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Generalized field-development optimization with well-control zonation

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Abstract Of concern in the development of oil fields is the problem of determining the optimal locations of wells and the optimal controls to place on the wells. Extraction of hydrocarbon resources from petroleum reservoirs in a cost-effective manner requires that the producers and injectors be placed at optimal locations and that optimal controls be imposed on the wells. While the optimization of well locations and well controls plays an important role in ensuring that the net present value of the project is maximized, optimization of other factors such as well type and number of wells also plays important roles in increasing the profitability of investments. Until very recently, improving the net worth of hydrocarbon assets has been focused primarily on optimizing the well locations or well controls, mostly manually. In recent times, automatic optimization using either gradient-based algorithms or stochastic (global) optimization algorithms has become increasingly popular. A well-control zonation (WCZ) approach to estimating optimal well locations, well rates, well type, and well number is proposed. Our approach uses a set of well coordinates and a set of well-control variables as the optimization parameters. However, one of the well-control variables has its search range extended to cover three parts, one part denoting the region where the well is an injector, a second part denoting the region where there is no well, and a third part denoting the region where the well is a producer. By this, the optimization algorithm is able to match every member in the set of well coordinates to three possibilities within the search space of well controls: an injector, a no-well situation, or a producer. The optimization was performed using differential evolution, and two sample applications were presented to show the effectiveness of the method. Results obtained show that the method is able to reduce the number of optimization variables needed and also to identify simultaneously, optimal well locations, optimal well controls, optimal well type, and the optimum number of wells. Also, comparison of results with the mixed integer nonlinear linear programming (MINLP) approach shows that the WCZ approach mostly outperformed the MINLP approach.

Keywords Well placement optimization · Generalized field development optimization · Well control zonation

1 Introduction

In oil field management, well locations and well-control specifications can be major deciding factors in the profitability of a waterflood project. As a result, these parameters are considered central to any waterflood optimization process. Traditionally, optimization of well placement and rates has been done using quality maps that indicate which regions of the reservoir have not been properly swept by the injected water [8, 20, 31]. While this method has been useful to some extent, it cannot properly place wells in locations that take advantage of long-term high oil saturation or respond to the dynamics of reservoir fluid flow over a long period of time. Thus, simply placing wells and adjusting well controls based on saturation or quality maps cannot guarantee long-term profitability of the project. Automatic well placement and rate optimization has been introduced to help optimize the placement and adjustment of well controls over the entire period of waterflood. A good amount of work

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has been done on optimization of well locations using local [17, 24, 35, 39, 41] and global [1, 7, 9, 10, 12–16, 22, 23, 25, 26, 32, 33, 40, 42] optimization methods. The application of global optimization algorithms to solve the well placement problem is increasingly becoming more popular due to its flexibility, ease of formulation, and wide applicability. Some works on optimizing only well rates using gradient-based methods have also appeared in the literature [2, 3, 19, 21, 29, 36]. However, a gradient-based strategy is local and would result in local optimum in the vicinity of the initial guess. Also, in situations where the gradient of the objective function cannot be obtained, gradient-based strategies would fail while global (stochastic) optimization algorithms would still work. Asadollahi et al. [4] proposed a workflow strategy that would significantly reduce the number of optimization parameters. However, such strategy relies on the enforcement of a voidage replacement ratio (VRR) of unity. Such enforcement, as shown in [7], may lead to significant loss of revenue. Optimizing the NPV, while maintaining an acceptable level of VRR, was proposed as a better alternative. Lien et al. [30] presented a multiscale regularization method for optimizing well controls in waterflood projects. The authors used the piecewise-constant function to limit the number of times the well controls are changed (fixed cycles) and placed wells into groups, with wells in the same group operated at the same control. At each cycle, the grouping is refined so that fewer wells are kept in each group as the iteration proceeded.

Recently, interest in well-control optimization and in the joint optimization of both well placement and well-control [11, 18, 24, 28] is on the rise. Joint optimization of both well placement and well-control is expected to provide better results than the exclusive optimization of either of these two. A recent paper [28], which initially motivated this work, addresses this issue to some extent. Isebor et al. [28] presented a paper on generalized field development optimization in which the authors included in the set of design variables, not only well placement and well control but also the number of wells and well type, in a joint optimization process. The authors posed the optimization problem as a mixed integer nonlinear program (MINLP) and proposed different methods to solve the problem. While their approach is expected to provide reasonably good solutions (for small-to-medium scale problems), the approach makes use of an additional variable per well that would otherwise be unnecessary. Also, this additional variable is an integer variable that represents the well type (an injector, a no-well situation or a producer), making the problem an MINLP problem requiring special solution methods.

We propose to estimate simultaneously, placement, control, type, and number of wells by using only well-control and well location variables as the optimization parameters. We consider only vertical wells and used the CWPO 1 method of Awotunde and Naranjo [6] to enforce minimum well spacing constraints. Also, we define two sets of parameters, one set for the well locations in the (x, y) plane and the other set for the well controls. The search range of the set of well controls is subdivided into three, one part indicating the presence of an injector, one part indicating the presence of a producer, and the third part indicating no well. The approach proves very useful in optimizing well placement, well rate, well type, and number of wells without substantially increasing the dimension of the problem space.

2 Objective function

Net present value (NPV) is used as the objective function in this work. NPV is the difference between the discounted cash inflow and the discounted cash outflow of a project. NPV is useful in measuring the profitability of an investment or in comparing between several mutually exclusive alternative projects. In waterflood projects, the NPV is an important metric in analyzing and gauging the profitability of different alternative scenarios of waterflood. Of importance in a waterflood project are the well controls and well locations. Consequently, these parameters are considered to be of primary interest in waterflood optimization. Different configurations of wells and/or different specifications of well-control will yield different NPVs. Thus, the NPV becomes a critical yardstick to assess the viability of the different waterflood scenarios and come up with the best alternative. In a well rate optimization procedure, the NPV serves as the objective function. The net present value of a waterflood project can be defined as

$$NPV(r, T) = \int_{t_0}^{T} (1+r)^{-t} R(t)\, dt, \tag{1}$$

where $R(t_0)$ is the initial capital outlay, t_0 is the time of start of investment (often zero) and $R(t)$ is the net cash flow at any other time t, T is the total time of waterflood project (investment period), and r is the discount rate. In a waterflood project, the initial capital expenditure is made up mainly of facility installation costs and the costs of drilling the injectors and producers. In the present work, all injectors and producers are assumed to be drilled at the beginning of the project. Thus, the capital expense, $R(t_0)$ is given by

$$R(t_0) = C_{\text{facility}} + n_p C_{\text{prod}} + n_i C_{\text{inj}}, \tag{2}$$

where C_{facility}, C_{prod}, and C_{inj} are the cost of facility installation, the cost drilling a producer, and the cost of drilling an injector, respectively. n_i and n_p denote the number of injectors and the number of producers, respectively. The rate of flow of net cash $R(t)$ is the total revenue less expenditure per unit accounting time t. $R(t)$ can be described by

$$R(t) = \text{Rvn}(t) - \text{Exp}(t). \tag{3}$$

In Eq. 3, Rvn is the revenue given by

$$\mathrm{Rvn}\,(t) = P^{o}\,(t)\,q_{o}^{\mathrm{prod}}\,(t) + P^{g}\,(t)\,q_{g}^{\mathrm{prod}}\,(t)\,, \tag{4}$$

where q_{o}^{prod} and q_{g}^{prod} are the rates of oil and gas production, respectively, and P^{o} and P^{g} are the unit prices of oil and gas at time t. As indicated in Eq. 4, q_{o}^{prod}, q_{g}^{prod}, P^{o}, and P^{g} can vary with time. Exp is the recurring expenditure defined as

$$\begin{aligned}\mathrm{Exp}\,(t) = C_{w}^{\mathrm{prod}}\,(t)\,q_{w}^{\mathrm{prod}}\,(t) + C_{w}^{\mathrm{inj}}\,(t)\,q_{w}^{\mathrm{inj}}\,(t) \\ + \frac{n_i + n_p}{N_{\mathrm{wells,max}}} C_{\mathrm{op}}\,(t)\,q_{\mathrm{fluid}}^{\mathrm{prod}}\,(t)\,,\end{aligned} \tag{5}$$

where $q_{\mathrm{fluid}}^{\mathrm{prod}}$ is the combined rate of fluid production in stb/day, C_{w}^{prod} is the unit cost of treating and disposing the water produced, C_{w}^{inj} is the unit cost of acquiring, treating, and injecting water, and C_{op} is the unit operating cost (excluding C_{w}^{prod} and C_{w}^{inj}) per barrel of fluid produced, all at time t. We note that the numbers of producers and injectors, n_i and n_p, vary from one solution to the other, as estimated by a candidate solution at any particular iteration in the optimization process. $N_{\mathrm{wells,max}}$ is the maximum number of wells the user has declared possible in the optimization program.

3 Generalized field development optimization

We define two sets of parameters, $\vec{\alpha}_{\mathrm{loc}}$ and $\vec{\alpha}_{\mathrm{contr}}$, representing well location and well-control variables, respectively. The first set $\vec{\alpha}_{\mathrm{loc}} \in D$, comprises the (x, y) coordinates of the wells to be placed in the reservoir while the second set $\vec{\alpha}_{\mathrm{contr}} \in E$, consists of the parameters describing the primary controls in the wells. $\vec{\alpha}_{\mathrm{contr}}$ can be further categorized into $\vec{\alpha}_{\mathrm{contr}}^{0}$ and $\vec{\alpha}_{\mathrm{contr}}^{1}$. $\vec{\alpha}_{\mathrm{contr}}^{0}$ contains a set of variables in $\vec{\alpha}_{\mathrm{contr}}$ that determine the well type while $\vec{\alpha}_{\mathrm{contr}}^{1}$ is the set of the remaining variables that give no indication of the well type. The length of $\vec{\alpha}_{\mathrm{loc}}$ is twice the maximum number of wells permissible because each well (vertical well) has two coordinates. The length of the vector $\vec{\alpha}_{contr}$ depends on the approach adopted. For example, if the constant rate approach is used, then the length of $\vec{\alpha}_{\mathrm{contr}}$ equals the maximum number of wells considered in the optimization scheme because only one control (an average value) is specified for each well throughout the entire waterflood period. Other approaches will have more parameters in $\vec{\alpha}_{\mathrm{contr}}$. Thus, the optimization problem can be stated as

$$\min_{\vec{\alpha}} \Phi\,(\vec{\alpha}) \tag{6}$$

such that

$$\vec{f}\,(\vec{\alpha}) = 0, \tag{7}$$

$$\vec{g}\,(\vec{\alpha}) \leq 0, \tag{8}$$

$$\vec{\alpha}_{\mathrm{loc}} \in D,\ \vec{\alpha}_{\mathrm{contr}}^{0} \in E^{0},\ \text{and}\ \vec{\alpha}_{\mathrm{contr}}^{1} \in E^{1}$$

where Φ is the objective function,

$$D = \left\{\vec{\alpha}_{\mathrm{loc}} \in \mathbb{R}^{2N_{\mathrm{wells,max}}} : l_{\mathrm{loc},i} \leq \alpha_{\mathrm{loc},i} \leq u_{\mathrm{loc},i} \forall i = 1, 2, ..., 2N_{\mathrm{wells,max}}\right\}, \tag{9}$$

$$E^{0} = \left\{\vec{\alpha}_{\mathrm{contr}}^{0} \in \mathbb{R}^{N_{\mathrm{wells,max}}} : l_{\mathrm{contr},j}^{0} \leq \alpha_{\mathrm{contr},j}^{0} \leq u_{\mathrm{contr},j}^{0} \forall j = 1, 2, ..., N_{\mathrm{wells,max}}\right\}, \tag{10}$$

and

$$E^{1} = \left\{\vec{\alpha}_{\mathrm{contr},j}^{1} \in \mathbb{R}^{(N_{\mathrm{wcvar}}-1)} : \vec{l}_{\mathrm{contr},j}^{1} \leq \vec{\alpha}_{\mathrm{contr},j}^{1} \leq \vec{u}_{\mathrm{contr},j}^{1} \forall j = 1, 2, ..., N_{\mathrm{wells,max}}\right\}. \tag{11}$$

$\vec{f}\,(\vec{\alpha}) = 0$ comprises the set of equality constraints and $\vec{g}\,(\vec{\alpha}) \leq 0$ are the set of inequality constraints. In Eqs. 9 to 11, l and u represent the lower and upper bounds, respectively, and N_{wcvar} is the number of unknown well-control variables per well. In the present work, no equality constraints were placed on the optimization problem. However, inequality constraints enforcing minimum well spacing [6] were imposed on the optimization problem. The method described here is applied to the piecewise-constant approach. In the piecewise constant approach, the well-control is held for some period of time before it is altered. Thus, the piecewise-constant (PWC) approach is made up of several time periods, each with its own constant control. The remainder of the discussion will focus on the implementation of the algorithm for the piecewise-constant approach.

3.1 Procedure/implementation for rate control in the piecewise constant approach

First, we describe the optimization procedure for rate control in the piecewise constant (PWC) approach and then extend the method to cases in which bottomehole pressure (BHP) is specified as the primary control. The two vectors, $\vec{\alpha}_{\mathrm{loc}}$ and $\vec{\alpha}_{\mathrm{contr}}$, are combined into a single vector $\vec{\alpha} = \begin{bmatrix}\vec{\alpha}_{\mathrm{loc}} \\ \vec{\alpha}_{\mathrm{contr}}\end{bmatrix}$ to form the vector of design variables. The parameters in $\vec{\alpha}_{\mathrm{contr}}$ are the actual well rates. In the PWC approach, the length of $\vec{\alpha}$ is $N_{\mathrm{wells,max}}\left(N_{\mathrm{cyc}} + 2\right)$ where $N_{\mathrm{wells,max}}$ is the maximum number of wells allowed and N_{cyc} is the number of cycles considered. In this work, a cycle is a period of time during which the well controls in all wells are held constant. Thus, for each well, we need to estimate only one well control variable per cycle. Any well declared has the possibility of existing, but may not necessarily be placed in the reservoir. To jointly optimize well rates, well placement, well type, and number of wells, we first identify suitable search ranges for well locations and controls. To implement the procedure on the piecewise-constant approach, we use the estimated values of well controls in all wells only at the *first* cycle to determine the well type. That is, the well controls estimated at the first cycle belongs to the set E^0 while the well controls estimated at other cycles belong to the set E^1. Therefore, the maximum limit of the rate is extended to cover the positive and negative portions of the real line only in the first cycle. The search range E^0 is then partitioned into three, with the first part completely on the negative real line, the second part covers some part of the negative real line and some part of the positive real line, and the third part is entirely on the positive real line. That is, E^0 becomes

$$E^0_{\text{ext}} = \left\{ \vec{\alpha}^0_{\text{contr}} \in \mathbb{R}^{N_{\text{wells,max}}} : -\vec{u}^0_{\text{contr}} + \vec{l}^{0,-}_{\text{contr}} \leq \vec{\alpha}^0_{\text{contr}} \leq \vec{u}^0_{\text{contr}} + \vec{l}^{0,+}_{\text{contr}} \right\}, \quad (12)$$

which is subdivided into three ranges E^-_{ext}, E^{-+}_{ext}, and E^+_{ext} for $\vec{\alpha}^0_{\text{contr}}$ in E^0_{ext}. These ranges are defined as

$$E^-_{\text{ext}} = \left[-\vec{u}^0_{\text{contr}} + \vec{l}^{0,-}_{\text{contr}}, \vec{l}^{0,-}_{\text{contr}} \right), \quad (13)$$

$$E^{-+}_{\text{ext}} = \left[\vec{l}^{0,-}_{\text{contr}}, \vec{l}^{0,+}_{\text{contr}} \right], \quad (14)$$

and

$$E^+_{\text{ext}} = \left(\vec{l}^{0,+}_{\text{contr}}, \vec{u}^0_{\text{contr}} + \vec{l}^{0,+}_{\text{contr}} \right], \quad (15)$$

respectively. Every component $\alpha^0_{\text{contr},j}$ (well control) in E^0_{ext} corresponds to two components (a pair of well coordinates) in D. During the search for optimum well configuration and optimum well control, if $\alpha^0_{\text{contr},j}$ falls in E^+_{ext}, then a well exists at the corresponding location in D and it is an injector. If $\alpha^0_{\text{contr},j}$ falls in E^-_{ext}, a well exists and it is a producer. However, if $\alpha^0_{\text{contr},j}$ falls in E^{-+}_{ext}, no well exists and the corresponding well location (pair of coordinates) in D is redundant. We term this procedure a well-control zonation (WCZ) approach. We note that the lower bound $\vec{l}^0_{\text{contr}}$ of the original search range (the range before extension to the negative real line) has been replaced with $\vec{l}^{0,-}_{\text{contr}}$ and $\vec{l}^{0,+}_{\text{contr}}$. These values must be specified by the user. In fact, the user can specify $\vec{l}^{0,-}_{\text{contr}} = -\vec{l}^{0,+}_{\text{contr}}$, as done throughout the remainder of this work. The interval between $\vec{l}^{0,-}_{\text{contr}}$ and $\vec{l}^{0,+}_{\text{contr}}$ is termed the no-well zone. From Eqs. 12 to 15, it is clear that the search is conducted between $-\vec{u}^0_{\text{contr}}+\vec{l}^{0,-}_{\text{contr}}$ and $\vec{u}^0_{\text{contr}}+\vec{l}^{0,+}_{\text{contr}}$. Thus, if an estimated rate falls in E^-_{ext}, the actual rate is obtained by subtracting $l^{0,-}_{\text{contr}}$ from the estimated rate. Also, if an estimate falls in E^+_{ext}, the actual rate is obtained by subtracting $l^{0,+}_{\text{contr}}$ from that estimate. While the difference between $\vec{l}^{0,-}_{\text{contr}}$ and $\vec{l}^{0,+}_{\text{contr}}$ can be zero so that the two values coincide, results in this work show that this situation is ineffective and would result in suboptimal well output.

Notice that E^1 contains only positive values of rates from zero to the maximum allowed rate since no-extension of the interval to the negative side is performed. For example, if the well rates in a well located in a reservoir undergoing waterflooding in which the project duration has been divided into six cycles are estimated to be −800, 600, 700, 400, 900, and 500 stb/day, respectively for the six cycles. Then, the well will be a producer producing 800 stb/day during the first cycle and producing at the other specified rates at their corresponding cycles. If the estimated rate for the first cycle had been 800 stb/day, then the well would be an injector injecting water at 800 stb/day at the first cycle and injecting at the other specified rates at their corresponding cycles. If the specified rate falls within the no-well zone, i.e., between $l^{0,-}_{\text{contr}}$ and $l^{0,+}_{\text{contr}}$, then no well will be placed at the corresponding location. By this, we are able to distinguish between a producer, an injector and a no-well scenario. We note that while only the rate in the first cycle is used here to determine the well type, the rate at any other specific cycle can be used. However, the cycle whose rate is used to determine the well type should be fixed throughout the search period (same cycle should be used in all members of the population and at all iterations of the optimization). We also note that other attributes such as the sum or average of rates in all cycles can be used but these would be less effective as this joint attribute will place an additional layer of constraint on the optimization. For instance, consider that the optimum well type at a certain well location is a producer and that the well type is to be determined by the sum or average of all rates in a well, then in order to achieve this, the optimizer will try to force the sum of all the rates of that well to be negative. This will affect the estimation of the cycle rates. Therefore, such joint attributes are not recommended.

3.2 Modifications for optimization of BHP

Because the search range of BHP for producers is different from that of injectors, the procedure described in the preceding section is modified for use in the optimization of BHP. If the lower BHP limit in any producer has been determined to be $p_{\text{wf,min}}$ and the upper BHP limit in any injector has been specified as $p_{\text{inj,max}}$, then the task of the optimizer will be to determine the operational BHP controls within these limits in all the wells. However, the BHPs imposed on the producers should be lower than those imposed on the injectors for effective production to take place. Also, the optimization procedure, requires separate search ranges be used for the producers and the injectors because they have different upper and lower bounds. While $p_{\text{wf,min}}$ can be specified as the lower bound for the producer and $p_{\text{inj,max}}$ specified as the upper bound for the injectors, there is still the need to specify the upper bound for the producers $\left(p_{wf,max}\right)$ and a lower bound for the injectors $\left(p_{\text{inj,min}}\right)$. In this work, we choose $p_{\text{wf,max}}$ and $p_{\text{inj,min}}$ to be close to the initial reservoir pressure and ensure that $p_{\text{wf,max}}$ is slightly lower than $p_{\text{inj,min}}$. Also, $p_{\text{wf,max}}$ may be chosen to coincide with $p_{\text{inj,min}}$. The choice of these values is solely the responsibility of the user, not the algorithm. Also, rather than search directly for pressure between the specified lower and upper bounds, we choose to search for a change in pressure, Δp, and then adding the estimated value of Δp to $p_{\text{wf,min}}$ if the well is a producer or adding it to $p_{\text{inj,min}}$ if the well is estimated to be an injector. With this description, we can now identify the bounds of the set E^0 as $l^0_{\text{contr},j} = 0$ and $u^0_{\text{contr},j} = \Delta p_{\max}$ where $\Delta p_{\max} = \max\left(\Delta p_{\text{wf,max}}, \Delta p_{\text{inj,max}}\right)$, $\Delta p_{\text{wf,max}} = p_{\text{wf,max}} - p_{\text{wf,min}}$, and $\Delta p_{\text{inj,max}} = p_{\text{inj,max}} - p_{\text{inj,min}}$. The extended search domain can then be created by using Eqs. 13 to 15 as $E^-_{\text{ext}} = \left[-\Delta\vec{p}_{\text{wf,max}} + \vec{l}^{0,-}_{\text{contr}}, \vec{l}^{0,-}_{\text{contr}}\right)$, $E^{-+}_{\text{ext}} = \left[\vec{l}^{0,-}_{\text{contr}}, \vec{l}^{0,+}_{\text{contr}}\right]$, and $E^+_{\text{ext}} = \left(\vec{l}^{0,+}_{\text{contr}}, \Delta\vec{p}_{\text{inj,max}} + \vec{l}^{0,+}_{\text{contr}}\right]$. The values in E^1 range from 0 to $\Delta p_{\max}$. The reason for this is that the search range in the producer is different from the search range in the injector. By this, it is possible to generate estimates of BHPs that are larger than the maximum value allowed in the well-type with the smaller search range. In that Case, the estimated value would be replaced by the maximum value (upper limit of that well-type).

As an illustration, consider an optimization problem in which the minimum pressure allowed in the producer is 2000 psi and the maximum pressure allowed in the injector is 6500 psi. Also, consider that we choose $p_{\text{wf,max}} =$ 3950psi, $p_{\text{inj,min}} =$ 4050psi, $\vec{l}^{0,+}_{\text{contr}} =$ 1000psi, and $\vec{l}^{0,-}_{\text{contr}} = -1000$psi. Then, $\Delta p_{\text{wf,max}} = 1950$psi, $\Delta p_{\text{inj,max}} =$ 2450psi, and the search range in E^0_{ext} would be from -2950 to 3450 psi while the search range in E^1 would be from 0 to 2450 psi. The no-well region would be from -1000 to 1000 psi. If the estimates of Δp for a particular well in a three-cycle scenario were -2100, 1500, and 2300 psi, this well would be identified as a producer producing at 1100 psi during the first cycle. The BHP of the well would be 3500 psi in the 2nd and 3950 psi in the 3rd because the estimated BHP is obtained by adding the estimated Δp to the lower bound (2000 psi in the case of a producer). The estimated BHP in the 3rd cycle (i.e. 2000+2300psi) was replaced with 3950 psi because it was out of range for a producer.

4 Practical considerations

In the generalized field development optimization presented, the maximum number of wells to use in the optimization scheme must be selected. The optimization scheme will then identify those wells that should be operated as producers, those to be operated as injectors and those that should not be drilled at all. Thus, the maximum number of wells selected should be more than or at least equal to the number that is required to optimize the waterflood project. While the maximum number of wells required can itself be considered as an optimization variable, we consider it a known variable in this work. Also, as it is evident in the WCZ, we do not assign separate operational variables that determine which wells are injectors and which wells are producers. The identification of an injector or producer is implicitly built into one of the parameters used to model the well controls, thus cutting down the size of the optimization problem. Therefore, this procedure uses fewer design variables than those used in the procedure proposed by Isebor et al. [28]. Also, the procedure makes the use of mixed integer nonlinear programming (MINLP) unnecessary for the solution of the optimization problem. More important is the fact that since there are no fixed integers to be determined, the definition of the no-well zone can be made according to the level of uncertainty one has in choosing the maximum number of wells allowed in solving the optimization problem. For instance, if a user of this algorithm is confident that the number of wells required to develop a field is between 50 and 60, he can set the maximum number of wells allowed in the algorithm to be 60 and set the no-well interval to be small. However, if the user believes the number of wells required to be between 30 and 80, then the user can set 80 and also set the no-well interval to be large. Setting the no-well interval to be large ensures that the algorithm is able to quickly push unneeded wells that were declared to this interval. Example 1 presented in a later section of this paper shows how the size of the no-well interval affects the NPVs obtained from the same problem but with different settings of $N_{\text{wells,max}}$.

5 Comparison with MINLP approach

To measure the advantage of using this methodology relative to the established method of using a separate variable for determining the well-type, we used the MINLP approach of [28] in finding the optimum parameters and compared the results with those from the proposed method. To do this, we considered the work of Isebor et al. [28] and focused on the ternary categorical MINLP formulation as described by the authors. In the ternary categorical MINLP formulation, a categorical variable z, was introduced and associated with each well. The categorical variable can take on the integer values -1, 0, and 1. In the authors' work, -1 corresponds to drilling an injector, 0 refers to not drilling any well, and 1 corresponds to drilling a producer. However, to solve this problem practically, the authors searched for the variable z within the bounds $[-1, 1]$ and rounded its value to the nearest integer. This formulation is somewhat similar to our formulation in this work except that the determination of well type is integrated into the search for the well-control so that we do not incur an additional optimization variable (z) per well. Note the reversal of nomenclature (i.e., the negative part corresponds to an injector in the authors' work while it corresponds to a producer in our work). Apart from this obvious difference in formulation, there is another difference in the actual search for well-type. In Isebor et al.'s ternary formulation [28], the region representing an injector is $[-1, -0.5]$, that representing a no-well situation is $(-0.5, 0.5)$, and the region representing a producer is $[0.5, 1]$. Thus, the round-off approach adopted by the authors means that each region has a fixed width for all problems. Also, a greater search space (i.e., from -0.5 to 0.5) is devoted to the no-well region. This is good if users do not have a good idea about the approximate number of wells needed to develop the field and would like to specify a maximum number of wells that is considerably large. In this case, many of the unneeded wells would easily end up in the no-well region. In the WCZ approach; however, the user can determine the size of the no-well region for different problems. As would be seen in the examples presented, the size of the no-well region relative to the other two regions affects the results obtained for different problems. Another significant difference between this work and that of Isebor et al. [28] is that in this work, a reducing tendency in the estimated value of a well control variable over iterations results in an increasing tendency towards a different well type. Take for example, a well that is estimated to be an injector but whose estimated rate (in the first cycle) is decreasing gradually as the search proceeds. As the value of the estimated rate reduces from one iteration to the next, there is an increase in the possibility that the well would change from being an injector to a non-existing well or even to a producer in later iterations if the trend continues. This tendency results from the implicit dependency of the well type on the estimated well control (of the first cycle).

6 Sample illustrations

Two sample optimization problems are presented. The problems were solved to estimate the optimum number and configuration of injectors and producers, and the controls to place on the wells. The first problem consisted of a small-sized reservoir model with heterogeneous permeability distribution while the second problem used a reservoir model having four facies. Relative permeability curves and PVT properties of reservoir oil are shown in Fig. 1. In all the examples, we have used Eclipse reservoir simulator and imposed economic limits of 100 stb/day minimum oil rate and maximum water cut of 97 % on every producer. This means that a producer would be shut-off if it produces at an oil rate less than 100 stb/day or at a water-cut greater than 97 %. Also, field-wide economic limits of 3000 stb/day minimum oil rate and a maximum water cut of 97 % are imposed on the reservoir. These field-wide limits ensure that all wells in the reservoir are shut down simultaneously and

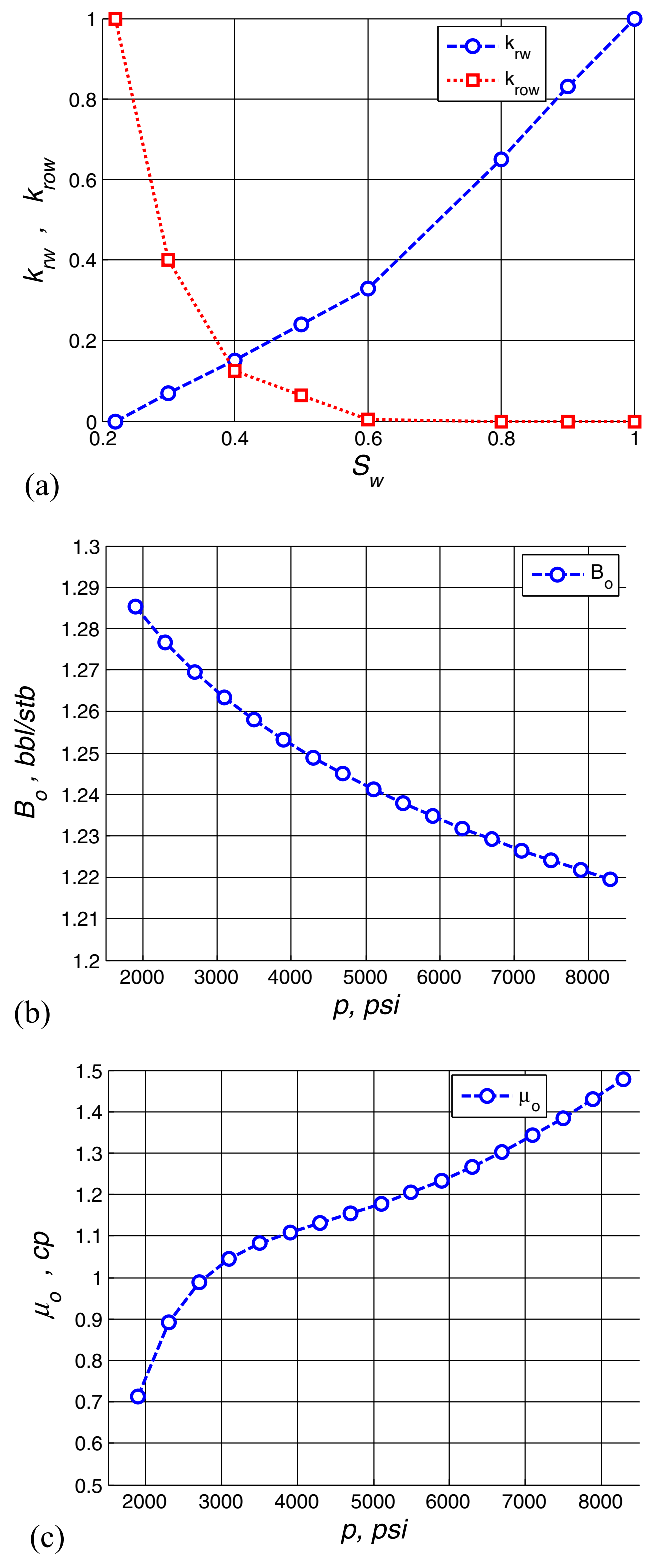

Fig. 1 Relative permeability and PVT properties of fluids. **a** Oil-water relative permeability curves. **b** Oil formation volume factor. **c** Oil viscosity

Table 1 Cases considered for well control zonation in the examples

Case	Rate		BHP	
	$\overline{l}^{0,-}_{\text{contr}}$	$\overline{l}^{0,+}_{\text{contr}}$	$\overline{l}^{0,-}_{\text{contr}}$	$\overline{l}^{0,+}_{\text{contr}}$
1	−100	100	–	–
2	−500	500	–	–
3	−1000	1000	–	–
4	−2500	2500	−975	1225
5	−5000	5000	−1950	2450
6	−7500	7500	−2925	3675

the simulation terminated if any of these global economic limits is violated.

In the examples, the performances of the WCZ and MINLP approaches were studied. The optimization problems were solved under both rate-control and BHP-control. Also, several cases involving different widths of the no-well zone were considered under each problem (Table 1). Under rate-control, six widths of the no-well region were considered while only three widths were considered under BHP-control. Cases 1 to 3 are considered to be of small widths and in all these three, the no-well zone is smaller than each of the other zones. In Case 4, the no-well zone covers one-third of the search space while in Case 5, the zone covers half of the search space. In Case 6, the no-well zone covers 60 % of the search space.

The performances of the approaches were studied using the NPV as the performance measure and DE [34, 38] as the optimizer. The same DE strategy and DE parameters were used for the two algorithms and all the scenarios/cases considered. The population size in the DE was obtained from

$$N_p = 4 + \text{floor}\,(3 \log M)\,, \tag{16}$$

where N_p is the population size and M is the problem dimension. Although, Eq. 16 was originally proposed to compute the population size in CMA-ES [27], it has been shown to work well for DE [5]. In the Appendix, we further compared the performance of this population size with those of larger population sizes to show its effectiveness. Five runs

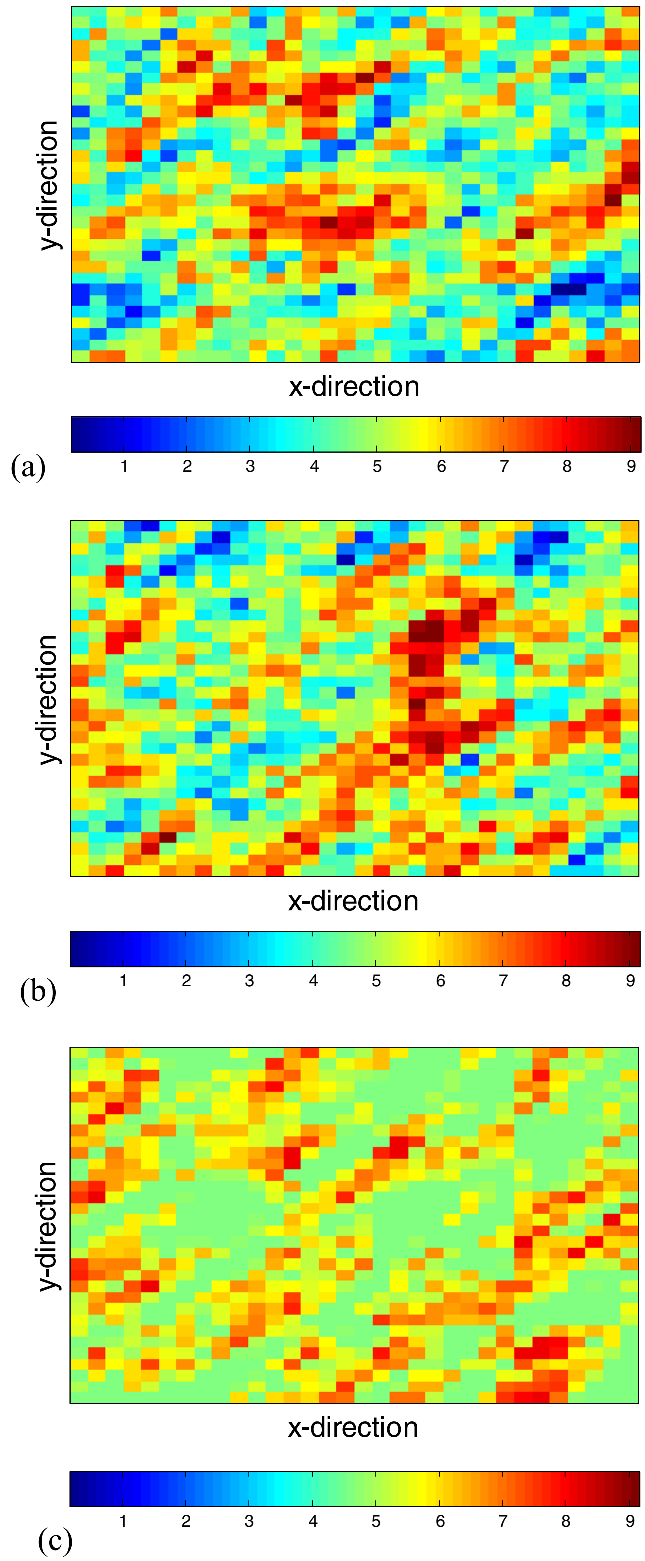

Fig. 2 *log* permeability distribution of 32 × 32 × 3 reservoir in Example 1. **a** Layer 1. **b** Layer 2. **c** Layer 3

Table 2 Values of variables used to compute NPV

Variable	Value	Unit
C_{facility}	50×10^6	USD
C_{prod}	7×10^6	USD
C_{inj}	7×10^6	USD
P^o	60	USD/bbl
C_w^{prod}	5	USD/bbl
C_w^{inj}	10	USD/bbl
C_{op}	8	USD/bbl
r	0.05	None

(realizations) of the optimization solutions were obtained in each problem/case considered and the solutions were ranked from the best (the run with the highest NPV) to the worst. A realization is obtained by solving the optimization problem once using a set of random numbers in the global optimizer. Thus, different realizations are obtained by using different sets of random numbers in solving the same problem. For the sake of fair comparison in stochastic-based approaches, it is necessary to make several runs and obtain several realizations of the optimization results because the performance of each method/case is different for different sets of random numbers.

Only vertical wells were used in the examples and the values of variables used in computing the NPV are presented on Table 2. The optimization algorithms were coded in a suitable programming language and a commercial reservoir simulator was used to simulate the reservoir performance. To ensure a fair comparison between the approaches/cases, the random seed in the programming software was set to 1 at the start of the first of the five runs in each approach/case. In this way, each algorithm/case was initialized with the same random seed and used almost the same set of random numbers.

6.1 Example 1

In this example, we consider a synthetic reservoir discretized into $32 \times 32 \times 3$ gridblocks, each grid of dimension

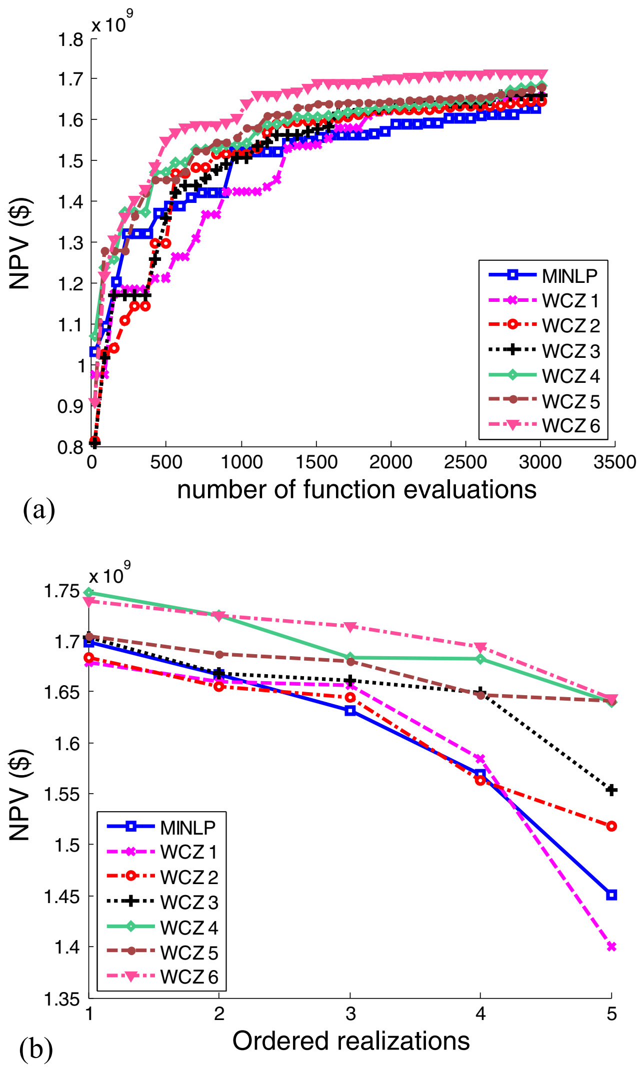

Fig. 3 Comparison of the NPVs attained by different algorithms on the optimization problem with rates as well controls. **a** Median realization. **b** Final optimized NPVs in all realizations (Scenario 1, Example 1)

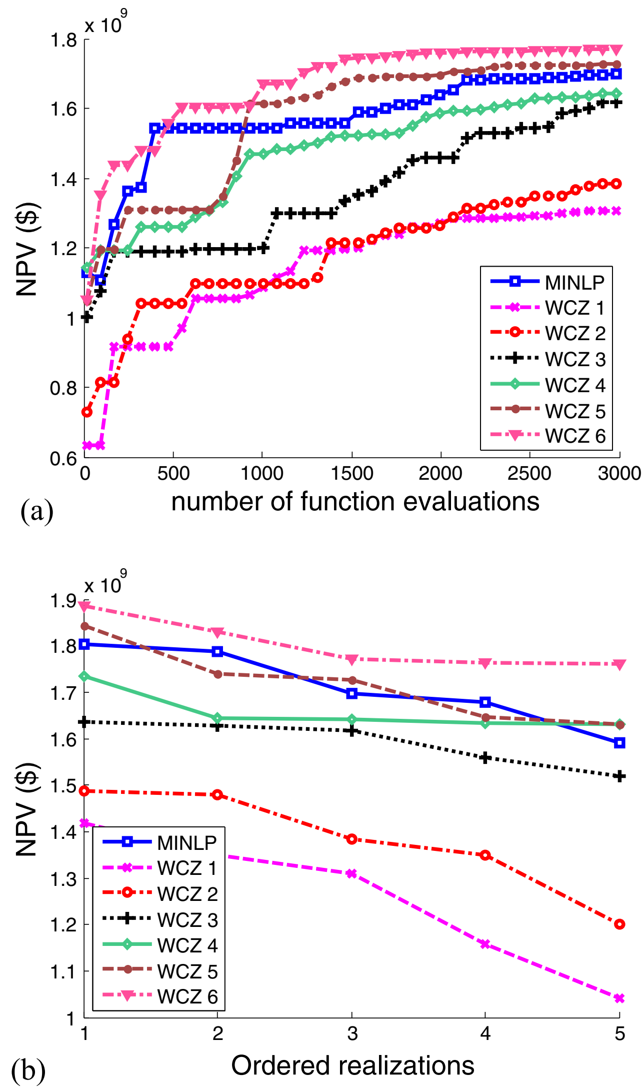

Fig. 4 Comparison of the NPVs attained by different algorithms on the optimization problem with rates as well controls. **a** Median realization. **b** Final optimized NPVs in all realizations (Scenario 2, Example 1)

150ft × 150ft × 70ft. The reservoir log permeability distributions in the three layers are shown in Fig. 2. The task is to optimize the well type, the well controls, and the number of wells simultaneously to maximize the NPV in a 20-year field development plan. In this example, we consider 5 cycles, each of 4 years, in optimizing the rates.

First, we start with rate constraints and consider three scenarios of this problem. In the first scenario, the maximum number of wells that can be declared, $N_{\text{wells,max}}$, is set to 15, in the second, $N_{\text{wells,max}} = 25$, and in Scenario 3, $N_{\text{wells,max}} = 40$. Using different scenarios would help us study how the size of the no-well interval affects the performance of the WCZ approach. This is because if $N_{\text{wells,max}}$ is much greater than the optimum number of wells required to develop the reservoir, the optimizer would need to move a large number of wells to the no-well region. In these scenarios, well rates, estimated by the optimizers, were the primary constraints placed on the wells. The secondary constraint was a minimum BHP of 2000 psi on a producer and a maximum BHP of 6500 psi on an injector. In the first scenario, where the maximum number of wells allowed in the optimization solution was 15, the number of variables representing the well location coordinates was 30 and the number of variables representing well rates was 75. Thus, the total number of optimization variables in the WCZ approach was 105. However, the MINLP approach has an additional variable per well to determine the well type. Thus, the number of design variables in the MINLP was 120. We have used approximately 3000 function evaluations for this example.

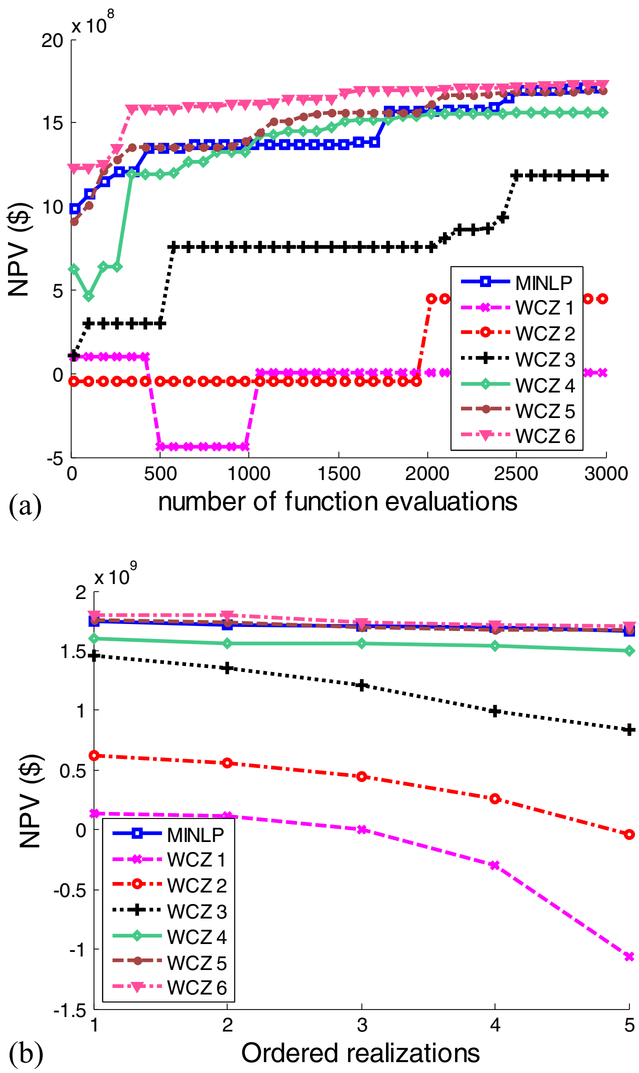

Fig. 5 Comparison of the NPVs attained by different algorithms on the optimization problem with rates as well controls. **a** Median realization. **b** Final optimized NPVs in all realizations (Scenario 3, Example 1)

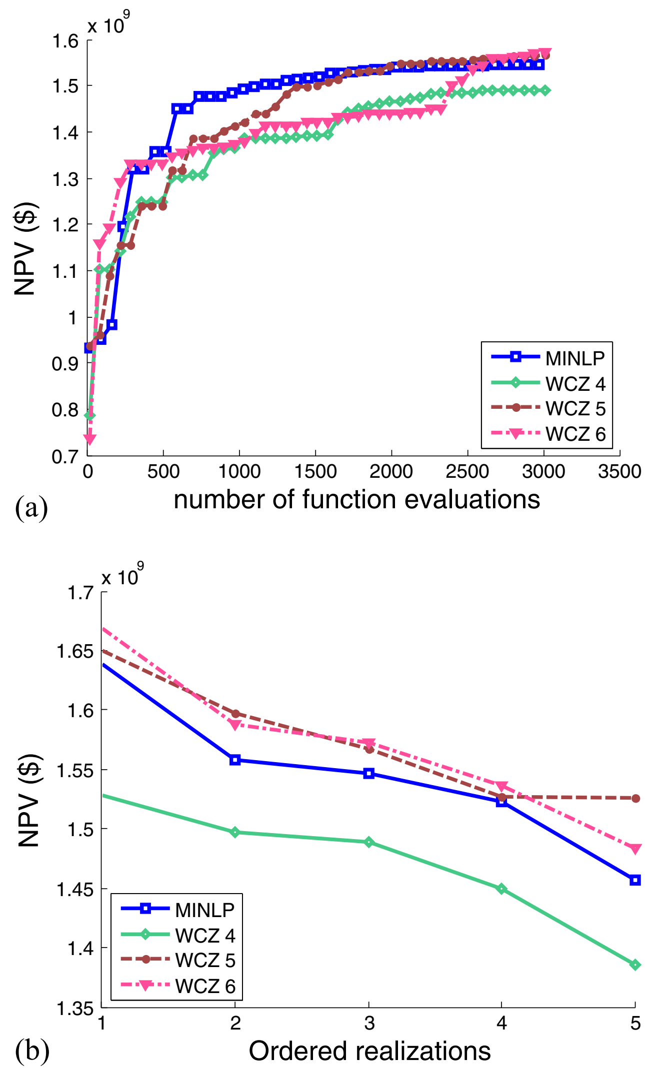

Fig. 6 Comparison of the NPVs attained by different algorithms on the optimization problem with BHPs as well controls. **a** Median realization. **b** final optimized NPVs in all realizations (Scenario 1, Example 1)

The NPVs obtained from the MINLP and the six cases of the WCZ are shown in Fig. 3a from the median realization. Figure 3b shows only the final NPVs attained by the methods in their respective five runs. The values of NPVs in Fig. 3b have been arranged in decreasing order for each method/case considered. From the figures, we observe that the performance of WCZ improved as the width of the no-well region became bigger (i.e., WCZ 4 to 6 were much better than WCZ 1 to 3). This shows that it is better to start with many wells in the no-well region and gradually push the wells into their appropriate zones than initially placing many wells in the production-well and injection-well zones and finding ways to push the unneeded wells out. We also observe that almost all the cases considered performed better than the MINLP in this scenario. In particular, WCZ 5 has the same width of the no-well zone with the MINLP and has performed much better under this scenario.

The optimized NPVs from the second scenario, where the maximum number of wells allowed was 25, are shown in Fig. 4a, b. We observe a general and more conspicuous increase in NPV with increasing width of the no-flow region. This is consistent with our expectation because as the maximum number of wells allowed in the solution to the optimization problem increases, the need for a larger no-well zone becomes more acute. In this case, WCZ 6 consistently produced the highest NPV in all the five runs. Also, the performance of WCZ 5 was only slightly better than that of MINLP in this case. In the third Scenario $\left(N_{\text{wells,max}} = 40\right)$ shown in Fig. 5, two of the five runs of WCZ 1 yielded negative NPVs while one of the five runs of

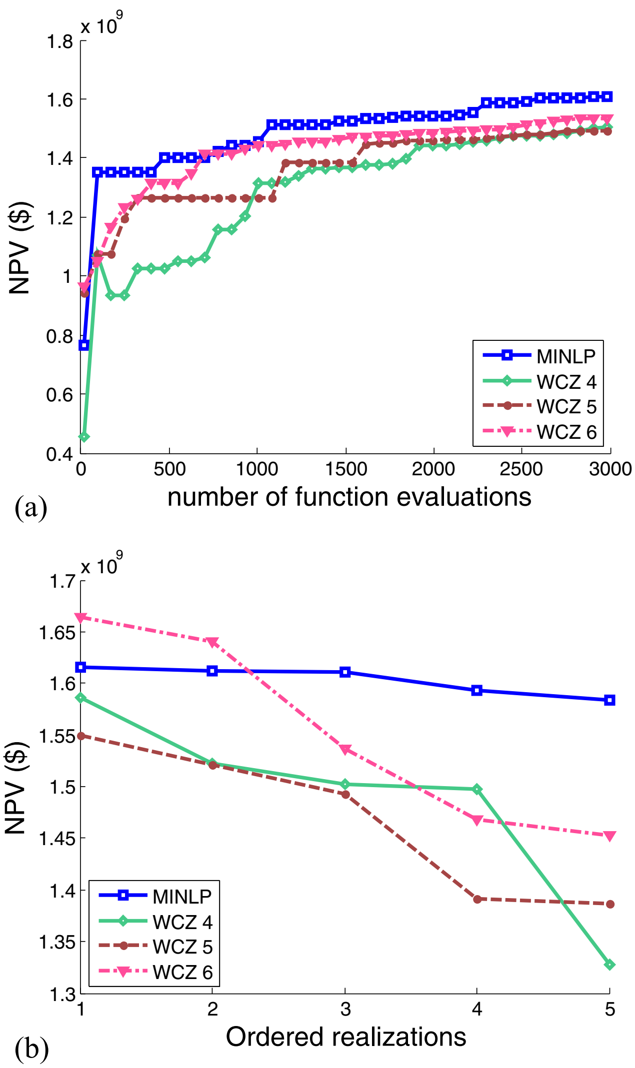

Fig. 7 Comparison of the NPVs attained by different algorithms on the optimization problem with BHPs as well controls. **a** Median realization. **b** Final optimized NPVs in all realizations (Scenario 2, Example 1)

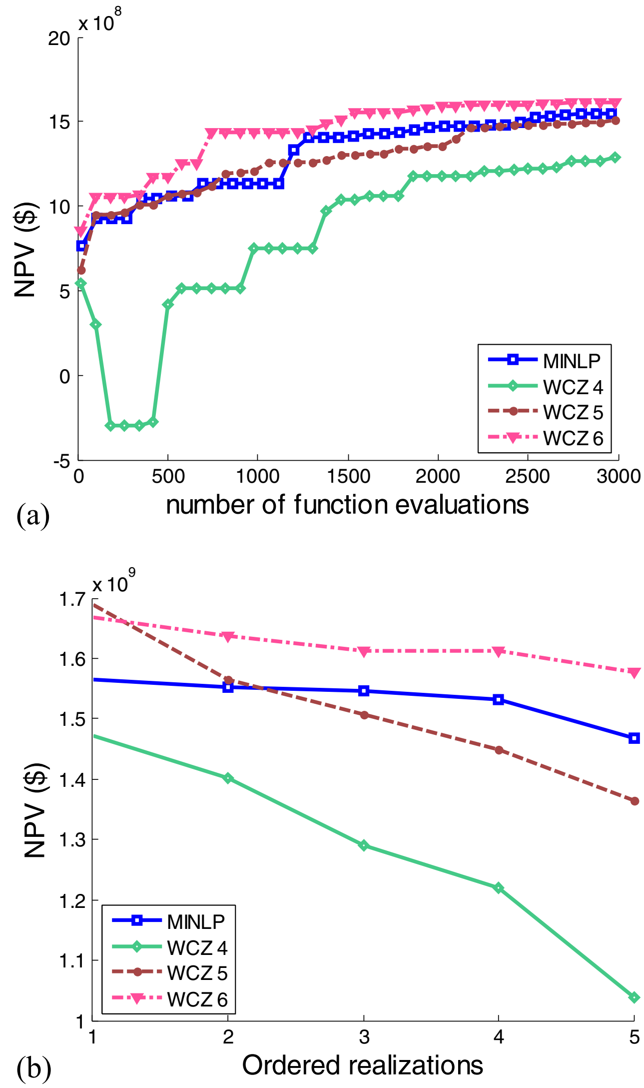

Fig. 8 Comparison of the NPVs attained by different algorithms on the optimization problem with BHPs as well controls. **a** Median realization. **b** Final optimized NPVs in all realizations (Scenario 3, Example 1)

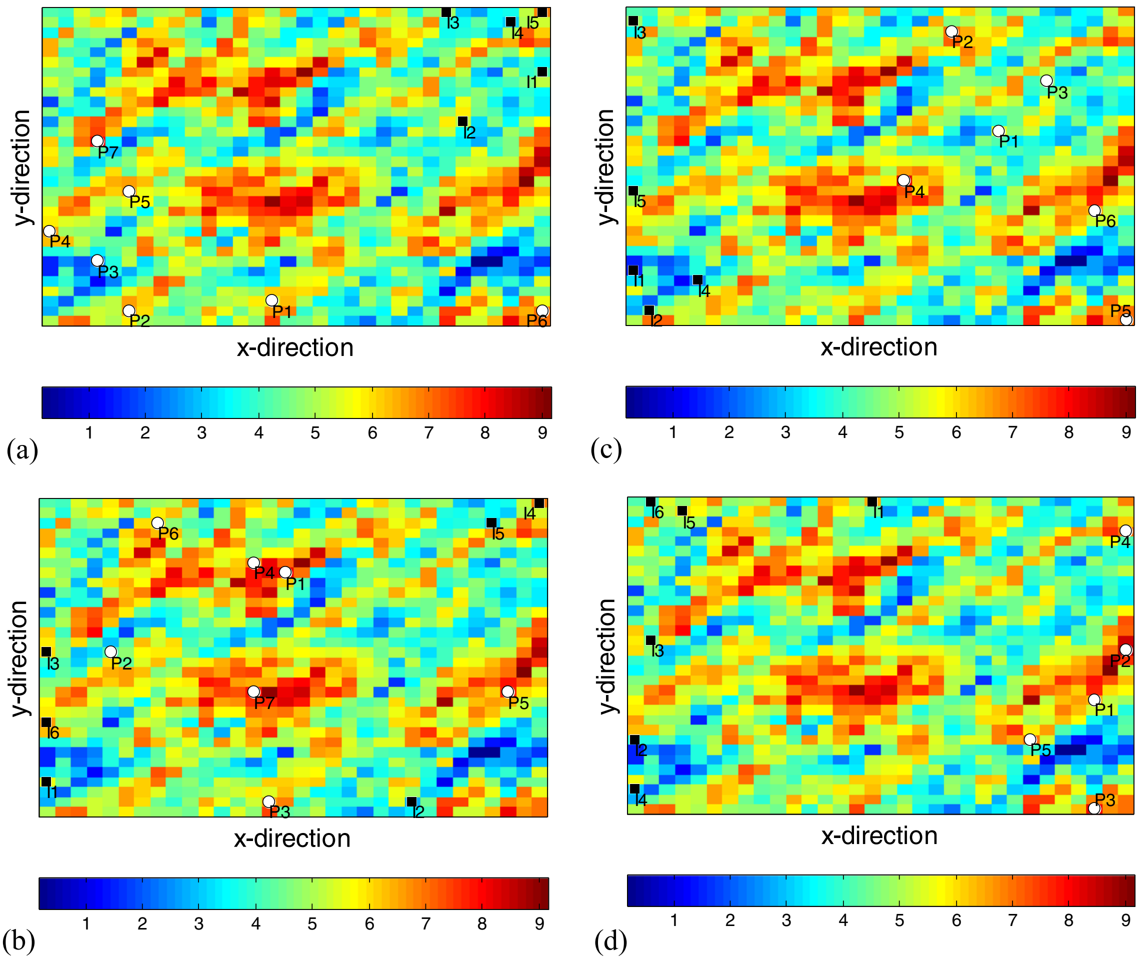

Fig. 9 Optimized well locations from Scenario 2 under rate-constraints. **a** MINLP. **b** WCZ 4. **c** WCZ 5. **d** WCZ 6 (Example 1). Note: producers are represented by *small white circles* while injectors are presented by small *black squares*

WCZ 2 produced a negative NPV. This situation occurred because the no-flow regions specified in these cases were too small to accommodate the null-wells that should result from the maximum of 40 wells specified under this scenario. This shows that a large width of the no-well zone is needed whenever the uncertainty in the maximum number of wells to be specified in solving the problem is high. Because of the large difference in the values of NPVs plotted in Fig. 5, the differences in some optimized results are not evident from the plot. In this 40-well scenario, WCZ 6 yielded the highest

Table 3 Estimated number of wells (Example 1)

	Rates						BHPs					
	15 Wells		25 Wells		40 wells		15 Wells		25 Wells		40 wells	
Approaches	n_i	n_p	n_i	n_p	n_i	n_p	n_i	n_p	n_i	n_p	n_i	n_p
MINLP	5	6	5	7	4	6	3	6	3	8	3	10
WCZ 1	4	8	6	15	13	25	–	–	–	–	–	–
WCZ 2	5	6	6	12	16	17	–	–	–	–	–	–
WCZ 3	4	5	5	10	8	18	–	–	–	–	–	–
WCZ 4	**3**	**5**	6	7	5	11	3	7	3	9	4	12
WCZ 5	4	6	5	6	6	9	3	6	3	8	**3**	**10**
WCZ 6	4	5	**6**	**5**	**6**	**7**	**3**	**5**	**3**	**5**	4	9

NPVs in all the five runs while WCZ 5 gave higher NPVs than MINLP in three of the five runs.

The performances of the algorithms under pressure-constraints were also tested. In this case, the optimized well controls are the bottomhole pressures placed on the wells. We intend to search for an optimum pressure between 2000 and 3950 psi in each producer and between 4050 and 6500 psi in each injector with a maximum well rate of 5000 stb/day placed on each well as secondary constraint. However, the search was made for optimum values of Δp in each well and the values were added to the lower bound of the producer or the injector as determined by the estimated well-type. In the WCZ, the first-cycle estimates of BHPs were used to determine the well-type while a separate variable was used in the MINLP approach. All the three scenarios (each scenario involving the specification of a different maximum allowable number of wells in the optimization problem) considered for the rate-constraints were also considered for the BHP-constraints. However, only cases 4 to 6 of the WCZ were considered under the BHP-constraints.

Figure 6 shows the optimized NPVs from Scenario 1 $\left(N_{\text{wells,max}} = 15\right)$. The figure shows that both WCZ 5 and WCZ 6 performed slightly better than MINLP while WCZ 4 exhibited the worst performance. The optimized NPVs from Scenario 2 are presented in Fig. 7. Although in this scenario, the two highest NPVs were obtained from WCZ 6, the MINLP approach performed much better than all the three cases of WCZ in three of the five runs. Figure 8 shows that WCZ 6 exhibited the best performance for the third scenario, even though the highest NPV obtained in this scenario was from WCZ 5. Also, the MINLP performed better than both WCZ 4 and WCZ 5 in this scenario.

From this example, our overall assessment of the methods studied is that the WCZ approach performs well when the no-well region is large enough and in fact as large as or larger than the combined width of the other two zones (production-well and injection-well zones). We observe that a larger width of the no-well region becomes indispensable when the maximum number of wells declared in the optimization problem is much larger than the optimum number of wells needed to develop the reservoir. Such a scenario may occur in situations where the user of the algorithm is uncertain about the rough estimate of the number of wells to declare and thus chooses to declare a large number. Specifically, we observe that in almost all the cases considered, the WCZ 6 (the case with the largest width of the no-well zone) performed much better than all the other cases considered. Furthermore, in all the scenarios considered under the rate-constraints, WCZ 5 performed better than MINLP. However, in two of the three scenarios under BHP constraint, MINLP performed better than WCZ 5. One other observation is that all the scenarios under rate-constraints yielded much higher NPVs than the scenarios under BHP-constraints. The overall highest NPV from this example was 1.88×10^9 from WCZ 6 in Scenario 2 $\left(N_{\text{wells,max}} = 25\right)$ under rate-constraints while the overall second highest NPV $\left(1.84 \times 10^9\right)$ was obtained from WCZ 5 of the same scenario under rate-constraints.

The well locations obtained from the best realizations of the approaches in Scenario 2 under rate-constraints are

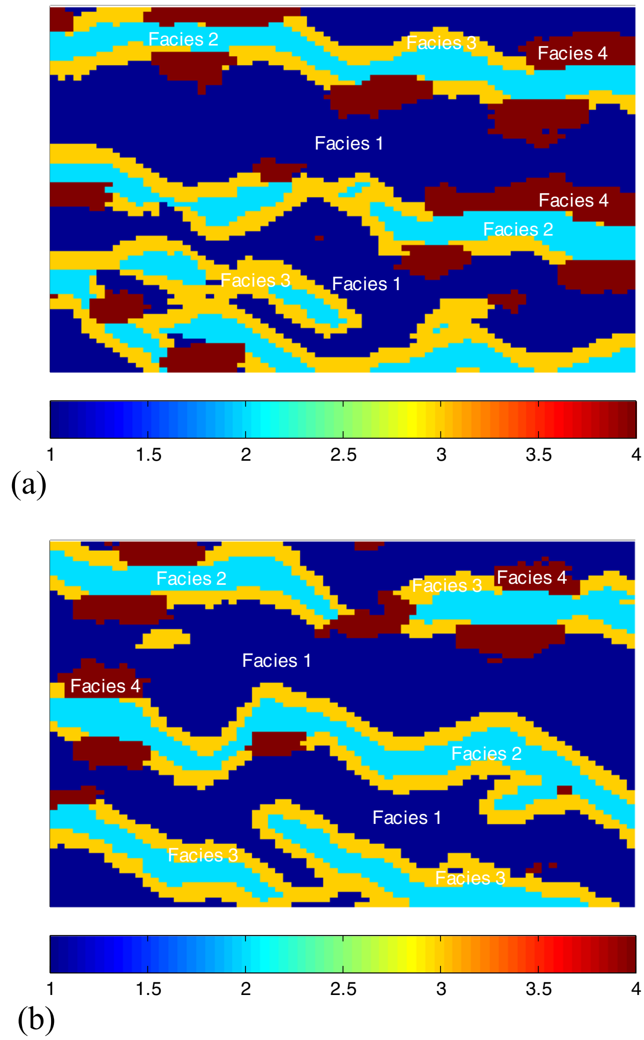

Fig. 10 Channel reservoir with four facies. **a** Layer 1. **b** Layer 2. *Color* represents facies type: Facies 0 has *deep blue color*, Facies 1 has *light blue color*, Facies 2 is *yellow* in color, and Facies 3 has *deep red color*

Table 4 Channel reservoir properties

Facies	Color	k (md)	ϕ
0	Deep blue	100	0.12
1	Light blue	2100	0.37
2	Yellow	750	0.23
3	Red	2	0.05

shown in Fig. 9a–d. A general observation from the figures is that wells of the same type tend to be placed in the same part of the reservoir. This observation had been made in a previous work [6]. The reason why this configuration may give higher NPV than the conventional belief of placing a producer in the neighborhood of an injector is that having the injectors together in a region of the reservoir may enhance the push of oil away from that region to an opposite region of the reservoir. Having producers in the opposite part will then allow such producers to benefit from the push of oil towards that part. This is reasonable considering that the field development process is for several years (20 years in this example) and early water breakthrough is prevented. The optimum number of injectors and producers as estimated by the approaches in their best realizations (runs) are presented on Table 3. The optimum numbers of injectors and producers from each scenario are shown in bold font and in brown color while the overall best numbers from all scenarios are shown in bold font and in blue color. We observe from the table that the optimum numbers of injectors and producers that yielded the highest NPV from Scenario 1 under rate-constraints are 3 and 4, respectively. These were achieved by WCZ 4. From Scenario 2 under rate constraint, the optimum numbers of injectors and producers were 6 and 5, respectively. These numbers, estimated by WCZ 6, also yielded the overall highest NPV for this problem. The optimum numbers for the other scenarios are presented on the table. The table shows that WCZ 6 yielded the optimum numbers leading to the highest NPV in four out of the six scenarios presented. Each of WCZ 4 and WCZ 5 yielded the highest NPV in only one scenario.

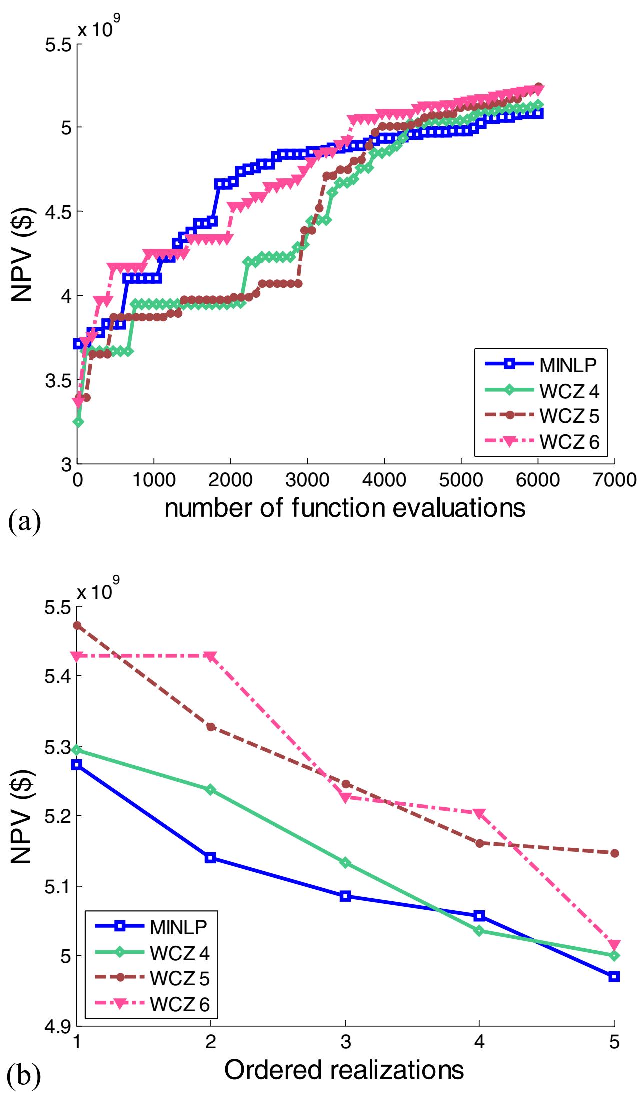

Fig. 11 Comparison of the NPVs attained by different algorithms on the optimization problem with rates as well controls. **a** Median realization. **b** Final optimized NPVs in all realizations (Example 2)

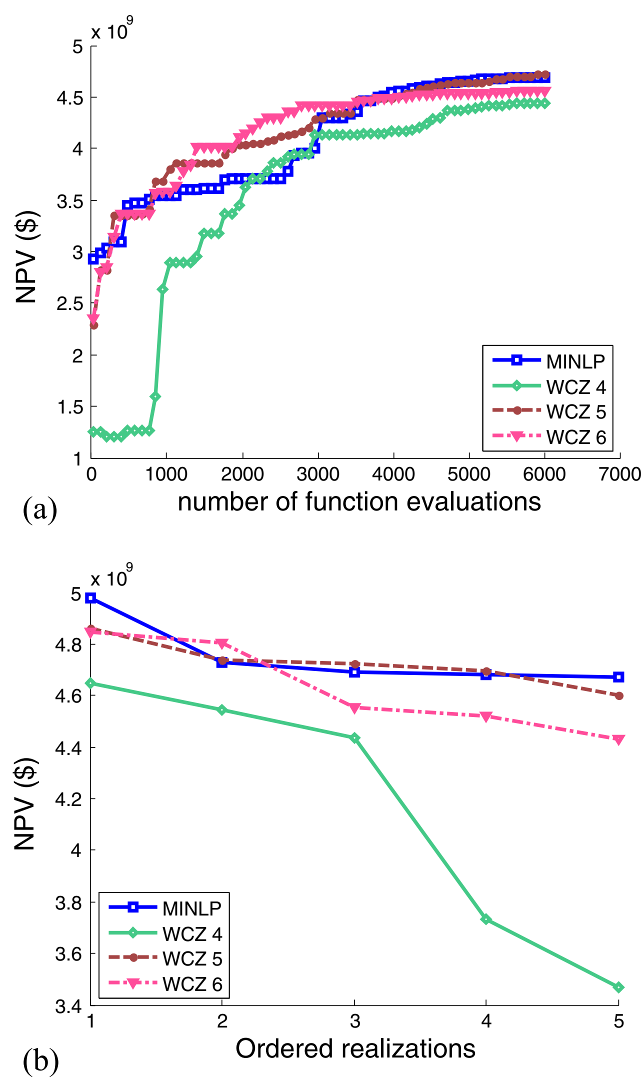

Fig. 12 Comparison of the NPVs attained by different algorithms on the optimization problem with BHPs as well controls. **a** Median realization. **b** Final optimized NPVs in all realizations (Example 2)

6.2 Example 2

In this example, we used a synthetic channel reservoir composed of four facies (Fig. 10) in two layers. The facies are represented by different colors with Facies 1 to Facies 4 progressing from deep blue color to deep red color. Each facies has distinct permeability and porosity as presented on Table 4. The reservoir is discretized into 75 × 75 × 2 gridblocks, each block of size 150ft × 150ft × 100ft. The NPV was computed for a 50-year operating period and approximately 6000 function evaluations were used in the search for the highest NPV.

We estimate the well controls, well locations, and number of wells simultaneously. A maximum of 60 wells were used in the optimization scheme. Ten cycles, each consisting of a 5-year period, were used. Thus, in this example, there were 780 variables in the MINLP approach and 720 variables in the WCZ approach. The optimized NPVs from the median realization of the algorithms are presented in Fig. 11a while the final NPVs from all the five runs are presented in Fig. 11b. The three cases (cases 4 to 6) of the no-well interval described in "Example 1" were also considered in this example. However, only one scenario $\left(N_{\text{wells,max}} = 60\right)$ was considered and the

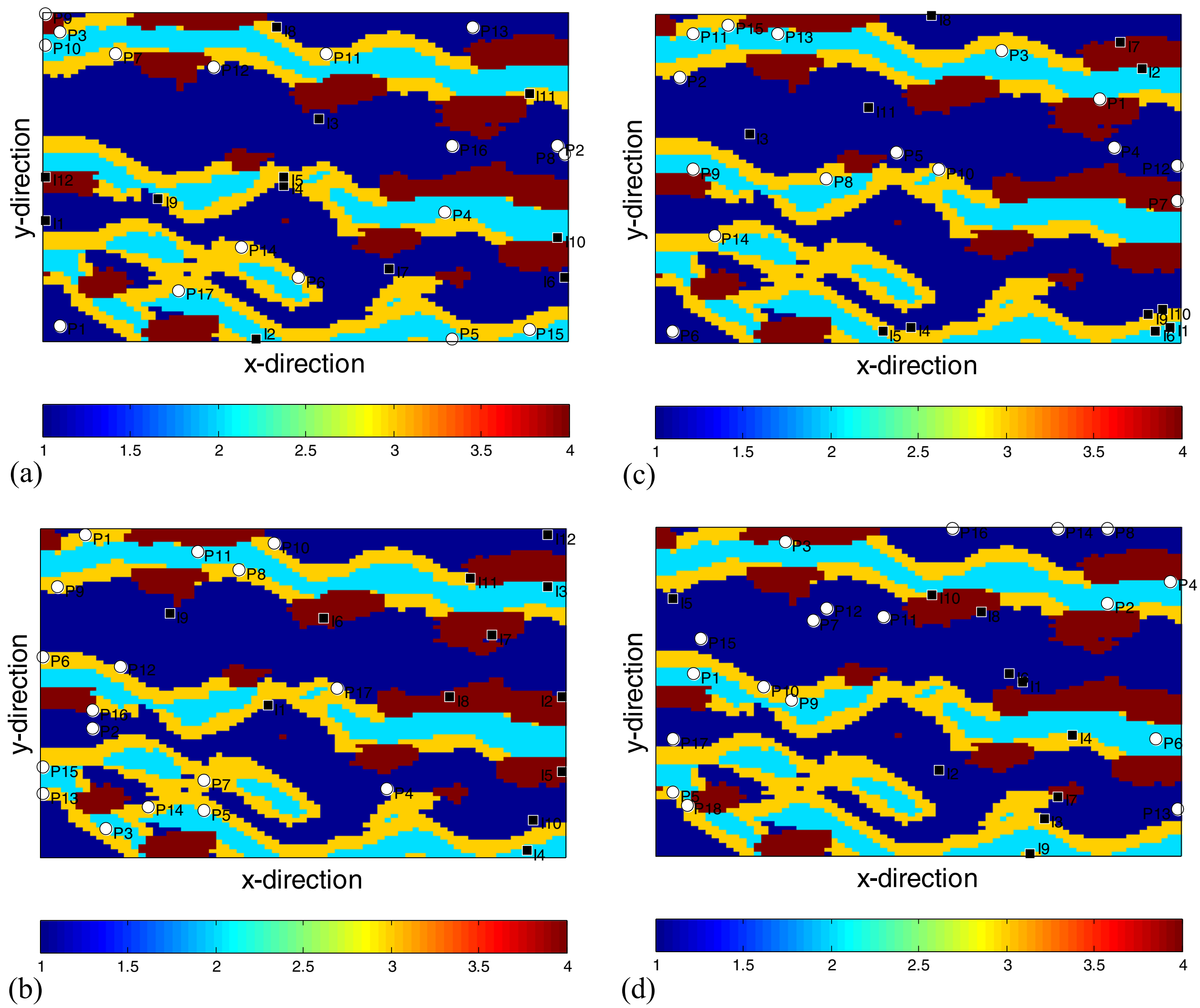

Fig. 13 Optimized well locations. **a** MINLP. **b** WCZ 4. **c** WCZ 5. **d** WCZ 6 (rates, Example 2). Note: producers are represented by small *white circles* while injectors are presented by *small black squares*

Table 5 Estimated number of wells (Example 2)

Approaches	Rates n_i	Rates n_p	BHPs n_i	BHPs n_p
MINLP	12	17	**7**	**15**
WCZ 4	12	17	8	26
WCZ 5	**11**	**15**	6	15
WCZ 6	10	18	7	14

problem was solved separately under rate-constraints and under BHP-constraints. The secondary constraints under the rate or BHP-constraints are the same as presented in "Example 1." Under rate-constraints, WCZ 5 showed the best overall performance while MINLP exhibited the worst performance (Fig. 11). Figure 12 shows the optimized NPVs under BHP-constraints. In this case, the MINLP exhibited the best performance while WCZ 5 came second. However, WCZ 5 produced slightly higher NPVs than MINLP in three of the five runs. Also, we observe that the optimized NPVs obtained under rate-constraints (Fig. 11) are much higher than those obtained under pressure constraints (Fig. 12). We conclude that the optimized NPVs from both the rate-constraints and the pressure-constraints in this example indicate that the optimum no-well width should be about half of the search space (as indicated by Case 5).

The optimized well locations are shown in Fig. 13a–d for the best runs of the algorithms/cases under rate-control. We again observe from the figures that wells of the same type cluster in the same region of the reservoir in similar fashion to those observed in Example 1. The estimated numbers of wells from the algorithms are presented in Table 5. The optimum numbers corresponding to the algorithm that yielded the overall highest NPV (i.e., WCZ 5) are 11 and 15 for the injectors and producers, respectively, indicating that only 26 wells were needed to develop the reservoir in this example. The second highest NPV was obtained from WCZ 6 and this indicated a need for 10 injectors and 18 producers (28 wells). This numbers are much less than the stipulated maximum number of wells (60) used in solving the problem, explaining the need for large no-well zone.

7 Conclusion

A well-control zonation (WCZ) approach was presented to optimize well controls, well type, and number of wells. The approach is based on the extension of the search space of the well-control variable (rates or BHPs) to create three well-control zones: the injection-well zone, the no-well zone, and the production-well zone. The approach helps to avoid the use of integer optimization variables in determining well types and also reduces the total number of optimization variables needed. The width of the no-well zone is not fixed but left to the user to determine. This makes it possible to adjust the width for different degrees of uncertainty associated with setting the maximum allowable number of wells in the optimization problem. The approach was used in conjunction with the piecewise constant approach and was compared to the MINLP method of handling the optimization problem [28]. Two example problems were designed to test the performance of the approach relative to the MINLP. The first example was used to test different scenarios involving different values of the maximum number of wells allowed $\left(N_{\text{wells,max}}\right)$. Results show that the proposed algorithm (well-control zonation) mostly outperformed the existing algorithm (MINLP). Also, we observed that the WCZ performed best when the width of the no-well zone is as large as or larger than the combined width of the two other zones.

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Appendix

In this appendix, we show the effectiveness of using Eq. 16 in computing the population size in DE algorithm. This is done by comparing the optimized NPV from this population size to those generated by using larger population sizes. In the literature, a population size 10 times the problem dimension [37] was suggested. However, this size is often too large and unnecessary for many practical problems. We have found that Eq. 16 yields a much smaller population size and we show that DE performs very well with this small population size by comparing with two larger population sizes. The larger sizes considered are a population size that is 3 times of that generated by Eq. 16 and another size that is 6 time that generated by Eq. 16. We have used Case 4 and Scenario 2 of Example 1 for this study. In order to allow the cases with larger population sizes to realize their full potential, we have allowed a total of 6000 function evaluation for those cases while keeping only 3000 function evaluations for the original case (Eq. 16). In Figs. 14 and 15, N_p represents the NPV obtained from the population size used in this work (i.e., computed from Eq. 16). The other two labels represent the NPVs obtained from population that are 3 and 6 times larger than N_p, respectively. Only the best, median and worst runs (of the five runs) are presented. It is evident from the figures that using Eq. 6 is much better than using the larger population sizes.

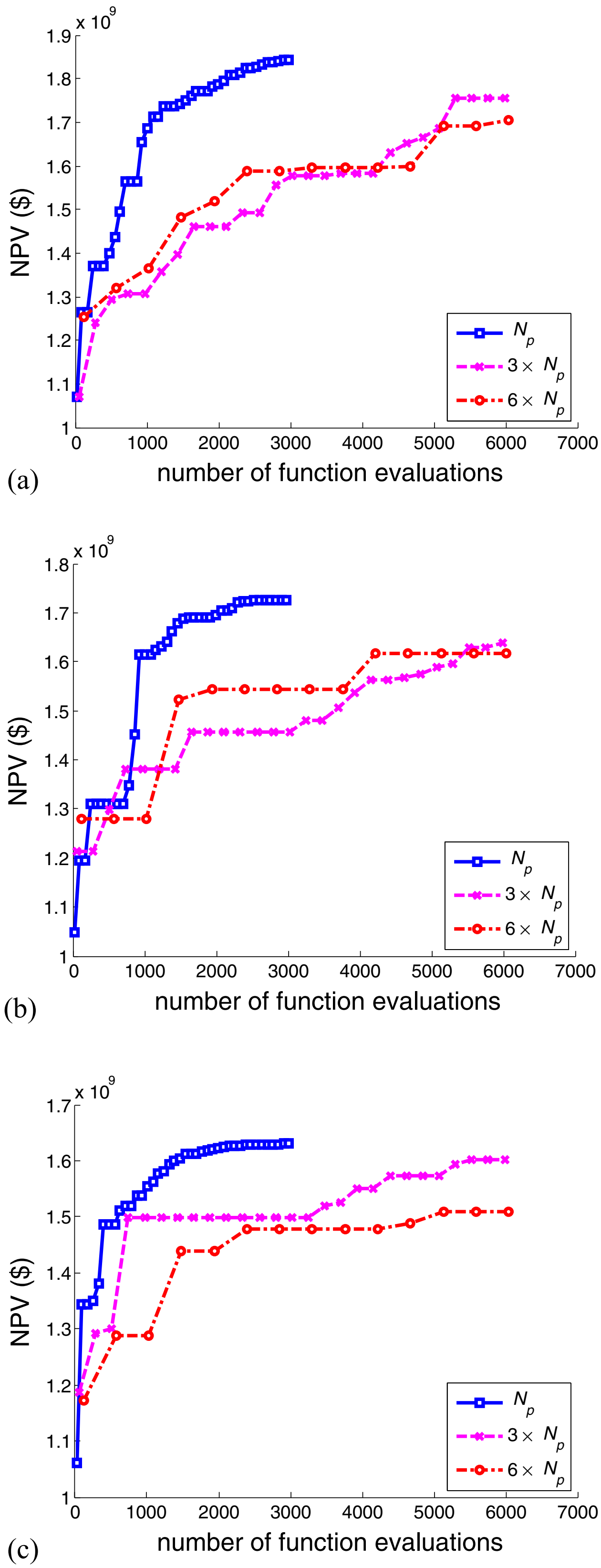

Fig. 14 Comparison of the NPVs attained by different algorithms on the optimization problem with rates as well controls. **a** Best. **b** Median. **c** Worst (Scenario 2 of Example 1)

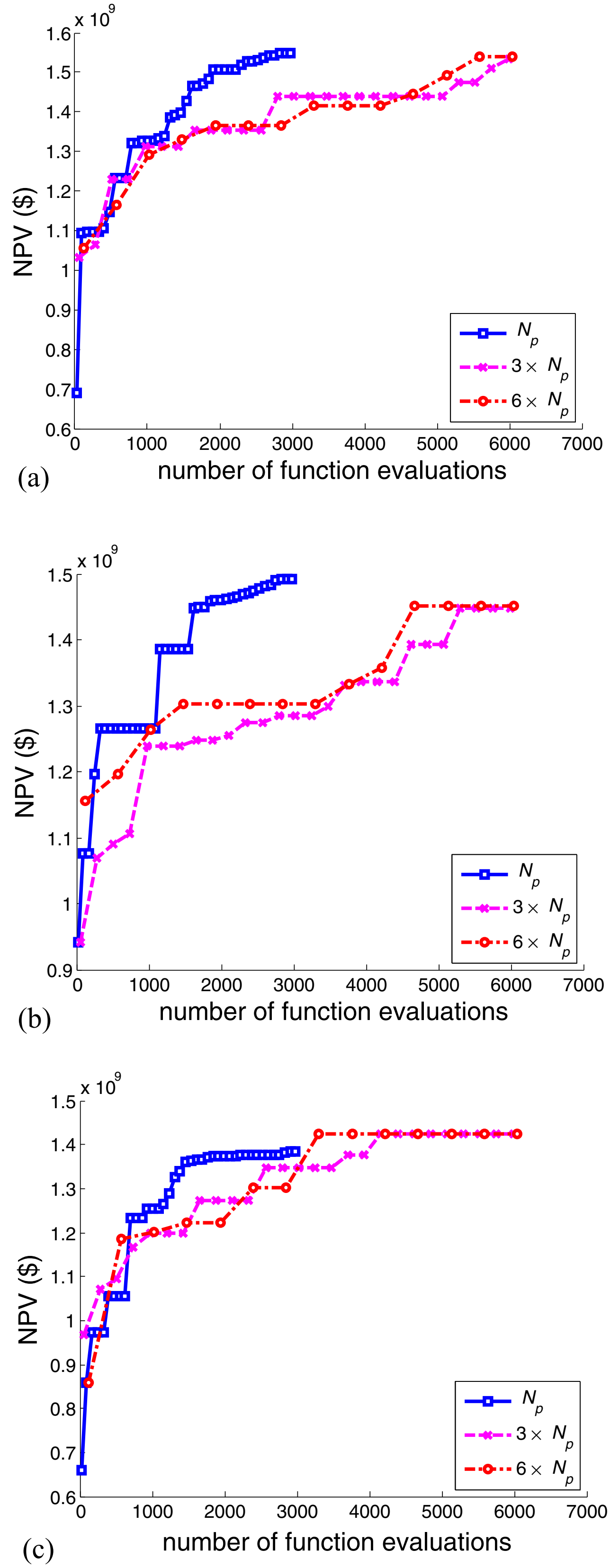

Fig. 15 Comparison of the NPVs attained by different population sizes on the optimization problem with BHPs as well controls. **a** Best. **b** Median. **c** Worst (Scenario 2 of Example 1)

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