattern, the reservoir is firstly approximated by a two-dimensional domain with a set of boundaries and then, the domain is subdivided into several quad-meshes. After the quad-meshes are acquired, injection wells are placed on the vertexes of quadrangles and production wells are placed on the centroid of them. Thus quadrangular well pattern is constructed.

To get an optimal well pattern, well spacing in different parts of it should be adjusted to fit the reservoir condition there. The algorithm above can adjust the density of quad-meshes by controlling side length of quadrangles. When defining the domain, every initial point possesses a value which controls the distance between it and the adjacent points. The bigger the value, the larger the distance will be. If the initial points have different values, the distance between the new inserted points will vary smoothly. When quad-meshes are generated according to these points, quadrangles will have gradually varied side length. Thus, a heterogeneous well pattern is established.

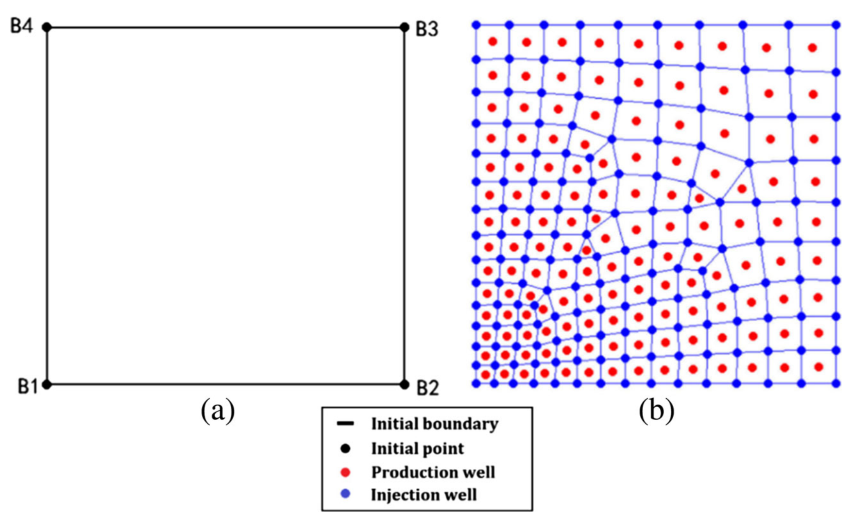


Fig. 6 Example of Delaunay quad generation with different boundary points' value

Table 1 Values of boundary points

Initial boundary points	Values
B1	100
B2	200
B3	300
B4	200

An example can be seen in Fig. 6. As shown in Fig. 6a, a simplified square domain (side length = 2000 m) with four initial boundary points is defined. These initial boundary points have different values (shown in Table 1), which control the distance between them and the nearest points. The well pattern generated based on these control values can be seen in Fig. 6b. It can be seen that well spacing is affected by values of initial boundary points significantly.

Besides boundaries, real reservoirs will usually have faults. And in some developed reservoirs, the existing well pattern consists of previously drilled injection and production wells which can not be ignored. So, in the process of generating well pattern, faults and existing wells in the domain should be taken into consideration as fixed factors. That is, regardless of the variation of well pattern, some of the sides and vertexes of the quadrangles in the domain are fixed. To account for these fixed factors, the faults are treated as special boundaries and existing producers and injectors are treated as special points. These special boundaries and points together with initial boundaries form a complete domain to be quadrangulated. The construction of well pattern with faults and existing injection and production wells will be shown in following subsections respectively.

2.2.1 Treatment of faults

An example of constructing well pattern in a reservoir with a fault is shown in Fig. 7a. A simplified square domain with four boundaries represents the reservoir, and a black line in the domain represents the fault. To subdivide the domain into quad-meshes, the fault shall be treated as a kind of special inner boundary which overlapped together. Thus, a quadrangular well pattern is acquired by the algorithm mentioned above, based on boundaries and faults. (shown in Fig. 7b).

2.2.2 Treatment of existing injection wells

Figure 8a shows a simplified model of reservoir with existing injection wells. As the model described above, the square represents the reservoir and the two points represent the existing injection wells. When dealing with this situation, these two points can also be treated as special inner boundaries which consist of only one point. Together with outer boundaries, the points are added to the initial definition of the domain. Similarly, Delaunay quadrangular meshes based on boundaries and points can be generated smoothly, quadrangular well pattern which fits the domain perfectly is acquired. It can be seen from Fig. 8b that vertexes of quadrangles lies on the fixed points precisely. So, the well pattern generated can be constrained by the existing injection wells.

2.2.3 Treatment of existing production wells

The treatment of existing production wells is more complicated than faults and injection wells, for the production wells located neither on the side nor the vertex of the

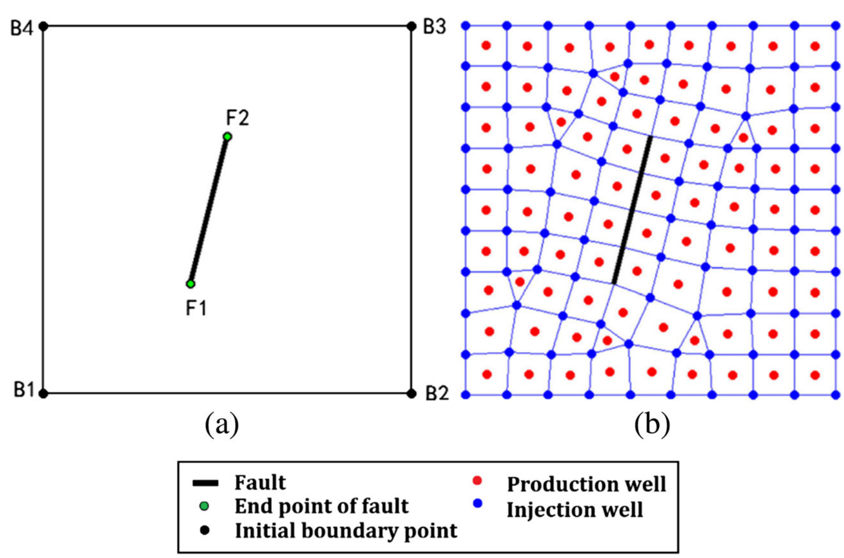


Fig. 7 Quadrangular well pattern generated on the basis of fault

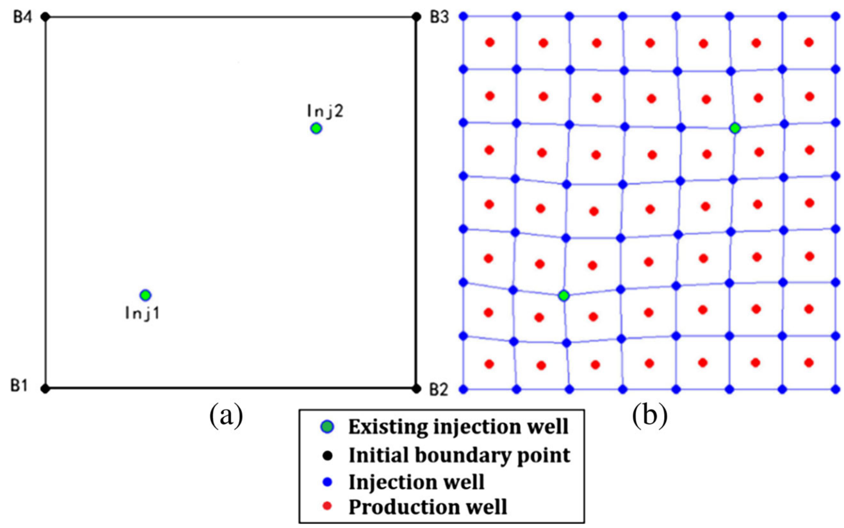


Fig. 8 Quadrangular well pattern generated on the basis of existing injection wells

quadrangles but inside them. So, they can not be treated the same as injection wells above. However, the production well can be regarded as the centroid of a quadrangle that consists of four injection wells. According to this point, we can define a quadrangle with four injection wells on the vertexes based on one production well. By this way, the existing production well can be transformed into four existing injection wells. As a result, the well pattern can be generated by the algorithm mentioned above.

Figure 9 shows the method to transform a production well into four injection wells. Fig. 9a shows a simplified domain within two points which represent the existing production wells. In Fig. 9b, two squares which regard the points of production wells as centroids are defined. Side lengths of the squares are decided by values of the production wells. To decide the orientation of each square, distances of each producer point to all the inner and outer boundaries are compared and the square is made parallel to the nearest boundary. Then, place four injection wells at the vertexes of the square. Thus, an existing production well is transformed into four existing injection wells. Shown in Fig. 9c, well pattern can be established smoothly in this domain.

An example of a domain is shown in Fig. 10a to examine the effect of quadrangular well pattern's generation in complicated reservoir. A fault with an existing injection well and a production well are inside the domain. The side length of the domain is 2000 m and the initial points (the boundary points, the endpoints of fault as well as the producer and the injector points) are assigned with different values (shown in Table 2). The well pattern generated based on the initial values is shown in Fig. 10b. Figure 10 shows that the well pattern generated adapts to the complicate condition perfectly. The fault, as well as the existing wells, is constrained precisely, and well spacing in different parts of it can be controlled by values of the initial points.

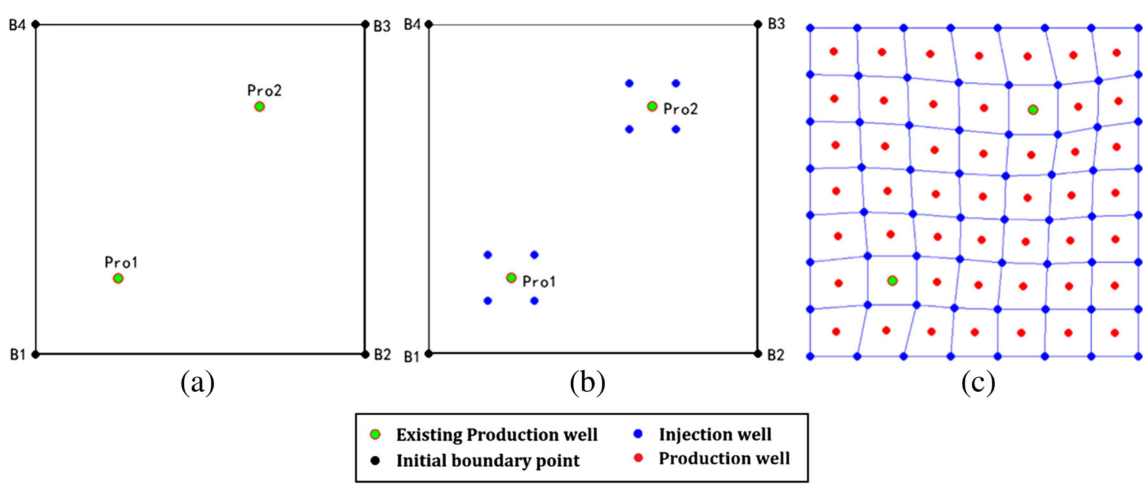


Fig. 9 Quadrangular well pattern generated on the basis of existing production wells

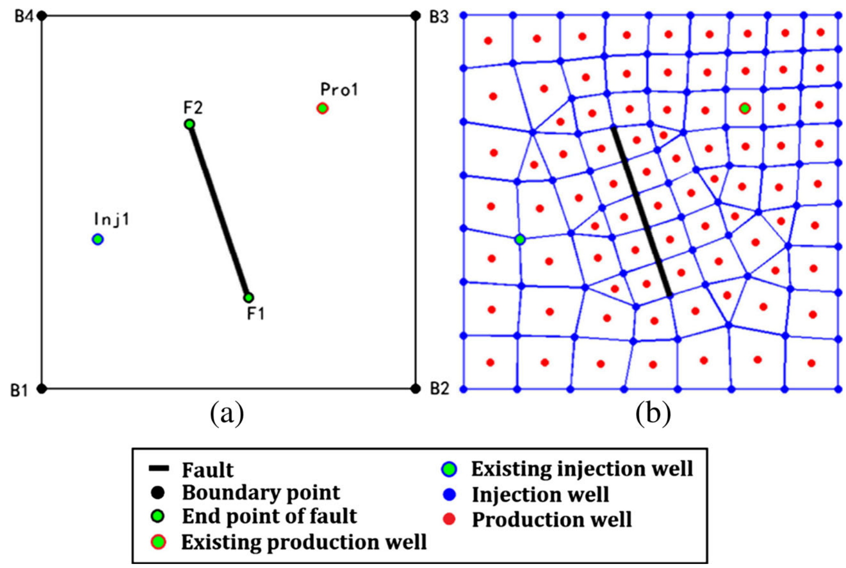


Fig. 10 Quadrangular well pattern generated in complicated reservoir

3 The Optimization model

In this study, the final purpose is to construct an optimum and adaptive quadrangular well pattern in a heterogeneous reservoir, achieving the optimal effect using the fewest wells. And the optimal well pattern can be acquired by assigning appropriate values to the initial points. To get this appropriate values, an optimization model of quadrangular well pattern is established. Details are described as follows.

3.1 Objective function

During the optimization, the net present value (NPV) [30] is chosen to be the objective function. NPV is an economic standard method for evaluating a developing plan. It is defined as the difference value between the net cash flow of a developing plan and its original investment. The larger the NPV is, the more profit the plan will get. A commercial numerical simulator is employed to generate water and oil production which is used as parameters of the function to calculate NPV. The function is defined as follows:

$$J^n(q_o^n, q_w^n, q_{\text{inj}}^n) = \sum_{n=1}^{N_t} \left\{ \sum_{j=1}^{N_{\text{prod}}} \left(\frac{r_o q_o^n, j - r_w q_w^n, j}{(1+b)^{t^n/365}} \right) - \sum_{j=1}^{N_{\text{inj}}} \left[\frac{r_{\text{inj}} q_{\text{inj}}^{n,j}}{(1+b)^{t^n/365}} \right] \right\} \times \Delta t^n - (N_{\text{prod}} C_o + N_{\text{inj}} C_w) \tag{1}$$

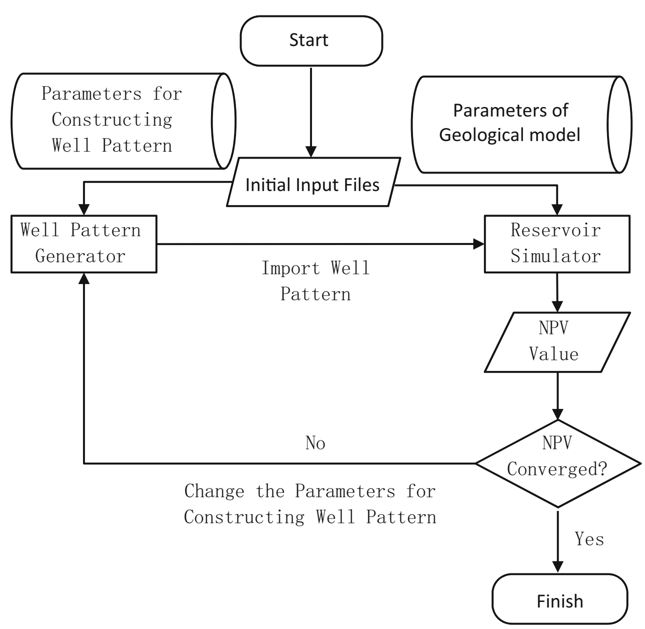


Fig. 11 Schematic diagram of optimization procedure

Table 2 Values of initial points

Initial points	Values
B1	300
B2	300
B3	150
B4	300
F1	150
F2	150
Inj1	300
Pro1	150

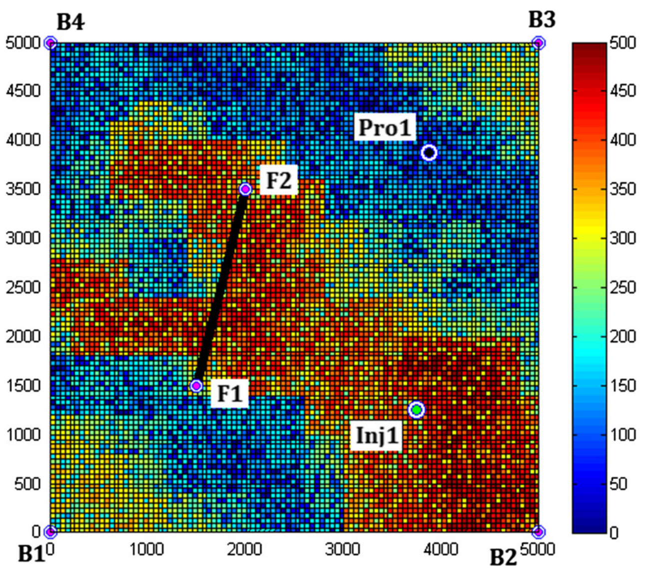


Fig. 12 Initial conditions of the reservoir

In the function above, J^n represents the NPV value of the nth simulation step; N_t represents the total number of reservoir simulation time step; N_{inj} represents the total number of injection wells; N_{prod} is the total number of production wells; r_o in US dollars/stock tank barrel (STB) is the revenue of oil per unit volume; r_w in US dollars/STB is the cost of water disposal per unit volume; r_{inj} in US dollars/STB is the cost of water injection per unit volume; $q_{o,j}^n$ and $q_{w,j}^n$ represents the average oil and water rates of the jth production well over the nth simulation time step; C_o and C_w represents the cost of drilling and completing of a production well and an injection well, respectively; and b represents the annual discount rate. It is assumed that no gas is produced during the production.

Table 3 Parameters of reservoir model and their values

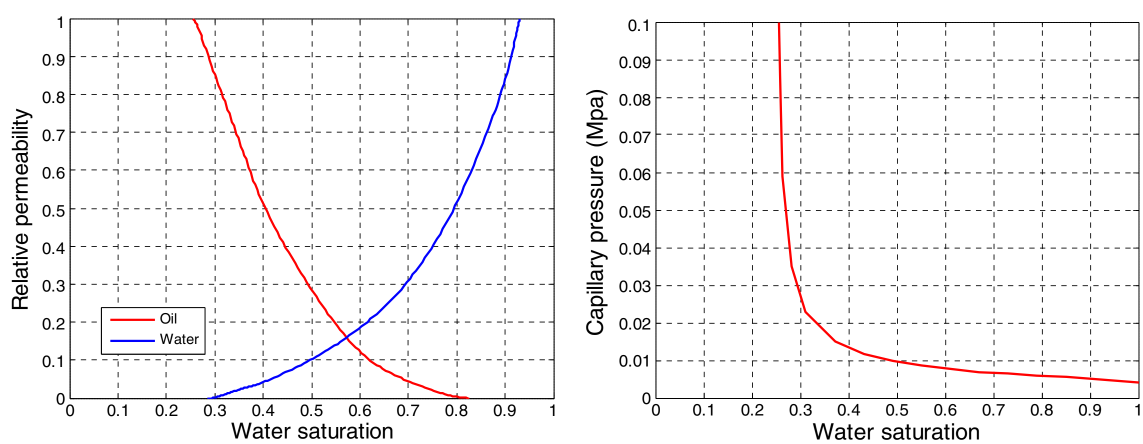
Parameters	Value
The price of crude oil	US$50
The cost of disposition of produced water	US$5
The cost of injecting water	US$5
The cost of drilling a well	US$5 × 10^5

3.2 Optimization variables

The variables to be optimized are the values assigned to the initial points. Just as the example shown in Fig. 6, values assigned to the initial points will affect the density of meshes generated around them; thus, the shape of well pattern will be affected accordingly. The whole reservoir will be taken into consideration when calculating these variables. In some cases, values of some initials points will be extremely small or large during the optimization, so the wells generated around the points will be too fine or too coarse. There is no inappropriateness in the perspective of function, but for a real reservoir, too fine or too coarse well pattern is unacceptable. So, upper and lower limits should be set for the variables, and they can be determined by practical experience.

3.3 Description of optimization algorithm

To do the optimization by computer, the analytical function above is transformed into numerical function, and the steepest ascent method is selected to be the algorithm to calculate the optimum variables. Suppose n initial points are defined in the domain, with the values of $x_1, x_2, \cdots x_n$. These values can be defined as an optimization vector $\mathbf{x} =$



(a) Relative permeability (b) Capillary pressure

Fig. 13 The relative permeability and the capillary pressure

Table 4 Initial points and their initial values

Initial points	Initial values
B1	425
B2	425
B3	425
B4	425
F1	425
F2	425
Inj1	425
Pro1	425

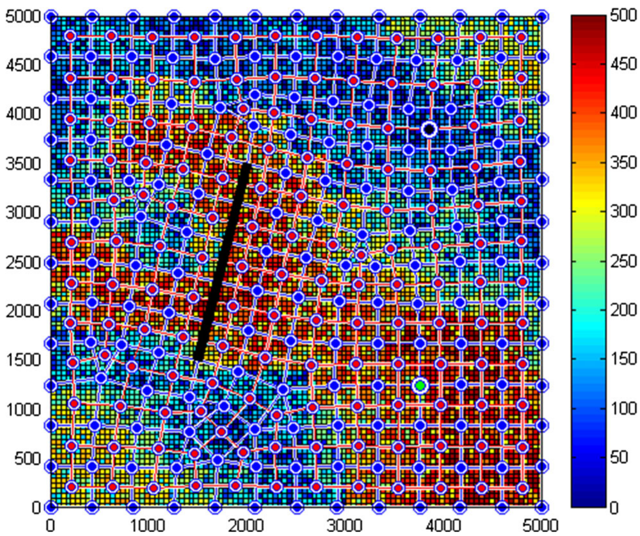


Fig. 14 Initial quadrangular well pattern

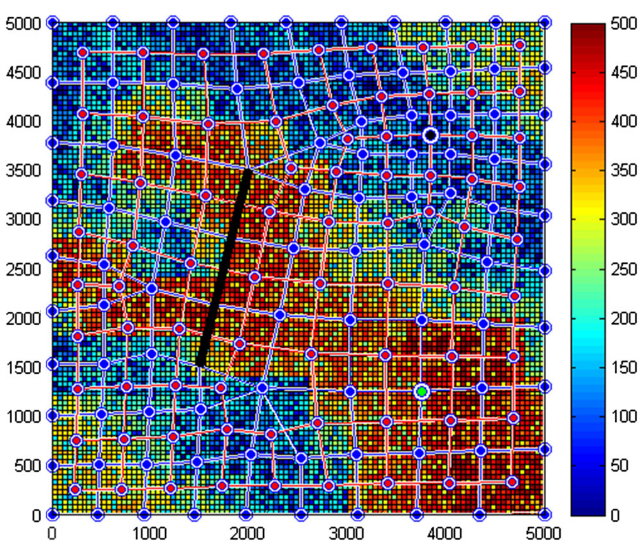


Fig. 15 Optimal quadrangular well pattern

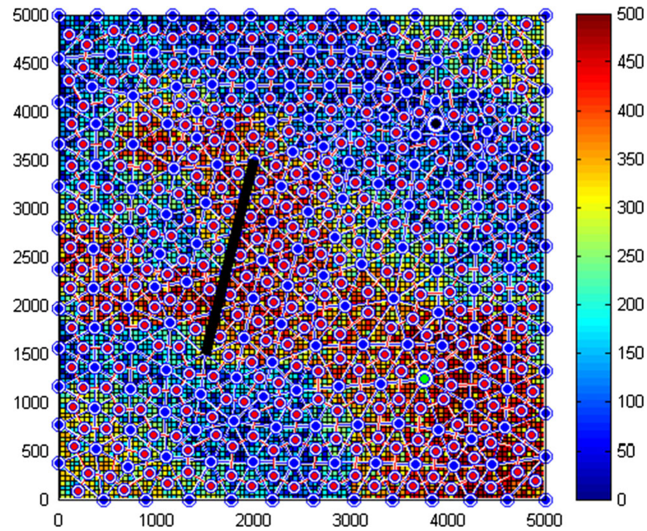


Fig. 16 Initial triangular well pattern

$(x_1, x_2, \cdots x_n)$. The objective function NPV can be written as $J = f(\mathbf{x})$, and finite difference method is used to calculate the gradient of this function. The derivative of the function J on variable x_i can be acquired by the following method:

$$\frac{\partial J}{\partial x_i} = \frac{J(x)\,|_{x_i+\delta x_i} - J(x)\,|_{x_i}}{\delta x_i}, \quad i = 1,\ 2,\ \cdots n \tag{2}$$

where δx_i represents the perturbation step.

After the gradient of objective, function J on $\mathbf{x}_k$ has been acquired (k represents the kth iteration); the steepest ascent method should be used to find a better location. The equation is as follows:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \nabla_{\mathbf{x}_k} J \tag{3}$$

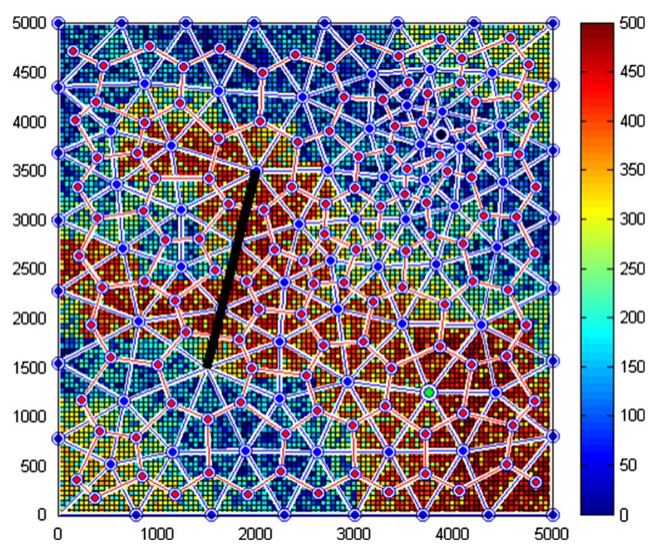


Fig. 17 Optimal triangular well pattern

Table 5 Values of optimization variables during optimization (Quadrangular well pattern)

Optimization variables	Limits of optimization variables	Initial values	Values in step 1	Values in step 2	Values in step 3	Values in step 4	Values in step 7	Values in step 9	Values in step 10
1 (B1)	(250,800)	425.00	470.71	496.03	498.13	498.26	498.28	498.25	498.25
2 (B2)	(250,800)	425.00	680.86	740.04	749.85	751.69	753.35	753.35	753.38
3 (B3)	(250,800)	425.00	461.94	467.22	493.58	503.30	506.20	506.20	506.20
4 (B4)	(250,800)	425.00	469.57	523.78	578.81	624.04	627.12	652.22	652.22
5 (F1)	(250,800)	425.00	443.47	559.64	619.19	628.67	648.82	650.10	650.10
6 (F2)	(250,800)	425.00	470.93	573.66	654.12	750.88	777.76	778.74	778.88
7 (Inj1)	(250,800)	425.00	452.18	452.03	521.01	700.45	719.94	720.62	720.62
8 (Pro1)	(250,800)	425.00	422.18	415.01	403.66	401.20	398.71	398.45	398.45

where α_k represents the step length for the kth iteration; $\nabla_{\mathbf{x}_k} J$ represent the gradient of function J on the variable vector $\mathbf{x}_k$, defined as $\nabla_{\mathbf{x}_k} J = \left[\frac{\partial J}{\partial x_1}, \frac{\partial J}{\partial x_2}, \cdots, \frac{\partial J}{\partial x_n}\right]_k^T$,

Log transformation will be adopted to eliminate the constraints when lower and upper limits exist. Suppose the lower and upper limits for variables $x_i,\ i = 1,\ 2,\ \cdots n$ are $x_i^{\text{floor}},\ i = 1,\ 2,\ \cdots n$ and $x_i^{\text{ceil}},\ i = 1,\ 2,\ \cdots n$; then, the log transformation of x_i can be done as follows:

$$s_i = \log_{10}\left(\frac{x_i - x_i^{\text{floor}}}{x_i^{\text{ceil}} - x_i}\right),\quad i = 1,\ 2,\ \cdots n \tag{4}$$

After the log transformation, the range of variable $s_i,\ i = 1,\ 2,\ \cdots,\ n$ becomes the whole real number space, and the gradient of function J on the transformed variables $s_i,\ i = 1,\ 2,\ \cdots,\ n$can be acquired by the similar equation:

$$\frac{\partial J}{\partial s_i} = \frac{J(x)\left|_{s_i+\delta s_i} - J(x)\right|_{s_i}}{\delta s_i},\quad i = 1,\ 2,\ \cdots\ n \tag{5}$$

The steepest ascent method can also be used to deal with the transformed variable vectors$\mathbf{s}_k$, $k = 1,\ 2,\ \cdots,\ n$, the equation is as follows:

$$\mathbf{s}_{k+1} = \mathbf{s}_k + \alpha_k \nabla_{\mathbf{s}_k} J \tag{6}$$

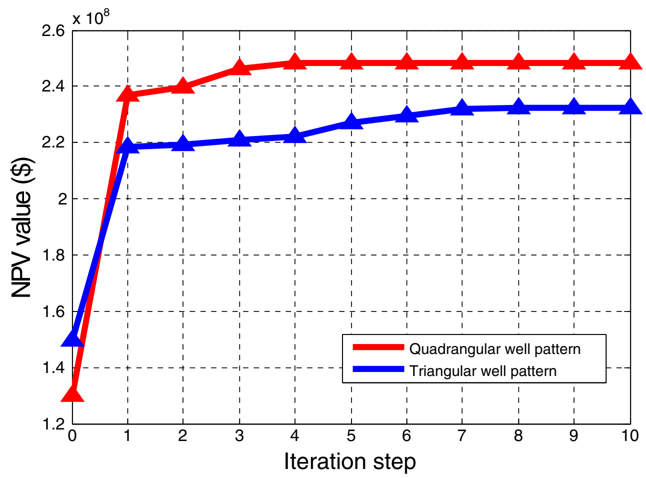
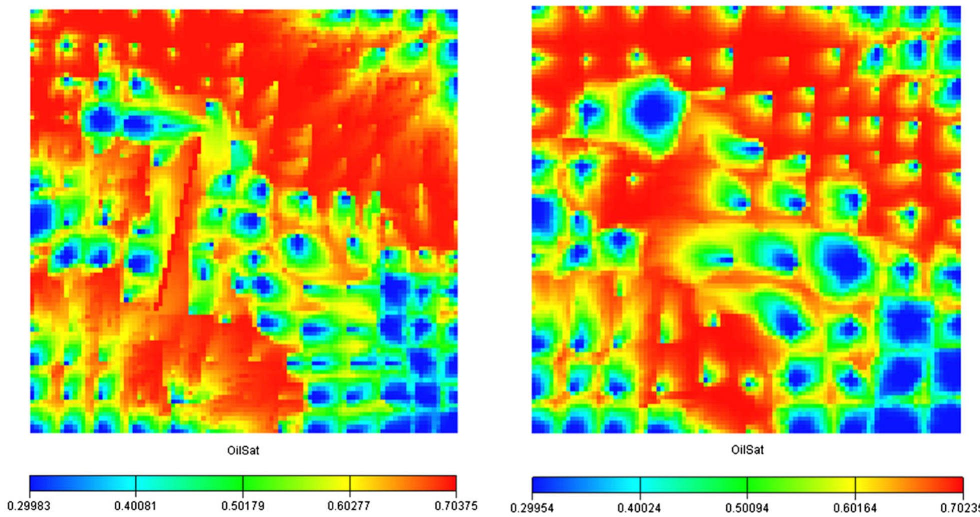


Fig. 18 NPV values during iteration steps

Table 6 Values of optimization variables during optimization (Triangular well pattern)

Optimization variables	Limits of optimization variables	Initial values	Values in step 1	Values in step 2	Values in step 3	Values in step 4	Values in step 7	Values in step 9	Values in step 10
1 (B1)	(250,800)	425.00	701.39	759.88	774.98	782.51	786.87	786.87	786.87
2 (B2)	(250,800)	425.00	726.41	765.34	800.00	800.00	800.00	800.00	800.00
3 (B3)	(250,800)	425.00	608.78	380.52	696.28	800.00	800.00	800.00	800.00
4 (B4)	(250,800)	425.00	695.09	746.68	783.55	799.88	800.00	800.00	800.00
5 (F1)	(250,800)	425.00	511.55	648.79	756.87	798.49	799.82	799.82	799.82
6 (F2)	(250,800)	425.00	499.99	524.87	543.72	693.76	762.15	762.15	762.15
7 (Inj1)	(250,800)	425.00	541.27	642.38	714.17	714.14	668.20	668.20	668.20
8 (Pro1)	(250,800)	425.00	400.00	399.96	399.81	399.65	399.72	399.72	399.72

Fig. 19 Distribution of residual oil saturation for quadrangular well pattern



(a) Initial well pattern (b) Optimal well pattern

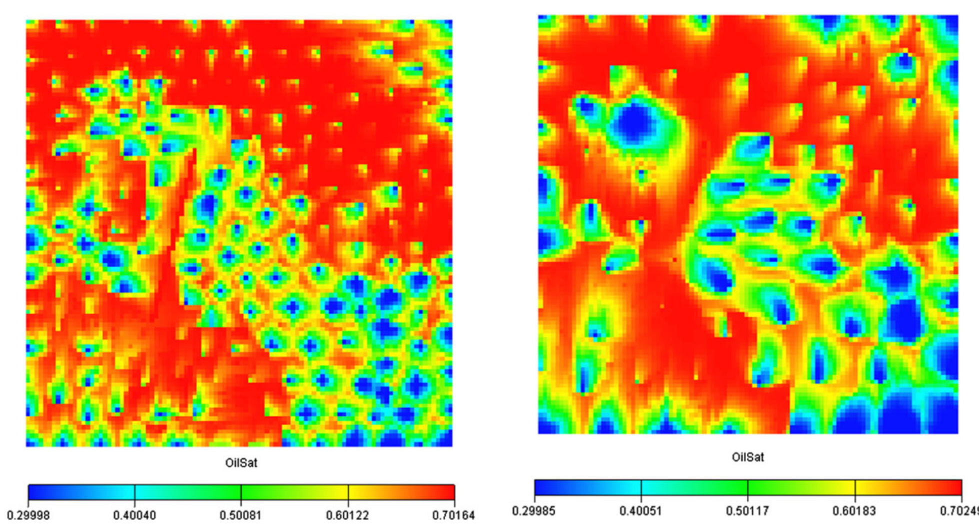
After the transformed variables are updated in an iterative step, they should be converted back to the values of real domain and then be used in the calculation of objective function.

3.4 The workflow of the construction and optimization of adaptive well pattern

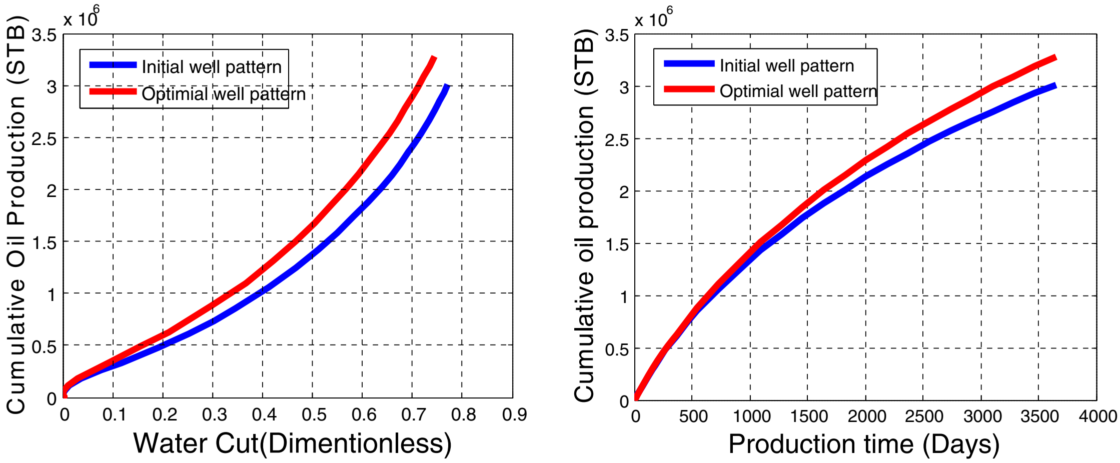
In this study, a set of computer programs in FORTRAN language are written to realize the function of constructing adaptive well pattern based on values of initial points as well as optimizing well pattern based on the objective function. After values of initial points are set, the optimization process will proceed automatically according to the instruction of the FORTRAN programs until the optimal values are obtained. The whole workflow of the optimization is as follows, and it is also shown in Fig. 11 visually.

1) Define a reservoir domain with initial points and boundaries as well as other necessary parameters, initial values x_i, $i = 1, 2, \cdots n$ are assigned to the initial points, respectively.
2) Establish an initial quadrangular well pattern based on these initial points and boundaries, well spacing in different parts of the well patter is controlled by the initial values.
3) Match the locations of wells into the grids of reservoir numerical model in the commercial numerical simulator, and calculate the initial NPV value J_1 based on the parameters obtained from the simulation.
4) Use the steepest ascent method to get a set of new values for the optimization variables x_i, $i = 1, 2, \cdots n$,

Fig. 20 Distribution of residual oil saturation for triangular well pattern



(a) Initial well pattern (b) Optimal well pattern



(a) Field cumulative oil production with respect to water cut (Quadrangular well pattern)

(b) Field cumulative oil production with respect to production time (Quadrangular well pattern)

Fig. 21 Production parameters for quadrangular well pattern

back to step 2, establish an new well pattern then calculate new NPV value $J_i, i = 2, \cdots n$.

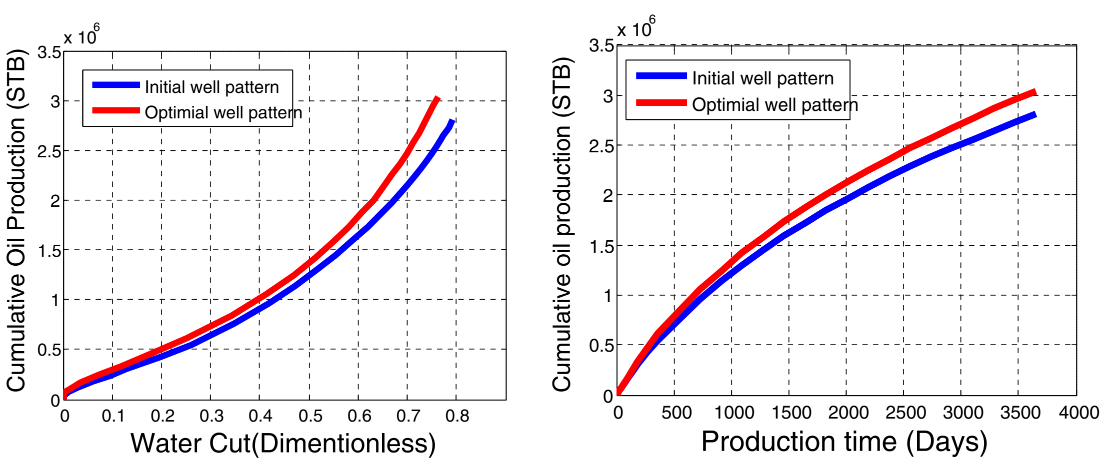
5) Repeat the previous steps until the NPV value converges.

4 Examples

Two examples are shown in this section to examine the reliability of the methodology established in this study. Well patterns will be generated and optimized on the basis of complicated heterogeneous reservoirs with faults and existing wells. Initial and optimal well patterns will be exhibited. The variation of NPV values and parameter values during the optimization will be shown and production parameters before and after the optimization will be compared.

4.1 Example 1: optimization of well pattern under the constraints of faults and existing wells

In this example, a two-dimensional heterogeneous reservoir is used to examine the reliability of the model. Shown in Fig. 12, the reservoir is a square domain, with the side-length of 5000 m. To facilitate the simulation, the domain is discretized into $100 \times 100 \times 1$ blocks, each of which is with the size of 50 m $\times$ 50 m $\times$ 30 m. The color of each block



(a) Field cumulative oil production with respect to water cut (Triangular well pattern)

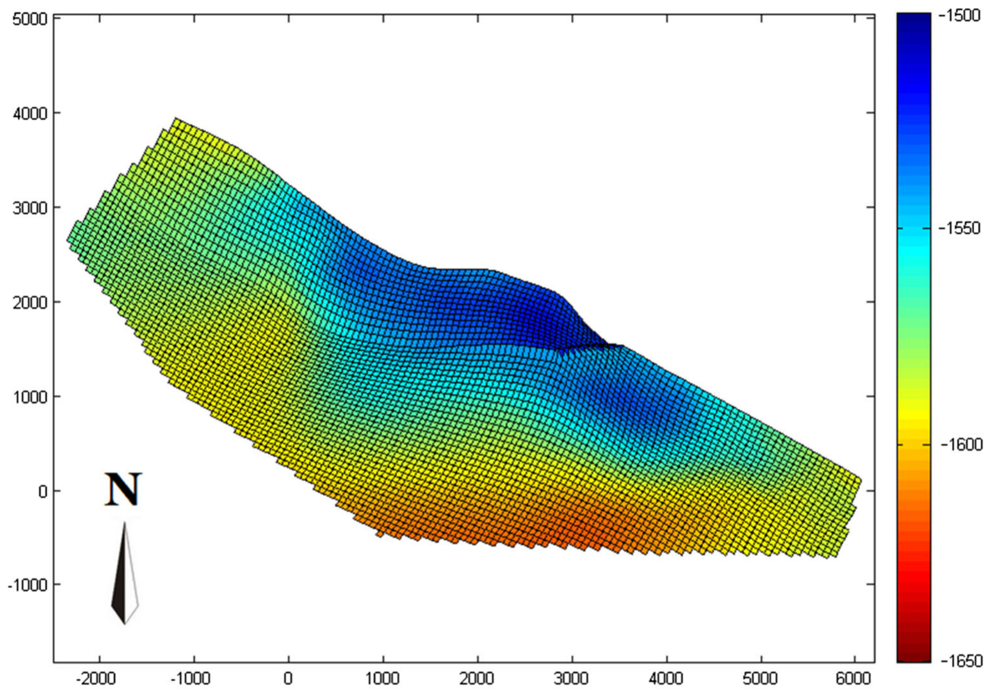
(b) Field cumulative oil production with respect to production time (Triangular well pattern)

Fig. 22 Production parameters for triangular well pattern represents the permeability (mD) of the unit. Besides one fault (represented by the black line) stretched across the reservoir, there is also one existing injection well (represented by the green point) and an existing production well (represented by the black point) in it.

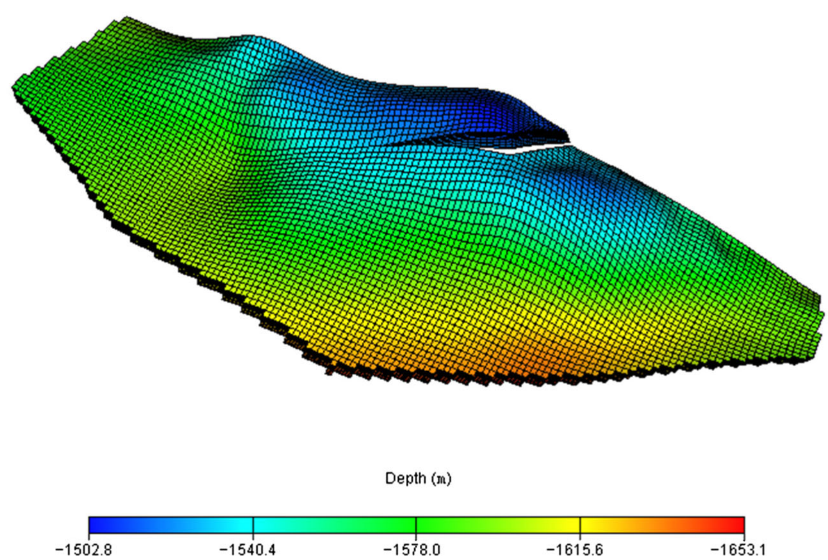
Initially saturated by water and oil homogenously, the initial oil saturation is 0.7. The porosity of the reservoir is 0.2 and the initial pressure is 30 MPa. The relative permeability of oil and water and the capillary pressure with water saturation can be seen in Fig. 13. The total injection rate is constant and the production wells also extract fluids at a constant rate. The period for simulation is set to 10 years. Other parameters are listed in Table 3.

The NPV value is chosen to be the objective function, and the widely used commercial reservoir numerical simulation software Eclipse 2006 is employed to do the simulation. During the simulation, the reservoir's total injection rate is fixed as 2000 STB/day and is distributed among all the

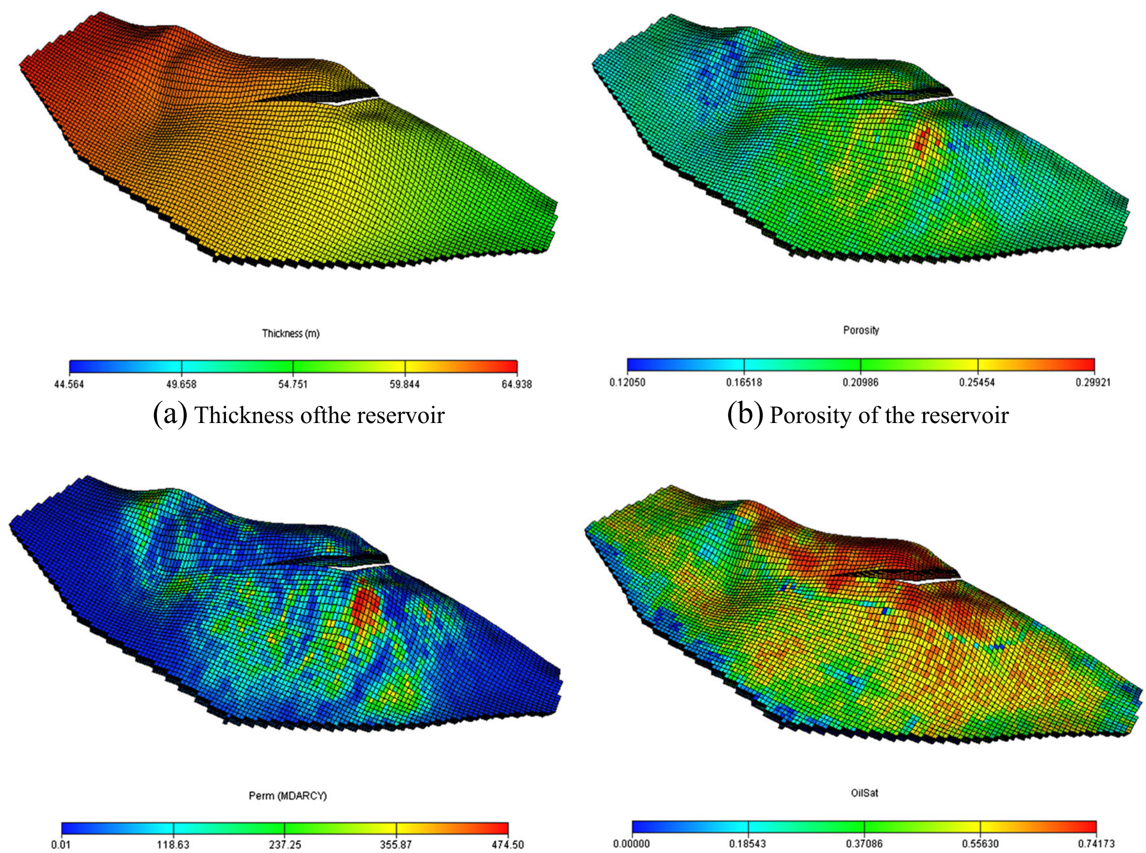
Fig. 23 Horizontal and three-dimensional views of the reservoir



(a) The horizontal view of the reservoir



(b) The three-dimensional view of the reservoir



(a) Thickness ofthe reservoir (b) Porosity of the reservoir

(c) Permeability of the reservoir (d) Initial oil saturation of the reservoir

Fig. 24 Parameters of the reservoir

injectors according to their injectivity index automatically by the simulator. The initial values for the variables to be optimized are listed in Table 4.

In order to verify the superiority of quadrilateral well pattern, we will simultaneously construct the triangular well pattern for the comparison, the method for triangular well

Fig. 25 Initial points and boundaries of the domain

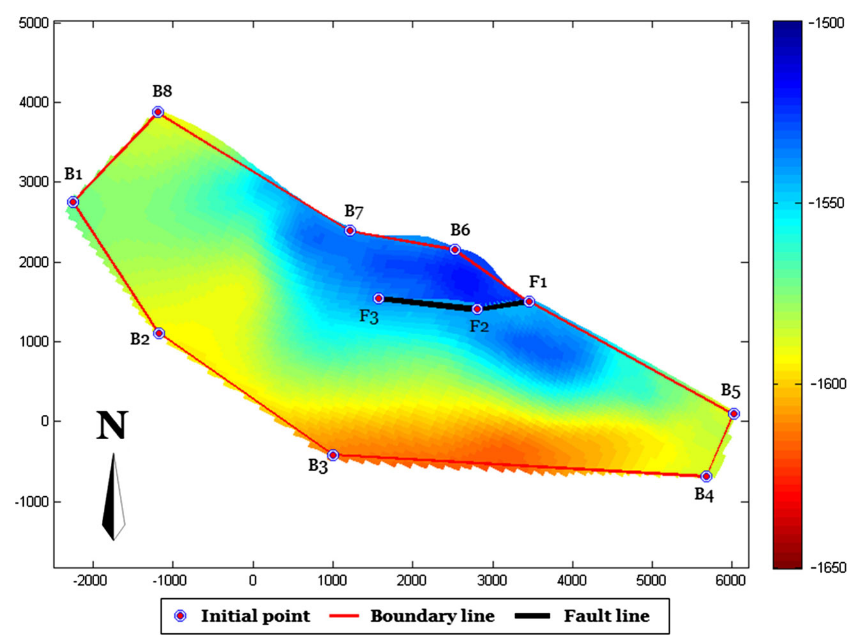


Table 7 Initial points and their initial values

Initial points	Initial values
B1	425
B2	425
B3	425
B4	425
B5	425
B6	425
B7	425
B8	425
F1	425
F2	425
F3	425

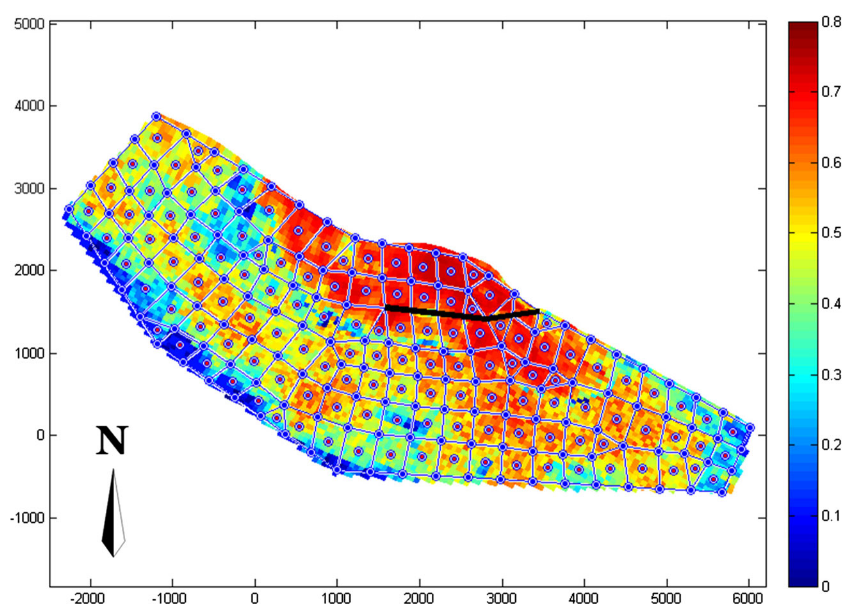
pattern's construction and optimization is proposed in our recent paper [17]. The quadrangular well pattern generated based on these initial values can be seen in Fig. 14 and the quadrangular well pattern after the optimization is shown in Fig. 15. The initial and optimal triangular well patterns are shown in Figs. 16 and 17, respectively. In these figures, the generated injection wells are denoted by blue points, while the generated production wells are denoted by red ones. Tables 5 and 6 exhibit the variation of variable values during the optimization.

The variation of NPV value with respect to the iteration steps is shown in Fig. 18. The curve shows that the NPV value increases rapidly at the beginning of the iteration; then, the increase gradually becomes slow and finally reaches a steady state, which indicates that the optimization has reached the optimum. It is also obviously shown in the figure that the recovery effect of the quadrangular well pattern is better than the triangular well pattern for the same reservoir.

The residual oil saturation after the simulation for both of the initial and optimal quadrangular well patterns is shown in Fig. 19, while Fig. 20 shows the same parameter for triangular well pattern. Combining with Figs. 14 and 16, we can see that, for the initial well pattern, wells are distributed evenly, but the development effect is not ideal. The low permeability areas of the reservoir have not been fully exploited, so the heterogeneity has not been alleviated. However, for the optimal well pattern, combining with Figs. 15 and 17, we can clearly recognize the improvement. The well spacing is changed with respect to the heterogeneity of the reservoir, it becomes larger in the high permeability area, which prolongs the water breakthrough time, while in the low permeability area, it becomes smaller, thus decreases the flowing resistance. The total number of wells is reduced after the optimizing which leads to the increase of the single well injection rate, so more oil is produced in low permeability area, and the heterogeneity is significantly alleviated.

The oil field production parameters (including cumulative oil production with respect to production time and water cut) for both of the quadrangular and triangular well patterns before and after the optimization can be found in Figs. 21 and 22, respectively. As the figures show, the cumulative oil production at the same time step increases significantly after the optimization for both of the quadrangular and triangular well patterns. Besides, the water cut decreases obviously at the same cumulative oil production after the optimization. When comparing the recovery effect quadrangular well pattern to the triangular well pattern, we can see that, for the

Fig. 26 Initial well pattern



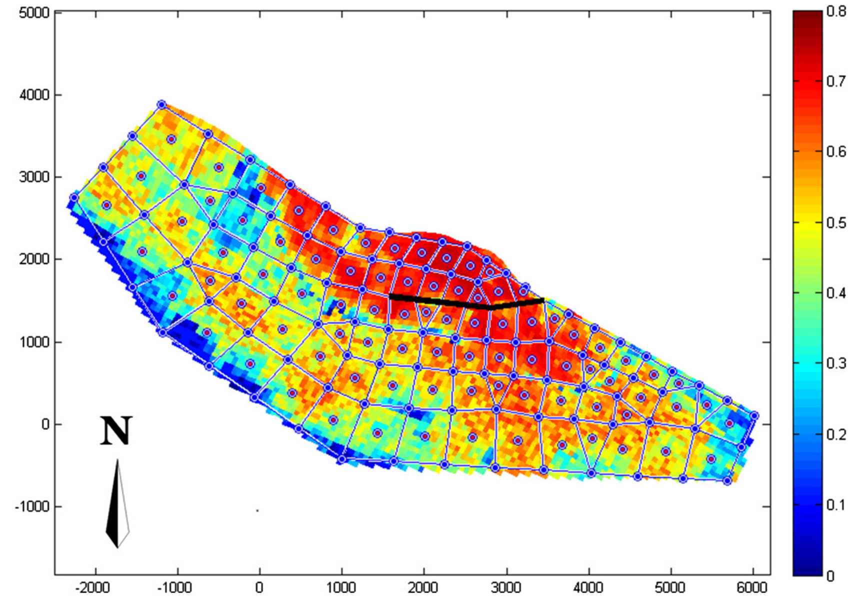


Fig. 27 Optimal well pattern

same reservoir, the final cumulative oil production for the quadrangular well pattern is higher than the triangular well pattern. All of these figures are sufficient to demonstrate the superiority of quadrilateral well pattern construction and optimization method proposed in this paper.

4.2 Example 2: construction and optimization of well pattern in real 3D reservoir model

In this example, the effect of quadrangular well pattern's construction and optimization in real 3D reservoir model will be tested. Figure 23 provides both the horizontal and three-dimensional views of the reservoir; it can be seen that the reservoir used in this example is a large-scale and irregularly shaped 3D reservoir model, which is about 8650-m long and 2940-m wide. The reservoir is discretized into 139 × 48 blocks (including some invalid blocks, not shown in the figure) in horizontal direction and divided into nine layers in vertical direction; the color of each block represents the depth there. As shown in Fig. 23b, the depth of the reservoir is inhomogeneous, which varies from about 1502 to 1653 m below surface. There is a fault throughout the reservoir from top to bottom in vertical direction and extends from the center to the boundary in horizontal direction. Parameters including thickness (m), permeability (mD), porosity, and initial oil saturation are shown in Fig. 24. At the depth of 1590 m, the initial pressure is 18 Mpa. Other parameters such as oil price, relative permeability, capillary pressure etc. are the same as example 1.

The figures above show that the reservoir has a strong heterogeneity, especially for the oil saturation, which mostly

Table 8 Values of optimization variables during optimization

Optimization variables	Limits of optimization variables	Initial values	Values in step 1	Values in step 2	Values in step 3	Values in step 4	Values in step 7	Values in step 9	Values in step 10
1 (B1)	(250,800)	425.00	458.86	583.76	667.79	701.76	703.14	703.65	703.65
2 (B2)	(250,800)	425.00	512.69	586.98	668.98	713.95	725.11	725.14	725.16
3 (B3)	(250,800)	425.00	443.99	574.75	623.65	643.68	645.13	647.62	647.62
4 (B4)	(250,800)	425.00	419.00	484.40	517.48	534.48	536.37	536.36	536.36
5 (B5)	(250,800)	425.00	420.67	418.86	412.85	412.51	412.51	412.51	412.51
6 (B6)	(250,800)	425.00	417.65	394.54	373.70	368.87	365.21	362.16	362.79
7 (B7)	(250,800)	425.00	431.40	453.76	462.76	472.78	473.19	473.21	473.21
8 (B8)	(250,800)	425.00	538.27	594.76	658.24	689.75	700.53	700.73	700.78
9 (F1)	(250,800)	425.00	390.18	385.49	381.69	380.36	379.85	379.38	379.38
10 (F2)	(250,800)	425.00	418.59	373.65	366.77	364.57	363.36	363.36	363.36
11 (F3)	(250,800)	425.00	421.65	418.75	417.53	417.46	417.48	417.45	417.45

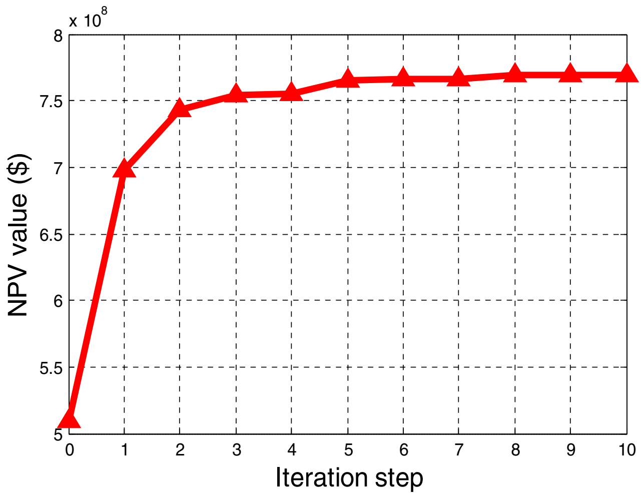


Fig. 28 NPV values during iteration steps

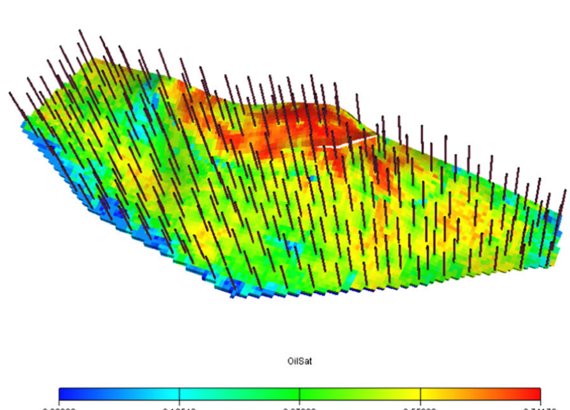
concentrated in the high area. Besides this, the reservoir has no regular boundaries. So before the well pattern's construction, we should first set the initial points and boundaries for the reservoir model to create a domain for the following procedures (shown in Fig. 25). All of these initial points are located in the corners of the reservoir, so that the generated domain can cover almost the entire reservoir. Initial values for the variables to be optimized are listed as Table 7. During the simulation, the total injection rate for this model is fixed as 10000 STB/day and is distributed among all the injectors according to their injectivity index automatically by the simulator.

The initial well pattern generated based on the initial variables is shown in Fig. 26. After the optimization, an adaptive optimal well pattern is generated (shown in Fig. 27). The background color in the two figures represents the initial oil saturation of the reservoir. Comparing the well patterns before and after optimization in these two figures, we can find that the well spacing of the initial well pattern is uniform and does not show any adaptability to reservoir heterogeneity. However, for the optimal well pattern, the well spacing is varied. Observing along with the initial oil saturation, we can see that the well spacing is smaller in areas with high oil saturation, whereas for areas of low saturation, the well spacing is larger, which shows a good adaptability to reservoir heterogeneity. Besides, the total number of wells in the optimal well pattern is less than the initial well pattern, which reduces the drilling cost.

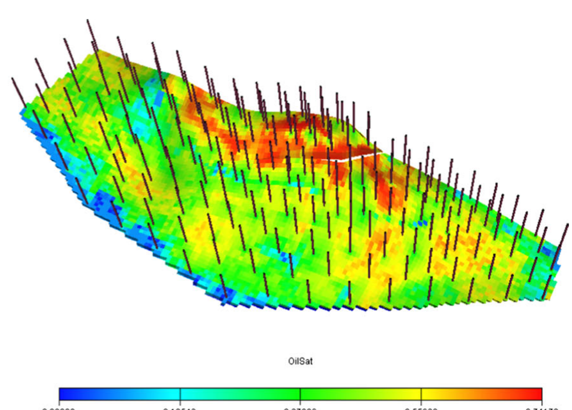
Table 8 shows the values of initial points in different iteration steps and Fig. 28 exhibits the variation of NPV value with respect to the iteration steps. Similar to the result in example 1, the NPV value increases rapidly at the beginning of the iteration; then, the increase rate gradually slows down and finally reaches a steady state, indicating that the optimization has reached the optimum.

The comparison of residual oil saturation after the simulation between initial and optimal well pattern can be seen in Fig. 29 (the black stick represents the well). It can be seen that, for the optimal well pattern, the oil recovery effect in high oil saturation areas is more obvious than the initial one. Figure 30a shows the cumulative oil production with respect to production time, and Fig. 30b shows the cumulative oil production with respect to water cut; it can be found that more oil is produced based on the optimal well pattern while less water is produced at the same time.

By comparing the simulation results before and after the optimization, same conclusion can be obtained as example 1. The quadrangular well pattern can also be constructed in large-scale 3D reservoir model and perfectly constrains the irregular boundaries. After the optimization, the adaptive quadrangular well pattern is obtained, which can recover more oil with fewer wells and decrease the water cut at the same time. The heterogeneity of the reservoir is alleviated and the development effect is improved. The example demonstrates the reliability of the method proposed in this paper significantly.

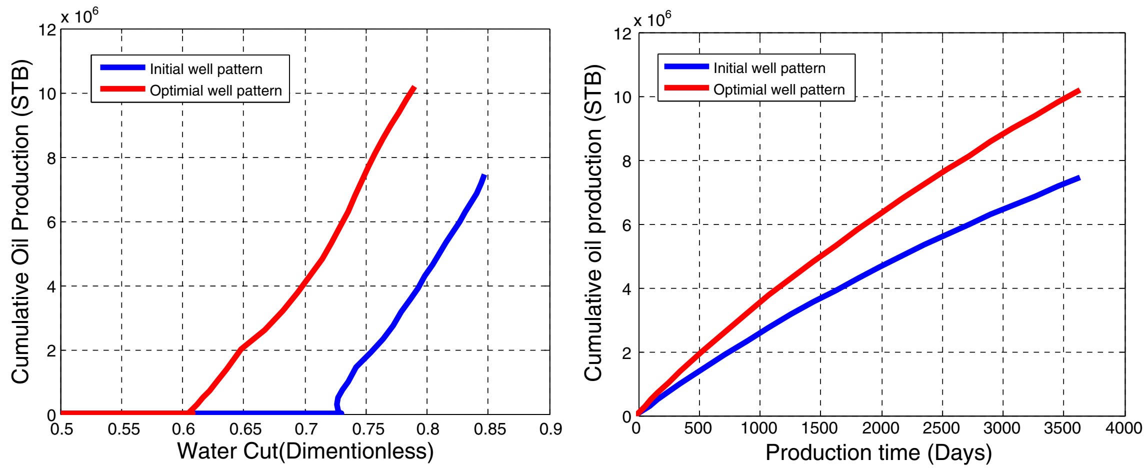


(a) Initial well pattern



(b) Optimal well pattern

Fig. 29 Distribution of residual oil saturation for initial and optimal well patterns



(a) Field cumulative oil production with respect to water cut

(b) Field cumulative oil production with respect to production time

Fig. 30 Production parameters for 3D reservoir

5 Conclusion

Well pattern optimization is an important way to improve reservoir development. Facing with the situation that quadrangular well pattern has not gain enough research, the paper makes an inter-disciplinary approach that incorporates frontal Delaunay quad-mesh generation and well pattern design. A new model for quadrangular adaptive well pattern's construction and optimization is proposed. The adaptive quadrangular well pattern can be constructed and adapts its shape to the heterogeneous of the reservoir. Together with the steepest ascent method and numerical simulator, an optimization algorithm with NPV value to be the objective function is successfully established. Examined by examples, the model is proved to be able to improve the development of reservoir significantly.

However, during the research, we found that our study is not enough. Considering the complexities of real reservoirs, there are still too much researches to be done, such as the well pattern generated in reservoir with complex inner and outer boundary conditions; the combination of triangular and quadrangular well pattern according to the condition of reservoir; and the local refinement and adjustment of well pattern during the production. All of these are topics of our future works.

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