Diffusion dynamics in small-world networks with heterogeneous consumers

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Abstract Diffusions of new products and technologies through social networks can be formalized as spreading of infectious diseases. However, while epidemiological models describe infection in terms of transmissibility, we propose a diffusion model that explicitly includes consumer decision-making affected by social influences and word-of-mouth processes. In our agent-based model consumers' probability of adoption depends on the external marketing effort and on the internal influence that each consumer perceives in his/her personal networks. Maintaining a given marketing effort and assuming its effect on the probability of adoption as linear, we can study how social processes affect diffusion dynamics and how the speed of the diffusion depends on the network structure and on consumer heterogeneity. First, we show that the speed of diffusion changes with the degree of randomness in the network. In markets with high social influence and in which consumers have a sufficiently large local network, the speed is low in regular networks, it increases in small-world networks and, contrarily to what epidemic models suggest, it becomes very low again in random networks. Second, we show that heterogeneity helps the diffusion. Ceteris paribus and varying the degree of heterogeneity in the population of agents simulation results show that the

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more heterogeneous the population, the faster the speed of the diffusion. These results can contribute to the development of marketing strategies for the launch and the dissemination of new products and technologies, especially in turbulent and fashionable markets.

Keywords Innovation diffusion · Threshold models · Word-of-mouth · Social networks · Heterogeneous markets

1 Introduction

Technological innovation drives the progress of societies. Any time a new technology, a new device, a new product appears into a society, its members have the chance to become aware of the innovation and to use it. In western societies people encounter new inventions and technologies on an almost daily basis. When these are consumed on an individual (or household) basis, single consumers (or households) can decide whether to adopt or not. The study of diffusion patterns of new products into society, from their launch to their successful adoption or failure to spread, closely involves managers and marketers whose interests are in disseminating new products into the society.

Recently marketers' attention has focused on the explosion of new fashions (Gladwell, 2000) and on the buzz that accompanies these explosions (Rosen, 2000). Especially in highly social susceptible contexts like clothes markets, many innovations emerge from minor events that are strongly related with the dynamics of local networks of friends. Then the new innovative fashion trend is adopted by some early adopters which are easily influenced by new trends and once the *critical mass* is reached, the diffusion and the number of adoptions get at their peaks. Almost all potential consumers decide to adopt and also laggards and skeptical consumers may decide to conform adopting the new product (Rogers, 1995). Throughout all this process, the social influence of other consumers' behaviors constantly affects the individual adoption. For example, somebody's decision of buying a cell phone partly depends on the number of friends and acquaintances already having one. If just a few of them have a cell phone, and she has no strong preference for using a cell phone, she would probably not feel an urgent need to buy one as well. However, if most of them use a cell phone, the social influence they have on her would become strong, and she may decide to buy one, despite her preference is not strong. Here we present an agent-based model that formalizes the consumer decision-making including the social influence as part of her utility. This agent-based model allows us to analyze how social influence is exerted into personal networks and how it shapes the macro diffusion of the innovation.[1]

Most studies on innovation diffusion modeling are rooted in the work of Bass (1969). The Bass model formalizes the aggregate level of penetration of a new product

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[1] The code of the agent-based model is written in C and it is available from the contact author.

emphasizing two processes of communication: (1) external influence via advertising and mass media, and (2) internal influence via word-of-mouth. The decision of a consumer is described as the probability to adopt the new product during time and it is assumed to depend linearly on these two forces. The first force is not related to previous adopters and it represents the external influence of mass media; the other force is related to the number of previous adopters and it represents the internal influence of word-of mouth:

$$\frac{f(T)}{(1 - F(T))} = p + qF(T) \tag{1}$$

$f(T)/(1 - F(T))$ is the hazard function defining the probability of a consumer to adopt at time t, p reflects the mass media influence and q reflects the influence due to word-of mouth. This basic Bass model fits very well to real data of durable goods, and many other variations of the model have appeared in order to explain different aspects of the diffusion (for overviews see Mahajan and Muller, 1979; Mahajan, Muller, and Wind, 2000). The model is able to represent a cumulative S curve of adopters and the fast growth is generated by the social interaction between early and late adopters (Rogers, 1995). However, the Bass model assumes all consumers to be homogeneous. It does not specify at the micro level how the consumer decision-making changes during time and how consumers communicate and influence each other. One of the rare examples of micro-level models of diffusion process in a traditional economic framework is the work of Chatterjee and Eliashberg (1990). This study presents an analytical model of innovation diffusion based on an individual decision-making that determines the adoption of agents one by one. The decision of adopting depends on the characteristics of the consumers, namely the perception of the innovation, the personal preference and the perceived reliability of information. The model introduces heterogeneity in the individual parameters of the population of potential consumers and these specific parameters are tested by a pilot study conducted in an experimental laboratory setting. Chatterjee and Eliashberg's model generated much interest on the impact of heterogeneity on diffusion models (Bemmaor and Lee, 2002) and it represents a complete framework that links individual decision-making and aggregate dynamics of innovation diffusion processes. However, the analytical tractability of the model obliges to limited analysis of aggregated variables and of consumers characteristics. This holds both for the estimation of the parameters at the aggregate level and for the estimation of individual parameters in the laboratory experiments. Our agent-based model can easily include heterogeneity in the population of consumers and it allows us to study how it affects the shapes of the diffusion curves.

Besides the work in line with the Bass model, much research on innovation diffusion has focused on computational models that investigate the patterns of innovation diffusion through social networks (Abrahamson and Rosenkopf, 1997; Goldenberg et al., 2000; Weisbuch and Stauffer, 2000). These models are based on the similarities between viral marketing dynamics and the diffusion of diseases (Moore and Newman, 2000; Newman, 2002; Dodds and Watts, 2005). These models include a network with nodes and links, and a virus infecting the nodes traveling through the links. The nodes

are consumers, links are the relations that consumers have among themselves and consumers are infected when they decide to adopt the innovation. Epidemic models explicitly define adoption rules and they are able to explain aggregate dynamics in terms of individual transmissibility. From a behavioral point of view, these models are extremely interesting because they permit to derive macro dynamics from micro hypothesis on individual decision making. However, in order to accept these models in social contexts, they need to be integrated with more realistic social processes like, as mentioned above, social influence and imitation. We propose a diffusion model that explicitly includes consumer decision-making affected by social influences and word-of-mouth processes. In fact the agents of our simulation model decide according to both their individual preference and the experienced social influence from other agents' behavior. This model allows us to study diffusion patterns in time for different markets. In particular, we focus our analysis on (very) turbulent and fashionable markets where consumers highly affect each others' behaviors. Examples are clothes markets, markets of electronic devices (e.g., cell phones, mp3 players) and entertainment industry (music, movies). Our model shows how in these markets the social structures and the heterogeneity of the consumers significantly determine the shape and speed of the diffusion.

The paper is structured as follows: in Section 2 we review epidemic models; in Section 3 we comment on threshold models in social science and how they are used in modeling herding behaviors; in Section 4 we present our agent-based model; Section 5 reports results of simulations and finally in Section 6 we report comments and conclusions.

2 Epidemics in social networks

A new product that invades a society is like a contagious epidemic that spreads in a population of humans or like a virus that is transmitted in a computers' network (Dodds and Watts, 2005). Thus, epidemic models can be very useful also in social and marketing contexts because they propose models that explain aggregate diffusion dynamics in terms of individual characteristics.

Most of the epidemic models are divided into two families: SIS (Susceptible, Infected, Susceptible) and SIR (Susceptible, Infected, Removed). The former assumes that nodes are initially susceptible and they become infected with probability ν if they are directly linked with one or more infected nodes. Then the infected node recovers and becomes susceptible again with probability δ. When $\delta = 0$, infected nodes cannot recover and the SIS model is converted into a SI (Susceptible, Infected) model. In the latter the same dynamics are assumed but once the node is infected, it just dies with probability γ and it never recovers. For social and marketing purposes, we will focus mostly on SIS and SI models because these are more relevant in social and marketing contexts: once somebody adopts a product she is not removed form the market; on the contrary, her decision of adopting affects other consumers.

In a SIS model, at the beginning of the spreading process, the diffusion of the disease involves only a few nodes of the network. These nodes infect each one of their direct neighbors with probability ν. It has been found in random graphs that if $\lambda = \nu/\delta$ overcomes a given threshold λ_c, then the diffusion speeds up, the rate

of diffusion increases in time infecting the majority of the network. Finally the rate of diffusion decreases only when almost all the population has been infected. If the rate of infection λ cannot overcome λ_c, then the diffusion dies out and the majority of the network is not involved in the process of diffusion (Anderson and May, 1992). The structure of the network (number of nodes, distribution of the links, clustering coefficients) determines the speed and the degree of diffusion. Watts (2002) showed that diffusion in random graphs does not depend on the amount of initially infected nodes but on the connectivity of the network. In highly connected random graphs, the disease spreads easily because when a node is infected, it is likely that among its neighbors, there is someone that decides to adopt as well and the diffusion continues spreading. At each time step there is always some new node that is infected. Then, the diffusion process depends on the critical mass as described in classical innovation diffusion marketing models (Rogers, 1962; Mahajan and Muller, 1979): if the early adopters (the nodes that are infected at the beginning of the diffusion) reach the critical mass, the diffusion will finally succeed in reaching the whole population.

However, social and artificial networks often have global structures that are not random, but display stylized characteristics like power law distribution of the links, high clustering coefficients and short paths between any couple of nodes (Barabasi and Albert, 1999; Watts and Strogaz, 1998). Both analytically and with computer simulations, Pastor-Sartorras and Vespignani (2002) showed that in scale-free networks λ approaches 0. With a multi-agent based model Delre, Jager and Janssen (2004) found a similar result for diffusion of innovations in a population of social susceptible consumers: innovations are more likely to spread and be adopted by more consumers when consumers are linked in a scale-free network than in a regular lattice.

Also the small-world network structure has been extensively investigated. Newman and Watts (1999) and Moore and Newman (2000) investigated epidemic dynamics in the small-world area and they describe how the percolation threshold depends on the number of shortcuts.[2] They found similar results when they vary the *transmissibility* of the disease (the probability that a disease is passed from an infected to a healthy and susceptible node). These studies show that diffusion dynamics in the small-world networks are the same of those in the random networks if the degree of randomness is big enough and the percolation threshold is reached. This result is relevant especially for diffusion of infectious diseases because it focuses on the transmissibility of diseases.

However, from an economic and consumer behavior point of view, there are two issues that appear to be problematic. The first is about the assumption of infectious contacts. It is not always convenient to assume that all nodes are equally susceptible during the time of the diffusion. People, in particular consumers, decide to adopt a new product partly according to how much they are exposed to the product. Consumer

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[2] A percolation threshold is the minimum probability for which an infinite regular lattice percolates (i.e. in a bi-dimensional regular lattice, cells are activated in such a way that a cluster reaches the borders of the lattice) (Stauffer, 1985). For this probability, the cluster scales extensively with the total number of the cells. Newman and Watts (1999) and Moore and Newman (2000) adopt the concept of percolation threshold to the small-world network graph. Here the percolation threshold is the minimum probability for which a giant component first form.

adoption partly depends on what other consumers do (Granovetter and Soong, 1986). When an innovation has vastly spread into a market, also those that were initially sceptical about the innovation may decide to adopt. Nowadays there is a strong social pressure to adopt a cell phone because almost all the people have one. The second problem concerns the results of random networks: for social scientists it is difficult to accept the idea that random networks are as efficient as small-world network in spreading fads and fashions. Having a high clustered group of friends is a crucial factor in determining the adoption of the group. Usually, if the network is highly clustered and a fashion emerges in a cluster, the social influence towards the non-adopters is very strong and it is very likely that the fashion involves the entire cluster. Contrarily, if all friends of a consumer belong to completely different groups (like relationships in a random graph), the consumer would not fell a strong social pressure to adopt. Moreover, because in the small-world networks clusters are connected through short-cuts, it can be hypothesised that a cluster that has already adopted affects connected clusters that have not adopted yet. Both problems derive from an oversimplification of the metaphor between disease spreading and innovation diffusion. While epidemic models can assume a unique virus to spread into the network, social scientists have to distinguish at least between two different processes: diffusion of the *information* about the product through friends' connections and *social influence* that takes place in local groups and in personal networks.

Although some studies have reported the high performances of small-world networks in diffusion of knowledge (e.g. Cowan and Jonard, 2004) and consumption (e.g. Janssen and Jager, 2003), to our knowledge there is not an economic model that formalizes the emergence and the diffusion of innovations in the small-world networks in terms of local interactions. Here we present an agent-based model that simulates the emergence of innovations in social networks. We conduct an extensive sensitive analysis of the model parameters and we draw the area of parameter space for which small-world network are more efficient in spreading the diffusion into the population of consumers.

3 Threshold models in social networks

Threshold models have a relevant tradition in social science, especially in modelling collective behaviors (Granovetter, 1978; Macy, 1991). They formalize situations in which there is a population of individuals that decide either to be involved or not in a group behavior. The focus of these models is on the social influence that adopters exert on those that have not adopted yet. Each individual has a *personal threshold* and if the size of the group is bigger than her personal threshold, then she will decide to adopt the behavior of the group. Threshold models formalize a positive feedback into the dynamics of the population: the more individuals are involved into the group behavior, the more others will feel the social pressure to adhere to the group behavior. If the group behavior is able to involve enough individuals, its diffusion will easily take off because of this positive feedback. Otherwise it likely dies out. Similar distributions of personal thresholds can derive very different results at the aggregate level (Shelling, 1978). Threshold models can be used to formalize many social phenomena, including innovation, rumours and disease spreading (Rogers, 1995). However threshold models

are usually extremely demanding with regard to the amount of information computed by individuals. When deciding what to do, individuals have a complete knowledge about what all others are doing. Granovetter (1978) suggests that "Social structure is one reason why the simple form of threshold models may not provide an adequate account of events. (...) The simple model makes an implicit assumption of complete connectedness which is often inappropriate: that each individual is responsive to the behavior of all the others, regardless of the size or special or temporal dispersion of aggregation" (p. 1431).

Interesting variation of threshold models have been proposed to solve this limitation focusing on the study on the local effects in the personal networks of each individual (Valente, 1996). Here also it is assumed that individuals have to face a binary decision: either to adopt the innovation or not. Valente draws a distinction between external influence of the social system and internal influence of the personal network. While external influence affects individuals through mass media and cosmopolitan links, internal influence affects individuals through the personal network and according to the level of *exposure*. Personal exposure to the innovation is defined as the proportion of adopters in an individual's personal network at a given time. Like in other threshold models, individuals decide to adopt when a personal threshold is surpassed but, despite classical threshold models, it is also possible to distinguish whether the threshold is reached because of external or internal influence.

We also include a threshold mechanism in our innovation diffusion model in order to focus on social influence effects. When deciding whether to adopt or not, our consumers are affected by other adopters of their local networks if and only if the exposure in their personal network is higher than a given personal threshold. In our model we use a parameter in order to vary the horizon of the local network and we show that adoption dynamics vary considerably according the definition of the local network. More precisely, we find that epidemic models (Newman, 2002; Newman and Watts, 1999; Dodds and Watts, 2005) can be used to predict the dissemination of products into a society of consumers when the local networks is relatively small (consumers observing only their friends) but they fail when the local network becomes a bit bigger (consumers observing also friends of friends).

4 The model

In our innovation diffusion model, agents are connected in a unique network. The nodes of the network are the consumers and each link between two nodes represents a relation of friendship between two consumers. Such network can vary from completely regular ($r = 0$) to completely random ($r = 1$) (Watts and Strogatz, 1998). On one hand, when the network is completely regular, agents are completely clustered and any information takes long time in order to travel from a node to another distant node. On the other hand, when the network is completely random, agents are not clustered at all and any information is spread to all other nodes within a very short time. However, in between these limits there is an area (the so called small world area) where the network is both still very clustered and information spreads very fast to all the clusters of the network. Our model studies how the penetration of the product in the population of consumers is affected by the structure of this network.

The decision to adopt the innovation depends on an internal word-of mouth process. Agents are involved in the word-of-mouth process if and only if they receive a message from some neighbor that has already adopted. This means that at each time step, each agent looks at its neighbors and it decides to adopt if and only if at least one of its neighbors has already adopted. If none of the neighbors has adopted yet, then it does not adopt either. When agent i is involved into the word-of-mouth process, the probability of agent i to adopt is:

$$a_{i,j} = P(U_{i,j} \geq U_{i,\text{MIN}}) \tag{2}$$

where

$$U_{ij} = \beta_j \cdot x_i + (1 - \beta_j) \cdot y_i \tag{3}$$

$$y_i = \begin{cases} q_j \geq p_i \Rightarrow 1 \\ otherwise \Rightarrow 0 \end{cases} \tag{4}$$

$$x_i = \begin{cases} A_i \geq h_i \Rightarrow 1 \\ otherwise \Rightarrow 0 \end{cases} \tag{5}$$

$U_{i,j}$ is the utility agent i has if it adopts innovation j and $U_{i,\text{MIN}}$ specifies i's minimum utility requirement. The utility has two components that are threshold functions: individual preference y_i and local social influence x_i of i's personal network; β_j weights these two components and it represents how strong the social influence effect is in the market of product j. Markets with high β_j are fashionable markets (e.g., clothes, electronic devices) whereas markets with low β_j are more stable (e.g., groceries). Concerning the individual part, p_i is the individual preference of agent i and q_j is the quality of the innovation j. Concerning the social influence part, h_i is a personal threshold which determines the individual agent's susceptibility to its neighbors' behavior and A_i is the fraction of adopters in the Lth order set of alters of agent i (personal network). Agents included in i's personal network are called alters. Direct friends are first alters (L = 1), friends of friends are second alters ($L = 2$) and so on. If the fraction of adopters in i's personal network is higher than h_i then the agent does feel social influence, otherwise it does not. The rationale of this formalization is the classical threshold mechanism of collective action: a consumer does not feel social pressure if just a few people around her behave in a particular way but once these people reach a certain number then she suddenly decide to change her mind and she behaves differently (Granovetter, 1978). Finally, diffusion is introduced in the population by external marketing effort e_1 that is assumed to be given and linear along the dynamics of the diffusion. During all the diffusion, any non-adopter agent is convinced to adopt with probability e_1. Once an agent has adopted, other agents connected to it through their personal network become also aware of the innovation and they are involved in the word-of-mouth process evaluating their utility according to (3).

In order to compare different speeds under different conditions, we report the variations in the ρ indicator defined as

$$\rho = \frac{1}{T} \cdot \frac{\sum_{t=0}^{T} D(t)}{\sum_{t=0}^{T} f(t)} \tag{6}$$

where T indicates the total cycles of the simulations, $D(t)$ is the cumulative function of adopters at time t, and $f(t)$ is the number of adopters at time t (Arenas et al., 2000). The ρ indicator allows us to compare different diffusions that reach the same number of adopters. In this model if the external marketing effort e_1 is positive, a complete diffusion always occurs. Then, because the external marketing effort e_1 is also constant during all the diffusion process, the speed of the diffusion is also a good indicator of how strong the word-of-mouth process is in the market.

In our analysis (Section 5.3) we investigate how the speed of the diffusion changes when consumers have very similar or very different personal thresholds. Then we use a beta distribution in order to vary heterogeneity respect to the threshold h_i of agents in the population[3]:

$$f(x) = \frac{x^{a-1} * (1-x)^{b-1}}{\frac{(a-1)!*(b-1)!}{(a+b+1)!}} \tag{7}$$

with mean $\mu = \frac{a}{a+b}$ and variance $\sigma^2 = \frac{a*b}{(a+b)^2*(a+b+1)}$.

Notice that the beta distribution allows us to model the heterogeneity of the agent population from the homogeneous case (very high value for a and b) for which all agents have the approximately the same threshold until the uniform distribution ($a = b = 1$) for which thresholds can vary randomly around the mean value.

5 Results

We implement our model as an agent-based model. Here we present simulation results for a population of one thousand agents and where on average each agent has 4 neighbors. Each set of simulations contains twenty runs. Here we report the average of the runs and when it is relevant the standard deviation of the runs.

5.1 Effects of social influence in different network structures

We begin investigating a very social susceptible society ($\beta_j = 1$, $h_i = 0.3$) representing a fashionable market where agents have a large personal network (L $= 2$). Letting the external marketing effort being low and constant ($e_1 = 0.001$) we observe changes in ρ. In Fig. 1, each point represents the speed of diffusion in a network with a different degree of randomness (r).

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[3] The beta distribution (http://mathworld.wolfram.com/BetaDistribution.html) is defined between 0 and 1 and it allows specifying the degree of heterogeneity of random drawings (Garcia-Diaz and Witteloostuijn, 2005).
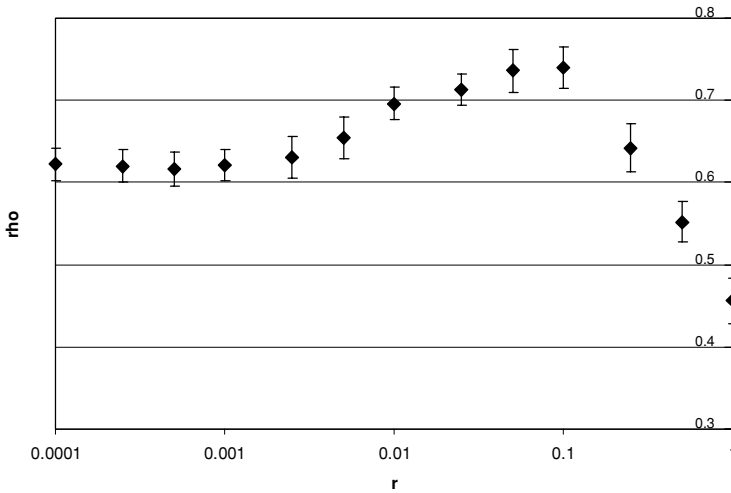
Fig. 1 The speed of diffusion ρ (after 250 time steps) varying the degree of randomness in the network

When the network is completely clustered ($r = 0.0001$), a group of innovators that start the diffusion can influence only local neighbors. Such influence is strong because the more clustered the group of adopters, the higher its influence on non-adopters neighbors (high exposure). Thus, the diffusion can travel along the network but it is slow: it cannot be spread in another distant region of the network. Consequently if some agents decide to not adopt the innovation, the word-of-mouth process dies and the only way to set the diffusion process again is by external influence. Then the time needed to convince all agents of the network to adopt is relatively large. The process changes when adding a little randomness into the network. Then shortcuts allow the innovation to emigrate in different parts of the network; diffusions can succeed easily and they spread very fast. Agents can see the spreading of diffusion in other clusters and they can import the fashion in their own cluster. At the same time, social influence is still very strong because the network is highly clustered. We observe the maximum values of ρ for this small-world area. Finally, when the randomness becomes very high, social influence is dimmed. In random network, agents are not clustered, the portion of adopters in their neighborhood is very low (low exposure). Consequently there is no social influence that presses them to adopt. During the initial part of the diffusion, innovators may decide to adopt only because of external influence. Because the external influence is low, then the critical mass is reached very late and, only then, the rest of the population will be suddenly convinced to adopt.

The parameter L plays an important role in this result. In Fig. 2 shows how the speed of diffusion varies in clustered, small-world and random networks if we vary L. When L is equal to 1, agents have a very small personal network because they are affected only by first alters' behavior and when L is equal to 2, agents have a large personal network because they are affected both by first alters and by second alters' behavior (see the social component of the utility function, i.e., Eq. (5). When L is in between 1 and 2, then agents are affected by first alters' behavior plus a proportion of second alters' behavior as indicated in the decimals. (For example, when L is equal to
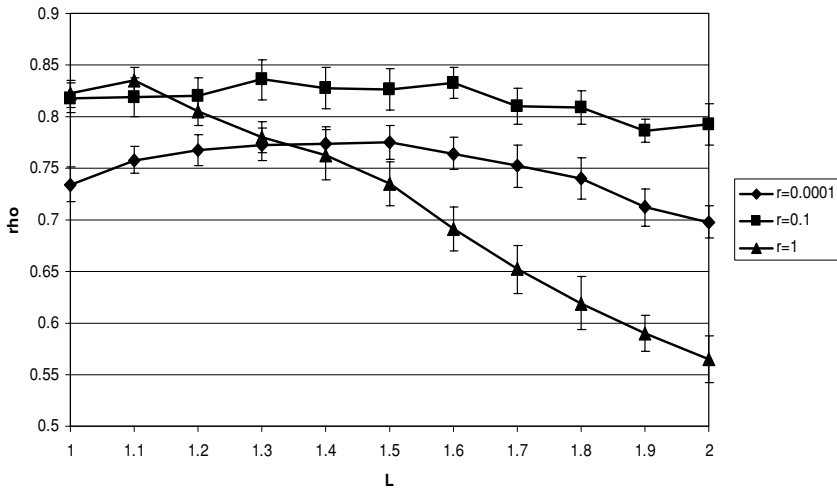
Fig. 2 The speed of diffusion ρ (after 250 time steps) in different networks varying the horizon of agents' personal network

1.2, agents include in their personal network all first alters plus 20% of their second alters.)[4]

It is not trivial to foresee what happens to the speed of the diffusion when varying personal networks because there exists a trade-off between social influence and availability of information. On one hand, when personal network of agent i is small (for example L = 1), just a few adopters in i's personal network may represent a high percentage and i's personal threshold can easily be reached. Then the innovation diffusion easily sets up and it can spread into the group. However in this case agent i is affected only by those adopters that are very close and it may ignore clusters of adopters that are just 2 steps far. On the other hand, when i's personal network is large (for example L = 2), i is affected by more friends' behaviors and just a few adopters into its group may not be sufficient to reach its personal threshold. However, having a larger social network allows i to perceive the social influence of other clusters of adopters. Figure 2 shows what happens enlarging the personal network parameter (L). For the values of our simulation runs, the trade-off is quite balanced in clustered networks like the regular one ($r = 0.0001$) and the small-world network ($r = 0.1$). But the situation changes in random networks ($r = 1.0$). Here, the absence of clusters does not permit the social influence to take place at all. Then, enlarging agents' personal network highly increases the time of the diffusion. Compared to the situation in which agents have a small personal network, the critical mass is reached later and it takes more time for the innovation to penetrate into the population.

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[4] It is important to mention that varying the personal network does not directly affect the word-of mouth process. What L does is to move the borders of personal networks when evaluating the social influence (Eq. (5)). For example, when L = 2, an agent can observe a friend's friend behavior and include it in the computation of its utility, but it cannot receive from it information about the innovation. In this way we can study the effect of L on the speed of diffusion without varying the altering the word-of mouth process.
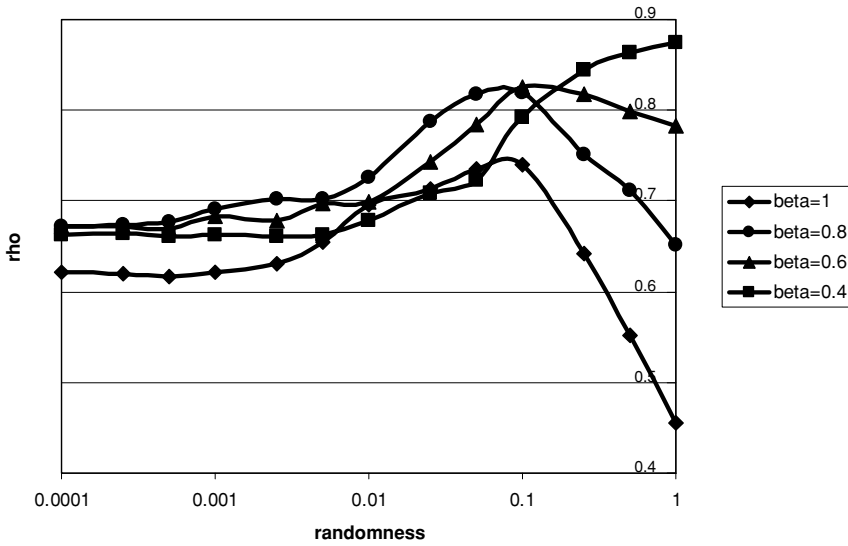
Fig. 3 The speed of diffusion ρ (after 250 time steps) varying β_j

5.2 Different markets

In the following set of simulations we control the robustness of our previous results investigating other values for the parameters β_j and h_i. When we decrease (increase) the value of β_j, we simulate more (less) individualistic markets because agents decide more (less) according to their personal preferences. When we decrease (increase) the value of h_i, we simulate a more (less) turbulent market because agents are more (less) reactive to what other agents do in their personal networks. To investigate different kinds of market, from very individualistic ($\beta_j \approx 0$) to very social susceptible ($\beta_j \approx 1$), we let L = 2.0, $h_i = 0.3$ and we set $p_i = [0, 1]$ and $q_j = 0.5$ assuming that agents have equal probability for positive or negative individual preference towards the innovation.[5] In Fig. 3 we show results for different markets.[6] Here it can be seen that the effects of network structures decrease when markets are more individualistic. Decreasing the value of β_j, the value of ρ depends less on the topology of the social network and, more importantly, when $\beta_j = 0.4$ we see that diffusion in random networks is as fast as in small-world networks. This confirms the idea that epidemic models are suitable for individualistic markets but fail in markets with high social influence. In these fashionable markets diffusions are basically driven by social influence and having a clustered network is fundamental in order to spread the innovation fast.

Figure 4 shows diffusion dynamics in different turbulent markets. For these simulation runs, we set $\beta_j = 1.0$ and let the other parameters' values as before. However,

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[5] For an analysis of different personal preferences on hits and flops of innovations, see Delre, Jager and Janssen, 2004.

[6] For these runs at the end of the simulations the fraction of adopters in the population, f, (average among the 20 runs) varied depending on the value of β_j. For $\beta_j = 1.0$, $0.998 < f < 1.0$; for $\beta_j = 0.8$, $0.903 < f < 0.917$, for $\beta_j = 0.6$, $0.782 < f < 0.837$; for $\beta_j = 0.4$, $0.674 < f < 0.746$.
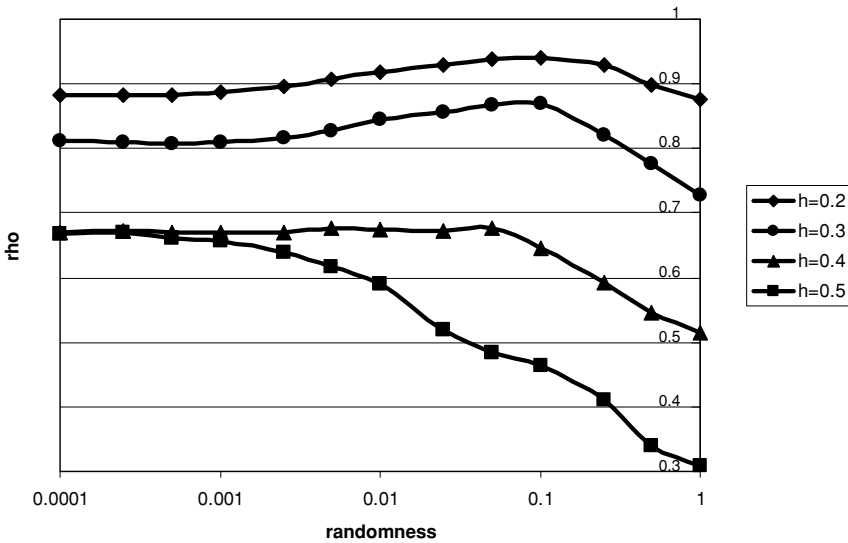
Fig. 4 The speed of diffusion ρ (after 400 time steps) varying h_i

especially in less turbulent markets, complete diffusions occurred after more than 300 time steps. Thus we collected values of ρ after 400 time steps for each simulation run.[7] Obviously, the speed of the diffusion is lower when personal thresholds are higher. More interestingly, it can be noted that when agents have high personal thresholds, the small-world networks is not the fastest in spreading the diffusion anymore. The only thing that counts here is how clustered the agents are: the more clustered they are, the more social influence adopters exert on non-adopters, the sooner the high personal threshold can be reached and the faster the diffusion disseminates.

5.3 Heterogeneous populations of consumers

In the last set of simulations, we include heterogeneity in the populations. With the same parameter values as before ($L = 2, \beta_j = 1.0, r = 0.1$) we observe a very high difference in the value of ρ between the homogeneous case and the uniform distribution case ($h_i = 0.3 \rightarrow \rho = 0.792$ and $h_i = [0, 0.6] \rightarrow \rho = 0.892$ after 250 time steps). Then we draw the value of h_i from beta distributions (Eq. (7)) and we vary the values of a and b in order to maintain the average $<h_i>$ fixed and to obtain different variance in the population representing, in this way, different degrees of heterogeneity. In Fig. 5, we show three sets of simulations for three different turbulent markets ($< h_i >= 0.2; < h_i >= 0.3, < h_i >= 0.4$). For each market we distribute agent's personal thresholds changing the variance into the population. In all three cases we find that more heterogeneity always causes a faster diffusion speed. When the populations become more heterogeneous there are more agents with lower and higher

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[7] Notice that the ρ indicator is a mean of the speeds of the diffusion at each time step and varying the number of steps causes variations in ρ. However, as long as we compare simulation runs with the same time steps, differences among different simulation runs are not altered.
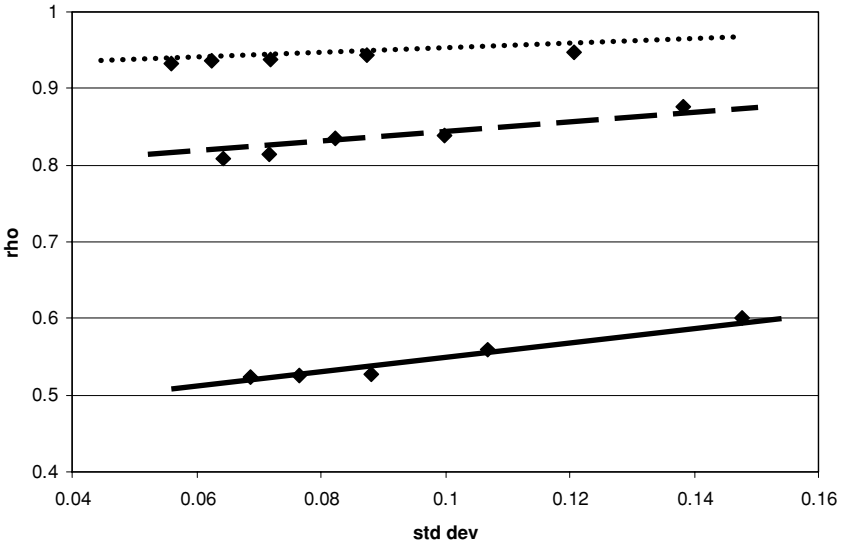
Fig. 5 The speed of diffusion ρ (after 250 time steps) varying the degree of heterogeneity in the population. The continuous line touches the points for which $<h_i> = 0.4$, the continuous line touches the points for which $<h_i> = 0.3$ and the pointed line touches the points for which $<h_i> = 0.2$
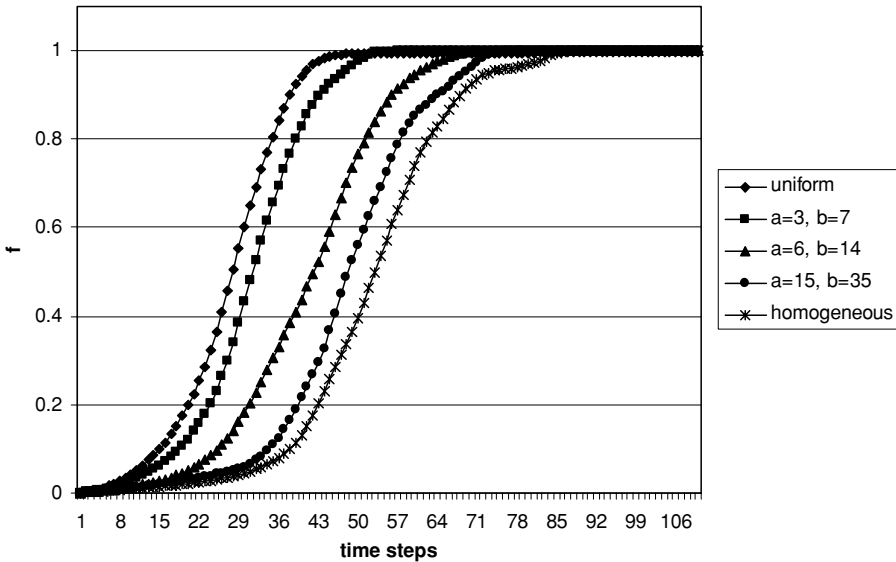

Fig. 6 The S curves of diffusion varying the degree of heterogeneity in the population

personal thresholds. Those that have a lower personal threshold are influenced sooner to adopt and they anticipate the ignition of the diffusion. Figure 6 shows five S curves of diffusion for different degrees of heterogeneity in the population of agents (homogeneous population, $h_i = 0.3$; $a = 3$, $b = 7$, $\sigma^2 = 0.138$; $a = 6$, $b = 14$, $\sigma^2 = 0.1$; $a = 15$, $b = 35$, $\sigma^2 = 0.064$; uniform distribution, $h_i = [0, 0.6]$). It is clear how the time

needed to complete the diffusion is much smaller as the population becomes more heterogeneous.

6 Conclusions and discussion

Epidemic models propose a new interesting methodology in order to study diffusion dynamics in biological, artificial and social networks (Dodds and Watts, 2005). The high relevance of these models stays in their generality which permits them to give insights and be applicable in many different fields and in the clear connection between micro specifications of individual characteristics and aggregate macro dynamics. There exists a population of nodes which are connected through links into a global network. The nodes are in a given state and diffusion dynamics are modeled as a penetration of a new state into the network: it can be a virus that flows because of infection and it can be a product that penetrated because of word-of-mouth. Epidemic models assume some individual characteristics like transmissibility (usually homogeneously into the population) and, either analytically or via computer simulations, they derive diffusion dynamics. However social contexts may need different assumptions about human behavior and decision making. In this paper we propose a new model in order to formalize innovation diffusions. Our model still belongs to the epidemic framework but it includes two strictly *social* concepts: (1) social influence in personal networks and (2) heterogeneity in decision-making. Simulation results show that in high social susceptible contexts the speed of the diffusion depends on how clustered groups are. Surprisingly, in high clustered networks innovations diffuse faster than in random networks. This is due to the fact that in clustered groups, individuals are exposed to more social influence and they may decide to adopt sooner. Especially in random networks, the dimension of personal networks also affects the diffusion: the bigger personal networks are, the slower the diffusion. Social influence explains this result. The bigger i's personal network, the higher the number of adopters that are necessary in order to exert social influence on i. Then it takes longer for the diffusion to be set up. Finally we find that heterogeneity in consumer population helps the speed of the diffusion. In more heterogeneous population the critical mass is reached sooner than in homogeneous ones because there are more individuals that adopt at the beginning and they ignite the diffusion sooner.

The success of epidemic models in social studies depends on how adaptable these models are and how they are translated in social contexts that can include relevant behavioral and social aspects. Especially in contexts where the decisions are interconnected and interdependent it is necessary to reproduce more realistic decision-making rules. In fashionable markets, promotion and marketing strategies have to take these aspects into consideration. Because the success of the diffusion depends on the internal dynamics of groups of consumers, it is crucial to identify the right consumers (targeting those consumers that occupy strategic positions in the social networks) at the right time (finding the most efficient periods for promotion), in the right way (starting the diffusion with clustered, cohesive, visible groups that can influence others' behavior).

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