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Treasury Management Model with Foreign Exchange Exposure

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Abstract. In this paper we formulate a model for foreign exchange exposure management and (international) cash management taking into consideration random fluctuations of exchange rates. A vector error correction model (VECM) is used to predict the random behaviour of the forward as well as spot rates connecting dollar and sterling. A two-stage stochastic programming (TWOSP) decision model is formulated using these random parameter values. This model computes currency hedging strategies, which provide rolling decisions of how much forward contracts should be bought and how much should be liquidated. The model decisions are investigated through ex post simulation and backtesting in which value at risk (VaR) for alternative decisions are computed. The investigation (a) shows that there is a considerable improvement to "spot only" strategy, (b) provides insight into how these decisions are made and (c) also validates the performance of this model.

Keywords: scenario generation, stochastic programming, fx currency hedging, value at risk, vector error correction model, treasury management, forward price, spot price, mark to market

1. Introduction and background

Foreign Exchange (FX) markets have gone through a turbulent period since 1973 (after the collapse of Bretton Woods). More recently since 1999 with the emergence of the euro as well as increased globalisation of trade a spectacular amount of currency movement has been recorded. In her recent book Taylor [34] reports that more than 1.2 trillion USD change hands daily on the foreign exchanges. It is therefore only natural that FX management has become an important topic especially so over the last decade.

The FX participants can be grouped into four categories. (i) The first participants are domestic and international banks, which act on their own behalf and for their customers. (ii) The second group comprise the Central banks, which may intervene in the market in order to support or suppress the value of the domestic currency for reserve management purposes. (iii) The third group is made up of multinational firms (MNFs) who are the customers of banks and buy physical currency in the spot or forward FX market for the purposes of facilitating trade. These MNFs buy and sell foreign currency. (iv) The fourth group includes the individual or corporate speculators or traders. In general FX decisions can be seen from two perspectives, such as: (a) hedgers and (b) speculators or traders. In this paper we use the term trader and speculator interchangeably from now on.

The currency management undertaken by multinational firms (MNF) constitutes only a small fraction (5–10%) of total FX transactions. Yet for the purpose of treasury management

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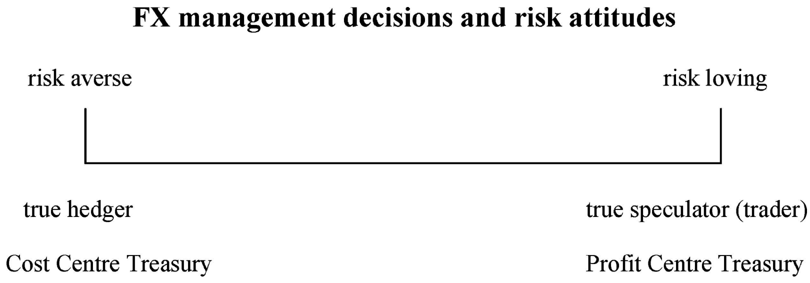

Figure 1. FX decisions and risk attitudes.

hedging and limited trading are of vital importance to the corporations and FX decisions can be categorised as shown in figure 1, Taylor [34]. Whereas introducing some element of FX trader (speculator) approach may lead to a better FX decision making there are natural pitfalls for an MNF should it move too far to the right of the scale shown in figure 1. The well-known case of Metallgesellschaft A.G. is one of a few notorious examples of the plight of MNFs who ventured into FX trading activities largely from the position of a speculator. In this paper we are concerned with risk exposure of a multinational firm (MNF) and treasury risk management requirement in respect of FX exposure.

The traditional foreign currency exposure represents a certain (known in advance) volume of foreign currency cash flows exchanged to the domestic currency at an uncertain future exchange rate. The optimal hedge ratio represents the ratio of the amount of foreign currency cash flow covered by forward contracts to the uncovered future foreign currency cash flow, such that this ratio minimises the risk (measured by variance) of the portfolio formed by future cash flows and a position in forward contracts. The optimal hedge ratio can be calculated by creating a portfolio of two assets: an unhedged future foreign currency cash flow and a position in a forward currency market. Then it can be shown that the minimum variance portfolio is achieved when the optimal hedge ratio takes the value $[-\text{cov}(s_t, f_t)/\text{var}(f_t)]$, where s_t, f_t are the spot and forward exchange rates respectively. Provided the future cash flow stream is known with certainty it is very likely for the value of the optimal hedge ratio to be in the region of 0.9 or higher [14, 25, 33] for most of the currencies.

Adler and Dumas [2], Eaker and Grant [13], and Shapiro [29] have addressed various implications of uncertain cash flows on hedging decisions. Eaker and Grant study the effect of new information on the optimal hedge, while Shapiro examines the case of multiple hedging tools. Adler and Dumas show that the optimal hedge ratio is the coefficient of a regression of the cash flow (expressed in home currency) on the exchange rate. First the treasury manager specifies a number of future states of nature regarding cash flows, exchange rates, and their respective probabilities. Then the regression coefficient is estimated from a linear regression across the states of nature. Rolfo [28], Stiglitz [32], Britto [8], and Hirshleifer [20] have examined the problem of hedging uncertain production and hedging in macro-market frameworks.

A more realistic setting, where an MNF has to hedge both uncertain FX exposure and uncertain future foreign currency cash flows simultaneously was investigated by Kerkvliet and Moffett [21]. They show that the optimal hedging decisions will be firm-specific and depend on the extent of correlation among the cash flows, spot and futures exchange rates.

FX risk hedging in a static, single-period framework is a straightforward decision problem. The variance-minimising hedge involves taking a position in forward FX market equal in size but opposite in sign to the particular future foreign currency cash flow exposure. It can be shown that this exposure represents the regression coefficient of the cash flow on the exchange rate.

In a multi-period setting optimal hedging is less straightforward. The hedging decision taken at an early stage may be revised many times due to new information being revealed to the market. These frequent revisions may themselves constitute additional risks to the MNF. Dumas [12] investigates the timing when it is optimal to initialise a hedge. He examines the case of deliberately leaving the cash flows unhedged for some time, initiating the hedge at some appropriate time and then leaving the hedge unchanged until the cash flow is received or paid. He states that the appropriate timing of the optimal hedging decision depends on whether the cash flow to be hedged is correlated with the changes in the exchange rates or with its level.

Sharda and Musser [30] used a multi-objective goal programming model for bond portfolios. Their approach is to dynamically hedge interest rate risk using futures contracts. In 1993 Sharda and Wingender [31] reapplied the same model with some modifications to hedging foreign currency accounts receivables using foreign exchange futures. Wingender and Sharda [37] in their later paper modified their original model in several ways. They examined a portfolio of Treasury Notes, incorporation of priorities and the previous week's futures position. The above three studies improve on the static framework by allowing the treasury manager to re-estimate and re-adjust the optimal hedging decisions every time period of the multi-period time horizon. Although these are otherwise comprehensive optimum decision models, the main shortcomings of these studies are that they consider neither stochastic cash flows nor stochastic future exchange rates.

In many real world problems, the uncertainty relating to one or more parameters can be modelled by means of probability distributions. In essence, every uncertain parameter is represented by a random variable over some canonical probability space; this in turn quantifies the uncertainty. *Stochastic Programming (SP)* enables modellers to incorporate this quantifiable uncertainty into an underlying optimisation model. Stochastic Programming models combine the paradigm of dynamic linear programming with modelling of random parameters, providing optimal decisions which hedge against future uncertainties.

Two-stage and multistage SP framework provides a logical extension of the deterministic approach to optimum decision models. SP incorporates uncertain parameters into the model, and the optimal decisions recommended by the model take into account a multi-period time horizon. There have been numerous applications of SP methodology to real life problems over the last two decades. Kusy and Ziemba [24] formulated a multistage SP to balance a bank's revenues from a set of assets against a set of liabilities. The assets consist of investments and loans with uncertain returns and varying risk levels, whereas the liabilities represent depositor's withdrawals from demand accounts. Klaassen et al. [22] use a multistage SP model to select a minimal cost currency option portfolio to hedge FX exposure faced by an MNF. The portfolio guarantees an acceptable level of dollar revenues subject to a certain (known) quantity of a foreign currency to be exchanged in the future. Carino et al. [9] modelled a problem of asset management for a property insurance company as a multistage linear SP model. Golub et al. [17] developed a two-stage SP model for money management using mortgage-backed securities. Beltratti et al. [5] formulated an SP model for portfolio management in the international bond markets. In Topaloglou et al. [35] an integrated simulation and optimisation framework for multicurrency asset allocation problems is reported. The authors examine empirically the benefits of international diversification and the impact of hedging policies on risk-return profiles of portfolios. In Beltratti et al. [5] the authors develop a scenario based optimisation model that simultaneously makes optimal asset allocation and hedging decisions, They contrast selective hedging with complete hedging and no-hedging strategies. Wu and Sen [38] used SP approach to develop currency option hedging models, which addresses a problem with multiple random factors in imperfect markets. Kouwenberg [23] developed a multi-stage SP model for pension fund asset liability management using rolling horizon simulations. The use of two stage stochastic programming model to determine the natural oil buying policy of an MNF taking up a forward position is discussed in Poojari et al. [27].

A number of different hedging instruments are available to the treasury managers [1] but in this paper we only consider forward currency contracts since they are the simplest and one of the most popular hedging products available to MNFs. The specification of the contract can be tailored to the requirements of the customer such as maturity date and size of the contract. Also the forward FX market is very liquid for major currencies and for maturities under two years, which makes it a perfect choice for the problem at hand.

In this study we have applied two-stage Stochastic Program (TWOSP) with recourse as a decision model. By using an SP framework we are able to take into account both time and uncertainty in our ex ante decision model. We also apply an ex post results analysis, which is based on backtesting with historical data.

The rest of this paper is organised in the following way. In Section 2 (with more details in Appendix A) we describe how the random exchange rate (forward and spot) fluctuations over future time periods are modelled. The forward rate and spot rate are modelled together using a vector error correction model (VECM).

In Section 3 we introduce a two stage stochastic programming (TWOSP) model, which is used for optimum (hedged) decision making under uncertainty. The decision model uses the random parameter values computed by the VECM model and presented as a scenario tree to the TWOSP. In addition to SP formulation the model incorporates a goal programming structure such that (a) the revenue in GBP after conversion is maximised (b) possible margin account "top ups" (virtual) for the "forward positions" are minimised and (c) target deviations from exact cash flow[1] matching by deviational variables are minimised. A full description of the model formulation is given in this section.

In Section 4 the TWOSP model is embedded as a rolling decision model within a simulation framework. Using historical data backtesting is carried out and the actual revenues achieved are tabulated and displayed in the form of a histogram. The results are analysed for the purpose of model validation and also to compute the Value-at-Risk (VaR) exposure. Finally, we draw our conclusions in Section 5 and discuss the scope of future work.

2. Modelling stochastic processes

2.1. Scenario generation

The econometric model used for generating scenarios of future exchange rates is described in Appendix A.

The data set used in our empirical analysis consists of 241 observations of the spot and forward (thirty-day rate) Sterling/Dollar exchange rate for the period from January 1984 to January 2004. The spot and forward exchange rates were found to be integrated of order one and to cointegrate.[2] This is in agreement with other studies.

Scenarios are generated using a series of recursive forecasts, which are computed in the following way. For a given bivariate time series $\{w_t = (f_t, s_t)'\}_{t=1}^{T}$ a VEC model is fitted to the subseries $\{w_t = (f_t, s_t)'\}_{t=1}^{T-n}$, where n is the desired number of forecasts and 12 is the longest forecast horizon under consideration, and $f_{t,t'}$ and $s_{t'}$ are the relevant forecasts.

Using $t = T - n$ as the forecast origin, 100 sequences of t'-step-ahead forecasts are generated from the fitted models for $t' \in \{1, \ldots, 12\}$, by drawing $\{u_t^*\}$ randomly from the bivariate normal distribution of $\{u_t\}$. To ensure the relevance of our artificial time series, the values of the parameters σ_s^2, σ_f^2 and $\sigma_{s,f}$ are chosen on the basis of the empirical variance-covariance matrix obtained with the Sterling/Dollar exchange rate data.

The forecast origin is then rolled forward one period to $t = T - n + 1$, the parameters of the forecast models are re-estimated and other 100 sequences of one-step-ahead to 12-step-ahead forecasts are generated. The procedure is repeated until 100 forecasts are obtained for each $t' \in \{1, \ldots, 12\}$, which are used as an input to the SP decision model.

2.2. The scenario tree

Consider a probability space, $(\Omega, \mathcal{F}, P)$ where $\omega \in \Omega$ denotes parameter realisations with probability $p(\omega) \in P$ and $\mathcal{F}$ is a σ-field on Ω. For the current time period $t = 0$: $f_{0,t'}$ and s_0 are known with certainty. Whereas, the data paths are depicted by the triplet $(f_{t',t'}(\omega), s_{t'}(\omega), p(\omega))$, $t = 1..T$, $t' = 1..T'$, $\omega = 1..|\Omega|$, which provide all the necessary information about the forward and spot rates in the future. In out model $p(\omega) = \frac{1}{|\Omega|}$, that is, all the data paths are equiprobable.

The model introduced and validated in Appendix A represents a stochastic process. For the purpose of visualisation and a simple description, the behaviour of the parameters $s_{t'}(\omega)$ and $f_{t,t'}(\omega)$ over time, is illustrated by a tree of alternatives of possible parameter values with corresponding probability weightings. Each expected path in this tree from the origin to the end of the time horizon T' is a "data path". The scenario tree is illustrated in figure 2.

3. The problem setting

3.1. Two stage SP model with recourse

Stochastic Programming Problems with recourse are dynamic LP models characterised by uncertain future outcomes for some parameters. For decisions made under uncertainty it is a natural extension of the LP model.

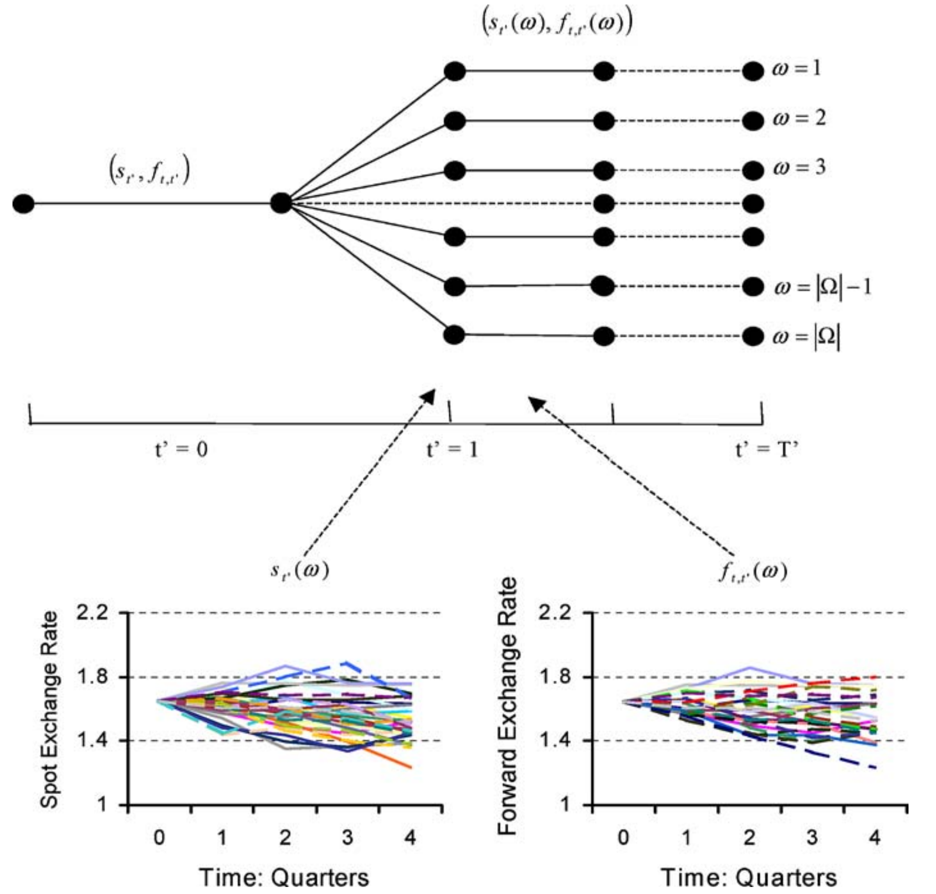

Figure 2. Scenario tree.

Consider the deterministic LP problem

$$z = \text{Min } cx \tag{3.1}$$

$$s.t.\ Ax = b \tag{3.2}$$

$$x \geq 0 \tag{3.3}$$

$$\text{where} \quad A \in \Re^{m \times n}, \quad c, x \in \Re^n, \quad b \in \Re^m \tag{3.4}$$

Let $(\Omega, \mathcal{F}, \text{P})$ be the probability space, $\omega \in \Omega$ the realisations of the uncertain parameters, $\mathcal{F}$ is a σ-field and $\text{P}(\omega)$ the probability of such realisations, and let $\xi(\omega) = (A, b, c)_\omega$ denote the vector of random model parameters which depends on the realisation of ω, also called a scenario. Let $C^{\omega} = \{x \mid Ax = b, x \geq 0\}$ for $(A, b, c)_{\omega}$ define the feasibility set for a given scenario (realisation) ω.

In general, a stochastic programming model is used to make a “Here and Now” (HN) ex-ante decisions and is formulated as set out in (3.5) to (3.11).

The classical stochastic linear program with recourse makes the dynamic nature of SP explicit, by separating the model’s decision variables into the first stage strategic decisions, which are taken facing future uncertainties and the second stage recourse (corrective) actions, taken once the uncertainty is revealed. The formulation of the classical two-stage SP model with recourse is as follows:

$$Z = \min cx + E_{\omega} Q(x, \omega) \tag{3.5}$$

$$\text{subject to } Ax = b \tag{3.6}$$

$$x \geq 0, \tag{3.7}$$

where:

$$Q(x, \omega) = \min\ f(\omega)y(\omega) \tag{3.8}$$

$$\text{subject to}\quad W(\omega)y(\omega) = d(\omega) + T(\omega)x \tag{3.9}$$

$$y(\omega) \geq 0 \tag{3.10}$$

$$\omega \in \Omega \tag{3.11}$$

The matrix A and the vector b are known with certainty. The function $Q(x, \omega)$, referred to as the recourse function, is in turn defined by the linear program defined by (4) to (7). The recourse matrix W(ω), the right-hand side $d(\omega)$, the technology matrix T(ω), and the objective function coefficients $f(\omega)$ of this linear program are random. For a given first stage decision x and a given realisation ω, the corresponding recourse action $y(\omega)$ is obtained by solving the problem set out in (3.8) to (3.11).

The stochastic properties of the recourse model are characterised by analysing three alternative problems:

(a). The first problem is defined using the expected value over the set Ω. In this approach the stochastic parameters are substituted by their expected values. The “*Expected-Value*” model becomes:

$$z_{ev} = \min f(x, \bar{\xi}) \tag{3.12}$$

The expectation of the expected value problem over the given scenarios is defined as:

$$z_{eev} = \mathrm{E}_{\xi}[f(\bar{x}(\bar{\xi}), \xi(\omega))] \tag{3.13}$$

(b). A second approach that relies on perfect information is called the "*Wait-and-See*" model:

$$z^{\omega} = \min f(x, \xi(\omega)), \quad x \in C^{\omega}, \forall\omega \tag{3.14}$$

Therefore

$$z_{ws} = \mathrm{E}(z^{\omega}) = \sum_{\omega\in\Omega} p(\omega) z^{\omega} \tag{3.15}$$

(c). A third approach so-called "*Here-and-Now*" where the decision-maker makes the decision "now":

$$z_{hn} = \min \mathrm{E}[f(x, \xi)] = \min \mathrm{E}[cx], \quad x \in C = \bigcap_{\omega\in\Omega} C^{\omega}, \tag{3.16}$$

where the optimal solution $x^* \in C$ hedges against all possible (known) contingencies $\omega \in \Omega$ that may occur in the future.

To assume the expected value scenario will occur and accept the solution of the expected value problem, is not always the right decision, since the expected value might be far from the scenario that actually takes place. The Wait-and-See problem cannot be implemented in reality, as the decision-maker must wait to take the decision only when the uncertainty is resolved. This is not realistic, since the decision-maker needs to decide before hand. Thereafter, to consider at a time all the possible (known) scenarios, so-called the Here-and-Now approach, is the most appropriate, since it hedges against all the uncertain future outcomes. The solution that this model provides is not optimum for any one outcome, but is the best for many outcomes considered altogether.

To verify whether the stochastic approach is better than any other, some SP analysis must be carried out. The analysis of stochastic programming models requires that we

(i) investigate the underlying expected value (EV) problem, and
(ii) compute stochastic information, such as the Expected Value of Perfect Information (EVPI), and the Value of the Stochastic Solution (VSS) defined below:

(a) *Expected value of perfect information (EVPI)*:

EVPI measures the maximum amount a decision-maker would be ready to pay in return for complete (and accurate) information about the future.

Let $z_{hn} = \min_x \mathrm{E}_{\xi} f(x, \xi)$, and $z_{ws} = \mathrm{E}_{\xi} \lfloor \min_x f(x, \xi) \rfloor$ then:

$$EVPI = z_{hn} - z_{ws} \tag{3.17}$$

(b) *Value of the stochastic solution (VSS)*:

Let $EV = \min_x z(x, \bar{\xi})$, where $\bar{\xi} = \mathrm{E}(\xi)$, and $\bar{x}(\bar{\xi})$ an optimal solution to the EV problem. Let

$$EEV = \mathrm{E}_{\xi}[f(\bar{x}(\bar{\xi}), \xi)] \tag{3.18}$$

Then, VSS measures the cost of ignoring uncertainty in choosing a decision and is defined as:

$$VSS = z_{eev} - z_{hn}. \tag{3.19}$$

It can be shown that the three objective function values z_{eev}, $z_{hn,}$ z_{ws} are connected by the following ordered relationship:

$$z_{ws} \le z_{hn} \le z_{eev} \tag{3.20}$$

The inequality:

$$z_{hn} \le z_{eev} \tag{3.21}$$

can be argued in the following way: any feasible solution of the average value approximation is already considered in the Here and Now model, therefore the optimal Here and Now objective must be better.

Bounds on EVPI and VSS

Some useful bounds on the EVPI and VSS are presented below:

$$0 \le EVPI \le z_{hn} - z_{ev} \le z_{eev} - z_{ev} \tag{3.22}$$

$$0 \le VSS \le z_{eev} - z_{ev} \tag{3.23}$$

These can help in estimating the relative benefit of implementing the computationally costly Stochastic Programming solution, as opposed to approximate solutions obtained by processing the Expected Value LP problem.

3.2. The SP decision model

The problem under investigation is to determine a strategy for employing forward exchange rate contracts to hedge against fluctuations in the spot rate between the US dollar and UK sterling. Currently, the Company receives a positive cash flow stream of US dollars every month that they convert into UK sterling using the available spot rate. Although the spot rate is uncertain for future time periods the Company has not engaged in using forward contracts. We wish to determine a policy of hedging against such uncertainties by allowing the Company to engage in forward contracts on exchange rates. Given the inherent risks in speculative trading in foreign exchange we include limits to reduce the risks of speculation on forward exchange rates.

The uncertainties involving forward exchange rates have been modelled as a discrete set of scenarios based on our work in VEC forecasting. We now develop a stochastic optimisation model for determining the best "hedged" investments in forward contracts of exchange rates. The first stage decisions represent the contracts on the forward exchange rates that should be purchased while the second stage decisions are of two types: goal deviational variables and future decisions about purchases of forward exchange rate contracts. We have adopted a similar approach to that of Sharda and Wingender (1991). They formulate a goal-programming model to dynamically hedge accounts receivables with futures currency contracts. We extend this method as follows:

Our objective function has three **main** components: (i) minimizing deviations from treasury targets, similar to that of Sharda's Goal Programming Model; (ii) minimizing transaction costs and (iii) we maximise the company's expected GBP-equivalent total income over the next 4 quarters. The treasury manager specifies the weights attached to each of these main goals. In addition they also set the level of risk exposure in achieving the third component of the objective.

By varying the weights assigned to different goals and varying the maximum forward exposure limit, the treasury manager has the flexibility to choose their preferred strategy. The two-stage stochastic programming decision model is formulated below.

Indices

$t = 1, 2, 3$: the set of future time periods in the planning horizon, corresponding to the end of each of the next 3 quarters from now.

$t' = 1, 2, 3, 4$: the forward currency contract maturity dates.

$t'' = 1, 2, 3, 4$: the set of time periods in the planning horizon when we assess the cumulative forward position.

$$\omega = 1, \ldots, |\Omega| : \text{ set of scenarios. } |\Omega| = 100.$$

Data

Transaction cost:

TransCost: The transaction cost of acquiring/selling a forward currency contracts. Theoretically there is no charge for entering a forward agreement though the bank could charge for selling back the outstanding forward contract (closing out a forward position). Transaction cost can also reflect an ask-bid spread. In this study we set the value of transaction costs at 0.1% of the contract value.

Exchange rates:

$USDFwdRateCurrent_{t'}$: Currently available forward exchange rate on selling USD with maturity t'.
$USDFwdRateFuture_{\omega,t,t'}$: Forecasted forward exchange rate on selling (buying) USD at time t with maturity t' under scenario ω, for $\omega \in \Omega, t \in [1, 2, 3], t' \in [2, 3, 4]$ and $t' > t$.
$USDSpotRate_{\omega,t'}$: Forecasted spot exchange rate on selling USD at a future time t' under scenario ω, for $\omega \in \Omega, t' \in [1, 2, 3, 4]$.
USDSpotRateCurrent: Current spot exchange rate on selling USD for GBP.

Cash Flows:

$NetCashFlow_{t'}$: Amount of USD revenue less expenses in month t', for $t' \in [1, 2, 3, 4]$.

Initial data:

$FwdPrev_{t'}$: Number of forward currency contracts brought forward from the previous quarter with maturity month t', for $t' \in [1, 2, 3]$.

Other data:

UpperLimitOnHedge: Treasury set upper limit on the proportion of the net cash flows to be offset by taking a position in the forward exchange rate contracts.

Prob_{ω}: Probability of scenario ω.

$W_1 W_2$ and W_3: The weights assigned to the three main components of the objective.

$w_{1,1}$, $w_{1,2}$, $w_{1,3}$: The weights assigned to the 1-st, the 2-nd and the 3-rd goal associated with deviation from treasury targets.

First-Stage Decision Variables

$XFwdHold_{t'}$: The total amount of current USD forward currency contracts held with maturity date t', for t' $\in [1, 2, 3, 4]$.
$XFwdBuy_{t'}$: The number of forward currency contracts acquired at the beginning of the current month with maturity date t', for $t' \in [1, 2, 3, 4]$.
$XFwdSell_{t'}$: The number of forward currency contracts settled (sold) at the beginning of the current month with maturity t', for $t' \in [1, 2, 3, 4]$.

Second-Stage Variables

$YFwdBuy_{\omega,t,t'}$: The amount of USD to buy forward at the beginning of month t with maturity date t' under scenario ω where $t < t'$, for $\omega \in \Omega$, $t \in [1, 2, 3], t' \in [2, 3, 4]$.

YFwdSell$_{\omega,t,t'}$: The expected amount of USD to sell forward at the beginning of month t with maturity date t' under scenario ω where $t < t'$, for $\omega \in \Omega$, $t \in [1, 2, 3]$, $t' \in [2, 3, 4]$.

ExpectedGBPValFromSpot: A reporting variable representing the expected GBP-converted income from future net cash flows for quarters: 1, 2, 3 and 4 using spot exchange rates at the same time as when the income stream is received.

ExpectedGBPValFromForward: The expected GBP-converted income e.g. gain or loss, from the positions taken in forward currency contracts made over the next 4 quarters.

ExpTransCost: The expected transaction costs for the positions taken in forward currency contracts over the next 4 quarters.

UnderHedge$_{\omega,t'}$: A goal variable representing the GBP amount by which the change in cash flows with maturity t' is larger than the change in forward currency position for quarter t' under scenario ω, for $\omega \in \Omega$, $t' \in [1, 2, 3, 4]$.

OverHedge$_{\omega,t'}$: A goal variable representing the GBP amount by which the change in cash flows with maturity t' is less than the change in forward currency position for quarter t' under scenario ω, for $\omega \in \Omega$, $t' \in [1, 2, 3, 4]$.

TopUp$_{\omega,t'}$: A goal variable representing the speculative loss, i.e. the amount of GBP top-up to the "virtual margin account" during quarter t' under scenario ω, for $\omega \in \Omega$, $t' \in [1, 2, 3, 4]$.

Objective Function

The objective function represents a trade-off between the immediate potential losses in the first quarter, the expected transaction costs and the value of the wealth over the whole planning horizon. These are now described.

Goal 1: Minimize deviations from the treasury targets (imminent losses)

By assigning different (respective) weights and summing into a linear form this goal is decomposed into three sub-goals summarized below.

1. Minimize the expected change in the cash flow over the change in the forward position.
2. Minimize the expected change in the forward currency position over the change in the cash flows.
3. Minimize the expected top-up amount on the "virtual margin account" i.e. speculative loss.

Goal 2: Minimize expected transaction costs over the next four quarters

Goal 3: Maximize expected cumulative GBP-equivalent income over the next four quarters

This goal is achieved by maximizing the two sub-goals shown below:

Future net cash flows over t', where $t' \in [1, 2, 3, 4]$, converted to GBP at the spot exchange rates at the time income is received.

Expected cumulative GBP-equivalent gain or loss made over t', where $t' \in [1, 2, 3, 4]$ from taking positions in forward exchange rate contracts.

The algebraic representation of the complete objective function is as follows:

$$\begin{aligned} \text{MinZ} = {} & W_1^* \Bigg(w_{1,1}^* \sum_{\omega \in \Omega} \text{Prob}_\omega^* \sum_{t'=1}^{T'} \mathit{UnderHedge}_{\omega,t'} \\ & + w_{1,2}^* \sum_{\omega \in \Omega} \text{Prob}_\omega^* \sum_{t'=1}^{T'} \mathit{OverHedge}_{\omega,t'} + w_{1,3}^* \sum_{\omega \in \Omega} \text{Prob}_\omega^* \sum_{t'=1}^{T'} \mathit{TopUp}_{\omega,t'} \Bigg) \\ & + W_2^* \mathit{ExpTransCost} - W_3^* (\mathit{ExpectedGBPValFromSpot} \\ & + \mathit{ExpectedGBPValFromForward}) \end{aligned} \tag{3.24}$$

subject to:

(1) *Constraints related to treasury target*

$$\begin{aligned} & \mathit{XFwdHold}_1(1/\mathit{USDFwdRateCurrent}_1 - 1/\mathit{USDSpotRate}_{\omega,1}) + \mathit{UnderHedge}_{\omega,1} \\ & -\mathit{OverHedge}_{\omega,1} = \mathit{NetCashFlow}_{\omega,1}(1/\mathit{USDSpotRateCurrent} \\ & - 1/\mathit{USDSpotRate}_{\omega,1}) \quad \forall \omega \end{aligned} \tag{3.25}$$

Equation (3.25) seeks to establish the necessary forward position for maturity $t' = 1$ in order to offset the changes in the net cash flows in three months time. The $\text{UnderHedge}_{\omega,1}$ and $\text{OverHedge}_{\omega,1}$ represent under and over achievements of this goal. Although the model is developed over four (quarters) time periods, we are more concerned with potential losses in the next quarter. This constraint emphasises that this can be hedged against by committing to more forward contracts.

$$\begin{aligned} & \mathit{XFwdHold}_{t'}(1/\mathit{USDFwdRateCurrent}_{t'} - 1/\mathit{USDFwdRateFuture}_{\omega,1,t'}) \\ & + \mathit{UnderHedge}_{\omega,t'} - \mathit{OverHedge}_{\omega,t'} = \mathit{NetCashFlow}_{\omega,t'}(1/\mathit{USDSpotRateCurrent} \\ & - 1/\mathit{USDSpotRate}_{\omega,1}) \quad \forall \omega, t' > 1 \end{aligned} \tag{3.26}$$

Equation (3.26) similarly considers the same issues as for Eq. (3.25) but for contracts with maturities $t' > 1$. The $\text{UnderHedge}_{\omega,t'}$ and $\text{OverHedge}_{\omega,t'}$ represent under and over achievements of this goal.

$$\mathit{XFwdHold}_1(1/\mathit{USDFwdRateCurrent}_1 - 1/\mathit{USDSpotRate}_{\omega,1}) + \mathit{TopUp}_{\omega,1} \geq 0 \quad \forall \omega \tag{3.27}$$

Equations (3.25) and (3.26) encourage investing in forward contracts, while constraint (3.27) accounts for possible losses incurred in these forward contracts and as such, conflicts with constraint (3.25). By changing the weights in the objective of these deviational variables it is possible to investigate different strategies for controlling speculation in the forward market.

$$XFwdHold_{t'}(1/USDFwdRateCurrent_{t'}-1/USDFwdRateFuture_{\omega,1,t'}) + TopUp_{\omega,t'} \geq 0 \quad \forall\omega, t'>1 \tag{3.28}$$

Similarly, the set of Eq. (3.28) account for losses in forward contracts maturing in later time periods.

(2) *Expected transaction cost sonstraint*

$$\begin{aligned} ExpTransCost = {} & TransCost * \sum_{t'=1}^{T'} (XFwdBuy_{t'} + XFwdSell_{t'}) \\ & + TransCost * \sum_{\omega\in\Omega} \text{Prob}_{\omega} * \sum_{t=1}^{t'-1} \sum_{t'=1}^{T'} (YFwdBuy_{\omega,t,t'} \\ & + YFwdSell_{\omega,t,t'}) \end{aligned} \tag{3.29}$$

Equation (3.29) measures the expected transaction costs for the next 4 quarters from purchasing (selling) forward currency contracts.

(3) *Constraints related to the final wealth objective*

The following constraints are all related to measuring the final wealth of the revenues converted to sterling. The first set of constraints are balance constraints for forward contracts with maturity t' quarters ahead

$$XFwdHold_{t'} = FwdPrev_{t'} + XFwdBuy_{t'} - XFwdSell_{t'} \quad \forall t' \tag{3.30}$$

Equation (3.30) represents a balance constraint on the activities with forward exchange rate contracts, which actually take place at the beginning of the current time period.

$$\begin{aligned} & XFwdHold_{t'} + \sum_{t=1}^{t''} (YFwdBuy_{\omega,t,t'} - YFwdSell_{\omega,t,t'}) \\ & \quad \leq UpperLimitOnHedge * NetCashFlow_{\omega,t'} \quad \forall\omega, t' > 1, t'' = 1..t'-1 \end{aligned} \tag{3.31}$$

Equations (3.31) represent an upper limit on the expected cumulative forward exchange rate position at any future time period t''.

$$XFwdHold_{t'} + \sum_{t=1}^{t''} (YFwdBuy_{\omega,t,t'} - YFwdSell_{\omega,t,t'}) \geq 0 \quad \forall\omega,\ t'>1,\ t''=1..t'-1 \tag{3.32}$$

Equation (3.32) states that no short sales are allowed on forward contracts.

$$YFwdBuy_{\omega,t,t'} + YFwdSell_{\omega,t,t'} \le UpperLimitOnHedge * NetCashFlow_{\omega,t'} \quad \forall \omega, t' > 1, t = 1..t' - 1 \tag{3.33}$$

Equation (3.33) defines the upper limit on the future expected forward exchange rate trades.

$$XFwdHold_{t'} \le UpperLimitOnHedge * NetCashFlow_{\omega,t'} \quad \forall \omega, t' > 1 \tag{3.34}$$

Equation (3.34) represents an upper limit on the actual forward exchange rate position opened during the current time period.

$$ExpectedGBPValFromSpot = \sum_{\omega \in \Omega} \text{Prob}_{\omega} * \sum_{t'=1}^{T'} NetCashFlow_{\omega,t'} / USDSpotRate_{\omega,t'} \tag{3.35}$$

Equation (3.35) reports the expected GBP-valued cumulative net cash flows for the 4 quarters using spot exchange rates at the time when the cash flow is received.

$$\begin{aligned} &ExpectedGBPValFromForward \\ &= \sum_{\omega \in \Omega} \text{Prob}_{\omega} * \sum_{t'=1}^{T'} XFwdHold_{t'} (1/USDFwdRateCurrent_{t'} \\ &\quad -1/USDSpotRate_{\omega,t'}) + \sum_{\omega \in \Omega} \text{Prob}_{\omega} * \sum_{t=1}^{t'-1} \sum_{t'=1}^{T'} (YFwdBuy_{\omega,t,t'} - YFwdSell_{\omega,t,t'}) \\ &\quad \times (1/USDFwdRateFuture_{\omega,t<t',t'} - 1/USDSpotRate_{\omega,t'}) \end{aligned} \tag{3.36}$$

Equation (3.36) measures the expected marginal benefit from using forward currency contracts for the planning horizon, e.g. the company either obtains a speculative gain or a loss from entering forward currency contracts with various maturities.

4. Backtesting and rolling forward the SP decision model

Our modelling framework has three aspects (a) calibration of the VECM model, which is used for scenario generation, (b) a decision model, (c) a simulation model to evaluate the decisions (backtesting). In our case there are three decision models, namely, Here-and-Now (HN), Expected Value (EV) and Perfect Information (PI) models (see Section 4.3). It is widely accepted by practitioners that out-of-sample simulation (see Michaud), which includes backtesting is an important step in validating practical financial models. For the purpose of simulation and backtesting we split the historical data into two parts. The Part 1 comprises first 169 monthly observations (Jan. 1984 to Jan. 1998) and the Part 2 comprises 60 months (Feb. 1998 to Jan. 2003) of the remaining 72 months[3] (Feb. 1998 to Jan 2004).

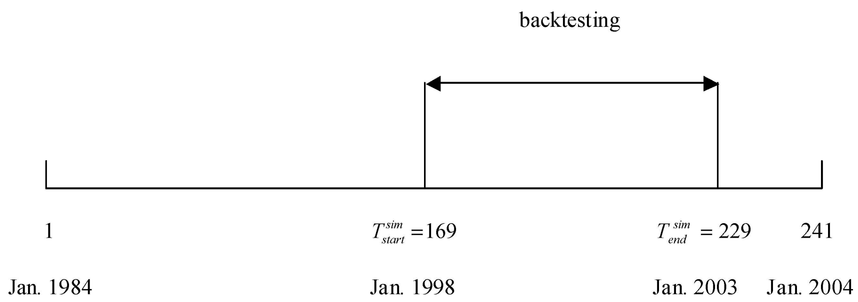

Figure 3. Breakdown of the historical data sample into a training sub-sample and backtesting sub-sample.

In figure 3 we explain this using a time line where $T^{\rm sim}_{\rm start} = 169$ and $T^{\rm sim}_{\rm end} = 229$ indicate the months, when backtesting starts and ends respectively.

The experimental set up is progressively described in the following sections: in Section 4.1 we discuss how the databases are updated for the decision model as we step through time (month at a time), Section 4.2 explains the rolling of the decision model, then in Section 4.3 we contrast three different decision models on the basis of Risk and Return (Income) and in Section 4.4 we compare different treasury strategies on the basis of stochastic measures.

4.1. Dynamic data model

The role of the historical market data, the organisational data, their interaction with the decision model and backtesting are illustrated in figure 4. The experimental set up requires that we dynamically:

(i) Use market data in order to revalue the forward positions, a well-known "mark to market" procedure.
(ii) We also record the decisions made in the current step of the model as an input of the starting position of the next "roll" of the model.

Whereas in futures currency contracts there is an external requirement for "marking to market", for forward positions there is no such obligation. As an "internal good practice procedure", however, we have introduced this in our "Forward currency contract" decision model so that we are able to compute the "moneyness" of the current positions to give some indication of ongoing performances.

Thus for each time movement the model database is updated with the most currently available forward rates and spot rates. By accessing our current forward commitments along with their current marked to market forward rate from the model database, we adjust our forward rates to be the same as the current month forward rates. The process of "marking to market" of our currently held forward contracts involves realigning the contracts by one time

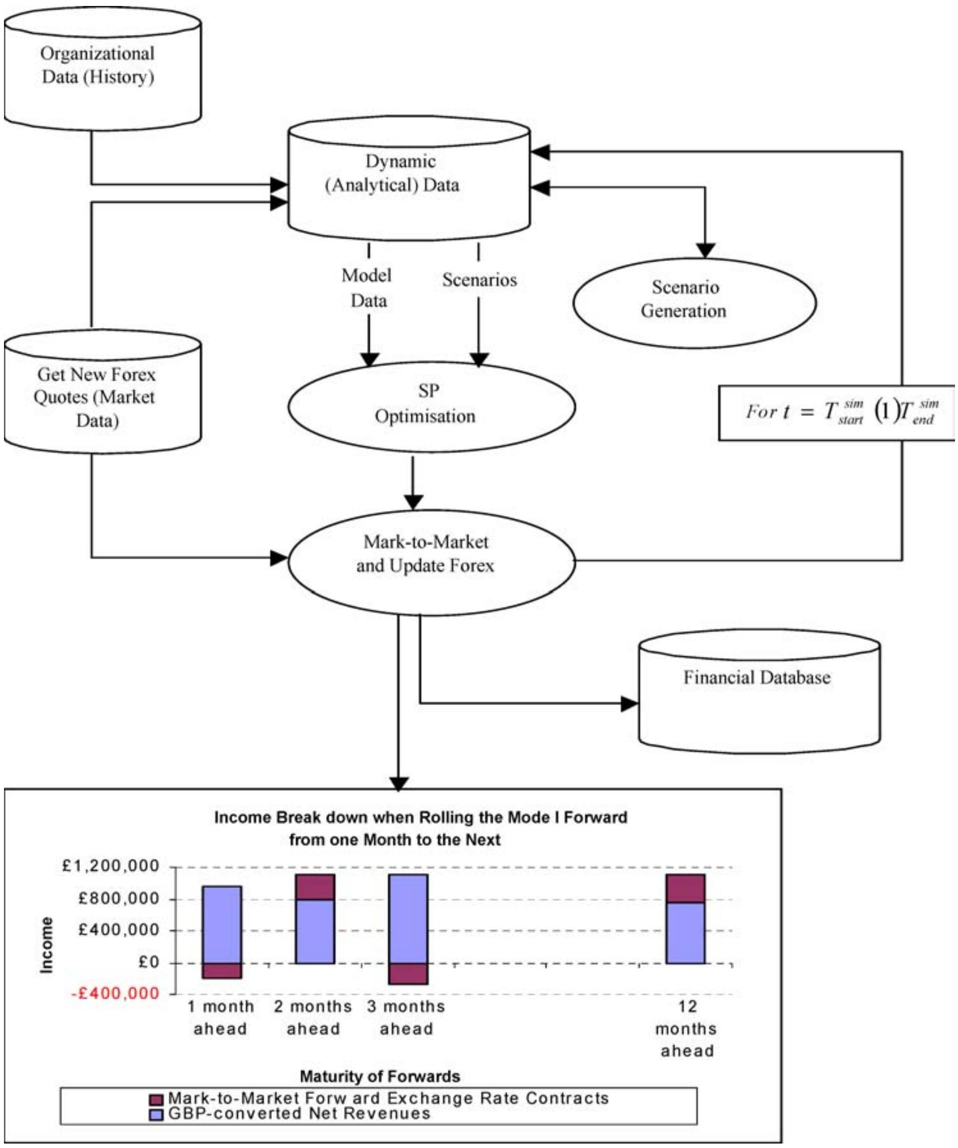

Figure 4. Rolling the model forward and "Mark to Market" process.

period as well as determining the financial losses or gains made on our forward positions. Similarly, we close out the opening income stream using a combination of currently maturing forward contracts and the current spot rate. All these cash transactions, namely the marking to market of forwards contracts and conversion of the current income revenue are recorded in our financial database.

4.2. The rolling decision model

The decision model uses data sets, which are updated every month. The scenario generator uses the historical spot rates and historical forward rates data having stepped through by one month $t = t + 1$. Thus the scenario generator creates a completely new set of scenarios looking ahead over a time horizon of $T = 12$ months. In respect of revenues we have developed a scenario generator for representing alternative realisations of the future income but in our current analyses we have assumed that the income stream remains constant.

Given that the decisions are made altogether 60 times by stepping through the time line $t = T^{\text{sim}}_{\text{start}}(1)T^{\text{sim}}_{\text{end}}$ we process the corresponding TWOSP model 60 times using the SPInE system. The SPInE system has the dual capability of SP decision modelling and simulation (see Valente et al. 2004; Di Domenica et al. 2004).

When rolling the model forward we mark to market positions held in forward contracts, which may have two outcomes. Firstly, in realigning our forward contracts to the current rates we either make some profit on our currently held forward contracts or our speculation has led to a loss, these are represented by the red bars. Secondly, in processing the current month's revenue we use the spot rate thus the income revenue is marked to market and is represented by the blue bars.

4.3. Simulation 1: Risk and return analysis

In this paper we estimate risk exposure of each treasury strategy by calculating Value-at-Risk (VaR) measure. The meaning of VaR in our paper is slightly different from the conventional one. When dealing with returns on investments VaR represents the maximum loss incurred with certain probability (e.g. 95%). In our case, since we are dealing with revenues, not returns, VaR represents the lowest monthly revenue achieved with certain probability. As a result, in the case of returns one aims at a smaller VaR, i.e. smaller loss but in our case we are better off having larger VaR, i.e. larger monthly revenues at a certain probability level.

We have assumed constant revenue throughout the planning horizon, also the financial decisions made prior to each optimisation run are independent; we run the rolling decision model and create a histogram with appropriate bins and compute VaR values for different probability levels for the monthly income.

In order to evaluate the impact of randomness on (optimal) decision we backtest the three rolling models over 60 months of the historical sample, Feb. 1998 to Jan. 2003.

- Here-and-Now (HN): represents the TWOSP model described in Section 3.2.
- Expected Value (EV): is the deterministic representation of our treasury model for foreign exchange rate exposure with the uncertain parameters replaced with by their expected values.
- Perfect Information (PI): is the deterministic representation of our treasury model for foreign exchange rate exposure with the true realised data for the uncertain parameters (historical data as scenarios). This model is the true upper bound on the overall optimisation problem.

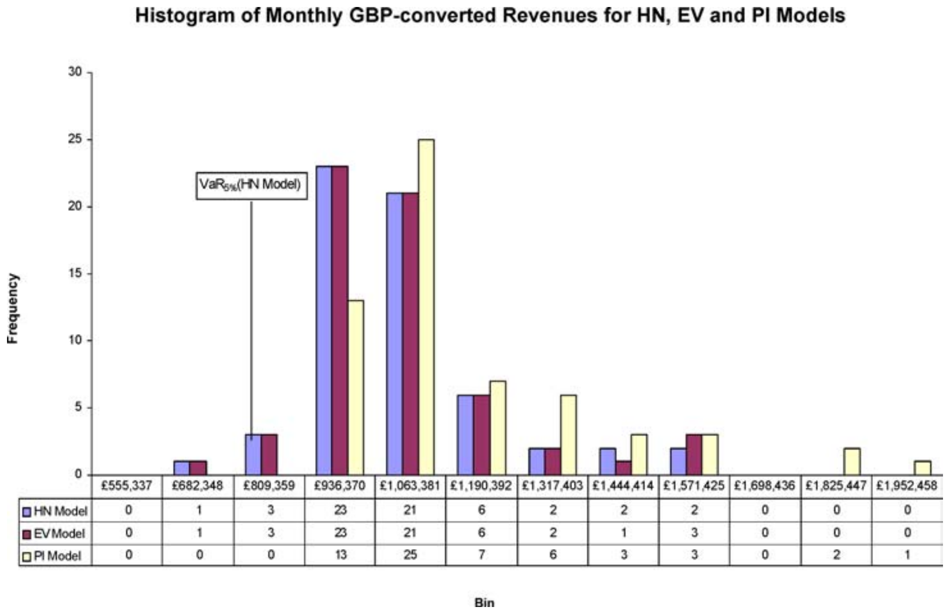

Figure 5. Histograms of monthly revenues associated with the output of each of the 3 models (HN, EV, PI), backtested to identify the revenue that those models (decisions) would have yielded.

We backtest the above models for all the strategies outlined earlier. In figure 5 we show the performance of our models over the 60 months period, using the following strategy: UpperLimitOnHedge $= 3$, $W_1 = 0$, $w_{1,1} = 0.3$, $w_{1,2} = 0.3$, $w_{1,3} = 0.4$, $W_2 = 1$ and $W_3 = 1$.

Figure 5 shows that the EV model has a marginally longer right tail than that of the HN model. It also shows that for this strategy the distribution for the EV decision is preferable to that of HN. However, if we consider a range of strategies as in Table 1 we see that this is not always the case.

Table 1 summarises the risk and returns measures for the 3 models investigated: PI, HN and EV over various treasury strategies. Each simulation run provides a possible realisation of the financial income received for any given time period. Experiments have been carried out on a number of different strategies each representing a particular upper limit on the forward exchange rate position and a combination of different penalties for not meeting the treasury targets.

As we can see from the above table, $\text{VaR}_{5\%}$ there is a trade-off between VaR and Revenues. Surprisingly, EV Model did not do worse that HN Model, which reflects the quality of the scenario generator. At the same time if we could "perfectly" foresee the future realisations of exchange rates we would receive higher monthly revenues. This also shows the potential for achieving better results by improving scenario generation model.

In the results displayed, the base strategy represents the current company practice of spot only conversion and not entering into forward contracts. The results indicate that there is

Table 1. Risk and return measures for HN, EV and PI models over various treasury strategies.

	Strategy 1 (Base)	Strategy 2 (Naïve Hedge)	Strategy 3	Strategy 4	Strategy 5	Strategy 6	Strategy 7	Strategy 8	Strategy 9	Strategy 10	Strategy 11
UpperLimitOnHedge	0	Hedge Ratio = 1	3	3	3	3	3	2	2	1	1
W_1 (Treasury targets goal weight)	0.5	0.5	0	1	0.5	0.5	0.2	0	1	0	1
$w_{1,1}$ (UnderHedge subgoal weight)	0.3	0.3	0.3	0.3	0.3	0.8	0.1	0.3	0.3	0.3	0.3
$w_{1,2}$ (OverHedge weight)	0.3	0.3	0.3	0.3	0.3	0.1	0.1	0.3	0.3	0.3	0.3
$w_{1,3}$ (Top-up weight)	0.4	0.4	0.4	0.4	0.4	0.1	0.8	0.4	0.4	0.4	0.4
W_2 (Expected transaction costs goal weight)	1	1	1	1	1	1	1	1	1	1	1
W_3 (Final wealth goal weight)	0.5	0.5	1	0	0.5	0.5	0.8	1	0	1	0
Historical cumulative GBP-income over 60 months (***HN*** model)	£58,406,737	£58,402,262	£59,715,890	£59,067,405	£58,451,172	£59,224,038	£58,341,578	£59,279,506	£59,067,405	£58,843,122	£59,091,799
s.d.	£55,349	£117,970	162,588	£72,826	£82,235	£123,414	£98,752	£111,932	£72,826	£68,834	£72,773
min	£879,611	£785,634	£682,348	£861,016	£800,581	£759,875	£785,754	£790,221	£861,016	£888,481	£861,016
Mean Monthly GBP-Income: ***HN*** Model	£973,446	£973,371	£995,265	£984,457	£974,186	£987,067	£972,360	£987,992	£984,457	£980,719	£984,863
$VaR_{5\%}$: ***HN*** Model	£898,150	£827,690	£780,047	£897,700	£840,225	£814,435	£840,225	£861,698	£879,611	£894,292	£897,700
Mean Monthly GBP-Income: ***EV*** Model	£973,446	£973,371	998,929	£972,533	£976,937	£990,137	£992,560	£990,434	£972,533	£981,940	£972,533
$VaR_{5\%}$: ***EV*** Model	£898,150	£827,690	785,914	£898,150	£853,277	£853,144	£821,438	£867,294	£898,150	£894,292	£898,150
Mean Monthly GBP-Income: ***PI*** Model	£973,446	£973,371	£1,111,140	£1,012,396	£1,110,708	£1,111,015	£1,111,085	£1,065,242	£1,012,396	£1,019,344	£1,011,559
$VaR_{5\%}$: ***PI*** Model	£898,150	£827,690	£900,410	£897,995	£900,410	£900,410	£900,410	£902,364	£897,995	£902,667	£897,995

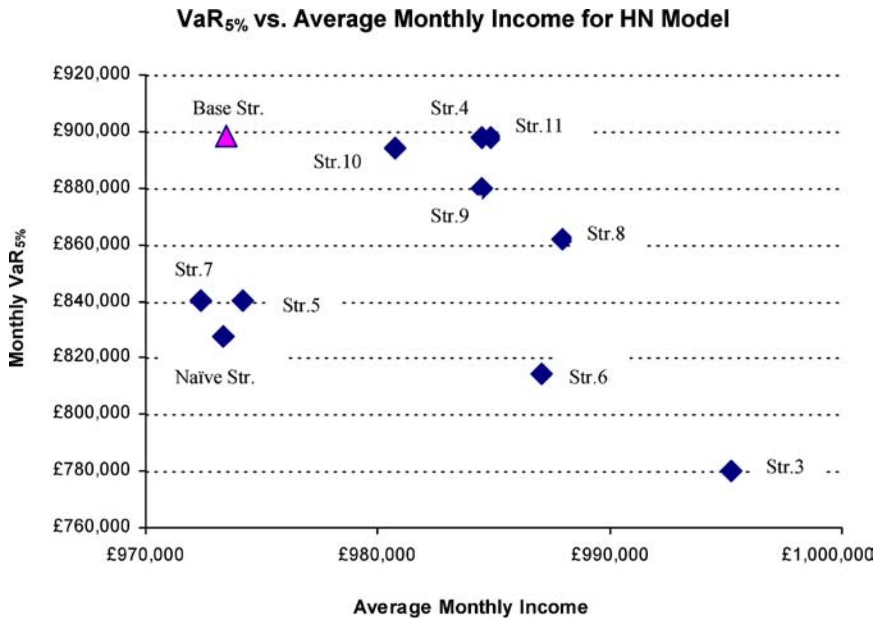

Figure 6. Income relationship for HN model.

scope of making more (marginal) income by using forward contracts. Although the risk increases with increased speculation as seen by the VaR values. The treasury manager can plot VaR against return (total terminal income) to determine their preferred strategy as shown in the figure 6.

By analysing the graph above a frontier can be plotted whereby strategies with points below this frontier are sub-optimal with respect to either return or VaR. For any inefficient strategy we can find an efficient point with more return for this level of VaR, or higher VaR for this given level of return. Our current investigations are looking at how easy it is to determine the various strategies to create points on the "efficient" frontier.

4.4. Simulation 2: Stochastic measures

In SP models the expected value of perfect information (EVPI) indicates, how far the stochasticity impinges on the decision-making. EEV stands for the expectation of the expected value (EEV) problem. The methods of computing these measures are summarised in Section 3.1. For a full discussion of stochastic measures including EVPI and EEV we refer the reader to Birge and Louveaux [16], p. 138–142.

We first solved HN, WS and EV models and then computed EEV and EVPI measures. The computations are conducted for October 2002 across all treasury strategies. The results are summarised in Table 2.

We have also investigated the dynamic behaviour of EVPI over 60 months time period. Figures 7 and 8 show respectively the dynamics of EVPI and the histogram over

Table 2. Stochastic measures computed for October 2002 for a representative four quarter model across all treasury strategies.

Strategies	WS	HN	EEV	EVPI	VSS
Strategy 1 (Base)	−1,937,463	−1,937,463	−1,937,463	0	0
Strategy 2 (Naïve Hedge)	−2,036,976	−2,036,976	−2,036,976	0	0
Strategy 3	−4,287,880	−4,137,954	−4,137,252	149,926	702
Strategy 4	25,699	36,271	39,849	10,572	3,578
Strategy 5	−2,103,658	−2,040,550	−2,035,255	63,108	5,295
Strategy 6	−2,123,410	−2,051,963	−2,039,159	71,447	12,804
Strategy 7	−3,421,978	−3,302,915	−3,302,915	119,063	0
Strategy 8	−4,163,512	−4,063,561	−4,063,093	99,951	468
Strategy 9	25,726	36,271	39,849	10,545	3,578
Strategy 10	−4,039,144	−3,989,168	−3,988,934	49,976	234
Strategy 11	27,016	36,271	39,514	9,255	3,243

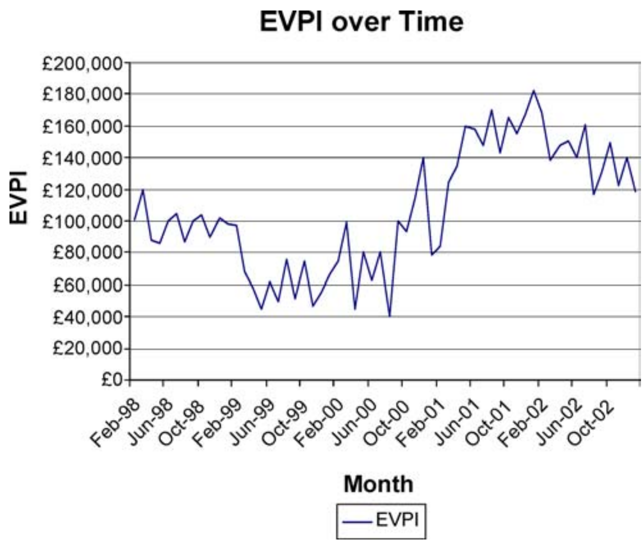

Figure 7. EVPI over time for strategy 3.

60 months of historical sample (Feb-1998 to Jan-2003) for just one treasury strategy: UpperLimitOnHedge $= 3$, $W_1 = 0$, $w_{1,1} = 0.3$, $w_{1,2} = 0.3$, $w_{1,3} = 0.4$, $W_2 = 1$ and $W_3 = 1$.

The average EVPI over 60 months[4] of historical sample equals £106, 818. In our context it could be interpreted as the average upper bound on the price of insurance to protect company's foreign currency revenues against uncertain exchange rates.

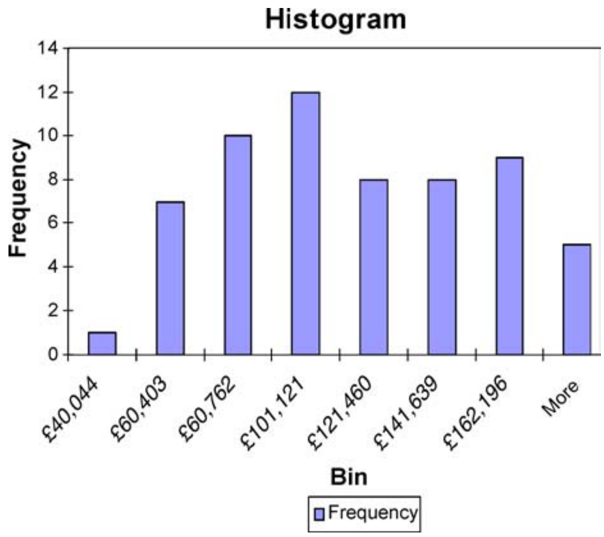

Figure 8. Histogram of EVPI for strategy 3.

5. Conclusions

We have considered an FX trading problem and proposed decision models, which can be used to develop and test alternative trading strategies. We have proposed a novel approach using SP as an ex-ante decision tool, which can be used by MNFs for the purpose of treasury management. The tool can be used both for decision making and simulation evaluation.

The VECM model (Appendix A) has been adopted in an innovative fashion to create exchange rate (forward and spot) scenarios, which are used in an SP decision model. We have applied ex post analysis of our decisions through backtesting (simulation).

Given that we roll the decision model over 60 months we have calculated the EVPI and VSS for each of these strategies 60 times. Although VSS is very small for all strategies, it cannot be deduced at the start of the investigation that the stochastic optimisation is redundant. However, for our application we can conclude that the HN and the EV models both provide good quality hedging decisions. This finding is also supported by backtesting both the HN model and the EV model over 60 months; both models produced similar results.

These results are for a constant cash (revenue) stream. These models can be easily extended to the situation when the revenue stream is random provided we know, how it relates to the fundamentals (interest rate, dividend rate, etc.), which also affect the FX rates. For random revenue stream it is likely that the VSS and EVPI might have larger values and HN might perform better than EV. The strategies for which EVPI has a relative high value may indicate that there is scope for performance improvement.

In summary we can state that:

(i) The effects of stochasticity on our hedging decisions are limited. Thus an EV model (LP) works nearly as well as an HN model.

(ii) For all the strategies considered both models provide much better results than "no hedging" spot only base strategy.
(iii) Backtesting PI model results in higher return and lower risk than both HN model and EV model for most of the strategies. This is only expected given the nature of PI model and can be taken as a benchmark or upper bound on the model performance.

Appendix A: Modelling exchange rates

The exchange rates: A model of cointegration between spots and forwards

The future realisation of the exchange rates, in particular their mean and variability over time, has the most important impact on the choice of currency hedging strategy. The two exchange rates (forward and spot) are forecast by suitable time series models. These appear as random parameters in our time staged currency hedging SP decision model, which is introduced in Section 3.

Our forecasting model is based on economic theories, which suggest the existence of long-run equilibrium relationships among variables. The idea is that even though short-run deviations from the equilibrium point are most likely, these deviations are bounded since stabilizing mechanisms tend to bring the system back to the equilibrium. Granger [18, 19] introduced and Engle and Granger [15] developed what can be regarded as the statistical counterpart of this idea: the concept of cointegration.

Cointegration allows individual time series to be stationary in first differences, while some linear combinations of the series are stationary in levels. By interpreting such a linear combination as a long-run relationship or an "attractor" of the system, cointegration implies that deviations from this attractor are stationary, even though the series themselves have infinite variance.

Granger [19] showed that there is a natural connection between the concept of cointegration and error-correction models. The latter may be thought of as providing an adjustment process through which deviations from a long-run equilibrium relationship (or an attractor) are corrected for.

The long-run relationship between the spot and the forward exchange rate has been studied by several authors, and the exchange rates are usually modelled by means of a vector error correction (VEC) model (see Zivot [39], for a survey). [5] In what follows we first review the statistical properties of such a model.

Consider the following model for the observed bivariate time series $\{w_t = (f_t, s_t)'\}_{t=1}^T$, where f_t is the one-period forward exchange rate and s_t is the spot exchange rate:[6]

$$w_t = A_0 + A_1 w_{t-1} + u_t \tag{A1}$$

and,

$$u_t \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_f^2 & \sigma_{s,f} \\ \sigma_{s,f} & \sigma_s^2 \end{bmatrix}\right). \tag{A2}$$

Equation (A1) is a bivariate vector autoregressive (VAR) model, which can also be written as:

$$\Delta w_t = A_0 + \Pi w_{t-1} + u_t, \tag{A3}$$

where Δ denotes the first-difference operator defined by $\Delta w_t = w_t - w_{t-1}$ and $\Pi = A_1 - I$.

Granger's representation theorem asserts that under the assumption of cointegration, and therefore if the coefficient matrix Π has rank 1, then there exist 2×1 vectors α and β such that $\Pi = \alpha\beta'$, and $\beta' w_t$ is stationary.

Therefore, using the normalisation $\beta = (1, -\beta_s)$, Equation (A3) becomes the VEC model:

$$\begin{aligned} \Delta s_t &= A_s + \alpha_s(f_{t-1} - \beta_s s_{t-1}) + u_{st}, \\ \Delta f_t &= A_f + \alpha_f(f_{t-1} - \beta_f s_{t-1}) + u_{ft}, \end{aligned} \tag{A4}$$

where $f_{t-1} - \beta_s s_{t-1}$ represents deviation from long-run equilibrium at time $t-1$, and the alphas are the adjustment parameters.

Note that if there is no cointegrating relation between f_t and s_t (i.e. as $\alpha_s = \alpha_f = 0$), standard time series analyses, such as the (unrestricted) VAR, may be applied to the first-differences of the data as the levels of the series each follows a random walk process with drift.

Comparing alternative models and model validation

We now examine the forecast performance of the VEC model and compare it with the benchmark random walk (RW) model, which is obtained by setting $\alpha_s = \alpha_f = 0$ in Eq. (A4).

We calculate traditional accuracy measures defined on the forecast errors $e_{t+h} = w_{t+h} - \hat{w}_{t+h}$, $h \geq 1$, where $\hat{w}_{t+h}$ denotes the h-step-ahead forecast average of the 100 sequences at the forecast origin t. Given n forecast errors $\{e_{t+h,i}\}_{i=1}^n$,popular measures of accuracy, such as the mean squared error, $MSE(h) = (1/n)\sum_{i=1}^n e_{t+h,i}^2$, and the mean absolute error, $MAE(h) = (1/n)\sum_{i=1}^n |e_{t+h,i}|$, are calculated for both the VEC and the RW. Furthermore, to assess whether $MSE(h)$ and $MAE(h)$ from the two competing models are statistically different, we use a test of equal forecast accuracy due to Diebold and Mariano [10]. If $\{d_i(h)\}_{i=1}^n$ are the loss differentials associated with the h-step-ahead forecasts from VEC model and RW, the test is based on the statistic $DM(h) = [\sum_{i=1}^n d_i(h)]/\sqrt{n\hat{\tau}_h^2}$ where $\hat{\tau}_h^2$ is a consistent estimator of $\tau_h^2 = \lim_{n\to\infty}(1/n)\mathrm{Var}\lfloor\sum_{i=1}^n d_i(h)\rfloor$. Under the null hypothesis of equal forecast accuracy (which entails $E[d_i] = 0$), $DM(h)$ has a standard normal asymptotic distribution.

A related criterion, widely used in evaluations of ex post forecasts, is Theil's inequality coefficient, $U(h)$, which, by construction, satisfies $0 \leq U(h) \leq 1$. If $U(h) = 0$, there is a perfect prediction; if, on the other hand, $U(h) = 1$, the forecast performance of the model is as bad as it can be.

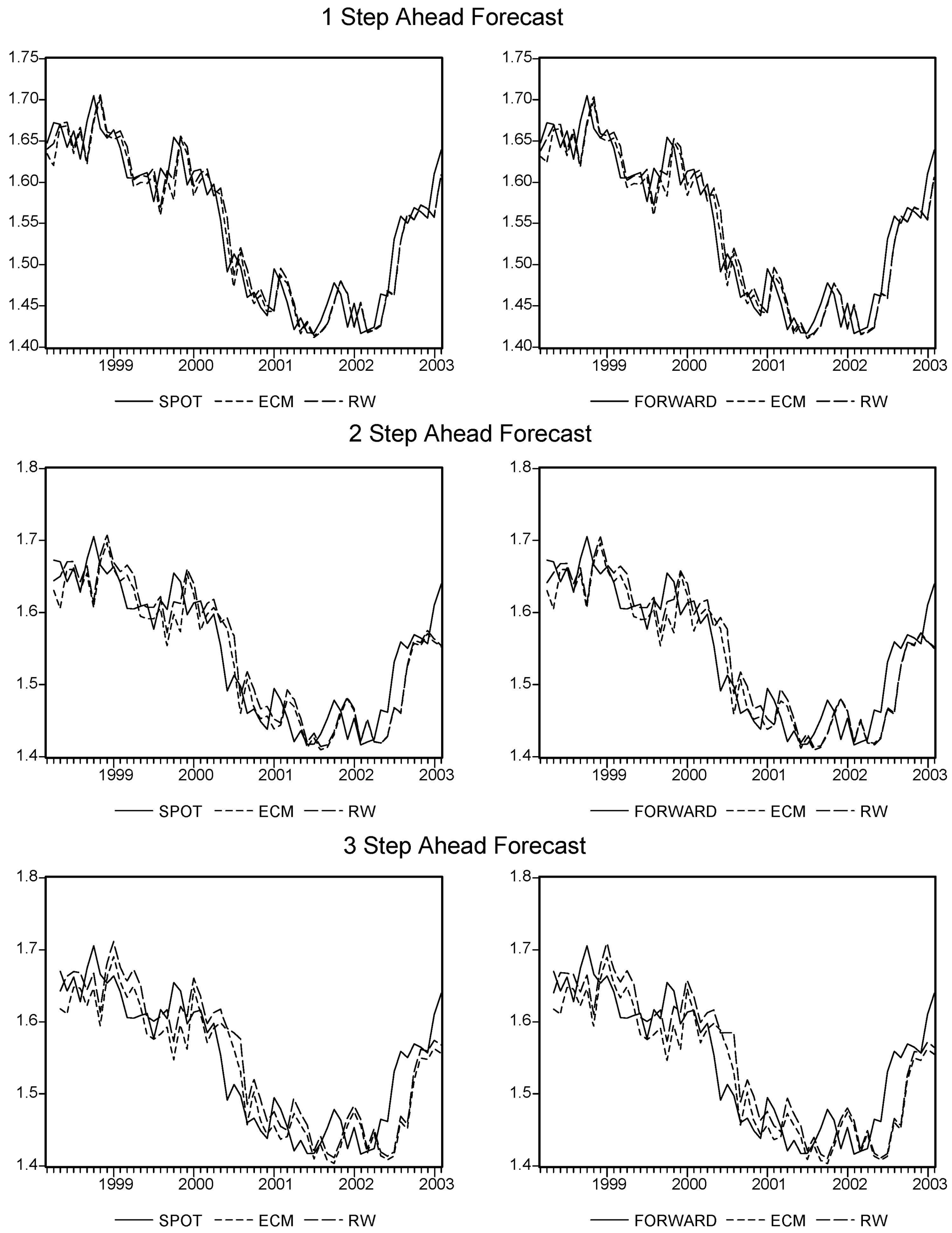

Figure A.1. Random Walk vs. Error Correction Model out-of-sample forecasts comparison.

On the basis of the various criteria that are used to evaluate the forecast accuracy, the results are clearly in favour of the VEC model specification. Therefore the VEC model is used for simulating scenarios of future exchange rates. Figure A-1 demonstrates the one-, two- and three-step-ahead out-of-sample forecasts of RW and ECM of exchange rates. The ECM follows more closely both spot and forward exchange rates.

Notes

1. In this Paper we assume cash flows are deterministic, since we want to focus on the effect of exchange rates on the optimal treasury decisions. More general model would account for the possibility that the cash flows follow a stochastic process, which could be easily accommodated in our setup. We leave such an extension as a topic for future investigation.
2. See note 2.
3. We do not conduct backtesting of the decision model over the remaining 12 months of the historical sample, Feb. 2003 to Jan. 2004 because when solving PI model we need 12 months of future actual realised exchange rates. Thus, the last roll of the decision model should be 12 months before the end of the historical sample of exchange rates. In order to compare PI model with HN and EV models on like-for-like bases we use the same backtesting time period for all the models.
4. It should be reiterated that we roll the model forward, at every time period we hedge (speculate) only the revenues expected in 3, 6, 9 and 12 months from the current time period.
5. A vector error correction model is a restricted vector autoregressive model that it is designed for use with nonstationary series that are known to be cointegrated. It restricts the long-run behaviour of the variables to converge to their cointegrating relationships while allowing for short-run dynamics.
6. Note that we will also consider model of cointegration between spot and 3, 6, 9 and 12-month forward rates. However, for explanation purposes, our attention will be restricted to the 1-month forward rate.

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