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MANUSCRIPT

Measuring and monitoring diversity in organizations through functional instruments with an application to ethnic workforce diversity of the U.S. Federal Agencies

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Abstract The role of diversity in organizations has been widely discussed in recent decades; nevertheless, both theoretical perspectives and empirical results appear conflicting and inconsistent. Scholars identify many possible reasons such as the definition of diversity, theoretical perspectives, variables, and methodological approaches; this study focuses on the methodological issue of assessing variety. To evaluate the role of diversity, most studies adopt static approaches and refer to the classical univariate indices; this research shows their limitations and stresses the importance of treating diversity with a multivariate dynamic approach. Taking advantage of functional data analysis and some recent ecological studies, this dual gap of the organizational literature is addressed by proposing a new methodological approach for measuring and monitoring diversity in organizations. We illustrate an application of this method by using a real dataset concerning the workforce diversity of the "Corporation For National And Community Service Overview" within the project "Federal Equal Opportunity Recruitment Program (FEORP)" of the Government of the United States of America. The goal of this research is to provide human resources specialists, policy makers, and scholars with additional techniques to improve the understanding of the dynamics of workforce diversity and minority employment within organizations.

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1 Introduction

Organizational units have become progressively diverse with respect to various cultural, demographic, informational, and personality attributes (van Knippenberg and Schippers 2007). This is inevitable due to increasingly globalized markets, greater mobility, and demographic developments which are bringing more people to work with others who differ in their backgrounds (Jackson et al. 2003; Joshi and Roh 2008; Biemann and Kearney 2010; Flache and Mäs 2008). Therefore, "managing diversity and promoting inclusion has become part of the business world's strategic agenda as a response to a more diversified society where knowledge and innovation are essential for obtaining competitive advantages in a globalised economy" (European Commission 2015). Furthermore, also institutions (e.g. the European Commission) welcomes these initiatives and has initiated the "Project Support for voluntary initiatives promoting diversity management at the workplace across the EU" in which it is remarked that "The business case for diversity shows that diversity management—whereby employers recognise, value and include women and men of different ages, abilities, ethnic origin, religion or sexual orientation—makes good business sense" (European Commission 2011). Particularly in the corporate world, the concept of "Diversity and Inclusion" has become almost a standard practice, and the voluntary initiatives in this direction combine with the legal requirements based on the EU anti-discrimination directives (Treaty of Amsterdam 1997) and their implementation in the member states (European Commission 2012). Thus, in recent decades, diversity is increasingly considered as a significant mechanism of good corporate governance.

Therefore, diversity has become "a fact of life and a main challenges of today's organizations" (Mayo et al. 2016) and thus a widely discussed topic in organizational studies. Hence, research on organizational diversity, heterogeneity, and similar concepts has proliferated in the past decades; however, few consistent findings have emerged because the organizational literature regarding diversity is confusing and difficult to understand and synthesize for four different reasons (Harrison and Klein 2007).

The first reason is certainly that the precise meaning of "diversity" is not clear; indeed, some scholars adopt this term as synonymous of "variability", incorporating different concepts such as variety, separation, and disparity (Harrison and Klein 2007). Contrary, many researchers consider "diversity" only as synonymous of "variety", and thus in principle could concern any attribute on which people may differ, e.g. disability, ethnicity, cultural, background, gender, religion, and sexual orientation (van Knippenberg and Schippers 2007; Miner et al. 2003; Ricotta et al. 2003; Mayo et al. 2016).

The second cause is that several different theoretical perspectives have been used to guide diversity research; hence, the role of diversity in organizations (particularly in firms) is ambiguous because different theories and empirical results encompass both positive and negative association between workforce diversity and performance. The main theoretical studies that are in favor of diversity in organizations have suggested that diversity within work groups increases their effectiveness (Cox et al. 1991), and enriches the supply of ideas and knowledge enhancing creativity and quality of decision making (Williams and O'Reilly 1998). Therefore, units with greater diversity will make more effective decisions and deliver more creative products than units whose members draw from the same pool of resources. On the contrary, the main theories against diversity in organizations, which are based on similarity attraction (Byrne 1971) and social categorization (Tajfel 1981; Turner 1985), suggest that diversity limits within-unit behavioral and social integration, fosters conflict and turnover, and diminishes morale, cohesion, and performance (Grow and Flache 2011). Moreover, according to social identity theory, cultural homogeneity in management groups may increase satisfaction and cooperation and decrease emotional conflict (Turner 1985; Williams and O'Reilly 1998), and positive social associations and in-group social contacts are fostered (Blau 1977). Empirical findings also do not help understand the role of diversity because results are contradictory. For example, recent studies have found a positive and non-linear relationship between demographic diversity and performance, mediated by the board's monitoring efforts (Ararat et al. 2015). Contrary, other researches suggest that the effect of diversity on performance is task-specific, nonlinear and contingent upon the context (Bell et al. 2011; Johnson et al. 2013). Furthermore, structural diversity appears to be generally weak or ineffective in most emerging markets (Young et al. 2008).

The third reason of why the role of diversity in organization is contradictory, is that the diversity literature itself is very diverse. Indeed, diversity is multidimensional concept and many different variables (both demographic and non-demographic) have been considered by scholars in recent studies: age (Pelled 1996), gender (O'Reilly et al. 1999), race and ethnicity (Riordan and Shore 1997; O'Reilly et al. 1999), pay (Pfeffer and Langton 1993), attitudes (Harrison et al. 1998), and individual performance (Doerr et al. 2002).

The fourth problem is that many indices have been proposed but no one is universally accepted for measuring and monitoring diversity (Ricotta et al. 2003; Di Battista et al. 2016b). In the field of organizational studies, scholars are used to focus on the classical univariate indices of diversity: proportions, richness, Shannon-Wiener (Shannon 1948) and Blau-Simpson (Simpson 1949) indices. However, these indices depict different aspects of variety, and thus, in certain conditions, may lead to very different results (Di Battista et al. 2016a). Particularly, the latter issue is the focus on this research.

Many recent studies have highlighted the managerial relevance of diversity and its impact on performance, decision making, creativity, and ideas (Adkins 2016; Roberson et al. 2017; Farndale et al. 2015). In effect, diversity management is considered a key to growth in today's competitive global marketplace, and thus organizations that seek global market relevancy should embrace variety. Indeed, the so-called value-in-diversity hypothesis (Ely and Thomas 2001; Shore et al. 2009; Podsiadlowski et al. 2013) suggests that a diverse workforce drives innovation and will more easily fulfill the needs of a broad customer base.

Furthermore, for decades, scholars have stressed the importance of considering a dynamic approach in organizational studies. According to the law of requisite variety, maintaining or creating a required level of internal variety within an organization is crucial for its success or survival in the face of changing environments (Ashby 1956). March has demonstrated that "without personnel turnover, which produces variability in an organization, the contribution of learning to organizational knowledge degenerates in the presence of environmental turbulence" (March 1991). Also recent studies have emphasized time as a key component of organizational theory (Ancona et al. 2001; George and Jones 2000) or have stressed that, to understand work organization, researchers should investigate how organization's dynamics develop and change over time (Gully 2000; McGrath and Tschan 2007). Therefore, it is widely held by scholars that no organizational theory is truly time-independent because time is a necessary dimension for understanding the complexity of the real world (Jebb and Tay 2017).

Because of the growing importance that diversity is taking in the field of management, it is necessary that future empirical studies will use a methodological approach that ensures the comparability of results (e.g. it is not possible to compare studies that measure diversity with the richness index with studies that instead adopt the Blau-Simpson index because they are measuring different aspects of the multivariate concept of variety). For these reasons, the main contribution of our proposal is to provide scholars and practitioners, in the field of organizational studies, with a novel methodological approach for measuring diversity and overcoming the limits of the classical indices; in addition, the insertion of the variable "time" is considered for taking into account diversity changes within organizations.

The proposed methodological approach is inspired by the ecological literature on the concept of "biodiversity". Conceptually, the problem of measuring biodiversity in ecological communities is identical to the problem of assessing diversity in organizations. Indeed, from a methodological perspective, the focus is on the variety of species (categories) within a community (organization). Many researchers agree that (bio)-diversity is a multivariate concept, and thus should consider both richness (the number of different species) and evenness (the degree to which abundances are equitably divided among species) (Blau 1977; Ricotta et al. 2003; Chao et al. 2014; Maturo et al. 2015). Therefore, scholars have proposed the use of a multivariate approach for taking into account both these aspects of diversity. However, as we show in Sect. 2), also these approaches suffer from some limitations, and in addition, most researches use a static approach which neglects the importance of considering diversity changes.

For these reasons, taking advantage of the functional data analysis approach (FDA) and some studies which have been developed in the field of ecology (Maturo and Di Battista 2018; Di Battista et al. 2017), this paper aims to address this dual methodological gap by proposing an original approach for measuring (considering both richness and evenness) and monitoring (assessing changes over time) diversity in organizations. These research introduces many original functional tools, which are based on diversity profiles, i.e. "the area under the beta diversity profile", "the beta diversity surface", "the volume under the beta diversity surface", "the relative contribution of the diversity of one period", "the relative change of diversity index based on the volume", "the total change of diversity index based on the volume'', and finally "the total change of diversity index based on the area". This functional multivariate approach solves the limitations of existing methods, and allows us addressing diversity by considering both richness and evenness, and all of the shades of the main classical diversity indices. In addition, we develop this method in a temporal perspective in order to consider the variations of diversity over time.

This article is structured as follows. The introduction presents the main issues which have caused, for many decades, inconsistent results of the literature regarding diversity. The second section discusses the prior instruments of measurements and illustrates the limits of existing indices, also taking into account some methodological advances, which Statisticians and Ecologists have proposed for studying the similar concept of bio-diversity. The third part of the paper proposes some new functional tools for overcoming the limits of classical indices. In the forth section, we propose an application of our method by using a real dataset concerning the workforce diversity of the "Corporation For National And Community Service Overview" within the project "Federal Equal Opportunity Recruitment Program (FEORP)" of the Government of the United States of America. The paper ends with our conclusions and perspectives of research.

2 The classical measures of diversity in organizational and ecological studies

In the field of Ecology, researchers distinguish between *alpha*-diversity, *beta*-diversity, and *gamma*-diversity: *alpha*-diversity refers to diversity within a particular sample (within-habitat diversity), *beta*-diversity refers to diversity associated with changes in sample composition along an environmental gradient (between-habitat diversity), and *gamma*-diversity refers to differences across samples when they are combined into a single (landscape diversity) (Whittaker 1972). This study focuses on both alpha and beta-diversity. First, we introduce the diversity profile for measuring diversity at each instant of time (alpha-diversity). Then, we use different time observations based on diversity profiles for monitoring diversity over a period (beta-diversity).

In approaching to diversity, researchers deal with two main methodological issues. First, the multidimensionality due the fact that diversity could refer to different attributes; thus, it is important to choose the variable according to which diversity is computed. Recent studies have tried to solve this issue by considering a composite index with age, gender, education, nationality, and independence (Ararat et al. 2015); however, even if they have used standardized Blau's values for each attribute to compose their board diversity index, the limits to give the same weight to each attribute and blend quantitative and qualitative data still remain (age is classified into five categories with a consequent inevitable loss of information on the variability of the data). The second serious methodological problem is that many indices have been proposed but no universally accepted measure for measuring and monitoring diversity has yet been established (Ricotta et al. 2003).

Scholars agree that two different aspects are generally accepted to contribute to the intuitive concept of diversity of a population: richness and evenness (Peet 1974). Richness is a measure of the total number of categories whereas evenness expresses how evenly the individuals in a community are distributed over the different categories. The problem of the traditional indices is that some of them measure only richness or evenness whereas others measure both aspects but with different shades. Indeed, different indices consider different aspects of diversity, and consequently, they can lead to different results and diverse ranking among groups when it is necessary to compare them according to their diversity (or similarly to compare diversity of one group at time t with diversity of the same group at time t+1, or again to compare diversity of a group to an ideal benchmark which an organization aims to reach).

In the literature, the most frequently used diversity indices are the richness, Shannon-Wiener (Shannon 1948) and Blau-Simpson (Simpson 1949; Blau 1977) indices. The richness index is the simplest one because it is the sum of the modalities (total number of categories) but it does not take into account evenness (the distribution of the different species).

The Blau-Simpson index (see Eq. 1) is given by:

$$\triangle_K = 1 - \sum_{i=1}^{k} f_i^2 \tag{1}$$

where f_i is the relative frequency of the i-th category and k is the total number of categories (modalities). The Blau-Simpson index is also known as Herfindahl (1950) index and Hirschman (1964) index, but it was originally proposed by Simpson (1949) as a measure of species diversity in an ecosystem. It can range from zero to $(k-1)/k$. Its maximum is a function of the number of categories and occurs when members of an organization are equally distributed among the existing categories (evenness) whereas its minimum (maximum homogeneity) happens when units belong to only one category. Both evenness and richness contribute to a higher Blau's index and it is a good indicator of the dominance of one or few categories on the others; however, it is not a good predictor of richness because it is particularly sensitive to changes in the relative abundances of the most dominant categories.

The Shannon-Wiener index (see Eq. 2) (Shannon 1948) is given by:

$$\triangle_{Sh} = -\sum_{i=1}^{k} f_i \log(f_i) \tag{2}$$

The Shannon-Wiener index is also known as Teachman's index (Teachman 1980). It can range from zero to $\log(k)$, thus its maximum is a function of the number of categories and its interpretation is similar to the Blau-Simpson index. The Shannon-Wiener index is affected by both the number of categories and their evenness; however, it is particularly sensitive to the presence of rare categories in a group.

According to the characteristics of the classical indices, scholars agree that the use of a single index greatly reduces the complexity of diversity (Patil and Taillie

1979; Gove et al. 1994). For this reason, first Hill (1973) and then Patil and Taillie (1979) have proposed a coherent system for diversity estimates, which are usually referred to as Hill's numbers (Hill 1973) and diversity profiles (Patil and Taillie 1979), respectively. They provide numbers that reflect both evenness and richness, and include variants of the richness, Shannon-Wiener and Blau-Simpson indices. These approaches are similar with regard to their basic idea and interpretation but their formulations are slightly different; thus, in this study, we focus only on the latter.

They consist of a sequence of measurements allowing different aspects of "community" (organization) structure to be encompassed in a single diversity spectrum. The "beta diversity profile" (Patil and Taillie 1979) is a measure of *alpha*-diversity (within-"habitat" diversity) and is given by Eq. 3:

$$\triangle_\beta = \sum_{i=1}^{k} \frac{(1 - f_i^\beta)}{\beta} f_i \quad \beta \geq -1 \tag{3}$$

where f_i is the relative abundance of the category i, k is the total number of categories, β denotes the relative importance of richness and evenness, and the restriction that $\beta \geq -1$ assures that $\triangle_\beta$ has certain desirable properties (Patil and Taillie 1979).

The plot of $\triangle_\beta$ versus β provides the diversity profile which is a decreasing curve in the domain of β. The curve, for $\beta > 1$, is not considered because the β diversity profile quickly converges to zero after this point (Chao et al. 2014; Maturo et al. 2016). We highlight that the most common indices of diversity are special cases of Eq. 3: for $\beta = -1$ we get the richness index minus one, for $\lim_{\beta \to 0}$ we obtain the Shannon-Wiener diversity index (Eq. 2) (Shannon 1948), and, for $\beta = 1$, we achieve the Blau-Simpson index (Eq. 1) (Simpson 1949). Therefore, diversity profiles are functions dependent on a parameter that reflects the sensitivities to rare and abundant categories; they provide a continuum of possible diversity measures (Ricotta et al. 2003) and yield a faithful graphical representation of organizations diversity.

For example, we suppose to analyse the national diversity of the workforce of an organization whose members are of five different nationalities with the following distribution: 54 Spanish, 20 Italian, 17 English, 6 Canadian, 3 French. Figure 1 shows the β-diversity profile of this organization. The main advantage of the β diversity profile is that it considers different shades of richness and evenness, and its interpretation is very easy: high curves correspond to great diversity whereas lower ones are characterized by more homogeneity (minimum diversity generates a straight line which coincide with the x-axis). However, this method is not without limitations because difficulties arise when we need to compare intersecting profiles.

To show the limitations of the classical indices and also diversity profiles, we suppose to repeat the measurement of the national diversity of our organization at the times t_2 and t_3 because we aim to understand if it has increased, decreased or unchanged (e.g. because the organization aims to guarantee a certain benchmark of diversity and inclusion for its voluntary strategy or legal requirements). At this purpose, we suppose that, at time t_2, we observe 35 Spanish, 35 Italian, 30 English

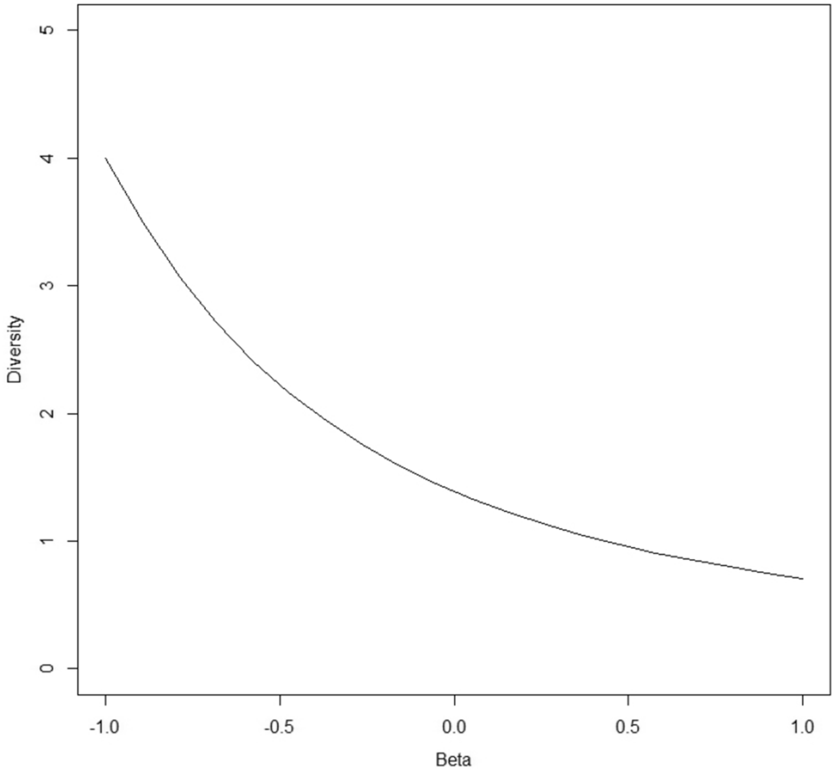

Fig. 1 β diversity profile of an organization with regard to the nationality of its workforce

whereas, at time t_3, we count 40 Spanish, 20 Italian, 10 English, 30 Canadian. Therefore, we point out that the composition of this organization has changed in its dimension, richness, and evenness over the period of observation. Figure 2 illustrates the β-diversity profiles of the organization regarding the nationality of its workforce with three different measurements: t_1 (solid black line), t_2 (dashed red line), and t_3 (dotted green line). It is immediate to understand that diversity has increased from t_2 to t_3 because the dotted green line is always higher than the dashed red line. However, problems arise in comparing t_1 to t_2 and t_1 to t_3 because the curves intersect. Focusing on the first part of the domain (when $\beta = -1$ diversity is equal to $k - 1$, i.e. richness index minus one), we can conclude that the organization is more diverse in t_1 (the solid back line is higher than the other curves). Considering the Shannon-Wiener index ($\lim_{\beta \to 0}$), we state that in t_3 the organization is more diverse than in t_1 and t_2, and in t_2 is more homogeneous. Instead, according to the Blau-Simpson index (when $\beta = 1$), we can say that there is greatest diversity in t_3 and lowest diversity in t_1.

In summary, this is a simple example of why the classical indices are unreliable and could lead to conflicting results; thus, diversity should be treated with a multivariate approach which is able to consider the whole *beta*-domain because a

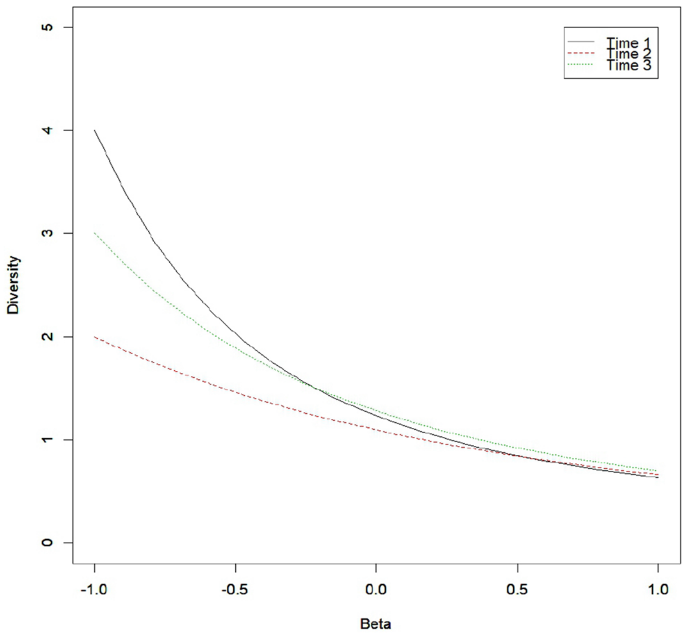

Fig. 2 β diversity profiles of an organization with regard to the nationality of its workforce with three different measurements

single index (the ordinate value corresponding to a single point of the *beta*-domain) greatly reduces the complexity of diversity. In organizational studies, any scholar that aims to evaluate the effect of diversity on performance (or other dependent variables) by using a univariate index, must be aware that he is neglecting the multivariate aspect of diversity; therefore, by using different univariate indices, it may happen that results are different (or even opposed) and lose of statistical significance.

Regarding the existing monitoring techniques, both organizational and ecological studies have focused for decades on changes in proportions, trends in the abundance, changes of Shannon-Wiener or Blau-Simpson indices. Some recent advances in the field of ecology have proposed species intactness indices based on occurrence or abundance (Buckland et al. 2005; Loh et al. 2005; Nielsen et al. 2007) but, even if these indicators have the advantage of being relatively easy to understand and calculate, they retain only a small portion of the available information that describes the concept of diversity (Magurran 2004). Thus, existing methods for measuring diversity with a single temporal observation and techniques for monitoring an organization with multiple points in time, suffer from the same identical limits. For this reason, we present a new method for assessing diversity changes in organizations taking advantage of the functional data analysis approach and diversity profiles.

3 Overcoming the issues of the classical indices using functional diversity tools

3.1 Diversity profiles and functional data analysis (FDA)

Because the β diversity profile is not simply a sequence of observations but a function in a fixed domain, it is possible to analyse the intrinsic structure of the data through the FDA approach (Gattone and Di Battista 2009; Di Battista et al. 2014, 2017). FDA addresses problems in which the observations are described by functions rather than finite dimensional vectors (Ramsay and Silverman 2005; Ferraty and Vieu 2006; Di Battista et al. 2016a). The functional datum should be regarded as a single entity instead of a sequence of observation and, in this context, it is expressed by a specific function known in advance (i.e. the diversity profile). The observations, in fact, belong to a parametric family of functions, called S, with s real parameters (De Sanctis and Di Battista 2012), that is:

$$S = \{f(\boldsymbol{\theta}; \beta)\} \tag{4}$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_s)'$ represents a vector of unknown parameters taking values in a parameter space $\boldsymbol{\Theta}$, and β is the functional domain.

In organizational studies, S could be the family of diversity profiles and for each i-th organization, $i = 1, 2, \ldots, N$, each relative abundance vector can be assumed as a single parameter, $\boldsymbol{f}_i = (f_{i1}, \ldots, f_{is}) = \boldsymbol{\theta}_i$, so that, $\boldsymbol{f} = \boldsymbol{\theta}$.

The FDA approach applied to the β diversity profile allows us overcoming the measurement issue of the univariate approach; indeed, the function takes into account the whole β domain by considering diversity as a multivariate concept. Thus, reflecting the sensitivities to rare and abundant categories, the parameter β provides the main three classical indices and their possible shades, which they are potentially infinite but, for computational needs, we can fix them with a loss of information that is negligible.

3.2 The area under the beta profile

Also diversity profiles (Eq. 3) suffer of some limitations; indeed, they allow only a graphic interpretation of diversity by using the whole domain (Fig. 1) but do not allow to extrapolate an unambiguous and multivariate measurement of diversity. In addition, when curves intersect, the comparison between them does not lead to a unique ranking (Fig. 2).

In this section, we aim to extend recent methodological advances of the ecological literature to organizational studies. Because the concept of biodiversity of an ecological community is perfectly adaptable to the notion of diversity in organizations, we start our contribution by taking advantage of a recent new functional tool which has been proposed in the ecological context, and is called the “area the beta profile” (Di Battista et al. 2017). Therefore, we adapt this instrument to organizational diversity and develop it for improving existing monitoring techniques. It is easy to perceive that, instead of having different species or species with different taxonomic characteristics, in this context, we have people which differ in some categorical variables (demographic or non-demographic).

The area under the beta profile considers the whole *beta* domain and thus it is an indicator of diversity that takes into account both richness and evenness. It is defined by:

$$A = \int_{-1}^{+1} \triangle_{\beta}\, d\beta \tag{5}$$

where $\triangle_{\beta}$ is the value of diversity (ordinate) corresponding to a specific value of β (abscissa).

Because this integral has no immediate and definitive solutions, it can be approximated by:

$$A \approx \sum_{j=1}^{p} (\beta_{j+1} - \beta_j) \triangle_{\left(\frac{\beta_{j+1}+\beta_j}{2}\right)} \tag{6}$$

where p is the number of the points of the *beta*-domain that we decide to consider for the computation and

$$A = \lim_{p \to \infty} \sum_{j=1}^{p} (\beta_{j+1} - \beta_j) \triangle_{\left(\frac{\beta_{j+1}+\beta_j}{2}\right)} \tag{7}$$

The area is directly proportional to both evenness and richness, and it does not attribute more importance to one or another aspect. In organizational studies, a higher value of the area denotes a greater diversity of an organization, and vice versa. It is obtained using FDA but has the advantage that, being a scalar measure, allows us to overcome the complexity of dealing with functions (e.g. if we aim to use it in a multiple regression model for assessing the impact of diversity on a depend variable, we do not need to refer to functional regression models).

3.3 The "beta diversity surface" for an immediate interpretation of changes in organization's diversity over time

The area under the beta profile (Sect. 3.2) is a useful tool for overcoming the limits of the classical indices and diversity profiles, and measuring diversity in an instant of time by starting from a multivariate functional approach. However, it gives a static view of the composition of an organization whereas diversity is a dynamic concept and continuously changes over time. Therefore, in this context, we aim to develop a new functional instrument for assessing changes in diversity (e.g. for reaching a given benchmark or simply understanding if diversity has improved, worsened or remained the same over time). The basic idea is to propose an index that is able to overcome the limits of the existing monitoring indicators (e.g. change in proportion, richness, Blau-Simpson or Shannon-Wiener indices).

To monitor diversity changes, the first step is to take into account also the variable "time". Thus, the β profile function can be seen also as function of time with $-1 \leq \beta \leq +1$ and $1 \leq t \leq T$, where T is the number of years that we aim to analyse.

$$\triangle(\beta, t) \tag{8}$$

This new function generates what we now define the “beta diversity surface”, which can be imagined as a sequence of points in a three-dimensional space or can be approximated by a continuous surface. For example, Table 1 illustrates simulated relative frequencies of an organization whose members belongs to five different nationalities and have been counted once a year for six years. Instead, Table 2 displays the values of beta profiles (Eq. 3) for each point of the *beta*-domain (rows) and for each year (columns), which are computed using Eq. 3. In this case, we have approximated the *beta*-domain $[-1, 1]$ by using 20 points.

The graphical representation of our simulated data generates the beta diversity surface of Fig. 3, which allows us obtaining an immediate representation of the national workforce diversity of the organization over time. For more clarity of notation, in representing the beta diversity surface, we refer to a new parameter F, called “sensitivity” (to rare and abundant categories in computing diversity), which is obtained by translating the original β parameter (Eq. 3) by one unit as follows:

$$F = \beta + 1$$

Thus, because $\beta \in [-1, 1]$, the new parameter $F \in [0, 2]$. Hence, to refer to the classical indices, we need to consider that:

- if $F = 0$ (first point of the domain of Table 2), we have the richness index minus one;
- if $F = 1$, (middle point of the domain of Table 2), we obtain the Shannon-Wiener index (see Eq. 2);
- if $F = 2$, (last point of the domain of Table 2), we refer to the Blau-Simpson index (see Eq. 1).

Specifically, Fig. 3 displays us the evolution of diversity over the six instants of time considering its two different aspects: richness and evenness. The graph highlights that diversity in our organization is slightly changed over the period. The function, for $F = 0$, is always equal to four in the instants t_1, t_2, t_4, and t_6 whereas it decreases in t_3 and t_5 because it equal to 2 and 3, respectively. Evenness (i.e. the

Table 1 Simulated data (relative frequencies) of an organization whose members belongs to five different nationalities and have been counted once a year for six years

Year	Spanish	Italian	English	Canadian	French
1	0.35	0.35	0.27	0.01	0.02
2	0.54	0.20	0.17	0.06	0.03
3	0.35	0.35	0.30	0.00	0.00
4	0.51	0.31	0.07	0.10	0.01
5	0.40	0.20	0.10	0.30	0.00
6	0.20	0.20	0.20	0.20	0.20

Table 2 The values of beta profiles (Eq. 3) are computed using the simulated data of Table 1

Point of the domain	Years					
	1	2	3	4	5	6
1	4.00	4.00	2.00	4.00	3.00	4.00
2	3.29	3.40	1.87	3.32	2.70	3.60
3	2.76	2.93	1.75	2.81	2.44	3.25
4	2.37	2.54	1.64	2.41	2.22	2.93
5	2.07	2.22	1.53	2.10	2.02	2.66
6	1.83	1.96	1.44	1.85	1.85	2.41
7	1.65	1.75	1.35	1.64	1.69	2.20
8	1.49	1.57	1.27	1.48	1.56	2.00
9	1.37	1.42	1.20	1.34	1.44	1.83
10	1.26	1.29	1.13	1.22	1.33	1.68
11	1.17	1.18	1.06	1.12	1.23	1.54
12	1.09	1.08	1.01	1.04	1.15	1.42
13	1.02	1.00	0.95	0.96	1.07	1.31
14	0.95	0.93	0.90	0.90	1.00	1.21
15	0.90	0.86	0.85	0.84	0.94	1.13
16	0.85	0.81	0.81	0.79	0.88	1.05
17	0.80	0.76	0.77	0.74	0.83	0.98
18	0.76	0.71	0.73	0.70	0.78	0.91
19	0.72	0.67	0.70	0.66	0.74	0.85
20	0.68	0.64	0.66	0.63	0.70	0.80

The beta domain [– 1,1] is divided into 20 points

degree to which abundances are equitably divided among categories) is slightly different in the different periods; in particular, the maximum evenness is observable in t_6 because the curve tends to look like a straight line. The beta diversity surface allows us to have an immediate knowledge of changes in the distribution of the categories in the organization but do not permit us to precisely assess the total variation of diversity; for obtaining such type of measure, we are going to develop an analytical index.

The main characteristics of the beta diversity surface are the following:

1. the more high the surface, the more diverse is the organization workforce with respect to an attribute;
2. the three-dimensional function has no theoretical higher bound because this depends on the number of modalities that characterize the attribute being studied;
3. the function is lower bounded because diversity is always zero when only one category is present in the workforce. In this case, the surface becomes a plane;
4. fixed the number of modalities of an attribute, the surface tends to be more curved in case of high dominance of one or few categories;

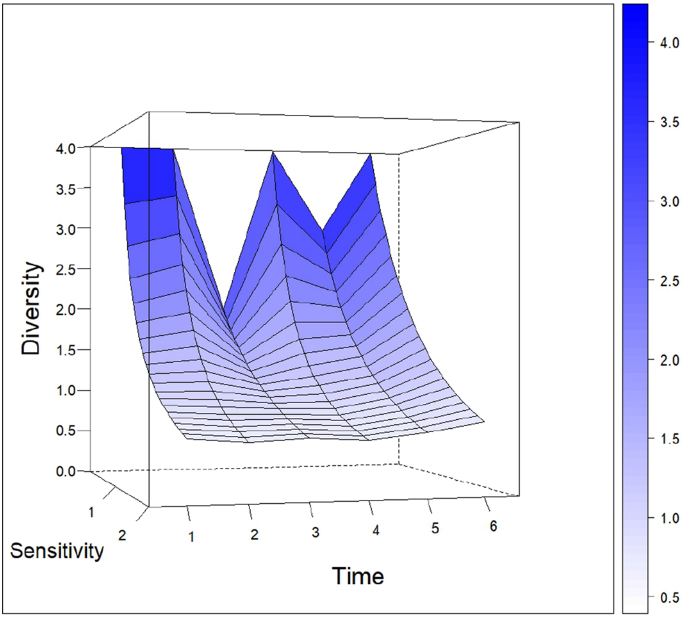

Fig. 3 Example of diversity surface generated by simulated data of an organization whose members belongs to five different nationalities and have been counted once a year for six years

5. given the number of categories of an attribute, the surface tends to be more flat in case of evenness, i.e. when the relative frequencies of all modalities are equal to $\frac{1}{k}$ (maximum diversity for a given k);
6. the maximum values of the function are: k, for $F = 0$; $log\ k$ for $F = 1$; and $\frac{k-1}{k}$ for $F = 2$, respectively;
7. identity: if all relative frequencies in a particular year are equal to those in another year, the surface graph is also equal;
8. absence of high sensitivity to appearing and disappearing categories. Changes in the surface are not dominated by new categories entering the workforce or categories that disappear from the organization such as the simple richness index.

The main advantage of the beta diversity surface is that it considers different shades of richness and evenness over a period of time, and its interpretation is very easy if considering the above properties.

3.4 Some extreme examples to show how to interpret the beta surface

In this section, we present some extreme examples to show how to interpret the beta diversity surface. Because to consider all the combinations of increase or decrease of both richness and evenness would require too many examples, we consider only some scenarios. First, we propose the case of a workforce organization that does not change over time; second, we present the diversity of a group that loses both richness and evenness; third, we suppose that richness increases but evenness remains maximum over the whole period; finally, we suppose that there is a loss of evenness (with constant richness) over the period.

Our examples take into account the national board workforce diversity of an organizational unit which has been monitored for six years. In these examples, the total number of nationalities varies between one and five, i.e. Spanish, Italian, English, Canadian, and French.

3.4.1 *Example no. 1: The case of an organization in which the distribution of nationalities of the workforce does not change over time*

The first example displays the simplest case of an organization in which the distribution of nationalities of the workforce does not change over time; thus, richness and evenness are constant over the period of observation. We highlight that we intentionally do not talk about "same diversity over time" because it may happen that the total amount of diversity does not change but richness and/or evenness vary). We suppose that there is maximum evenness (all relative frequencies are equal to $\frac{1}{k} = \frac{1}{5} = 0.2$) and the number of nationalities is always five (Table 3).

The data of Table 3 are used to compute the values of the beta profiles in an equivalent way of Table 2; however, we omit to display also these tables for our examples to unnecessarily burden the reading. According to the values of the beta profiles, which are computed using Eq. 3 for each year, we obtain the diversity surface (Fig. 4). The graph highlights that there are no jumps or variations in the distribution of nationalities over time.

3.4.2 *Example no. 2: The case of an organization in which there is a loss of richness of nationalities (whereas evenness changes as natural consequence of missing categories)*

The second example shows the case of an organization in which there is a loss of richness of nationalities of the workforce (whereas evenness changes as natural

Table 3 Data (relative frequencies) of the nationalities of the workforce observed at six instants of time: the case of an organization in which the distribution of nationalities of the workforce does not change

Year	Spanish	Italian	English	Canadian	French
1	0.2	0.2	0.2	0.2	0.2
2	0.2	0.2	0.2	0.2	0.2
3	0.2	0.2	0.2	0.2	0.2
4	0.2	0.2	0.2	0.2	0.2
5	0.2	0.2	0.2	0.2	0.2
6	0.2	0.2	0.2	0.2	0.2

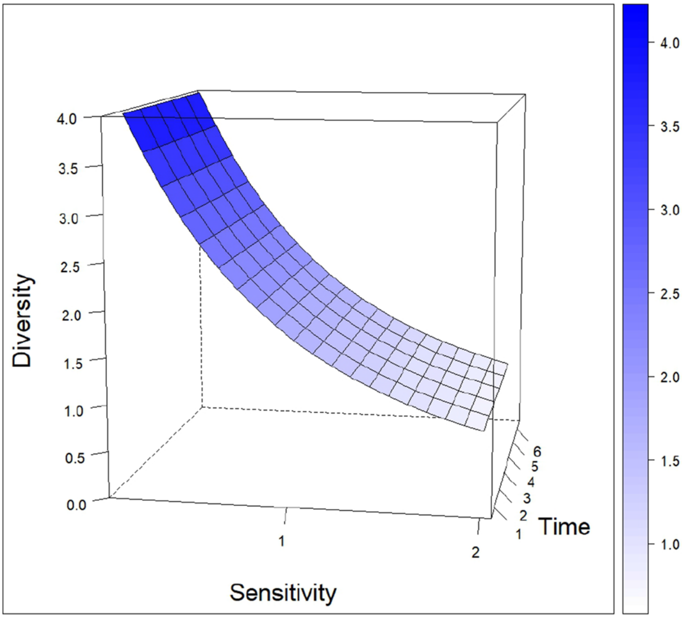

Fig. 4 Diversity surface in the case of an organization in which the distribution of nationalities of the workforce does not change over time

consequence of missing categories). Table 4 displays the relative abundance vectors for each year whereas Fig. 5 shows the diversity surface. We point out that there is no variation from t_1 to t_2 whereas, from the second year, richness start to decrease (for each year, the organization loses one category). In the last year (i.e. t_6), there is maximum homogeneity, thus, minimum diversity because only one category is present (Spanish); indeed, the beta diversity surface, in the sixth year, is parallel to the "sensitivity" axis and the value of the function is zero throughout the domain.

Table 4 Data (relative frequencies) of the nationalities of the workforce observed at six instants of time: the case of an organization in which there is a loss of richness of nationalities of the workforce (whereas evenness changes as natural consequence of missing categories)

Year	Spanish	Italian	English	Canadian	French
1	0.35	0.35	0.27	0.01	0.02
2	0.35	0.35	0.27	0.01	0.02
3	0.40	0.20	0.10	0.30	0.00
4	0.35	0.35	0.30	0.00	0.00
5	0.51	0.49	0.00	0.00	0.00
6	1.00	0.00	0.00	0.00	0.00

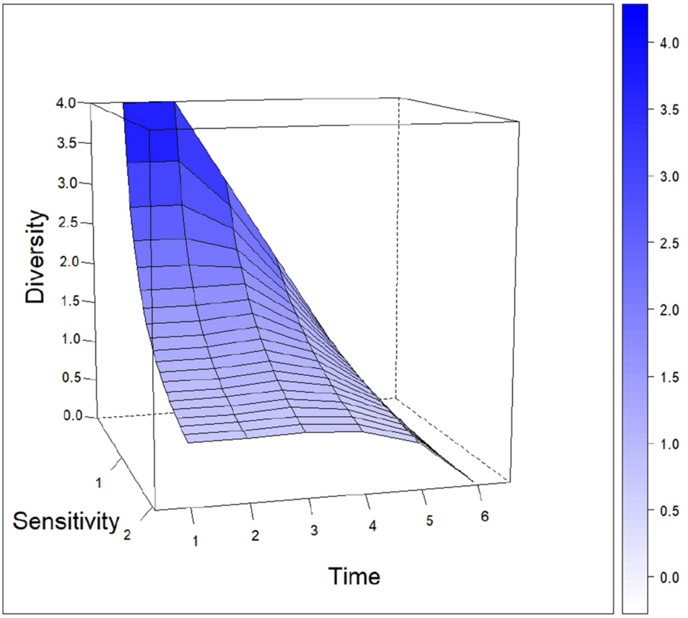

Fig. 5 Diversity surface in the case of an organization in which there is a loss of richness of nationalities of the workforce (whereas evenness changes as natural consequence of missing categories)

3.4.3 *Example no. 3: The case of an organization in which the workforce richness of nationalities increases but evenness remains maximum over the whole period*

The third scenario contemplates the case of organization in which the workforce richness of nationalities increases but evenness remains maximum over the whole period. Table 5 displays the relative abundance vectors during the whole period and Fig. 6 illustrates the diversity surface. The graph underlines that, for $F = 0$, the surface increases by one units for each year (except for the last period in which richness remains constant). Instead, evenness is maximum over the whole period because the relative frequencies f_i of the k present categories are always equal to $\frac{1}{k}$, i.e. $f_i = \frac{1}{1} = 1$ in year n.1, $f_i = \frac{1}{2} = 0.5$ in year n.2, ..., $f_i = \frac{1}{5} = 0.2$ in the last two years (see Table 5).

Recalling the property mentioned in Sect. 3.3, "the surface tends to be more flat in case of evenness, i.e. when the relative frequencies of all modalities are equal to $\frac{1}{k}$ (maximum diversity for a given k)", we stress the following further graphical behavior: the more the number of categories, the more the surface tends to be flat in case of perfect maximum evenness (e.g. in time n. 2, with only two categories, the surface is almost flat). Consequently, in the following years, even if there is perfect variety, the surface "tends" also to be flat but, due the increasing number of nationalities, this tendency starts to be less evident (see years n. 4-5-6 of Fig. 6).

Table 5 Data (relative frequencies) of the nationalities of the workforce observed at six instants of time: the case of an organization in which the workforce richness of nationalities increases but evenness remains maximum over the whole period

Year	Spanish	Italian	English	Canadian	French
1	1.00	0.00	0.00	0.00	0.00
2	0.50	0.50	0.00	0.00	0.00
3	0.33	0.33	0.33	0.00	0.00
4	0.25	0.25	0.25	0.25	0.00
5	0.20	0.20	0.20	0.20	0.20
6	0.20	0.20	0.20	0.20	0.20

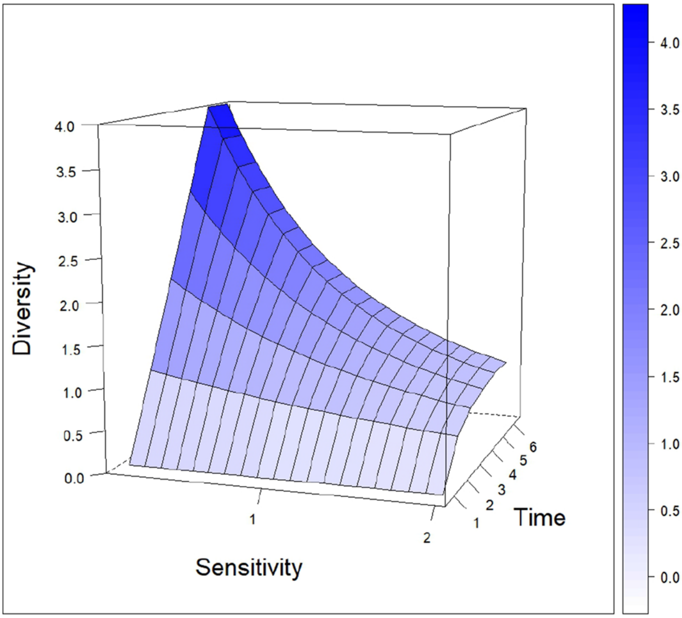

Fig. 6 Diversity surface in the case of an organization in which the workforce richness of nationalities increases but evenness remains maximum over the whole period

3.4.4 *Example no. 4: The case of an organization in which there is a loss of evenness of nationalities in the workforce whereas richness does not change*

The last example displays the case of an organization in which there is a loss of evenness of nationalities of the workforce whereas richness does not change. Table 6 presents the relative abundance vectors for each instant of time and Fig. 7 provides the diversity surface. We observe that, for $F = 0$, diversity is always equal to four; this means that the total number of nationalities is constant over the period

Table 6 Data (relative frequencies) of the nationalities of the workforce observed at six instants of time: the case of an organization in which there is a loss of evenness of nationalities in the workforce whereas richness does not change

Year	Spanish	Italian	English	Canadian	French
1	0.20	0.20	0.20	0.20	0.20
2	0.21	0.21	0.21	0.21	0.16
3	0.25	0.25	0.25	0.24	0.01
4	0.30	0.30	0.30	0.09	0.01
5	0.40	0.40	0.08	0.08	0.04
6	0.96	0.01	0.01	0.01	0.01

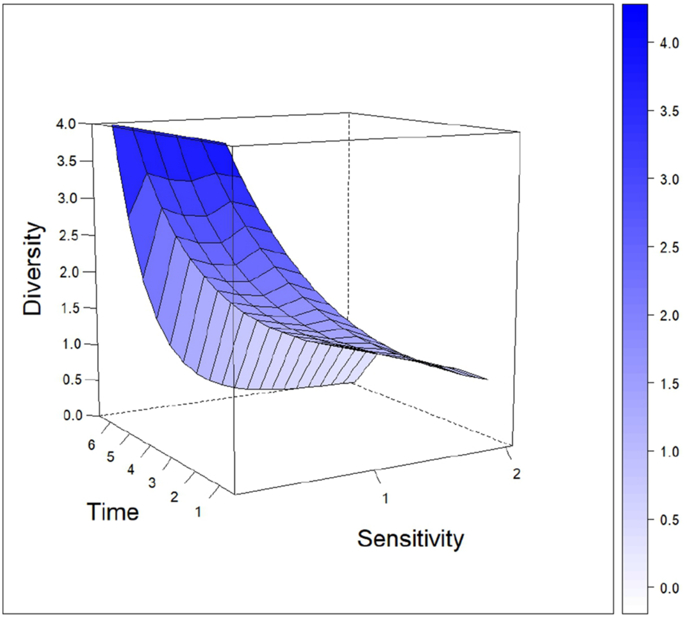

Fig. 7 Diversity surface in the case of an organization in which there is a loss of evenness of nationalities in the workforce whereas richness does not change

and is equal to five (i.e. $k = 1 + \Delta_F|(F = 0)$). Evenness changes over the period and causes a loss of diversity from t_1 to t_6, even if richness remains constant. Table 6 confirms that, for each year, there is an increase of dominance and decrease of evenness; in the first year, there is maximum heterogeneity because the nationalities are equally distributed whereas, in the last year, there is the dominance of Spanish people (however, it is not maximum homogeneity as in Fig. 5 because all the other categories are present even if inconsistent). Figure 7 highlights that the more the dominance of one or few nationalities, the more curved the surface.

3.5 The "Volume under the beta diversity surface" for assessing diversity changes over time

The beta surface gives an immediate view of variations in the distribution of categories but does not allow us to precisely measure the total variation of diversity; therefore, in this section, we present a functional tool for analytically measuring and monitoring diversity in organizations. Taking advantage of FDA, we can extend Eq. 4; considering also the variable "time", the observations belong to a parametric family of functions, called S_2, with s real parameters, that is given by:

$$S_2 = \{f(\boldsymbol{\theta}; F; t)\} \tag{9}$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2, \ldots, \theta_s)'$ represents a vector of unknown parameters taking values in a parameter space $\boldsymbol{\Theta}$, F and t provide the functional domain in which F is the sensitivity ("*feeling*") to rare and abundant categories in computing diversity whereas t represents time.

Therefore, in organizational studies, S could be the family of diversity profiles, and for each i-th organization, $i = 1, 2, \ldots, N$, and for each t-th instant of time, $t = 1, 2, \ldots, T$, each relative abundance vector can be assumed as a single parameter, $\boldsymbol{f}_{it} = (f_{it1}, \ldots, f_{its}) = \boldsymbol{\theta}_{it}$, so that, $\boldsymbol{f} = \boldsymbol{\theta}$. However, this should also generate a family of beta surfaces (one for each organization which is observed over time). Thus, in the case of only one organization that is monitored over a period, we can simplify with $\boldsymbol{f}_t = (f_{t1}, \ldots, f_{ts}) = \boldsymbol{\theta}_t$ and avoid the use of the subscript "i" that refers to other organizational units.

Thus, working in a three-dimensional space, we introduce a new functional tool, which we call the "Volume under the beta diversity surface". It can be defined as follows:

$$D_V = \int_0^2 \int_1^T \triangle(F, t)\, dF dt \tag{10}$$

where F is the *sensitivity* parameter and T is the total number of instants of time.

Because this integral has no immediate and definitive solutions, we provide the following approximation:

$$D_V \approx \sum_{j=1}^{I-1} \sum_{i=1}^{T-1} \triangle_{\alpha_j, \beta_i} \cdot (F_{j+1} - F_j) \cdot (t_{i+1} - t_i) \tag{11}$$

where I is the total number of points into which we decide approximate the F domain [0, 2], $I - 1$ is the number of intervals of the F domain that we consider consequently to the choice of I, $T - 1$ is the number of time intervals, $\alpha_j = \frac{F_{j+1}+F_j}{2}$, and $\beta_i = \frac{F_{i+1}+F_i}{2}$.

It is easy to point out that the total amount of diversity under the surface (the diversity of the whole period) can be approximated by the sum of all of the parallelograms that form the volume:

$$D_V = \lim_{I\to\infty,\triangle t\to 0} \sum_{j=1}^{I-1} \sum_{i=1}^{T-1} \triangle_{\alpha_j,\beta_i} \cdot (F_{j+1} - F_j) \cdot (t_{i+1} - t_i) \tag{12}$$

Recently, in the ecological literature, the use of the volume has been proposed to deal with Hill's numbers (Maturo and Di Battista 2018); however, in this context, the function and its domain are different with important consequence also on the computation and interpretation.

The main characteristics of D_V are the following:

1. the more high D_V, the more diverse is the organization workforce with respect to an attribute;
2. positivity: $D_V \in [0, +\infty)$; $D_V = 0$ when only a category is present over the whole period but D_V has no fixed maximum because it depends on the number of categories k;
3. fixing the number of categories, it is possible to normalize D_V because the maximum can be easily computed (when the relative frequencies of all modalities are equal to $\frac{1}{k}$);
4. absence of high sensitivity to appearing and disappearing categories. D_V are not strongly dominated by new categories entering the workforce or categories that disappear from the organization such as the simple richness index;
5. not monotony with respect to the number of categories: D_V does not always increases if new categories are added;
6. D_V always increases when the relative frequencies tends to $\frac{1}{k}$;
7. sub-additivity: $\forall a, b$, $D_V(a \cup b) \leq D_V(a) + D_V(b)$ with a and b two different organizations;
8. base year dependence: changes in D_V are sensitive to the choice of the base year chosen. If the base year is different by the unit (e.g. year or month), D_V must be divided by the different base;
9. independence from absolute frequencies: if all individual abundances are multiplied by a common factor, D_V does not change because it depends on the weight that each category has within the distribution.

Following this perspective, the "relative contribution of the diversity of one period" to the total variety is given by:

$$D_{V_{t\%}} = \frac{D_{V_{t,t+1}}}{D_V} = \frac{\sum_{j=1}^{I-1} \triangle_{\alpha_j,\beta_i} \cdot (F_{j+1} - F_j) \cdot \triangle_t}{\sum_{j=1}^{I-1} \sum_{i=1}^{T-1} \triangle_{\alpha_j,\beta_i} \cdot (F_{j+1} - F_j) \cdot \triangle_t} \tag{13}$$

where $\triangle_t$ is a fixed time unit which is constant in the whole domain, and $D_{V_{t,t+1}}$ is the volume under the surface of one interval with $D_{V_t} = (D_{V_1}, D_{V_2}, \ldots, D_{V_{T-1}})$.

In the same way, we can obtain the "relative change of diversity index based on the volume" considering the passage from the period $t \to t+1$ to the period $t+1 \to t+2$:

$$\pi_g = \frac{D_{V_{t+1}} - D_{V_t}}{D_{V_t}} \quad for\ D_{V_t} \neq 0 \tag{14}$$

where $g = (1, ..., T-2)$. We stress that no relative change can be computed for the first period because the previous one does not exist. Thus, if we have five instants of time, we deal with four intervals, and thus we can compute three relative changes among volumes. The more the instants of time, the more accurate the analysis. Thus, $\pi_g > 0$ if diversity increases, $\pi_g < 0$ if diversity decreases, and $\pi_g = 0$ if it does not change from the period $t \rightarrow t+1$ to the period $t+1 \rightarrow t+2$.

Similarly, it is possible to calculate the "total change of diversity index based on the volume" over the whole period:

$$\Pi_{TOT} = \frac{D_{V_T} - D_{V_1}}{D_{V_1}} \quad for\ D_{V_1} \neq 0 \tag{15}$$

Thus, $\Pi > 0$ if diversity increases, $\Pi < 0$ in case diversity decreases, and $\Pi = 0$ if it does not vary over the whole period.

To obtain a more refined value of the relative change of diversity over the whole period (not depending on the interval but on the extreme diversity profiles), we can also use the areas under the diversity profiles in $t = 1$ and $t = T$ (see Eq. 6) and obtain a second "total change of diversity index based on the area" as follows:

$$\Upsilon_{TOT} = \frac{A_T - A_1}{A_1} \quad for\ A_1 \neq 0 \tag{16}$$

The main characteristics of π_g, Π_{TOT}, and Υ_{TOT} are the following:

1. π_g, Π_{TOT}, and $\Upsilon_{TOT} \in (-\infty, +\infty)$;
2. identity: if all relative frequencies in a particular year are equal to those in another year, $\pi_g = \pi_{g+1}$ (whereas $\Pi_{TOT} = 0$, and $\Upsilon_{TOT} = 0$);
3. independence from absolute frequencies;
4. base year independence: changes in π_g, Π_{TOT}, and Υ_{TOT} are not sensitive to the choice of the base year chosen.

4 An application on a real dataset regarding the project "federal equal opportunity recruitment program (FEORP) of the Government of the United States of America

4.1 The "Diversity and Inclusion (D&I)" dashboard

In this section, we present an application of our method by using a real dataset concerning the workforce diversity of the "*Corporation For National And Community Service Overview* (CNCSO)" within the project "*Federal Equal Opportunity Recruitment Program (FEORP)*" of the Federal Government of the United States of America (U.S. Federal Government) (OPMgov 2017).

A primary goal of the U.S. Federal Government is to promote diversity by providing "Federal agencies concrete strategies and best practices to recruit, hire, include, develop, retain, engage and motivate a diverse, results-oriented, high-performing workforce" (OPMgov 2017). The basic idea is that the workforce should reflects the population to better understand and meet the needs of the American people. Moreover, a diverse workforce could improve individual and organizational performance and result in better value to customers, clients, taxpayers, and other stakeholders. The U.S. Federal Government strongly supports its agencies to create a more diverse, high performing workforce.

In this context, we focus on the "Corporation For National And Community Service Overview" that is an "executive branch federal agency that fosters service as a solution, administering national programs such as Senior Corps, AmeriCorps, and Learn and Serve America". The Corporation realizes the promise of the Serve America Act, by developing a roadmap to strategically manage rapid growth, thus ensuring that its organization remained focused on this critical service mission, as opposed to internal operations (Accenture Consulting 2017).

The data are collected from the "Diversity and Inclusion (D&I)" Dashboard that is a government instrument which has been created to provide agencies with demographic data about hiring, group attrition, employee inclusion perceptions, and overall accountability in regard to D&I efforts.

4.2 Analysis of the workforce diversity of the 'Corporation For National And Community Service Overview (CNCSO)' of the Government of the United States of America

This study analyses the evolution of the ethnic diversity in the workforce of the CNCSO during a period of five years (2010–2014). The data are composed by seven possible ethnicities: "Native Hawaiian or Pacific Islander", "American Indian or Alaskan Native", "Asian", "White", "Black", "More Than One Race", and "Hispanic".

Table 7 shows the percentages of each ethnicity in the workforce of the CNCSO during a period of five years (2010–2014).

Figures 8 and 9 illustrates the diversity surface of the ethnicity in the workforce of the CNCSO from two different perspectives for facilitating the interpretation.

Table 7 Percentages of each ethnicity in the workforce of the CNCSO during a period of five years (2010–2014)

Year	Native Hawaiian-Pacific Islander (%)	American Indian-Alaskan Native (%)	Asian (%)	White (%)	Black (%)	More than one race (%)	Hispanic (%)
2010	0.0	0.2	4.5	62.2	2.2	30.9	0.0
2011	0.0	0.4	4.4	61.1	2.8	31.0	0.4
2012	0.0	0.4	4.3	62.4	2.7	29.9	0.4
2013	0.2	0.3	3.9	64.9	2.7	27.3	0.7
2014	0.2	0.2	4.0	66.8	2.2	25.8	0.8

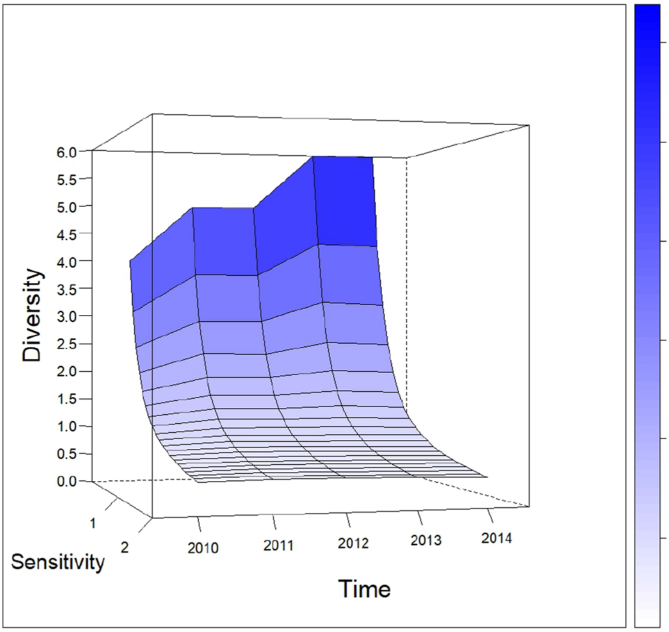

Fig. 8 Diversity surface of ethnicity in the workforce of the CNCSO during a period of five years (2010–2014)

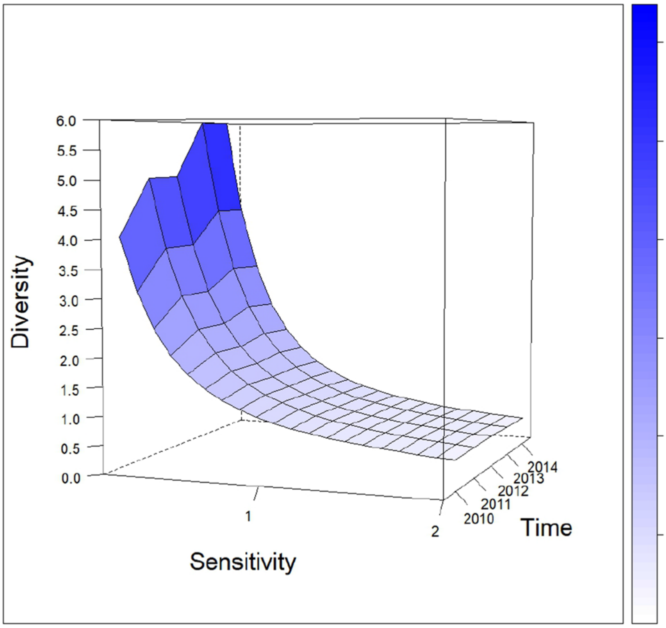

Fig. 9 Diversity surface of ethnicity in the workforce of the CNCSO during a period of five years (2010–2014)

The organization, in 2010, is composed only by five ethnicities because Native Hawaiian-Pacific Islander and Hispanic people are not present in the workforce. Thus, the diversity surface is equal to “4” when $S = 0$ and $t = 2010$. We observe that, for $S = 0$, the function increases to “5” in 2011 and then to “6” in 2013, respectively; this is because, during these years, the workforce gradually gains two new categories: Hispanic in 2011 and also Native Hawaiian-Pacific Islander in 2013. Figure 9 emphasizes that the surface reaches the highest value in 2013 and the lowest one in 2010. Comparing the years in which the richness is the same, we can observe that: first, in 2012, the workforce diversity of the CNCSO is lower than in 2011 because the surface tends to be more curved given the number of categories, i.e. the surface in 2011 tends to be more flat than in 2012. In 2012, it is more curved indicating more dominance of some categories whereas, in 2011, is more flat indicating more evenness; in 2013, there is more variety with respect to 2014 for the same reasons (the surface in 2014 is lower than the surface in 2013). Therefore, it is clear that the workforce diversity of the CNCSO increases from 2010 to 2011, decreases from 2011 to 2012, is similar in 2012 and 2013 even if there is one more ethnicity, increases from 2013 to 2014, and finally decreases from 2014 to 2015.

To exactly compare diversity among instants of time when richness is different, the volume is more accurate than a simple view of the graphs in Figs. 8 and 9. In addition, the volume is necessary to obtain an analytical assessment of the amount of diversity and its changes. Table 8 shows the volumes under each time interval computed using Eq. 11; hence, now our attention moves from the instants of time (i.e. the years) to the periods (i.e. the intervals between years). Table 8 illustrates that the maximum value of diversity is reached in the interval 2013-2014, and that diversity, on average, has grown over time. We underline “on average” because the volume under the surface of a period between two years is approximated by the mean of the two diversity profiles of the two instants of time; thus, it may happen that, observing many consecutive intervals, we note a continuous increase in the volumes but there can be also some little decreases in the areas that do not affect the volume’s trend.

Table 8 Values of the volume under the beta diversity surface of each period

$D_{V_{t,t+1}}$	Volume under the surface of each period
$D_{V_{2010,2011}}$	2.6114
$D_{V_{2011,2012}}$	2.7568
$D_{V_{2012,2013}}$	2.8353
$D_{V_{2013,2014}}$	2.8967

Table 9 Relative change of the volume between periods

π_t	Relative change of the volume between periods (%)
$\pi_1 = \pi_{2010-2011,2011-2012}$	0.0556
$\pi_2 = \pi_{2011-2012,2012-2013}$	0.0284
$\pi_3 = \pi_{2012-2013,2013-2014}$	0.0216

Table 10 Classical diversity indices computed for the ethnicity diversity in the workforce of the CNCSO computed at five different years from 2010 to 2014

Year	Richness index-1	Shannon-Wiener index	Blau-Simpson index
2010	4	0.9327	0.5151
2011	5	0.9903	0.5288
2012	5	0.9766	0.5196
2013	6	0.9708	0.5020
2014	6	0.9414	0.4851

Table 9 shows the relative changes in diversity (Eq. 14) from a period to the following using the volume under the beta diversity surface of each period.

In addition, we can compute the relative variation of diversity by using Eq. 15:

$$\Pi_{TOT} = \frac{D_{V_{2013,2014}} - D_{V_{2010,2011}}}{D_{V_{2010,2011}}} = 0.1092$$

Hence, in percentage, the workforce diversity has increases 10.92% from the period 2010–2011 to the period 2013–2014 in CNCSO with regard to the attribute "ethnicity".

To obtain a more precise assessment of the relative change of diversity over the whole period, not depending on the intervals but only on the instants $t_1 = 2010$ and $t_T = 2014$, we refer to Eq. 16 as follows:

$$\Upsilon_{TOT} = \frac{A_{2014} - A_{2010}}{A_{2010}} = 0.1685$$

Thus, in percentage, the workforce diversity has increases 16.85% from the instant 2010 to the instant 2014 in CNCSO.

Tables 10 shows the classical diversity indices computed for the ethnicity diversity in the workforce of the CNCSO computed at five different years from 2010 to 2014. For $S = 0$, we observe that diversity always increases (or at most remains equal to the previous year); diversity starts form 4 and end to 6, and thus the number of categories starts from 5 and ends to 7. To calculate how much diversity (for $S = 0$) has increased in percentage from 2010 to 2014, we can calculate the following ratio: $\frac{(6-4)*100}{4} = 50\%$. According to the Shannon-Wiener Index, we note a conflicting result with the previous one based on the "richness". Indeed, diversity increases from 2010 to 2011 but decreases from 2011 to 2014, and the same is for the Blau-Simpson Index.

The explanation is very simple and this is why we introduce our approach. Although the number of ethnic groups has increased, their distribution (from the diversity perspective) has worsened. In effect, the maximum variety occurs only when each category has a relative frequency equal to the ratio of one to the number of categories. In this context, however, we note that even if the number of categories increases, there are some of them that are clearly dominant over the others (hence, much homogeneity and few heterogeneity).

This circumstance demonstrates that, different classical indices of diversity may lead to conflicting results because diversity is a multivariate concept. One may state that variety increases from 2011 to 2014 or contrary can conclude that diversity decreases in the same period. Both conclusions are supported by the indices: richness index minus one, and Shannon-Wiener index, respectively.

Another absurd result due to the classical indexes can be noted by calculating the percentage change of the Shannon-Wiener index and the Blau-Simpson index from 2010 to 2014. To obtain the percentage change of the Shannon-Wiener index, we calculate the following ratio: $\frac{(0.9414-0.9327)*100}{0.9327}=0.9327\%$. Instead, the percentage change of the Blau-Simpson index is given by the ratio: $\frac{(0.4851-0.5151)*100}{0.5151}=-5.8241\%$.

We observe that the results of the three indices are very different in terms of size, and also provide opposite signs. This partly demonstrates why diversity studies often give contradictory findings. Indeed, from 2010 to 2014, the proxy of the richness index increases 50%, the Shannon-Wiener index increases 0.9327%, and the Blau-Simpson index decreases 5.8241%. On the contrary, our approach proposes a measure that incorporates infinite nuances of the classical indices, and thus it does not suffer the limits of the individual indices. It considers both richness and evenness, hence it is a synthetic measure of the overall diversity within the organization. In effect, we can observe that our method indicates that ethnicity diversity increases 16.85% from 2010 to 2014 (in an intermediate position compared to the extreme results that the classical indices provide).

5 Conclusions

Voluntary initiatives and legal requirements for promoting "diversity & inclusion" in organizations have recently stimulated the debate of scholars and institutions on this topic. In addition, today's organizations, especially business organizations, need to measure and monitor their internal diversity through appropriate instruments for understanding whether and how to use diversity as a strategic tool.

"Managing diversity and promoting inclusion has become part of the business world's strategic agenda" due to an increasing diversified society where "knowledge and innovation are essential for obtaining competitive advantages in a globalised economy" (European Commission 2015). Most scholars and insiders agree that diversity is "a fact of life" for every organization (Mayo et al. 2016), especially for companies. To date, in organizational studies, many theoretical discussions and empirical studies on the concept of diversity and its effects have been proposed. However, there is still a general consensus about the difficulty of synthesizing and comparing the results. This is due to several factors. First, there is not a general consensus on the definition of "diversity". Second, several different theoretical perspectives have been used to guide diversity research and, in addition, in the literature, many different variables have been considered. Finally, one of the greatest problem is that a unique accepted index for measuring and monitoring diversity does not exist. This study has partly focused on the question of the definition and mostly on the measurement issue.

After a critical review of the main methods for assessing diversity, this paper has proposed a new methodology for overcoming the issues of the classical indicators. We have shown that the richness index is too much sensitive to the inclusion of new categories without considering their weight within the distribution. The Shannon-Wiener index depends by both the number of categories and their evenness and is particularly sensitive to the presence of rare categories. Finally, the Blau-Simpson index, even if is influenced by both evenness and richness, is particularly sensitive to changes in the relative abundances of the most dominant categories.

Hence, before discussing about diversity management, we need adequate tools to measure diversity and consequently evaluate its impact on a diverse range of factors. At present, there are many tools for assessing different aspects of the multivariate concept of diversity, and thus the findings are often conflicting. Thus, the aim of this research is to introduce a new method for measuring and monitoring diversity in organization and overcoming the limits of the existing indices.

Taking advantage of functional data analysis and beta diversity profiles, we have developed some functional tools: the area under the beta diversity profile, the beta diversity surface, the volume under the beta diversity surface, the relative contribution of the diversity of one period, the relative change of diversity index based on the volume, the total change of diversity index based on the volume, and finally the total change of diversity index based on the area. The beta diversity surface allows us to obtain an immediate graphical representation of the changes of diversity over time, and as consequence, some general quick information about changes in diversity. Instead, "the volume under the beta diversity surface" and the other linked functional tools provide analytical information about the amount and variations of diversity over time.

In addition, this study has proposed an application of our method to a real dataset concerning the workforce diversity of the 'Corporation For National And Community Service Overview (CNCSO)' of the Government of the United States of America. This analysis has illustrated a comparison between our approach and the classical ones, and proved that the latter, in specific circumstances, may even lead to opposite results.

In the introduction, we have specified that we have focused on beta-diversity; a clarification on this is mandatory. The existing beta-diversity indices focus on diversity changes over time but at the category-level, i.e. comparing the relative frequencies of a category with the relative frequencies of the same category at the following instant of time. Our approach is slightly different because it calculates diversity using the diversity profile, and therefore it is a summary indicator of the whole variety. However, even if the profile, by nature, is an index of alpha-diversity, we consider its changes over time, and thus, conceptually, this study is positioned in the field of beta-diversity. Hence, this a limit of our approach because we lose information about the specific categories and obtain a synthetic measure.

A second limit of this study is that this focuses on the descriptive multivariate analysis of diversity trends over time without dealing with the problem of significance of variations. To evaluate if diversity changes are statistically significant, a functional analysis of variance (FANOVA) for repeated measures of the diversity profiles should be performed (e.g. starting from the functional non-parametric approach of Chao et al. Chao et al. (2014) or the functional parametric one (Di Battista et al. 2016b; Maturo et al. 2017).

A third aspect that we need to underline is that the concept "change" could be, in principle, seen as dynamic, i.e. it allows us to observe the degree of modification of some characteristic of the subject of study; however, the same change, being dynamic, needs to be analyzed at different times, and thus, our approach would be limited in observing only a kind of photograph of the change and the sensitivity of the aspect under consideration at a certain moment.

Finally, we remark that the scope of this research is based on the quoted literature. However, to the best of our knowledge, this is the first study that deals with developing a functional approach for the assessment of diversity in the organizational field.

In the introduction, we have pointed out that many indices have been proposed but no universally accepted measure for measuring and monitoring diversity has yet been established (Ricotta et al. 2003). On this point, we stress that a practical-philosophical discussion could be opened: is it convenient to find a method that will resolve the issues of all existing indices? In the field of science, the definitive solution to scientific problems does not always exist and knowledge is constantly changing; thus, we strongly believe that our approach does not pretend to replace the others methods but aims to provide an additional tool to the existing methods.

Being aware that functional data analysis can be a gold mine for organizational studies, we believe that this line of research can bring many future developments, and can help both organizations and policy makers in monitoring diversity and assessing its impact by using the proposed instruments as independent or response variables in many functional models (e.g. functional regression, clustering of functional data, functional analysis of variance).

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