Cluster Computing (2024) 27:6703–6772
https://doi.org/10.1007/s10586-024-04328-3

Hyperbolic Sine Optimizer: a new metaheuristic algorithm for high performance computing to address computationally intensive tasks

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Received: 12 October 2023 / Revised: 10 January 2024 / Accepted: 17 January 2024 / Published online: 7 March 2024

Abstract
In recent decades, the demand for optimization techniques has grown due to rising complexity in real-world problems. Hence, this work introduces the Hyperbolic Sine Optimizer (HSO), an innovative metaheuristic specifically designed for scientific optimization. Unlike conventional approaches, HSO takes a unique approach by engaging individual members of the population, ensuring a comprehensive exploration of solution spaces. Employing distinctive exploration and exploitation phases, coupled with hyperbolic *sinh* function convergence, the optimizer enhances speed, simplify parameter adjustment, alleviates slow convergence, and demonstrates efficiency in high-dimensional optimization. This approach is designed to tackle optimization challenges and enhance adaptability in unpredictable real-world scenarios. The evaluation of HSO's performance unfolds through four distinct testing phases. Initially, a set of 65 widely recognized benchmark functions is employed. These functions cover both unimodal and multi-modal varieties across dimensions of 30, 100, 500, and 1000, including fixed-dimensional functions, to comprehensively assess the exploration, exploitation, local optima avoidance, and convergence capabilities of the proposed algorithm. The results of the HSO algorithm are then compared to those of 15 state-of-the-art metaheuristic algorithms and 8 recently published algorithms. Secondly, HSO's performance is assessed in comparison with the benchmark suite from the Institute of Electrical and Electronics Engineers (IEEE) Congress on Evolutionary Computation (CEC). This suite includes 15 benchmark functions for CEC-2015 and an additional 30 benchmark functions for CEC-2017. During the third phase, HSO tackles seven real-world classical engineering design problems by addressing both the constrained and unconstrained optimization challenges of IEEE CEC-2020. Finally, HSO undertakes training for a multilayer perceptron, utilizing four distinct datasets. To qualitatively assess HSO's performance, two statistical analyses—the Friedman and T tests—are employed. The findings of HSO showcase its adaptability and effectiveness as a high-performing optimizer in engineering optimization challenges. Note that the source code of the HSO algorithm are publicly accessible via https://github.com/Shivankur07/Hyperbolic-Sine-Optimizer.git.

Keywords Metaheuristics · Hyperbolic Sine Optimizer · HSO · Optimization · Evolutionary algorithms · Nature inspired algorithms

1 Introduction

The rapid progress of science and technology over the past decade has elevated the complexity of real-world optimization challenges, driving the creation of efficient and effective optimization algorithms. In recent times, addressing optimization challenges has emerged as a compelling and dynamic subject across various research domains. The rapidly expanding domain of decision-making problems can be characterized as instances of optimization problems. The first step of optimization involves creating an objective function that can be either maximized or minimized. Subsequently, after formulating the optimization problem, it becomes necessary to employ an optimization algorithm to explore the optimal variables leading to the most favourable solution. Real-world optimization challenges are expressed in mathematical terms as outlined below:

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$minimize f(X).$

$s.t. g(X) \leq 0.$

$X \epsilon R^n$

where $X = \{x_1, x_2, \ldots, x_n\}$ is an $n-$ dimensional solution, R^n is the domain of definition, $f(X)$ is the objective function, and $g(X)$ is the constraint function.

Generally, optimization challenges can be addressed through deterministic or stochastic approaches. Deterministic methods such as linear and non-linear programming, which rely on gradient information, are commonly used to discover optimal solutions. However, these traditional deterministic approaches may converge to local optima. In order to address the constraints associated with conventional methods, stochastic approaches like meta-heuristic algorithms can be applied to tackle complex real-world optimization problems. Therefore, metaheuristic algorithms have gained significant popularity and are extensively employed to address practical optimization challenges across various domains, including cloud computing [1], scheduling [2], neural networks [3], feature selection [4], image segmentation [5], fuzzy control [6], photovoltaic models [7], civil engineering [8], reliability-based design [9], and more.

The primary advantages of meta-heuristic algorithms include their simplicity, flexibility, capacity to evade local optima, and mechanisms that operate without relying on derivatives. The search process of meta-heuristic algorithms involves two distinct phases: exploration and exploitation. During the exploration stage, the algorithm extensively covers the search space to identify promising regions that may lead to the optimal solution. Inadequate exploration may result in being trapped in local optima. The exploitation phase concentrates on searching around the promising regions identified during exploration. Failure to execute effective exploitation can substantially diminish solution accuracy. Striking a balance between exploration and exploitation stands as a significant challenge for meta-heuristic approaches. An effective meta-heuristic algorithm is characterized by: (i). Optimal balance between exploration and exploitation (ii). Achievement of a high level of accuracy. (iii). Convergence towards the optimal solution while avoiding local optima. (iv). Consistent and stable performance, ensuring minimal variation in results across independent runs.

In recent years, numerous metaheuristic algorithms have been developed and implemented in a wide range of fields. For an in-depth and comprehensive review of these algorithms, please refer to Table 1. Despite numerous recently proposed algorithms in the field of optimization, a fundamental question remains about the necessity of introducing more optimization techniques. The No Free Lunch (NFL) theorem logically establishes that no single algorithm can solve all optimization problems. Therefore, the success of an algorithm in addressing a specific set of problems does not ensure effectiveness across all optimization problems with varied types and characteristics. In simpler terms, most optimization methods perform similarly on average, even though they may be really good at solving specific types of problems. The NFL theorem encourages researchers to develop new optimization algorithms designed to specific subsets of problems in diverse fields. This motivation underlies the presented work, which introduces a simple yet effective optimization technique designed for real problems with unknown search spaces. However, the development of numerous metaheuristic algorithms aimed at discovering optimal solutions, significant gaps persist between the theoretical concepts and the practical implementation of these algorithms. Some of these gaps are as follows:

- Metaheuristic algorithms may struggle to fully explore solution spaces.
- Slow convergence can lead to premature settling for suboptimal solutions.
- High-dimensional problems pose challenges due to increased computational demands.
- Some methods require complex parameter tuning, making them resourcE−intensive.
- Performance varies based on specific optimization problem characteristics.
- Adaptation to real-world problems, especially in dynamic or uncertain environments, may be challenging for metaheuristics.

To address the problems in existing algorithms, researchers are encouraged to develop new and creative algorithms with practical solutions for current challenges. It is suggested to create algorithms that reliably reach a solution, have a strong system for controlling and adjusting parameters, and perform well within reasonable time limits. This study also demonstrates that a meta-heuristic doesn't necessarily require direct inspiration; instead, basic mathematical functions, such as the hyperbolic *sinh* function, can be employed to create optimization algorithms in this domain. The proposed algorithm utilizes the hyperbolic *sinh* function to navigate and exploit the search space, leading to improved solutions. This research introduces the Hyperbolic Sine Optimizer (HSO), a novel meta-heuristic optimization algorithm designed to address a diverse range of optimization challenges in scientific applications. The algorithm places a primary emphasis on mathematical concepts, specifically exploring mechanisms related to algebraic and hyperbolic functions within the context of population members. A distinctive aspect of this study lies in the development of exploration and exploitation phases

Table 1 Survey of related works of recently published metaheuristic algorithms

S.No	Algorithm	Inspiration	Year	References
1	Nutcracker optimizer(NO)	Inspired by nutcracker foraging behavior	2023	[94]
2	Snow ablation optimizer(SAO)	Inspired by snow ablation process	2023	[95]
3	Gannet optimization algorithm(GOA)	Inspired by gannet seabird foraging behavior	2023	[29]
4	Bacteria phototaxis optimizer(BPO)	Inspired by bacterial phototaxis and evolution	2023	[30]
5	Group learning algorithm(GLO)	Algorithm inspired by group dynamics, leader′simpact	2023	[71]
6	Skill optimization algorithm(SOA)	Inspired from human efforts to acquire and improve skills	2023	[70]
7	Language education optimization(LEO)	Inspired by the foreign language educationprocess	2023	[69]
8	Kepler optimization algorithm(KOA)	InspiredbyKepler′sLawofplanetarymotion	2023	[56]
9	Walrus optimizer(WO)	Inspiredbythebehaviourofwalruses	2023	[96]
10	Growth optimizer(GO)	Inspired by the growth processes in society	2023	[72]
11	AFOX optimizer(AO)	Inspired by the hunting behaviour of foxes	2023	[97]
12	Propagation search algorithm(PSA)	Inspired by the wave propgation of voltage and currents	2023	[58]
13	Dung beetle optimizer(DBO)	Inspired by the behaviour of dung beetles	2023	[98]
14	Cat hunting optimization Algorithm(CHOA)	Inspired by cat hunting strategy	2023	[99]
15	Success history intelligent Optimizer(SHIO)	Inspired by Intelligent systems	2023	[73]
16	Aquila optimizer(AO)	The foraging behavior of Aquila	2022	[28]
17	White shark optimizer(WSO)	The movement and foraging behavior of white sharks	2022	[100]
18	Honey badger algorithm(HBA)	The foraging behavior of honey badgers	2022	[101]
19	Snake optimizer(SO)	The foraging and reproductive behavior of snakes	2022	[34]
20	Reptile search algorithm(RSA)	The foraging behavior of crocodiles	2022	[35]
21	Ebola optimization search Algorithm	The mechanism of transmission of Ebola virus	2022	[37]
22	Prairie dog optimization Algorithm(PDO)	The survival behavior of prairie dogs	2022	[36]
23	Dwarf mongoose optimization Algorithm(DMOA)	The foraging behavior of dwarf mongooses	2022	[33]
24	weighted mean of vector(INFO)	The weighted mean idea	2022	[59]
25	Coati optimization algorithm(COA)	Inspired by coati foraging behavior	2022	[32]
26	Sea horse optimizer(SHO)	Inspired by sea horse behaviors in nature′smovementandbreeding	2022	[31]
27	Archimedes optimization algorithm(AOA)	Inspired fr omaninterestinglawofphysicsArchimedesPrinciple	2022	[55]
28	Beluga whale optimization(BWO)	Inspired by behaviour of beluga whales	2022	[102]
29	Fick′slawalgorithm(FLA)	Inspired by Fick′sfirstLawofdiffusion	2022	[57]
30	Child drawing developmentOptimization(CDDO)	Inspired by child′scognitivedevelopement.	2022	[74]

for metaheuristic algorithms, departing from conventional mechanisms and relying entirely on behavioural patterns associated with hyperbolic function convergence. The exploration and exploitation phases are pivotal in any meta-heuristic algorithm, and this research introduces a unique approach to these stages. Unlike many existing population-based meta-heuristic optimization algorithms, the HSO ensures that population members actively contribute to the exploration of the search region. This deviation from the usual practice, where the population commonly stays inactive during the exploration phase, underscores the importance of involving the contributions of individual population members to enhance the efficiency of investigating the search region. The researchers have addressed this gap by developing an optimizer that utilizes the collective impact of population members to effectively explore the search region. To delve deeper into the functioning of the optimizer, the subsequent subsection provides additional insights. The effectiveness of any meta-heuristic algorithm is contingent upon achieving a well-balanced interplay between exploration and exploitation. In this context, the authors have presented the exploration and exploitation stages using the hyperbolic *sinh* function, marking a notable advancement for the HSO. The application of the *sinh* hyperbolic function in behavior analysis adds an exciting dimension to the optimizer, showcasing its potential to expedite problem-solving processes and reach global minima or maxima. The exploitation phase, crucial in optimization, relies on a set of algebraic equations based on population elements, further underlining the algorithm's versatility and efficacy in handling complex optimization challenges. Overall, the Hyperbolic Sine Optimizer presents a promising avenue for meta-heuristic optimization, offering a fresh perspective on exploration and exploitation dynamics in the pursuit of global extrema for diverse problem domains.

The main contributions of this study, are as follows.

- HSO actively involves individual population members, deviating from conventional practices, ensuring a more comprehensive exploration of solution spaces.
- Utilizes distinctive exploration and exploitation phases with hyperbolic *sinh* function convergence to enhance convergence speed, mitigate premature convergence, and actively involve population members for addressing slow convergence
- Demonstrates efficiency in handling high-dimensional optimization problems, with comprehensive evaluations across dimensions (30, 100, 500, and 1000) showcasing applicability.
- Utilizes mathematical principles to simplify parameter adjustment, employing behavioural dynamics with algebraic and hyperbolic *sinh* functions. This potentially reduces resource demands compared to complex-tuned algorithms.
- Emphasizes mathematical concepts and unique exploration/exploitation phases, providing a robust, versatile approach to address variability in performance across different optimization problem characteristics.
- Active involvement of population members in exploration enhances adaptability, and unique dynamics introduced by hyperbolic functions contribute to improved adaptability in real-world and uncertain environments.

The main objective of this paper is to develop a robust optimization algorithm suitable for addressing various challenges encountered in real-world optimization scenarios. This research effectively addresses substantial challenges in the research field related to the theoretical foundation by presenting a clear and straightforward layout. In contrast to other metaheuristic algorithms, this study stands out for its precise mathematical and theoretical underpinnings, exemplified by the incorporation of the hyperbolic *sinh* function.

2 Literature review

The focus of current research is on using metaheuristic algorithms for complex engineering problems, driven by their proven effectiveness. This has led to a substantial growth in literature on metaheuristic techniques. The theoretical research within literature can be categorized into three primary avenues: enhancing existing techniques, combining diverse algorithms, and suggesting novel algorithms. In the initial approach, scholars endeavour to enhance algorithmic performance by incorporating various mathematical or stochastic operators. The second prevalent research direction involves merging different algorithms to enhance overall performance or address particular issues. Finally, proposing new algorithms stands out as a widely pursued research path. The inception of innovative algorithms often draws inspiration from evolutionary phenomena, the collective behaviour of creatures (utilizing swarm intelligence techniques), fundamental physical principles, and concepts related to human experiences. The literature is progressively adopting classification methods based on these sources of inspiration, leading to the identification of four main categories of algorithms: swarm-based, evolutionary-based, physics/chemistry-based, and social/human-based algorithms. Table 1 summarizes notable metaheuristic algorithms.

- Swarm intelligence based algorithms: Swarm Intelligence simulates labour division and collaboration among organisms, evolving the population through interactions. Examples of swarm intelligence based algorithms include: PSO [10], a widely recognized swarm-based metaheuristic algorithm introduced by Kennedy and Eberhart in 1995. Another classical algorithm is Ant Colony Optimization (ACO), proposed by Dorigo et al. [11], inspired by ants' foraging behaviours and the communication of chemical pheromone trails to find optimal paths. The Artificial Bee Colony (ABC) algorithm [12] is inspired by bee foraging, involving employed bees, onlooker bees, and scouts. Various swarm-based metaheuristic algorithms have gained attention, such as the Grey Wolf Optimizer (GWO) [13], simulating grey wolves' cooperative hunting with alpha, beta, delta, and omega groups. The Whale Optimization Algorithm (WOA) [14] mimics whale foraging, demonstrating strong optimization convergence. The Salp Swarm Algorithm (SSA) [15] is inspired by the salp chain, attracting attention across diverse fields. Harris Hawks Optimization (HHO) [16] excels in engineering optimization, simulating hawks' preying with distinct chasing patterns. The Marine Predator Algorithm [17] draws from predator–prey behaviours, utilizing velocity ratio, Levy, and Brownian movement. Seagull Optimization Algorithm (SOA) [18] mimics seagull foraging for optimization. Other notable algorithms include Monarch Butterfly [19], Lion [20], Pity Beetle [21], Squirrel Search [22], Butterfly [23], Slime Mould [24], Golden Eagle [25], Red Fox [26], Starling Murmuration Optimizer (SMO) [27], Aquila Optimizer (AO) [28], Gannet optimization algorithm (GOA) [29], Bacteria phototaxis optimizer (BPO) [30], Sea horse optimizer (SHO) [31], Coati Optimization Algorithm (COA) [32], Dwarf Mongoose Optimization Algorithm (DMOA) [33], Snake Optimizer (SO) [34], Reptile Search Algorithm (RSA) [35], Prairie Dog Optimization Algorithm (PDO) [36], Ebola Optimization Search Algorithm [37] and more.

- Evolutionary algorithms: The second class of metaheuristic algorithms falls under the evolutionary-based category. These algorithms imitate biological evolution through processes like crossover, mutation, and selection, achieving evolution by preserving highly adaptable individuals (solutions). For instance, GA, introduced by Holland in 1975 [38], stands as a pioneer in metaheuristics, drawing inspiration from Darwin's theory of natural competition. This approach proves suitable for a diverse range of optimization problems. Storn and Price developed Differential Evolution (DE) [39], a widely used algorithm for optimization. Biogeography-based Optimization [40] is derived from the migration and mutation of biological organisms, with the best solution obtained by updating the habitat suitability index through migration and mutation. Additionally, various variants of evolutionary-based metaheuristics have been introduced, including Evolution Strategy [41], Gene Expression Programming [42], and Memetic Algorithm [43, 44].
- Physics and chemistry based algorithms: Physics and chemistry-based algorithms, inspired by physical forces like electromagnetic and gravity, as well as chemical concepts, emphasize theoretical and mathematical aspects. This makes their convergence easier to demonstrate with field-specific theorems. Examples include simulated annealing [45], gravitational search algorithm [46], big-bang big-crunch algorithm [47], charged system search [48], ray optimization [49], stochastic fractal search [50], equilibrium optimizer [51], sine cosine algorithm [52], water cycle algorithm [53], thermal exchange optimization [54], Archimedes optimization algorithm (AOA) [55], Kepler Optimization Algorithm (KOA) [56], Fick' s Law Algorithm (FLA) [57], Propagation Search Algorithm (PSA) [58], weighted mean of vector (INFO) [59] and others. Simulated annealing [45], inspired by the physical law of metal cooling and annealing, is effective in solving complex optimization problems. The gravitational search algorithm [46], drawing inspiration from the law of gravity, attracts particles based on mass weight to achieve optimal solutions.
- Human based algorithms: The final category of metaheuristic algorithms is based on social or human behaviours. Brainstorm Optimization [60], developed by Shi, simulates intense ideological collisions among people. Each idea represents a candidate solution, and solution updates involve clustering and fusion. Teaching–Learning-Based Optimization [61] draws inspiration from the teaching and learning process in classrooms, where students learn not only from teachers but also from peers. TLBO is recognized as a high-quality algorithm in the metaheuristic field. Other notable social or human-inspired metaheuristics include Tabu Search [62], Harmony Search [63], Political Optimizer [64], Imperialist Competitive Algorithm [65], League Championship Algorithm [66], Interactive Autodidactic School [67], Arithmetic Optimization Algorithm [68], Language Education Optimization (LEO) [69], Skill Optimization Algorithm (SOA) [70], Group learning algorithm (GLO) [71], Growth Optimizer (GO) [72], Success History Intelligent Optimizer (SHIO) [73], Child Drawing Development Optimization (CDDO) [74] and more.

The constant progress and improvement of initial metaheuristic optimization algorithms represent an ongoing process of getting better and more innovative.

Researchers continuously work on refining these fundamental algorithms by incorporating inventive methods and strategies (Table 2). Hybrid approaches take advantage of how different algorithms complement each other, making the exploration of the search space more effective and efficiently using potential solutions. This blending of ideas not only pushes the limits of optimization performance but also creates adaptable and strong algorithms that can handle various complex optimization challenges more effectively. In recent years, various enhanced and hybridized versions of existing metaheuristic algorithms have emerged. These variants are essential for optimizing algorithm performance, overcoming limitations, and adapting to diverse problem characteristics, ensuring efficient solutions. For instance, The Conscious Neighbourhood-based Crow Search Algorithm (CCSA) [75] addresses crow search algorithm concerns with three novel strategies, achieving outstanding results in benchmarks and engineering problems, surpassing state-of-the-art swarm intelligence. The Quantum-based Avian Navigation Optimizer Algorithm (QANA) [76] refines differential evolution, introducing self-adaptive quantum elements, success-based distribution, and V-echelon communication. QANA proves superior to DE and other swarm algorithms. The Enhanced Whale Optimization Algorithm (E−WOA) [77] improves feature selection, outperforming existing WOA variants. Binary E−WOA (BE−WOA) excels in medical datasets like COVID-19 detection. The Improved Binary QANA (IBQANA) [78] enhances binary metaheuristics for medical data pre-processing, surpassing seven alternatives. Binary QANA (BQANA) [79] optimizes medical feature selection, outperforming alternatives. Binary Starling Murmuration Optimizer (BSMO) [80] excels in feature selection, surpassing rivals. The Levy Arithmetic Algorithm [81] enhances the Arithmetic Optimization Algorithm, proving superior in diverse benchmarks and real-world scenarios. E-mPSOBSA [82] integrates the modified Backtracking Search Algorithm (BSA) and Particle Swarm Optimization (PSO) to improve global exploration and local exploitation. The hybrid HSMA [83] combines quadratic approximation with SMA, while the novel m-SCBOA [84] merges a modified Butterfly Optimization Algorithm with the Sine Cosine Algorithm, enhancing both exploratory and exploitative searches. E-SOSBSA [85], a hybrid of SOS and BSA, addresses SOS limitations through adaptive mutation and crossover operators. NwSOS [86] modifies the symbiotic organisms search algorithm, enhancing exploration/exploitation balance. ImBSA [87] improves BSA with a multi-population approach, diverse mutation strategies, and adaptive control parameters. MISOS [88], an improved SOS variant, tackles local optima and weak convergence issues, demonstrating enhanced search performance on benchmarks. The modified DE algorithm in paper [89] addresses control parameter challenges in optimizing real-world problems. QRSMA [90] integrates SMA with quasi-reflection-based learning for improved diversity, convergence, and search efficiency. Similarly, gQR-BSA [91] modifies the backtracking search algorithm with quantum Gaussian mutations, adaptive parameters, and quasi-reflection-based jumping. mLBOA [92] is a BOA variant utilizing self-adaptive parameters and Lagrange interpolation, while m-DMFO [93] addresses premature convergence issues in the moth flame optimization algorithm.

The preceding paragraphs showcase a range of metaheuristics devised by researchers and the broad domains where these algorithms find application. In addition to, Table 25 presents a comprehensive overview of the advantages and disadvantages associated with various metaheuristic techniques, offering insights into their strengths and limitations for optimization tasks. The subsequent section presents the Hyperbolic Sine Optimizer (HSO), a novel metaheuristic optimization algorithm crafted to tackle a diverse array of optimization challenges in scientific applications.

3 The proposed algorithm: Hyperbolic Sine Optimizer

The Hyperbolic Sine Optimizer (HSO) is a novel metaheuristic optimization method for scientific problems. HSO stands out for its utilization of algebraic and hyperbolic functions, particularly focusing on population members. The algorithm comprises two phases: exploration and exploitation, uniquely relying on behaviour mechanisms associated with hyperbolic function convergence, departing from conventional methods. Highlighting the significant role of population members in efficient search region exploration, HSO diverges from traditional population-based meta-heuristic optimization algorithms. While most algorithms minimize the influence of population members in exploring the search region, HSO emphasizes their importance for effective exploration. To address this research gap, the study proposes an optimizer actively involving population members in the exploration process. The paragraph underscores the critical need for balancing exploration and exploitation in meta-heuristic algorithms, with the authors introducing the hyperbolic *sinh* function to accelerate convergence towards global minima or maxima. In the exploitation phase, algebraic equations based on population constituents are employed. The paragraph concludes by noting the proposal of a specific equation for position updates during the exploratory phase, emphasizing the crucial role of these mathematical tools in achieving efficient optimization results. Overall, HSO introduces a new approach that considers population dynamics, behaviour mechanisms, and mathematical functions to enhance the exploration and exploitation phases in meta-heuristic optimization.

3.1 Exploration phase

The exploration stage within the framework of the Hyperbolic Sine Optimizer (HSO) signifies a revolutionary transformation in the field of meta-heuristic optimization. Unlike traditional methodologies where populations remain passive during exploration, the HSO takes an active approach by involving individual members and acknowledging their distinct contributions. This approach enhances overall efficiency by tapping into the combined impact of population members, introducing dynamism to the exploration process. Emphasizing the vital interplay between exploration and exploitation, the HSO utilizes the hyperbolic *sinh* function during the exploration phase, providing a new outlook on shaping exploration dynamics. The incorporation of the *sinh* hyperbolic function in behaviour analysis introduces an inventive aspect, prompting problem-solving and facilitating the effective traversal of the search region. Closely connected to mathematical principles, the exploration phase capitalizes on algebraic and hyperbolic functions, marking a significant advancement. In conclusion, the exploration phase of the HSO, characterized by active involvement and guided by mathematical principles, sets it apart with an innovative approach, ensuring adept handling of intricate optimization challenges and pursuit of global extrema across various problem domains.

$$x^{\alpha}_{i,j} = random_{uniform}\left(\min\left(x^{p}_{i,j}, r_1\right), \max\left(x^{p}_{i,j}, r_1\right)\right), \tag{1}$$

$$where\ r_1 = x^{best}_{j} - \left(x^{best}_{j} * \sinh\left(r_2 * {}^{\pi}\!/_{2}\right)\right), \tag{2}$$

$$r2 = randomnumberbetweenlbandub$$

In the exploration phase, we evaluate a random number $x^{\alpha}_{i,j}$ based on uniform random distribution between population members $x^{p}_{i,j}$ and r_1 as of Eq. (1). The definition of r_1 is defined as on Eq. (2) and thereafter replaces original population members with these newly generated members. Here x^{best}_{j} represents the population local best.

Afterwards, replaces all population members, finding the best local solution $[x]^{\alpha_{best}}_{1\times n}$. Now the exploration phase ends, and the exploitation phase initialize.

3.2 Exploitation phase

The Hyperbolic Sine Optimizer (HSO) relies on a crucial exploitation phase for its meta-heuristic optimization. Unlike traditional methods, HSO prioritizes mathematical concepts, especially algebraic and hyperbolic functions within the population, recognizing the importance of exploration and exploitation in meta-heuristic algorithms. The HSO stands out by actively involving population members during search region exploration. In the exploitation phase, departing from common inactivity, HSO prioritizes individual contributions to enhance search region investigation efficiency, emphasizing the algorithm's commitment to harnessing the collective impact of population members. Researchers stress the exploitation phase's importance, relying on algebraic equations based on population elements. This underscores HSO's adaptability and effectiveness in handling complex optimization challenges, presenting a distinctive approach to meta-heuristic optimization.

During the exploitation phase, the following equations are used to update the population:

$$[X]^{\beta}_{m\times n} = \begin{cases} x^{\beta}_{i,j} = r_4/(1+\sinh(\rho/\delta^2))|r_3 \geq \rho \\ x^{\beta}_{i,j} = x^{\alpha}_{i,j}|r_3 < \rho \end{cases} \tag{3}$$

$$where \delta = x^{\alpha}_{i,j} \times \mu \tag{4}$$

$$and \mu = \left(x^{\alpha}_{i,j} * x^{p_{best}}_{j}\right)/(\rho * \pi) \tag{5}$$

$$where \rho = 0.5, wherer_3 isarandomnumberbetween 0 and 1$$

$$and\ r_4 isarandomnumberbetweenlbandub.$$

Hereafter evaluate the target objective function value $[f]^{\beta_{obj}}_{m\times 1}$ concerning each search agent of the population $[X]^{\beta}_{m\times n}$ Subsequently, finding the best-fit search agent $[x]^{\beta_{best}}_{1\times n}$ concerning the objective function value $f^{\beta_{obj}}_{i}$. Now Repeat these procedures over some number of iterations (Fig. 1).

Algorithm 1 Hyperbolic Sine Optimizer (HSO)

1. Begin.
2. Initialize the starting population using a uniform random distribution with a specified lower and upper bound on $m \times n$ dimensions: $[X]^{pop}_{m\times n} = random_{uniform}(lb, ub)$, here $m = search\ agents$ and $n = dimensions$.
3. Evaluate the target objective function value $[f]^{p_{obj}}_{m\times 1}$ concerning each search agent of the population $[X]^{pop}_{m\times n}$.
4. Evaluate the best-fit search agent $[x]^{p_{best}}_{1\times n}$ with respect to the objective function value $f_i^{p_{obj}}$. Where i represent each search agent.
5. $for\ k = (0\ to\ iter)\ do$

   Exploration Phase

   a. $for\ i = (0\ to\ m)\ do$
      i. $for\ j = (0\ to\ n)\ do$
         1. Initialize new population w.r.t these expressions $[X]^{\alpha}_{m\times n}$:
         $$x^{\alpha}_{i,j} = random_{uniform}(\min(x^{p}_{i,j}, r_1), \max(x^{p}_{i,j}, r_1)),$$
         $$where\ r_1 = x^{best}_j - (x^{best}_j * \sinh(r_2 * {}^{\pi}/_{2})),$$
         $$r2 = random\ number\ between\ lb\ and\ ub$$
   b. Evaluate the target objective function value $[f]^{\alpha_{obj}}_{m\times 1}$ concerning each search agent of the population $[X]^{\alpha}_{m\times n}$.
   c. Evaluate the best-fit search agent $[x]^{\alpha_{best}}_{1\times n}$ with respect to the objective function value $f_i^{\alpha_{obj}}$
   d. $if\ f_i^{\alpha_{obj}} < f_i^{p_{obj}}, then\ f_i^{p_{obj}} = f_i^{\alpha_{obj}}, [x]^{p_{best}}_{1\times n} = [x]^{\alpha_{best}}_{1\times n}$.

   Exploitation Phase

   e. $for\ i = (0\ to\ m)\ do$
      i. $for\ j = (0\ to\ n)\ do$
         1. $x^{\beta}_{i,j} = x^{\alpha}_{i,j} \times \mu$
         $$where\ \mu = (r_3 * x_j^{p_{best}})/(\rho * \pi)\ and\ \rho = 0.5$$
         $$r_3\ is\ a\ random\ number\ between\ 0\ and\ 1$$
   f. Evaluate the target objective function value $[f]^{\beta_{obj}}_{m\times 1}$ concerning each search agent of the population $[X]^{\beta}_{m\times n}$.
   g. Evaluate the best-fit search agent $[x]^{\beta_{best}}_{1\times n}$ with respect to the objective function value $f_i^{\beta_{obj}}$.
   h. $if\ f_i^{\beta_{obj}} < f_i^{p_{obj}}, then\ f_i^{p_{obj}} = f_i^{\beta_{obj}}, [x]^{p_{best}}_{1\times n} = [x]^{\beta_{best}}_{1\times n}$.
   i. $for\ i = (0\ to\ m)\ do$
      i. $for\ j = (0\ to\ n)\ do$
         1. $if\ r_4 \le 0.5\ do$
            a. $x^{\gamma}_{i,j} = x^{\beta}_{i,j}/(1 + \sinh(\rho/x^{\beta^2}_{i,j}))$
            $where\ \rho = 0.5$
         2. $else\ do$
            a. $x^{\gamma}_{i,j} = random_{uniform}(lb, ub)$
            $where\ r_4\ is\ a\ random\ number\ between\ 0\ and\ 1$
   j. Evaluate the target objective function value $[f]^{\gamma_{obj}}_{m\times 1}$ concerning each search agent of the population $[X]^{\gamma}_{m\times n}$.
   k. $if\ f_i^{\gamma_{obj}} < f_i^{p_{obj}}, then\ f_i^{p_{obj}} = f_i^{\gamma_{obj}}, [x]^{p_{best}}_{1\times n} = [x]^{\gamma_{best}}_{1\times n}$.
   l. $[X]^{p}_{m\times n} = [X]^{\gamma}_{m\times n}$
   m. End.

Table 2 Tuning hyper parameters of various comparable metaheuristics, including the proposed ones

S. No	Algorithms	Parameters
1	Genetic algorithms (GA)	$pc = 0.8, pm = 0.2$
2	Firefly algorithm (FA)	$\alpha = 0.2, \beta = 2, \gamma = 1, \delta = 0.05 \times (ub - lb), DampingRatio = 0.98$
3	BAT algorithm	$Loudness = 0.9, f_{min} = 0, f_{max} = 2, \alpha = 0.99, \gamma = 0.01$
4	TSA algorithm	$ST = 0.9$
5	Gravitational search algorithm (GSA)	$G_0 = 100, \alpha = 20, Rpower = 1$
6	Particle swarm optimization (PSO)	$w = 1, w_p = 0.99, c_1 = 1.5, c_2 = 2.0$
7	whale Optimization algorithm (WOA)	$b = 1, a_1 = 2 - \left(2 \times \frac{FEs}{MaxFEs}\right), a_2 = -1 - \left(\frac{FEs}{MaxFEs}\right)$
8	Grey wolf optimizer (GWO)	$\alpha = 2 - 2 \times \left(\frac{FEs}{MaxFEs}\right)$
9	Cuckoo search (CS)	$P_a = 0.25, \alpha = 1, \beta = 1.5, \sigma_v = 1$
10	Biogeography-based optimization (BBO)	$KeepRate = 0.2, \alpha = 0.9, Mutation = 0.1, \sigma = 0.02 \times (ub - lb)$
11	Sine cosine algorithm (SCA)	$a = 2$
12	Moth–flame optimization (MFO)	$r = -1 - \left(\frac{FEs}{MaxFEs}\right), t = r + (1 - r) \times rand, b = 1$
13	flower pollination algorithm (FPA)	$p = 0.8$
14	Arithmetic optimization algorithm (AOA)	$MOP_{max} = 1, MOP_{min} = 0.2, \alpha = 5, \mu = 0.499$
15	Hyperbolic sine optimizer (HOA)	$r_2, r_4 = randomnumberbetweenlbandub, \quad r_3 = randomnumberbetween0and1, \rho = 0.5$

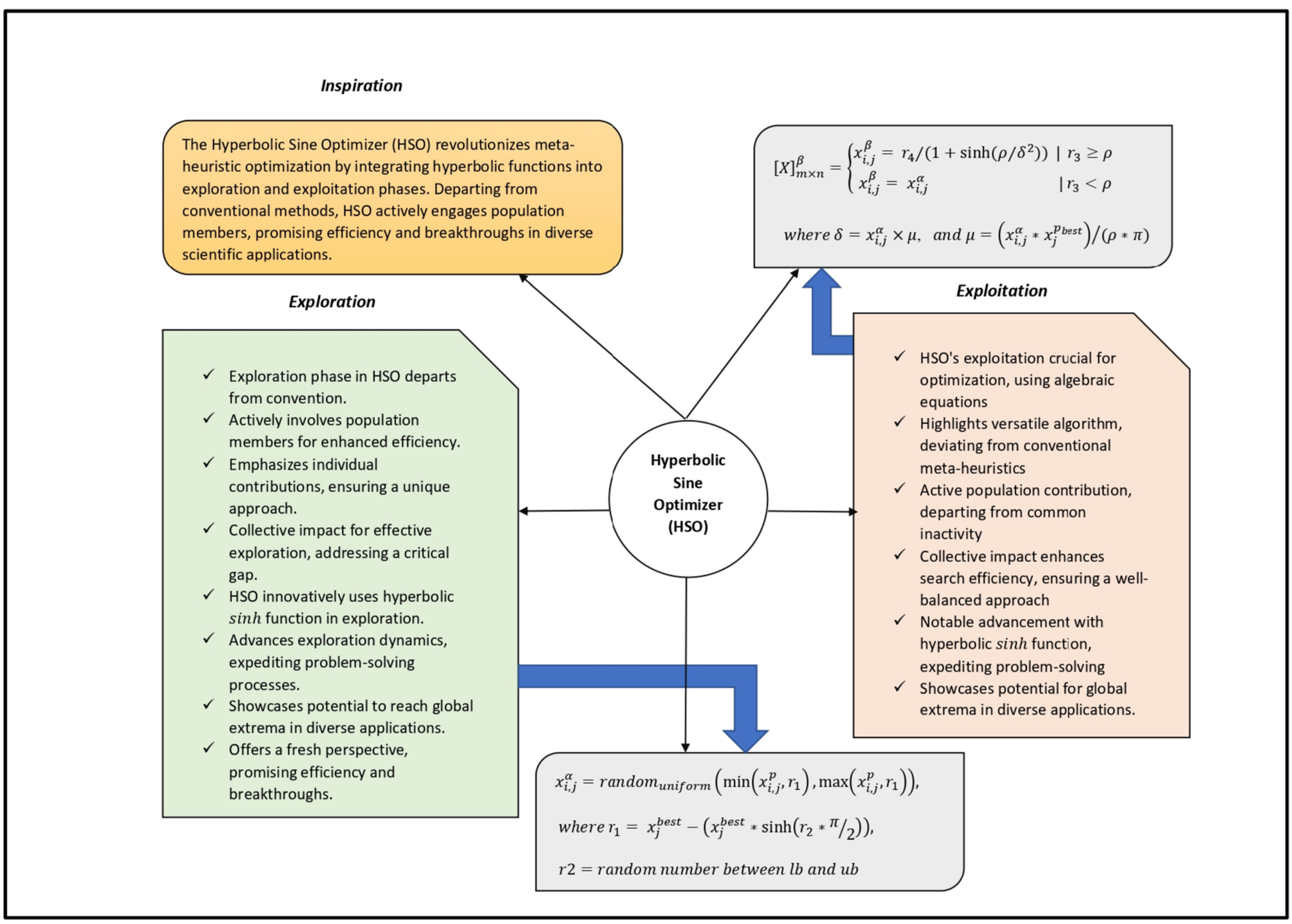


Fig. 1 Infographic of the proposed algorithm HS

3.3 Hypotheses about the HSO algorithm

For the following reasons, the HSO algorithm theoretically achieves the global optimum of optimization problems (Fig. 2):

- Effective search space exploration is guaranteed by the population dispersion behaviour of each generation around the population members.
- The population members' adaptively diverse boundaries guarantee efficient utilization of search space.
- Utilizing random adaptive variables and population update techniques greatly increases the likelihood of escaping the local optimum stagnation.
- A new exploration and exploitation phase based on hyperbolic functions gradually decreases the rates at which population members are modified over the course of iterations to assure convergence of the HSO algorithm.
- Each iteration's population diversity is increased by producing random population members within the search boundary for each search agent.
- Individual search agents are directed by the population to investigate more fruitful areas of the search space.
- Each iteration's best fitness value is noted and compared with the other previous values.
- HSO has a great ability to avoid local optima due to its population-based characteristics.
- There aren't many parameters to adjust when using the HSO algorithm.
- The HSO algorithm approaches the problem as a "black box," not using gradients.

4 Experimental results and discussions

The performance of the proposed HSO algorithm is assessed using 110 well-known standard benchmark functions, and the outcomes have been compared with those of other 15 metaheuristic algorithms, including GA [38], PSO [10], BBO [40], FPA [103], GWO [13], BAT [104], FA [105], CS [106], MFO [107], GSA [46], DE [39], TSA[108], SCA [52], WOA [14], AOA[68]. In addition to table 8 showcases the outcomes produced by the proposed algorithm (HSO) and contrasts them with the results derived from eight recently introduced algorithms. Following the experimental setup and comparing algorithms, details of the Benchmark problems are described first. The examinations for HSO's qualitative, quantitative, and scalability are then completed. Seven actual constrained and unconstrained optimization tasks are used to evaluate the performance of HSO

4.1 Benchmark problems and experimental setup

A set of 23 standards, well-known optimization benchmark functions, including unimodal, multimodal, and fixed-dimensional functions, as well as 42 additional functions based on various modalities, including many local minima, bowl-shaped, plate-shaped, valley-based, steep ridges and drops, and others, are used to evaluate the effectiveness of the proposed HSO. Next, all 15 benchmark suits from IEEE CEC-2015 and all 30 benchmark suits from IEEE CEC-2017 are applied. A summary of standard 23 and further 42 benchmarking problems are provided in Appendices A and B, respectively. The IEEE CEC-2015 and IEEE CEC-2020 test suite collections, respectively, are included in Appendices C and D. Each algorithm is subjected to 30 independent runs with 500 iterations per run and a population size of 30 in the experimental setup in order to make fair comparisons between the proposed algorithm HSO and all other fifteen algorithms. The best, worst, mean, and standard deviation values for each function are reported for each algorithm. Results are computed on an Intel(R)Core (TM) i5 7200U processor running at 2.5–2.71 GHz, with 8 GB of primary memory and 1 TB of secondary memory, respectively.

4.2 Exploitation efficiency of proposed HSO

The findings of the first experiment, which was carried out using 23 common benchmark functions suggested by Yao et al., are shown in Appendix Table 40. A set of seven unimodal and six multimodal problems are considered to have a maximum dimension of 30. The number of iterations (30×500) is 15,000, which is the maximum number of viable evaluations. Table 3 presents the results for a total of 13 functions, of which 6 are multimodal and 7 are unimodal. In addition, the fact that the unimodal functions have a single (global) optimum serves as evidence of the suggested algorithm's effectiveness in exploitation. Moreover, evidenced by the mean value in the results Table 3, the proposed HSO outperforms the other metaheuristics for unimodal functions F1, F2, F3, F4, and F7 in 30 dimensions. The proposed HSO successfully achieves the exact global optimum for the functions F1, F2, F3, and F4. Tables 4, 5, and 6 show the results for 100, 500, and 1000 dimensions, respectively. Even though the challenge was scaled up to 100 and 500 dimensions, significant progress was still apparent. It is obvious that the suggested HSO can locate the exact global optimum for each of the four functions we are considering (F1, F2, F3, and F4). The suggested algorithm's stability and predictability are further illustrated by the standard deviation. The Rosenbrock

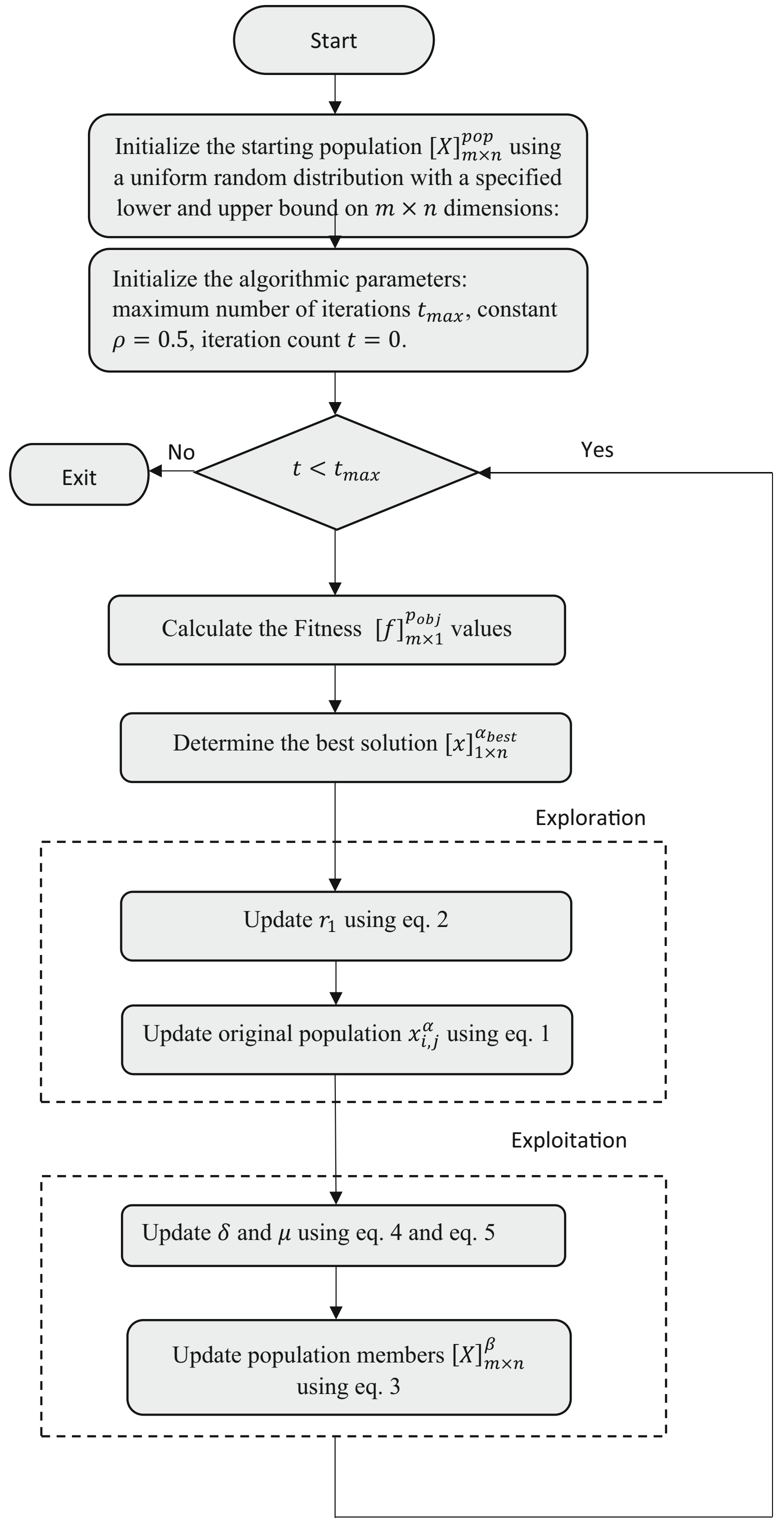


Fig. 2 Flow chart of the proposed algorithm HSO

function, a non-convex function used to evaluate optimization strategies, was created by Howard H. Rosenbrock in 1960. Other names include Banana Function or Rosenbrock's Valley. The global minimum is located in a long, thin, flat valley with a parabolic shape. It is simple to locate the valley. It is difficult to converge to the global optimum. F5 is the Rosenbrock function in AppendixTable 40. For F5, the suggested approach performed better than alternative metaheuristics in all dimensions (30, 100, 500, and

Table 3 Comparison results of HSO with other metaheuristics on 30 dimensions

F. No	HSO		GA		FA		BAT		DE		TSA	
	MEAN	STDEV	Mean	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	1.00E+03	4.62E+02	5.11E−05	2.11E−03	3.48E+04	5.31E+03	1.21 E−05	3.02 E−04	1.21 E−28	3.11 E−21
F2	0	0	2.28E+01	4.43E+00	3.19 E−02	2.62 E−01	1.62E+08	1.21E+09	4.61 E−03	1.05E−03	1.02E−13	1.42E−13
F3	0	0	1.42E+04	2.33E+03	1.44E+03	5.51E+02	1.11E+05	3.51E+04	2.72E+04	4.16E+03	1.41E−04	5.31E−04
F4	0	0	4.09E+01	4.00E−01	1.12E−01	3.42E−02	7.32E+01	1.74E+00	1.40E+00	1.25E+00	2.39E−01	1.92E−01
F5	27.03453	5.819422	7.90E+03	2.10E+03	6.78E+01	6.18E+01	1.18E+08	3.19E+07	1.50E+01	1.02E+02	6.20E+00	4.35E−01
F6	1.563701	1.479115	5.02E+02	1.69E+02	4.54E−03	1.52E−03	5.48E+04	4.51E+03	1.30E−04	3.17E−05	1.69E+00	6.19E−01
F7	0.000029	2.47E−05	8.10E−02	1.21E−01	5.51E−02	3.11E−02	3.35E+01	6.71E+00	3.21E−02	2.50E−03	8.00E−04	3.29E−03
F8	−12,569.5	0.018419	−1.26E+04	4.28E+00	−4.75E+03	1.15E+03	−2.23E+03	1.82E+02	−5.72E+03	2.72E+02	−4.37E+03	4.02E+02
F9	0	0	7.02E+00	3.57E+00	2.76E+01	2.00E−01	5.20E+01	2.31E+01	4.80E+01	1.61E+01	1.21E+02	2.15E+01
F10	4.44E−16	0	1.23E+01	1.21E+00	4.62E−02	1.22E−02	1.71E+01	2.19E−01	1.12E−02	2.19E−03	1.35E+00	1.38E+00
F11	0	0	1.02E+01	2.83E+00	4.01E−03	1.17E−03	5.11E+02	4.42E+01	2.53E−02	5.17E−02	3.09E−03	1.51E−02
F12	0.10493	0.194499	3.59E+00	2.28E+00	2.01E−04	6.70E−05	3.72E+07	3.10E+07	1.14E−05	1.42E−03	2.19E+00	2.23E+00
F13	0.731552	0.953763	1.41E+01	3.81E+00	1.09E−03	7.41E−04	6.21E+06	5.90E+07	7.01E−03	1.50E−03	2.78E+00	4.18E−01

F. No	HSO		GSA		PSO		WOA		GWO		CS	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	5.07E+02	4.54E+02	2.53E+02	2.03E+02	5.41E−78	1.37E−76	1.60E−29	2.10E−31	9.05E−08	3.45E−05
F2	0	0	1.39E+01	3.25E+00	3.27E+02	1.24E+02	9.62E−51	3.72E−52	7.51E−18	3.52E−17	1.38E−01	2.87E−04
F3	0	0	1.32E+05	3.75E+04	3.04E+04	7.19E+03	4.19E+04	1.36E+03	4.02E−06	1.02E−04	2.19E−01	4.79E−03
F4	0	0	7.67E+01	2.71E+00	4.28E+01	2.53E+00	4.98E+01	2.81E+01	2.10E−05	1.72E−05	8.64E−02	1.83E−02
F5	27.03453	5.819422	7.21E+02	7.61E+02	1.85E+07	5.14E+05	1.68E+01	5.79E−01	2.66E+01	6.62E−01	2.65E+01	3.42E−01
F6	1.563701	1.479115	2.07E+03	8.75E+02	1.69E+04	1.81E+03	3.39E−01	1.98E−01	7.21E−01	2.06E−01	3.12E−03	1.12E−04
F7	0.000029	2.47E−05	1.11E−01	2.59E−02	1.06E+01	2.04E+00	2.74E−03	3.21E−03	5.20E−04	2.00E−05	6.18E−02	2.19E−02
F8	−12,569.5	0.018419	−2.34E+03	3.71E+02	−3.75E+03	1.38E+02	−1.02E+04	1.71E+03	−7.98E+03	5.20E+02	−5.18E+01	6.40E+00
F9	0	0	3.12E+01	1.25E+01	2.76E+02	7.20E+00	1.36E+00	1.24E+01	1.07E+00	2.46E+00	1.42E+01	1.09E+00
F10	4.44E−16	0	3.63E+00	2.61E−01	1.69E+01	3.56E−01	3.71E−16	2.73E−15	1.00E−04	2.41E−12	2.18E−03	5.82E−03
F11	0	0	3.75E−01	3.97E−03	1.72E+02	2.07E+01	3.48E−02	2.41E−02	3.55E−03	6.19E−03	3.18E−05	1.09E−05
F12	0.10493	0.194499	4.62E−01	1.29E−04	1.42E+07	8.76E+06	2.19E−02	1.25E−03	2.61E−03	1.84E−02	5.46E−06	3.85E−05
F13	0.731552	0.953763	6.51E+00	1.13E+00	5.62E+06	2.57E+06	5.72E−01	2.77E−01	4.73E−01	1.07E−02	8.09E−03	5.53E−03

F.No	HSO		BBO		SCA		MFO		FPA		AOA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	6.48E+01	2.64E+01	1.34E+01	2.01E+01	2.00E+00	1.02E+02	6.06E+09	4.52E+02	1.61E−15	8.30E−15
F2	0	0	1.25E−04	7.32E−03	2.45E−02	3.42E−02	2.17E+00	2.04E+00	2.12E+01	4.58E+00	0.00E+00	0

Table 3 (continued)

F.No	HSO		BBO		SCA		MFO		FPA		AOA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F3	0	0	1.03E+04	2.58E+03	7.72E+03	5.19E+03	1.19E+03	2.60E+03	1.90E+01	4.36E+02	2.72E−03	0.010318
F4	0	0	3.01E+01	4.27E+00	2.75E+01	1.35E+01	5.03E+01	5.04E+00	1.18E+01	1.47E+00	2.76E−02	0.011242
F5	27.03453	5.819422	1.79E+03	9.39E+02	8.35E+04	1.78E+05	6.19E+03	1.48E+04	2.07E+05	1.52E+05	2.74E+01	0.261253
F6	1.563701	1.479115	6.61E+01	2.17E+01	2.01E+01	2.81E+01	1.41E+03	3.62E+02	1.66E+03	3.12E+02	3.12E+00	0.202616
F7	0.000029	2.47E−05	2.81E−03	1.72E−03	1.35E−01	1.86E−01	3.17E+00	7.01E+00	3.34E−01	1.10E−01	6.46E−05	5.72E−05
F8	−12,569.5	0.018419	−1.24E+04	2.52E+01	−3.69E+03	2.85E+02	−9.21E+03	4.72E+02	−6.44E+03	3.02E+02	−5.15E+03	394.6961
F9	0	0	0.00E+00	0.00E+00	4.52E+01	3.32E+01	3.70E+01	1.02E+01	1.71E+02	1.13E+01	0.00E+00	0
F10	4.44E−16	0	2.12E+00	2.42E−01	1.34E+01	8.01E+00	1.52E+01	3.74E+00	7.13E+00	1.07E+00	8.78E−16	3.83E−31
F11	0	0	1.35E+00	1.58E−01	1.13E+00	4.19E−01	3.01E+01	4.72E+01	1.62E+01	3.52E+00	2.23E−01	0.125162
F12	0.10493	0.194499	6.57E−01	1.51E−01	2.71E+04	1.07E+04	1.15E+01	1.90E+01	3.04E+02	1.03E+03	4.07E−01	0.048418
F13	0.731552	0.953763	1.71E+00	3.32E−01	1.84E+05	6.62E+04	3.51E+06	1.13E+07	9.48E+03	1.35E+05	2.68E+00	0.107213

1000). The proposed algorithm produces adequate results for functions F6 and F7.

4.3 Exploration efficiency of proposed HSO

Multimodal functions have more local optima than unimodal functions, which makes them harder to solve. In order to find a global optimum, it is, therefore, essential to avoid local optimal stagnation. Tables 3, 4, 5, and 6 present the results for the 30, 100, 500, and 1000 dimensions, respectively. When the proposed HSO is compared to various metaheuristics, it can be seen that it outperforms them significantly for functions F8, F9, F10, F11, F12, and F13. The results demonstrate that applied mechanisms for population distribution behaviour improve the HSO's exploration capability while exploitation mechanisms with hyperbolic function *sinh* strike a good balance between exploration and exploitation to improve convergence in the later stages of a generation.

4.4 Performance of proposed HSO over fixed dimension multimodal functions

These reference problems also exhibit multimodality in fixed dimensions. Each challenge is run through 500 iterations to arrive at the solution. Table 7 findings make it apparent that the suggested HSO can locate global optimum solutions for the specified functions F14, F15, F16, F17, F18, F21, F22, and F23. Additionally, the results for functions F19 and F20 are notably better than those of other metaheuristics. The proposed algorithm HSO's best, worst, mean, and standard deviation values are shown in Table 8 for 23 common benchmark functions with dimensions of 30, 100, 500, and 1000.

4.5 Performance of proposed HSO over some 42 extra functions with different modalities

To evaluate the effectiveness of the suggested HSO algorithm, 42 more supplementary functions (mentioned in Appendix Table 42) must be utilized, including various local minima, bowl- or plate-shaped features, valley-based features, steep ridges and drops, and others (Table 9). The 42 supplementary functions' best, worst, mean, and standard deviation results are shown in Table 10. The functions G1 through G15 can be found in numerous local minima categories. For the following functions: G3, G4, G6, G7, G11, G12, G13, and G15, HSO finds the exact global optimum. In addition to this, other operations would also result in positive effects. The bowl shape category takes into consideration seven functions that span G16–G22. In this class as well, HSO demonstrates its usefulness by generating exact global optimum levels for G16, G18, G19,

Table 4 Comparison results of HSO with other metaheuristics on 100 dimensions

F. No	HSO		GA		FA		BAT		‘DE		TSA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	4.32E+03	1.31E+03	2.04E−01	4.51E−02	2.61E+05	1.31E+02	7.15E+03	1.21E+02	2.73E−15	3.31E−14
F2	0	0	1.41E+02	2.29E+01	1.34E+01	5.62E+00	5.00E+43	1.17E+39	1.20E+02	2.29E+01	1.37E−06	2.16E−08
F3	0	0	1.42E+04	4.02E+04	4.52E+04	5.81E+02	1.32E+05	5.01E+05	4.02E+04	4.76E+04	1.18E+03	5.86E+03
F4	0	0	7.08E+01	2.04E+00	1.98E+01	2.71E+00	8.52E+01	1.38E+00	9.51E+01	1.01E+00	4.21E+01	1.42E+01
F5	96.1662	15.21704	1.29E+06	7.21E+06	7.35E+02	7.12E+02	1.19E+07	8.36E+06	1.81E+06	4.70E+04	7.62E+01	4.09E−01
F6	14.2404	8.266718	4.31E+03	1.08E+04	1.84E−01	4.23E−02	2.77E+05	1.24E+03	7.06E+02	1.52E+03	1.32E+01	1.22E+00
F7	3.51E−05	3.67E−05	1.69E+01	4.32E+01	4.52E−01	1.52E−01	2.02E+01	2.55E+01	1.96E+01	5.54E+00	4.82E−02	1.78E−02
F8	−41,898.2	0.177204	−4.10E+04	2.13E+02	−1.71E+04	6.12E+02	−4.06E+03	8.26E+02	−1.17E+03	5.79E+02	−1.21E+04	2.09E+03
F9	0	0	2.13E+02	4.16E+01	1.25E+02	1.52E+01	6.86E+02	5.21E+01	1.06E+02	3.02E+01	8.81E+02	8.16E+01
F10	4.44E−16	0	1.71E+01	3.24E−01	7.62E−01	2.42E−01	1.82E+01	5.42E−02	1.21E+01	7.29E−01	3.12E−05	2.49E−05
F11	0	0	4.23E+02	1.04E+02	1.05E−01	1.98E−02	1.36E+03	1.02E+02	6.32E+01	2.30E+01	4.54E−03	1.32E−02
F12	0.156306	0.214098	4.49E+06	7.12E+06	3.32E+00	2.10E−01	1.52E+09	1.58E+06	2.78E+07	1.76E+06	1.21E+01	5.27E+00
F13	5.580081	4.145827	4.15E+07	2.52E+07	3.42E+01	2.13E+01	4.02E+08	2.82E+07	7.18E+06	1.62E+06	2.23E+01	2.09E+00
F. No	HSO		GSA		PSO		WOA		GWO		CS	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	2.51E−01	3.14E−01	2.00E+03	6.28E+02	2.45E−73	1.21E−75	3.70E−18	2.42E−12	2.19E−01	4.19E−03
F2	0	0	3.26E−01	2.64E−01	4.01E+23	1.18E+21	3.12E−50	1.66E−61	2.18E−07	3.12E−08	3.04E+00	2.05E−04
F3	0	0	3.21E+04	7.20E+04	2.11E+05	5.06E+04	1.21E+06	2.05E+05	2.17E+02	1.58E+02	5.73E+00	1.03E+00
F4	0	0	1.25E−01	6.56E−01	4.15E+01	1.02E+00	6.82E+01	2.07E+01	6.62E−01	7.19E−01	1.47E−01	1.70E−02
F5	96.1662	15.21704	8.56E+02	6.62E+02	1.24E+08	3.15E+05	8.71E+01	2.32E−01	7.81E+01	4.52E−01	1.23E+02	6.23E+00
F6	14.2404	8.266718	1.19E−01	7.78E−02	1.06E+05	8.62E+02	3.02E+00	3.17E+00	1.02E+01	2.24E+00	1.54E+00	2.82E−02
F7	3.51E−05	3.67E−05	1.42E−01	4.28E−02	2.31E+02	7.63E++01	4.41E−03	5.46E−03	4.51E−03	1.51E−03	1.79E+00	1.42E−02
F8	−41,898.2	0.177204	−1.61E+04	2.32E+03	−6.42E+03	3.64E+02	−2.50E+04	4.29E+02	−1.46E+04	2.52E+03	−2.72E+18	5.82E+16
F9	0	0	1.01E+01	4.46E+01	1.15E+03	5.62E+01	2.68E−15	1.07E−17	1.02E+01	6.01E+00	1.75E+02	8.12E+00
F10	4.44E−16	0	1.86E−01	8.12E−01	1.82E+01	1.03E−01	3.86E−16	1.98E−16	1.31E−08	3.06E−07	2.07E−01	4.19E−06
F11	0	0	1.25E−01	1.53E−02	8.37E+02	5.00E+01	0.00E+00	0.00E+00	3.17E−04	5.00E−04	3.45E−03	4.62E−04
F12	0.156306	0.214098	3.02E+00	1.01E+00	2.42E+07	7.65E+07	3.57E−02	1.48E−02	1.62E−01	4.13E−02	1.36E−02	4.76E−03
F13	5.580081	4.145827	4.36E+01	7.23E+01	7.45E+06	1.41E+08	1.81E+00	7.18E−01	4.61E+00	1.07E−01	4.72E+00	1.19E+00
F.No	HSO		BBO		SCA		MFO		FPA		AOA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	1.72E+02	2.51E+01	1.15E+03	6.62E+03	4.13E+03	1.30E+03	1.28E+03	1.62E+03	2.41E−03	0.012355
F2	0	0	1.36E+01	2.18E+00	5.27E+00	3.41E+00	1.21E+02	2.29E+01	1.01E+02	8.25E+00	4.51E−58	2.39E−59

Table 4 (continued)

F.No	HSO		BBO		SCA		MFO		FPA		AOA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F3	0	0	1.52E+05	1.02E+03	2.51E+04	3.21E+04	3.02E+04	3.31E+03	1.78E+04	4.32E+03	5.14E−01	0.411305
F4	0	0	6.63E+01	2.19E+00	7.72E+01	2.19E+00	5.19E+01	1.42E+00	2.41E+01	3.26E+00	7.21E−02	0.012924
F5	96.1662	15.21704	3.36E+05	1.04E+05	1.21E+07	5.03E+07	2.12E+06	5.32E+06	3.52E+05	1.71E+06	7.78E+01	0.053464
F6	14.2404	8.266718	2.74E+03	3.06E+02	8.47E+02	6.02E+03	4.46E+03	1.30E+02	1.30E+02	1.21E+03	1.62E+01	5.27E−01
F7	3.51E−05	3.67E−05	1.14E+00	4.17E+00	1.31E+01	6.60E+01	1.23E+02	7.61E+01	3.61E+00	1.06E+00	7.29E−05	7.68E−05
F8	−41,898.2	0.177204	−3.75E+04	1.71E+02	−6.76E+02	4.71E+02	−1.32E+04	6.20E+01	−1.28E+04	2.42E+02	−9.64E+03	615.4816
F9	0	0	8.01E+00	1.52E+00	2.44E+02	3.01E+01	6.11E+02	5.02E+01	6.25E+02	2.19E+01	0.00E+00	0.00E+00
F10	4.44E−16	0	4.42E+30	3.61E−01	1.76E+01	4.07E+00	7.80E++00	6.11E−02	6.01E+00	1.10E−01	2.36E−04	0.000693
F11	0	0	1.13E+01	2.19E+00	1.62E+01	5.72E+01	4.11E+02	1.10E+01	1.07E+02	1.00E+01	5.02E+02	174.1632
F12	0.156306	0.214098	3.02E+02	2.76E+02	9.02E+05	1.44E+06	1.62E+06	1.20E+06	4.50E+03	1.64E+04	9.07E−01	0.021698
F13	5.580081	4.145827	5.71E+03	1.64E+03	4.13E+06	1.39E+06	5.42E+07	2.84E+06	1.56E+05	1.75E+05	9.961867	0.046137

G20, and G21. Other than these, HSO performs satisfactorily. For the platE−shaped categories (G23-G27), five functions are taken into account. For functions like G24 and G27, HSO generates the exact global optimum. Four functions (G28–G31) are considered in order to identify a valley-shaped function. For G28 and G29, HSO offers an exact global optimum. In addition to this, HSO performs well. Four functions (G32-G35) are taken into account while evaluating the shape of steep ridges and drops. For G33, HSO generates the exact global optimum. Except for them, HSO yields acceptable results. Seven further functions are considered to constitute other categories with diverse forms in the range G36-G42 aside from these. All functions in this range from (G36-G42) have exact global minima produced by HSO.

4.6 Performance of the proposed algorithm over IEEE CEC benchmark functions

4.6.1 Experimental set up for IEEE CEC benchmark problems

A set of 15 and 30 standard IEEE CEC-2015 and CEC-2017 benchmark functions, respectively, renowned for their optimization characteristics, is utilized to evaluate the effectiveness of the proposed HSO. These functions include variations with shifts, rotations, hybrids, and composites, and a brief summary is provided in Appendix Table 43. In the experimental setup, each algorithm undergoes 30 independent runs, involving 1000 iterations per run and a population size of 30. This thorough configuration ensures fair comparisons between the HSO algorithm and the remaining fifteen algorithms. Mean and standard deviation values for each algorithm are recorded for every function. The results are obtained from computations performed on an Intel(R) Core (TM) i5 7200U processor operating at speeds ranging from 2.5 to 2.71 GHz, equipped with 8 GB of primary memory and 1 TB of secondary memory, respectively.

4.6.2 Analysis of outcomes for IEEE CEC-2015 benchmark functions

Table 11. represents the performance evaluation of the Hyperbolic Sine Optimizer (HSO) algorithm on various CEC benchmark functions reveals noteworthy achievements, particularly when compared to competing algorithms such as MPA, TSA, WOA, GWO, TLBO, GSA, GA, and PSO. To assess the efficacy of HSO, the mean values obtained for each function were scrutinized, with an emphasis on achieving smaller mean values indicative of better performance. Among the benchmark functions, HSO

Table 5 Comparison results of HSO with other metaheuristics on 500 dimensions

F. No	HSO		GA		FA		BAT		‘DE		TSA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	5.04E+04	6.02E+04	5.21E+02	7.36E+03	1.41E+06	2.47E+03	6.21E+04	2.56E+03	2.28E−02	1.60E−02
F2	0	0	1.83E+03	5.02E+01	6.12E+02	2.65E+01	7.23E+09	1.66E+10	2.46E+06	1.62E+09	6.01E−03	2.89E−02
F3	0	0	4.68E+05	8.08E+04	1.18E+06	1.78E+05	2.19E+06	1.32E+06	2.28E+06	1.38E+05	1.35E+06	2.24E+03
F4	0	0	8.48E+01	1.21E+00	4.00E+01	1.72E+00	7.71E+01	3.19E−01	8.19E+01	2.32E−01	9.81E+01	2.07E−01
F5	498.8556	0.732828	1.67E+08	3.11E+07	2.45E+05	5.14E+05	5.53E+07	2.32E+07	3.06E+09	1.24E+06	1.35E+05	1.15E+05
F6	69.6401	44.40493	5.16E+04	6.21E+04	5.29E+03	7.71E+03	1.42E+05	3.36E+03	6.14E+04	3.27E+03	1.02E+02	1.78E+00
F7	4.13E−05	4.66E−05	8.10E+02	1.29E+03	2.62E+02	5.62E+01	2.12E+04	1.14E+02	2.28E+03	2.61E+02	2.31E+00	1.45E+00
F8	− 209,393	234.4447	− 1.31E+05	2.21E+00	− 7.26E+04	1.14E+04	− 9.03E+03	2.11E+02	− 2.67E+04	1.37E+02	− 3.05E+04	2.24E+02
F9	0	0	2.28E+03	1.85E+02	2.71E+03	1.31E+02	5.17E+03	2.21E+02	6.23E+03	1.98E+01	4.47E+03	5.21E+01
F10	4.44E−16	0	1.87E+01	1.03E−01	1.23E+01	3.45E−01	2.03E+01	2.24E−02	2.05E+01	2.44E−01	1.13E−02	5.24E−01
F11	0	0	4.31E+03	6.23E+02	4.72E+02	6.23E+01	1.29E+03	3.17E+02	5.64E+02	2.86E+01	1.05E−02	2.86E−02
F12	0.286441	0.364221	1.68E+08	1.13E+09	7.56E+05	6.14E+05	1.62E+09	5.18E+06	1.61E+01	2.23E+07	2.72E+06	2.31E+05
F13	20.19287	22.70836	7.63E+08	2.10E+08	1.28E+06	7.35E+04	3.16E+09	8.57E+07	2.31E+09	5.28E+07	8.07E+05	6.52E+03
F. No	HSO		GSA		PSO		WOA		GWO		CS	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	2.13E+05	1.84E+04	3.19E+04	1.77E+03	3.14E−79	1.47E−74	1.91E−05	2.72E−06	5.71E+00	3.82E−01
F2	0	0	2.29E+01	1.53E+01	3.07E+08	1.51E+10	7.68E−55	3.21E−50	1.78E−06	1.71E−02	3.46E+01	1.04E+00
F3	0	0	1.05E+04	3.71E+05	1.28E+05	1.21E+05	2.02E+06	1.17E+06	3.14E+05	6.79E+02	2.02E+03	1.61E+01
F4	0	0	3.01E+01	2.67E+00	7.15E+01	1.39E+00	7.47E+01	1.52E+01	4.13E+01	3.52E+00	3.05E−01	3.02E−06
F5	498.8556	0.732828	3.86E+04	3.06E+03	1.62E+08	1.01E+06	3.95E+02	3.78E−01	2.72E+02	3.11E−01	1.20E+03	6.03E+01
F6	69.6401	44.40493	6.71E+03	2.41E+03	5.31E+04	3.13E+03	3.32E+01	7.48E+00	8.52E+01	2.04E+00	7.16E+01	2.23E+00
F7	4.13E−05	4.66E−05	1.78E+01	4.70E+01	1.21E+04	1.23E+03	3.72E−03	4.82E−03	3.45E−02	1.02E−02	7.04E+01	1.36E+01
F8	− 209,393	234.4447	− 5.02E+05	1.00E+05	− 1.54E+04	7.71E+02	− 1.64E+05	3.21E+05	− 5.60E+04	2.12E+03	− 2.10E+14	1.12E+15
F9	0	0	3.47E+03	4.72E+03	4.82E+02	1.18E+01	0.00E+00	0.00E+00	7.62E+01	3.11E+01	2.42E+02	4.11E+01
F10	4.44E−16	0	6.51E+00	2.21E+00	2.75E+01	2.27E−01	5.26E−18	2.21E−18	2.61E−04	3.43E−06	1.06E+00	5.02E−03
F11	0	0	5.41E+02	5.25E+02	2.62E+03	2.67E+02	0.00E+00	0.00E+00	1.41E−02	3.52E−02	2.65E−02	2.21E−03
F12	0.286441	0.364221	3.52E+03	2.31E+01	3.41E+09	3.15E+08	8.03E−02	2.69E−02	5.32E−01	1.29E−02	3.68E−01	2.36E−02
F13	20.19287	22.70836	3.67E+03	8.86E+03	5.71E+08	7.19E+08	1.62E+01	4.36E+00	3.21E+01	1.90E−01	5.00E+01	1.12E+00
F. No	HSO		BBO		SCA		MFO		FPA		AOA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	4.20E+03	7.51E+02	2.13E+05	6.79E+04	1.40E+04	2.24E+03	7.15E+03	1.21E+02	5.32E−01	3.53E−02
F2	0	0	4.72E+02	2.91E+01	1.09E+02	4.18E+01	3.02E+08	1.48E+08	4.13E+02	4.72E+01	1.15E−03	0.001326
F3	0	0	1.65E+05	3.83E+04	5.57E+06	1.54E+05	4.82E+06	1.32E+05	4.29E+04	1.29E+04	2.17E+01	14.54408

Table 5 (continued)

F. No	HSO		BBO		SCA		MFO		FPA		AOA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F4	0	0	7.11E+01	8.13E−01	9.89E+01	2.21E−01	9.65E+01	3.02E−02	5.41E+01	3.17E+00	1.67E−01	0.013171
F5	498.8556	0.732828	1.05E+07	1.16E+06	1.82E+09	3.79E+08	5.11E+08	1.49E+08	2.23E+05	7.27E+05	4.89E+02	5.18E−02
F6	69.6401	44.40493	2.18E+04	6.18E+04	2.06E+04	4.18E+03	1.04E+06	3.36E+05	6.02E+05	7.12E+02	1.15E+02	1.03E+00
F7	4.13E−05	4.66E−05	1.42E+02	2.47E+01	1.42E+04	3.23E+03	2.62E+04	1.12E+02	1.31E+02	4.16E+01	8.06E−05	5.42E−06
F8	−209,393	234.4447	−1.62E+05	1.78E+04	−1.55E+04	1.16E+03	−6.17E+03	4.51E+03	−3.13E+03	1.10E+01	−2.33E+04	1.33E+02
F9	0	0	7.65E+02	2.12E+01	1.07E+03	4.42E+02	6.72E+03	1.36E+02	3.72E+03	5.42E+01	6.23E−06	4.85E−06
F10	4.44E−16	0	1.21E+01	2.18E−01	1.65E+01	2.81E+00	2.11E+01	2.80E−02	2.31E+00	6.14E−01	7.02E−03	0.00036
F11	0	0	1.25E+02	7.32E+01	3.02E+03	7.63E+02	1.20E+03	3.32E+03	5.78E+01	4.11E+01	1.01E+04	2456.695
F12	0.286441	0.364221	1.52E+07	3.12E+06	4.82E+08	1.12E+09	1.13E+09	5.16E+08	4.49E+05	2.15E+06	1.07457	0.010838
F13	20.19287	22.70836	5.13E+06	5.36E+06	8.66E+08	1.66E+08	1.19E+09	1.22E+08	3.84E+06	1.56E+06	50.18791	0.027988

demonstrated its superiority in terms of mean values in several instances, outperforming its competitors.

For CEC-3, CEC-6, CEC-7, CEC-9, CEC-12, and CEC-15, HSO secured the best mean values compared to MPA, TSA, WOA, GWO, TLBO, GSA, GA, and PSO. Notably, in CEC-3, HSO exhibited an average of 3.04E+02 with a standard deviation of 1.30E+00, showcasing its efficiency in minimizing the objective function. Similarly, for CEC-6, HSO obtained an average of 6.00E+02 with a minimal standard deviation of 7.40E−02, signifying its robustness in handling the complexities of this function. The consistent outperformance across various functions underscores the effectiveness of HSO in achieving superior mean values. It is worth highlighting that these results signify HSO's capability to converge towards optimal solutions, as reflected by the smaller mean values. The standard deviations, indicating the variability of results, were also competitive, emphasizing the stability of HSO across different functions. This comparative analysis establishes HSO as a promising optimization algorithm for addressing complex optimization problems, especially in scenarios where achieving lower objective function values is critical. The robust performance of HSO across these CEC benchmark functions positions it as a noteworthy candidate for diverse optimization applications.

4.6.3 Analysis of outcomes for IEEE CEC-2017 benchmark functions

In this section, we evaluate the effectiveness of the proposed method in optimizing the CEC 2017 test suite. This suite consists of thirty benchmark functions, categorized into three unimodal functions (C17-F1 to C17-F3), seven multimodal functions (C17-F4 to C17-F10), ten hybrid functions (C17-F11 to C17-F20), and ten composition functions (C17-F21 to C17-F30). The exclusion of C17-F2 from the simulation studies is due to its unstable behaviour, and a comprehensive description of the CEC 2017 test suite can be found in Appendix Table 43. To conduct a scalability analysis, we employed HSO and competitor algorithms to solve the test suite for problem dimensions (number of decision variables) set at 10, 30, 50, and 100. The results of implementing the proposed HSO approach and competitor algorithms on the CEC 2017 test suite are presented in Tables 12, 13, 14 and 15. The simulation outcomes indicate that, for dimensions equal to 10, HSO emerges as the most effective optimizer for handling C17-F1, C17-F3, C17-F5, C17-F6, C17-F8, C17-F9, C17-F10, C17-F19, C17-F20, C17-F21, C17-F22, C17-F24, C17-F27, and C17-F30. For dimensions equal to 30, HSO stands out as the best optimizer for solving C17-F1 to C17-F9, C17-F13 to C17-F15, C17-F18, C17-F23, C17-F25, and C17-F28. Likewise, for dimensions equal to 50, HSO

Table 6 Comparison results of HSO with other metaheuristics on 1000 dimensions

F.no	HSO		GA		FA		BAT		DE		TSA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	1.25E+06	1.68E+05	3.21E+05	2.01E+04	3.11E+06	3.52E+03	2.15E+05	3.28E+04	3.72E+00	3.81E+00
F2	0	0	3.18E+03	8.75E+01	1.68E+10	1.68E+07	1.68E+10	1.68E+09	1.68E+09	1.68E+09	1.69E−02	1.62E−02
F3	0	0	1.17E+07	2.82E+06	3.84E+06	7.18E+04	1.34E+07	3.66E+06	4.02E+06	4.23E+05	4.63E+05	8.71E+04
F4	0	0	8.65E+01	6.25E−01	6.05E+01	2.69E+06	8.78E+01	2.21E−01	7.84E+01	1.32E−01	9.85E+01	1.12E−01
F5	998.9967	0.012528	3.62E+08	9.52E+08	2.36E+07	2.13E+00	1.32E+10	2.29E+07	1.38E+09	3.05E+01	3.35E+06	2.23E+06
F6	124.0889	71.39418	1.41E+05	1.73E+05	2.17E+05	2.36E+04	3.01E+06	5.18E+03	2.03E+05	2.35E+04	2.24E+01	4.05E+00
F7	3.85E−05	0.000034	4.32E+03	7.32E+03	4.47E+03	4.01E+02	1.24E+04	3.82E+02	2.26E+06	3.41E+03	4.11E+01	2.09E+02
F8	−418,758	504.1048	− 1.94E+05	9.62E+03	− 1.08E+04	1.58E+04	− 1.37E+04	3.12E+02	− 3.72E+04	1.22E+02	− 4.49E+04	1.67E+02
F9	0	0	7.01E+02	2.02E+02	7.16E+03	1.87E+02	1.32E+04	1.74E+02	1.42E+04	1.68E+01	9.52E+02	2.12E+03
F10	4.44E−16	0	1.84E+01	2.45E−01	1.65E+01	2.32E−01	2.06E+01	2.13E−02	1.05E+01	1.06E−01	1.26E−01	1.41E−01
F11	0	0	1.25E+04	1.52E+02	2.76E+03	1.76E+01	2.72E+04	4.22E+02	1.74E+03	2.26E+02	3.71E−01	2.52E−01
F12	0.203335	0.271501	1.13E+01	1.26E+08	5.65E+06	1.89E+05	3.52E+09	1.21E+09	3.61E+09	7.56E+06	5.83E+08	2.85E+07
F13	52.4088	55.42051	1.82E+01	3.11E+08	3.31E+06	7.82E+06	6.58E+09	1.43E+08	6.55E+10	2.15E+08	3.47E+07	7.32E+07
F.no	HSO		GSA		PSO		WOA		GWO		CS	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	2.62E+01	6.56E+01	1.18E+05	5.12E+03	1.42E−74	7.62E−72	1.18E−01	2.71E−02	1.54E+01	1.26E+00
F2	0	0	1.68E+02	1.68E+02	1.37E+09	1.28E+09	5.57E−51	1.51E−54	5.01E−01	2.74E−01	1.01E+02	3.38E+00
F3	0	0	7.52E+03	1.21E+01	3.65E+06	1.06E+06	1.21E+06	3.51E+06	2.10E+05	1.89E+04	7.56E+02	1.10E+01
F4	0	0	1.97E+01	5.24E+01	7.63E+01	2.27E+00	7.72E+01	2.59E+01	4.79E+01	1.58E+00	4.32E−01	2.23E−03
F5	998.9967	0.012528	9.86E+02	2.02E+02	2.42E+08	2.14E+07	9.83E+02	9.18E−01	1.89E+02	1.15E+01	2.57E+03	1.26E+02
F6	124.0889	71.39418	2.82E+02	2.44E+01	1.58E+05	6.18E+03	6.56E+01	1.65E+01	1.39E+02	1.09E+00	1.87E+02	4.11E+00
F7	3.85E−05	0.000034	1.72E+03	5.68E+02	6.12E+03	3.22E+03	4.05E−03	3.84E−04	2.21E−01	1.18E−02	4.19E+02	7.21E+01
F8	−418,758	504.1048	−6.44E+04	1.81E+03	−2.12E+04	1.32E+02	−3.24E+04	5.31E+03	−8.54E+04	6.10E+03	−9.34E+04	1.11E+03
F9	0	0	3.21E+04	3.55E+02	1.21E+03	1.68E+01	0.00E+00	0.00E+00	1.19E+02	3.61E+01	6.04E+02	1.31E+01
F10	4.44E−16	0	4.08E+01	1.83E+01	1.51E+01	1.17E−01	4.45E−18	2.61E−17	1.22E−02	1.53E−02	1.17E+00	4.98E−01
F11	0	0	1.05E+02	1.02E+04	1.81E+03	4.11E+02	3.61E−20	1.02E−17	4.26E−02	6.12E−04	2.81E−04	2.47E−04
F12	0.203335	0.271501	4.72E+02	1.81E+03	5.44E+08	6.51E+06	1.00E−01	3.26E−02	1.03E+00	1.82E−02	6.42E−02	2.34E−03
F13	52.4088	55.42051	8.75E+03	3.21E+03	1.39E+09	1.82E+07	2.65E+01	1.48E+01	1.18E+02	1.01E+01	1.71E+02	1.37E+00
F.no	HSO		BBO		SCA		MFO		FPA		AOA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F1	0	0	5.42E+04	2.27E+03	4.32E+04	1.51E+04	1.62E+05	3.61E+03	1.69E+04	1.98E+03	1.6187183	0.082063
F2	0	0	1.85E+02	2.17E+01	1.79E+02	2.65E+01	2.58E+09	1.85E+09	8.23E+02	8.75E+01	0.013231	0.003553

Table 6 (continued)

F.no	HSO		BBO		SCA		MFO		FPA		AOA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F3	0	0	8.81E+05	1.37E+05	3.02E+06	7.41E+05	1.84E+07	3.58E+05	1.84E+05	3.21E+04	141.953	62.25748
F4	0	0	9.62E+01	7.51E−01	9.85E+01	1.07E−01	9.86E+01	1.38E−01	4.02E+01	4.29E+00	0.213082	0.018838
F5	998.9967	0.012528	1.28E+08	6.25E+06	4.23E+08	8.85E+08	1.24E+09	3.14E+07	6.16E+06	1.73E+06	989.3551	0.128164
F6	124.0889	71.39418	5.29E+04	1.71E+03	5.16E+04	1.46E+04	2.62E+05	4.45E+03	1.65E+04	1.82E+03	212.4288	1.180283
F7	3.85E−05	0.000034	3.72E+03	2.81E+02	6.41E+03	1.24E+03	1.85E+04	5.18E+02	1.07E+02	3.42E+01	0.000206	7.02E−05
F8	−418,758	504.1048	−2.29E+05	3.65E+02	−2.18E+04	1.72E+02	−9.00E+04	6.21E+02	−4.25E+04	1.77E+02	−32,434.2	1637.113
F9	0	0	2.75E+02	8.02E+01	1.82E+03	7.02E+01	1.54E+03	1.83E+01	1.19E+03	1.67E+01	6.24E−05	1.24E−05
F10	4.44E−16	0	1.56E+01	7.52E−04	1.97E+01	2.72E+00	2.03E+01	2.15E−01	2.51E+00	9.12E−01	0.009254	0.000137
F11	0	0	5.64E+02	1.67E+01	3.11E+02	1.47E+02	2.37E+03	3.42E+01	1.42E+02	2.56E+01	27,280.6	309.7502
F12	0.203335	0.271501	1.45E+08	1.25E+07	1.31E+09	2.48E+08	3.03E+09	8.61E+07	8.21E+05	3.36E+04	1.11659	0.003955
F13	52.4088	55.42051	3.26E+08	2.42E+07	2.39E+10	4.34E+09	5.52E+10	1.65E+08	8.85E+06	3.95E+06	100.4007	0.044823

proves to be the most efficient optimizer for handling C17-F1 to C17-F5, C17-F6, C17-F7, C17-F9, C17-F11, C17-F12, C17-F13, C17-F15, C17-F16, C17-F18, C17-F21, C17-F26, C17-F27, and C17-F30. Finally, for dimensions equal to 100, HSO excels as the optimal optimizer for solving C17-F1 to C17-F7 and C17-F9 to C17-F23, C17-F26, C17-F29, and C17-F30.

4.7 Analysis of exploration and exploitation using diversity graph

This section uses 23 standard benchmark functions with 30 dimensions to analyse the behaviour of exploration and exploitation in HSO. The aforementioned experiments examine the effectiveness of HSO using measures like standard deviation and mean value. The HSO algorithm can outperform its competitors on a variety of 23 benchmark functions, as explained in this section. Hussain et al. presented a method to evaluate the ability for exploitation and exploration in metaheuristic algorithms in [109]. This method expands on the mathematical model of dimension-wise variety introduced in [109]. The following formulas are presented as equivalents:

$$Divs_j = \frac{1}{N}\sum_{i=1}^{N} median(z^j) - z_i^j, Divs = \frac{1}{Dim}\sum_{j=1}^{Dim} Divs_j, \tag{6}$$

where the jth dimension's median is represented by the notation $median(z^j)$. Following that, the formulas used to determine the exploration percentage and exploitation % are outlined as follows:

$$Epl\% = \frac{Divs}{Divs_{max}} \times 100, Ept\% = \frac{|Divs - Divs_{max}|}{Divs_{max}} \times 100, \tag{7}$$

where $Divs_{max}$ stands for the highest level of diversity. The terms exploration percentage and exploitation percentage, respectively, are $Epl\%$ and $t\%$. The HSO algorithm can enable the preservation of the metaheuristics diversity and successfully prevent premature convergence when applied to handle difficult benchmarks like unimodal, multimodal and fixed dimensional functions because it can also achieve an appropriate level of exploration percentage. In conclusion, the HSO algorithm effectively recognizes a tradE−off between exploration and exploitation as illustrated in Table 24. Figure 3 depicted diversity graph of 23 standard benchmark functions.

4.8 Sensitivity analysis

The proposed HSO algorithm exhibits the capability to address optimization problems through a process centred

Table 7 Comparison results of HSO with other metaheuristics on fixed dimensions

F.no	HSO		GA		FA		BAT		DE		TSA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F14	0.998004	1.67E−16	9.98E−01	4.52E−16	3.51E+00	2.16E+00	1.19E+01	5.81E+00	1.23E+00	9.23E−01	8.13E+01	5.79E+01
F15	0.000552	0.000297	3.33E−02	2.70E−02	1.01E−03	4.01E−04	3.01E−02	3.21E−02	5.63E−04	2.81E−04	7.51E−02	1.73E−02
F16	−1.03163	2.67E−06	−3.78E−01	3.42E−01	−1.03E+00	6.78E−16	−6.57E−01	8.17E−01	−1.03E+00	6.78E−16	−1.03E+00	9.65E−03
F17	0.397888	3.68E−10	5.24E−01	6.06E−02	3.98E−01	1.69E−16	3.82E−01	1.47E−03	3.98E−01	1.69E−16	3.98E−01	6.75E−05
F18	3	1.11E−07	3.00E+00	0.00E+00	3.00E+00	0.00E+00	1.32E+01	1.01E+01	3.00E+00	0.00E+00	2.46E+01	3.43E+01
F19	−4.13306	0.050837	−3.42E+00	3.03E−01	−3.86E+00	3.16E−15	−3.73E+00	1.32E−01	−3.86E+00	3.16E−15	−3.86E+00	2.33E−03
F20	−2.80386	0.265719	−1.61E+00	4.60E−01	−3.28E+00	6.36E−02	−3.16E+00	5.71E−02	−3.27E+00	5.89E−02	−3.26E+00	7.22E−02
F21	−10.1531	5.96E−05	−6.66E+00	3.73E+00	−7.67E+00	3.51E+00	−4.19E+00	2.42E+00	−8.65E+00	1.52E+00	−6.31E+00	3.11E+00
F22	−10.4029	0.000106	−5.58E+00	2.61E+00	−9.64E+00	2.29E+00	−5.52E+00	3.17E+00	−9.75E+00	1.99E+00	−6.40E+00	3.66E+00
F23	−10.5363	0.000167	−4.70E+00	3.26E+00	−9.75E+00	2.35E+00	−3.82E+00	3.12E+00	−1.05E+01	8.88E−15	−7.70E+00	3.50E+00
F. No	HSO		GSA		PSO		WOA		GWO		CS	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F14	0.998004	1.67E−16	9.98E−01	4.52E−16	1.39E+00	4.60E−01	2.70E+00	3.29E+00	4.17E+00	3.61E+00	1.27E+01	1.81E−15
F15	0.000552	0.000297	1.03E−03	3.66E−03	1.61E−03	4.60E−04	7.59E−04	4.89E−04	6.24E−03	1.25E−02	3.13E−04	2.99E−05
F16	−1.03163	2.67E−06	−1.02E+00	6.78E−16	−1.03E+00	2.95E−03	−1.02E+00	1.87E−09	−1.02E+00	5.69E−16	−1.03E+00	6.78E−16
F17	0.397888	3.68E−10	3.71E−01	1.69E−16	4.00E−01	1.39E−03	3.98E−01	1.13E−05	3.87E−01	5.80E−17	3.98E−01	1.69E−16
F18	3	1.11E−07	3.02E+00	0.00E+00	3.10E+00	7.60E−02	3.00E+00	3.27E−04	3.01E+00	3.16E−05	3.00E+00	0.00E+00
F19	−4.13306	0.050837	−3.75E+00	3.16E−15	−3.86E+00	1.24E−03	−3.85E+00	2.19E−02	−3.75E+00	3.21E−03	−3.86E+00	3.16E−15
F20	−2.80386	0.265719	−3.12E+00	1.51E−01	3.11E+00	2.91E−02	−3.23E+00	1.39E−01	−3.15E+00	6.32E−02	−3.32E+00	1.78E−15
F21	−10.1531	5.96E−05	−8.51E+00	1.77E+00	−4.15E+00	9.20E−01	−7.85E+00	2.91E+00	−8.13E+00	2.43E+00	−5.06E+00	1.78E−15
F22	−10.4029	0.000106	−1.13E+01	7.27E−03	−6.01E+00	1.96E+00	−7.19E+00	3.14E+00	−1.13E+01	5.62E−04	−5.09E+00	8.88E−16
F23	−10.5363	0.000167	−1.24E+01	1.70E+00	−4.72E+00	1.74E+00	−7.70E+00	3.38E+00	−1.12E+01	1.61E+00	−5.13E+00	1.78E−15
F.No	HSO		BBO		SCA		MFO		FPA		AOA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F14	0.998004	1.67E−16	9.98E−01	4.52E−16	1.01E+00	8.72E−01	2.74E+00	1.32E+00	9.98E−01	2.00E−04	9.98E−01	5.54E−01
F15	0.000552	0.000297	1.66E−02	8.60E−03	1.12E−03	3.12E−04	2.35E−03	4.82E−03	6.88E−04	1.55E−04	3.12E−04	2.64E−04
F16	−1.03163	2.67E−06	−8.30E−01	3.16E−01	−1.13E+00	4.13E−05	−1.02E+00	6.18E−16	−1.03E+00	6.78E−16	−1.03E+00	5.48E−05
F17	0.397888	3.68E−10	5.49E−01	6.05E−02	3.12E−01	2.72E−03	2.89E−01	1.58E−16	3.98E−01	1.69E−16	3.88E−01	2.54E−06
F18	3	1.11E−07	3.00E+00	0.00E+00	3.12E+00	1.13E−04	3.00E+00	0.00E+00	3.00E+00	0.00E+00	3.12E+00	1.00E−02
F19	−4.13306	0.050837	−3.78E+00	1.26E−01	−3.72E+00	2.51E−03	−3.86E+00	1.32E−03	−3.86E+00	3.16E−15	−3.75E+00	4.29E−04
F20	−2.80386	0.265719	−2.71E+00	3.58E−01	−2.81E+00	3.62E−01	−3.24E+00	6.51E−02	−3.30E+00	1.95E−02	−3.12E+00	1.25E+01
F21	−10.1531	5.96E−05	−8.32E+00	2.88E+00	−2.63E+00	2.29E+00	−6.89E+00	3.29E+00	−5.22E+00	8.15E−03	−8.74E+00	1.25E+00

Table 7 (continued)

F.No	HSO		BBO		SCA		MFO		FPA		AOA	
	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV	MEAN	STDEV
F22	−10.4029	0.000106	−9.38E+00	2.60E+00	−3.61E+00	1.49E+00	−8.26E+00	3.17E+00	−5.34E+00	5.37E−02	−1.19E+01	2.21E+00
F23	−10.5363	0.000167	−6.24E+00	3.78E+00	−4.17E+00	1.67E+00	−7.66E+00	2.49E+00	−5.29E+00	3.56E−01	−1.05E+01	1.02E+00

on repetitions, utilizing examinations of the search space by members within the population. Consequently, the performance of HSO is subject to the influence of both the population size (N) and the number of iterations (T) performed by the algorithm.

Within this specific section, a sensitivity analysis of HSO with regard to parameters (N) and (T) is undertaken. To investigate the sensitivity to parameter (N), the proposed HSO is employed with various population sizes, including 20, 30, 50, and 100, to address problems F1 to F23. The simulation results of the sensitivity analysis regarding parameter (N) are presented in Table 16. The analysis of these simulation results indicates that an increase in the population enhances the search capabilities of HSO, resulting in a reduction of the values associated with the objective functions. For an examination of sensitivity to parameter (T), the proposed HSO algorithm is executed with diverse values of (T), specifically 100, 500, 1000, and 2000, applied to benchmark functions F1 to F23. The outcomes of the HSO sensitivity analysis under changes in parameter (T) are documented in Table 17. Clearly evident from the simulation results of the sensitivity analysis is that an escalation in the values of (T) prompts the algorithm to converge towards improved solutions, thereby decreasing the values associated with the objective functions.

5 Statistical measures

5.1 Student *t* tests

The student *t*-test [110] is a statistical method used to compare the means of two groups and determine if their differences are statistically significant. Named after William Sealy Gosset, who published under the pseudonym "Student," this parametric test is widely employed in research. It assumes that the data follow a normal distribution and calculates a t-statistic based on sample means, standard deviations, and sample sizes. A smaller *p*-value indicates a higher likelihood of rejecting the null hypothesis, suggesting a significant difference between the group means. The *t*-test is valuable in experimental design, allowing researchers to assess the significance of observed differences and make informed conclusions about the populations from which the samples are drawn. In this experimental configuration, the proposed algorithm HSO underwent a *t*-test in comparison to 15 competitor metaheuristics, including GA, FA, BAT, DE, TSA, GSA, PSO, WOA, GWO, CS, BBO, SCA, MFO, FPA, and AOA. Evaluations were conducted on 23 standard benchmark functions across 30 dimensions, with a focus on noting

Table 8 Comparison of the proposed algorithm (HSO) with newly published algorithms across 30 dimensions

Func	HSO		SCSO		GLA		SHO		SHIO	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
F1	0.00E+00	0.00E+00	2.42E − 97	7.22E − 97	2.4703E−323	0.00E+00	3.86E−141	8.58E−141	3.3347E − 77	5.7486E − 77
F2	0.00E+00	0.00E+00	1.16E − 52	2.55E − 52	1.58E−190	0.00E+00	6.6299E − 78	1.64E−77	1.6481E − 45	2.7742E − 45
F3	0.00E+00	0.00E+00	7.84E − 81	3.49E − 80	9.8813E−324	0.00E+00	1.05E−98	4.28E−98	3.7199E − 20	1.1707E − 19
F4	0.00E+00	0.00E+00	4.57E − 46	9.98E − 46	5.12E−190	0.00E+00	6.82E−57	1.09E−56	9.5353E − 05	9.2258E − 05
F5	2.70E+01	5.82E+00	2.80E+01	8.73E − 01	3.11E−01	1.14E−01	2.81E+01	5.64E−01	2.69E+01	1.27E+00
F6	1.56E+00	1.48E+00	2.15E+00	4.47E − 01	1.76E−07	8.04E−08	3.34E+00	5.00E−01	1.99E+00	4.05E−01
F7	2.90E−05	2.47E−05	1.51E − 04	1.33E − 04	8.25E−05	5.91E−05	1.12E−04	8.74E−05	1.66E−03	9.72E−04
F8	−1.26E+04	1.84E−02	− 1.01E+04	1.70E+03	−3.75E+03	1.26E+02	− 5.8448E+03	4.61E+02	− 5708.5464	6.92E+02
F9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.27E−14	4.72E−14	0.00E+00	0.00E+00	3.01E+01	9.95E+00
F10	4.44E−16	0.00E+00	8.77E − 16	0.00E+00	3.61E−15	3.69E−15	4.32E−15	6.49E−16	9.4147E − 15	2.9959E − 15
F11	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.07E−16	4.77E−16	0.00E+00	0.00E+00	4.86E−03	5.50E−03
F12	1.05E−01	1.94E−01	1.25E − 01	5.41E − 02	1.04E−07	8.37E−08	2.48E−01	1.10E−01	1.05E−01	7.63E−02
F13	7.32E−01	9.54E−01	1.99E+00	2.51E − 01	1.71E−05	1.39E−05	2.16E+00	2.42E−01	1.42E+00	2.37E−01
F14	9.98E−01	1.67E−16	1.73E+00	9.00E − 01	9.98E−01	3.02E−16	1.21E+00	1.34E−01	9.99E−01	1.93E−03
F15	5.52E−04	2.97E−04	4.82E − 04	3.22E − 04	3.08E−04	3.35E−08	2.53E−03	2.49E−03	4.69E−04	4.69E−04
F16	−1.03E+00	2.67E−06	− 1.0316	1.15E − 09	−1.03E+00	6.07E−14	−1.03E+00	2.19E − 12	− 1.0316	1.25E − 09
F17	3.98E−01	3.68E−10	3.98E−01	1.55E − 07	3.98E−01	5.86E−10	3.97E−01	5.29E − 14	3.98E−01	4.74E − 08
F18	3.00E+00	1.11E−07	3.00E+00	1.67E − 05	3.00E+00	3.78E−15	3.00E+00	1.32E − 11	3.00E+00	1.78E − 06
F19	−4.13E+00	5.08E−02	− 3.86	1.49E − 03	−3.86E+00	1.71E−14	−3.75E+00	2.51E − 08	− 3.8611	3.23E−03
F20	−2.80E+00	2.66E−01	− 3.25	9.95E − 02	− 3.3000E+00	5.3200E − 02	−3.39E+00	5.14E−02	− 3.2655	6.25E−02
F21	−1.02E+01	5.96E−05	− 7.1500E+00	4.11E+00	− 1.0013E+01	1.7800E − 11	−9.12E+00	2.19E+00	− 9.4793	1.78E+00
F22	−1.04E+01	1.06E−04	6.47E+00	3.68E+00	− 1.0400E+01	1.0400E − 11	−9.52E+00	−2.01E+00	− 10.4027	1.24E−04
F23	−1.05E+01	1.67E−04	− 1.0500E+01	1.5400E − 15	− 2.0900E+00	1.90E+00	−9.21E+00	−3.19E+00	− 9.9114	1.71E+00

Func	HSO		DBO		CDDO		PSA		CHOA	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
F1	0.00E+00	0.00E+00	2.02E − 104	1.11E − 103	1.3E − 57	3.6E − 57	3.507E − 81	1.568E − 80	1.09E − 09	2.17E − 09
F2	0.00E+00	0.00E+00	1.51E − 55	8.28E − 55	1.9E − 30	6.3E − 30	2.814E − 42	8.774E − 42	2.20E−03	4.66E−03
F3	0.00E+00	0.00E+00	1.37E − 80	7.48E − 79	2.8E − 39	1.4E − 38	5.964E − 81	2.536E − 80	4.17E − 05	6.63E − 05
F4	0.00E+00	0.00E+00	7.54E − 56	2.26E − 55	7.8E − 33	3.8E − 32	6.223E − 43	1.474E − 42	1.32E−03	1.31E−03
F5	2.70E+01	5.82E+00	2.57E+01	2.00E−01	3.30E+01	2.10E+01	2.507E − 02	4.152E − 02	3.05E+00	2.98E+00
F6	1.56E+00	1.48E+00	3.90E − 04	6.47E − 04	1.50E+00	1.30E+00	4.303E − 02	1.902E − 01	1.60E−03	9.04E−04
F7	2.90E−05	2.47E−05	1.40E−03	1.19E−03	1.2E − 03	1.1E − 03	6.207E − 05	6.558E − 05	4.74E−03	3.28E−03
F8	−1.26E+04	1.84E−02	−8.86E+03	8.70E+02	− 1.1E+04	1.80E+03	− 1.257E+04	2.810E − 02	− 4.3E+86	1.35E+87

Table 8 (continued)

Func	HSO		DBO		CDDO		PSA		CHOA	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
F9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.00E+01	2.10E+01	0.00E+00	0.00E+00	1.14E−03	3.40E−03
F10	4.44E−16	0.00E+00	8.88E − 16	0.00E+00	7.5E − 15	3.1E − 15	4.441E − 16	0.00E+00	3.86E−04	1.18E−03
F11	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.5E − 02	2.5E − 01	0.00E+00	0.00E+00	4.55E − 08	1.42E − 07
F12	1.05E−01	1.94E−01	5.53E − 07	3.02E − 06	3.8E − 02	3.8E − 02	1.433E − 05	2.472E − 05	2.17E−04	2.14E−04
F13	7.32E−01	9.54E−01	5.94E−01	5.35E−01	6.8E − 01	4.5E − 01	1.076E − 02	4.679E − 02	2.35E−04	1.85E−04
F14	9.98E−01	1.67E−16	1.06E+00	2.52E−01	1.00E+00	2.7E − 03	1.15E+00	4.857E − 01	4.14E+00	4.23E+00
F15	5.52E−04	2.97E−04	6.89E − 04	3.14E − 04	1.9E − 03	2.5E − 03	4.438E − 04	2.274E − 04	7.7400E − 04	4.5600E − 04
F16	− 1.03E+00	2.67E−06	− 1.03E+00	5.97E − 16	− 1.0E+00	4.7E − 03	− 1.032E+00	2.783E − 06	− 1.0300E+00	1.1800E − 09
F17	3.98E−01	3.68E−10	3.98E−01	0.00E+00	4.1E − 01	1.4E − 02	3.979E − 01	3.393E − 05	3.9800E − 01	1.9800E − 04
F18	3.00E+00	1.11E−07	3.00E+00	4.96E − 15	3.20E+00	3.2E − 01	3.00E+00	2.115E − 04	3.00E+00	1.5500E − 04
F19	− 4.13E+00	5.08E−02	− 3.86E+00	2.99E−03	− 3.7E+00	1.0E − 01	− 3.862E+00	1.400E − 03	− 3.8500E+00	4.4600E − 03
F20	− 2.80E+00	2.66E−01	− 3.23E+00	8.64E−02	− 3.3011E+00	1.3453E − 01	− 3.223E+00	8.357E − 02	− 3.2600E+00	9.9200E − 02
F21	− 1.02E+01	5.96E−05	− 8.00E+00	2.51E+00	− 7.5258E+00	2.73E+00	− 1.015E+01	6.068E − 04	− 8.0700E+00	2.75E+00
F22	− 1.04E+01	1.06E−04	− 8.51E+00	2.75E+00	1.03E+01	2.73E+00	− 1.040E+01	8.270E − 04	− 5.5600E − 01	2.7200E − 01
F23	− 1.05E+01	1.67E−04	− 9.01E+00	2.61E+00	− 7.6143E+00	2.76E+00	− 1.054E+01	2.631E − 03	− 9.4200E+00	2.42E+00

Table 9 HSO results on F1-F23 based on various parameters on 30, 100, 500 and 1000 dimensions

Functions	Best	Worst	Mean	STD	Best	Worst	Mean	STD
30 dimensions					100 dimensions			
F1	0	0	0	0	0	0	0	0
F2	0	0	0	0	0	0	0	0
F3	0	0	0	0	0	0	0	0
F4	0	0	0	0	0	0	0	0
F5	5.134839	29	27.03453	5.819422	14.22005	99	96.1662	15.21704
F6	0.17745	7.5	1.563701	1.479115	0.331293	25	14.2404	8.266718
F7	3.05E−07	0.000106	0.000029	2.47E−05	9.79E−07	0.000162	3.51E−05	3.67E−05
F8	− 12,569.5	− 12,569.4	− 12,569.5	0.018419	− 41,898.3	− 41,897.4	− 41,898.2	0.177204
F9	0	0	0	0	0	0	0	0
F10	4.44E−16	4.44E−16	4.44E−16	0	4.44E−16	4.44E−16	4.44E−16	0
F11	0	0	0	0	0	0	0	0
F12	0.00409	0.949056	0.10493	0.194499	0.008891	0.915722	0.156306	0.214098
F13	0.014344	5.22422	0.731552	0.953763	0.594422	14.65848	5.580081	4.145827
500 dimensions					1000 dimensions			
F1	0	0	0	0	0	0	0	0
F2	0	0	0	0	0	0	0	0
F3	0	0	0	0	0	0	0	0
F4	0	0	0	0	0	0	0	0
F5	494.9106	499	498.8556	0.732828	998.9411	999	998.9967	0.012528
F6	1.164765	125	69.6401	44.40493	23.52256	250	124.0889	71.39418
F7	3.10E−06	0.000221	4.13E−05	4.66E−05	8.08E−07	0.000119	3.85E−05	0.000034
F8	− 209,491	− 208,653	− 209,393	234.4447	− 418,983	− 416,469	− 418,758	504.1048
F9	0	0	0	0	0	0	0	0
F10	4.44E−16	4.44E−16	4.44E−16	0	4.44E−16	4.44E−16	4.44E−16	0
F11	0	0	0	0	0	0	0	0
F12	0.004268	1.20755	0.286441	0.364221	0.001604	1.128611	0.203335	0.271501
F13	0.87594	85.50367	20.19287	22.70836	1.486109	170.9073	52.4088	55.42051

p-values. Here, *p*-values less than 0.05 indicate more statistically significant results.

5.1.1 Discussion of the results of the proposed algorithm (HSO) using *t*-tests across 30 dimensions

The provided Table 18 contains *p*-values resulting from Student *t*-tests, which were conducted to compare the performance of different metaheuristics across a range of functions (F1 to F23). The table is organized with each row corresponding to a specific function, and each column representing a distinct metaheuristic such as GA, FA, BAT, DE, TSA, GSA, PSO, WOA, GWO, CS, BBO, SCA, MFO, FPA, and AOA. The reported p-values are in scientific notation, and the symbols (+), (−), and (=) accompanying them signify whether the p-value is less than 0.05 (indicating statistical significance), greater than 0.05 (indicating lack of statistical significance), or equal to 0.05, respectively. To interpret the results, p-values in parentheses approaching zero (e.g., 3.49E−15) signify highly statistically significant outcomes. The presence of a symbol (+) next to a p-value indicates that the corresponding metaheuristic demonstrates statistically significant performance compared to others on the specific function. Conversely, a symbol (−) suggests that the metaheuristic in question does not exhibit statistically significant performance on that particular function. A symbol (=) indicates that the result is not statistically significant, with the *p*-value being greater than or equal to 0.05. For instance, in function F1, all metaheuristics show *p*-values less than 0.05, signifying that they perform significantly well on F1. In contrast, in function F2, all metaheuristics, except GWO, have *p*-values less than 0.05. Furthermore, in function F5, GA, FA, BAT, DE, TSA, GSA, PSO, WOA, and CS demonstrate *p*-values less than 0.05, suggesting statistically significant performance on F5. It is crucial to

Table 10 HSO results on Forty-two extra functions based on various modalities

Function	Best	Worst	Mean	STD
G1	4.44E−16	4.44E−16	4.44E−16	0
G2	0.000627	0.056959	0.023104	0.013613
G3	− 2.06261	− 2.06261	− 2.06261	5.55E−11
G4	− 1	− 1	− 1	0
G5	− 956.908	− 891.822	− 934.605	8.837029
G6	0	0	0	0
G7	− 19.2086	− 19.2086	− 19.2086	2.44E−07
G8	− 4.15581	− 2.32752	− 4.05973	0.327953
G9	0.312289	2.019355	1.100165	0.345556
G10	1.84E−11	1.09E−08	2.17E−09	2.93E−09
G11	0	0	0	0
G12	0	0	0	0
G13	0.292579	0.292579	0.292579	2.68E−17
G14	0.000411	0.066145	0.013335	0.018785
G15	− 186.731	− 186.697	− 186.729	0.006262
G16	0	0	0	0
G17	8.43E−05	7.386125	1.345673	2.435742
G18	0	0	0	0
G19	0	0	0	0
G20	0	0	0	0
G21	0	0	0	0
G22	− 2293.14	− 1347.1	− 1705.82	226.355
G23	3.89E−13	2.13E−08	2.28E−09	4.74E−09
G24	0	0	0	0
G25	− 1.91322	− 1.91302	− 1.9132	3.87E−05
G26	0	0	0	0
G27	0	0	0	0
G28	− 1.03163	− 1.03151	− 1.03161	2.35E−05
G29	0.666667	0.999955	0.774681	0.153383
G30	28.72501	28.98971	28.87728	0.103542
G31	0.998004	0.998004	0.998004	1.62E−16
G32	− 0.99994	− 0.99955	− 0.99984	8.77E−05
G33	− 8.85621	− 6.80064	− 7.856	0.534894
G34	4.13E−09	2.42E−05	2.74E−06	4.93E−06
G35	0.397888	0.397889	0.397888	2.10E−07
G36	0.001852	0.635223	0.199904	0.185339
G37	3	3	3	2.37E−08
G38	− 3.86063	− 2.80014	− 3.66863	0.236079
G39	− 3.11638	− 1.32868	− 2.75146	0.358908
G40	− 3.21673	− 2.59341	− 2.10723	0.031023
G41	− 10.5364	− 10.5353	− 10.5363	0.000222
G42	0	0	0	0

note that a lower p-value generally indicates stronger evidence against the null hypothesis, emphasizing the statistical significance of the observed differences. Additionally, considering the practical significance and the context of the study is essential when interpreting these results.

5.2 Friedman ranking tests

The Friedman test [111] is a non-parametric statistical test used to determine if there are significant differences among the means of three or more related groups. Named after economist Milton Friedman, this test is applicable when the data are measured on an ordinal scale and the assumptions for parametric tests cannot be met. The procedure involves ranking the data within each group, calculating average ranks, and then computing a test statistic based on the differences between the observed and mean ranks. The null hypothesis assumes no significant differences among the groups. If the resulting test statistic is statistically significant, it indicates that there are variations among the groups. Post-hoc tests can be applied to pinpoint specific group differences. The Friedman test is particularly valuable for analyzing repeated measures data where normality assumptions may not hold, providing a robust method for assessing differences in central tendencies across multiple related groups. In this experimental configuration, the proposed algorithm HSO underwent Friedman ranking tests in comparison to 15 competitor metaheuristics, encompassing GA, FA, BAT, DE, TSA, GSA, PSO, WOA, GWO, CS, BBO, SCA, MFO, FPA, and AOA. Evaluations were carried out on 23 standard benchmark functions across varying dimensions, specifically 30, 100, 500, and 1000.

5.2.1 Discussion of the results of the proposed algorithm (HSO) using Friedman ranking tests across 30 dimension

Table 19 displays the results of Friedman ranking tests conducted on the proposed algorithm (HSO) in comparison to 15 competitor metaheuristics across 30 dimensions. The HSO ranking algorithm, assessed against fifteen optimization methods across benchmark functions (F1 to F13) on 30 dimensions, secured the top position with an average rank of 1. Outperforming algorithms like GA, FA, BAT, and more, HSO consistently demonstrated effectiveness in individual functions, asserting the top rank in F1, F2, F3, F4, F6, F7, F9, F10, F11, F12, and F13. With an average rank of 2.69, HSO showcased superiority, addressing a diverse range of functions and emerging as a promising choice for optimization tasks. In conclusion, HSO stands out as a robust and effective optimization method, consistently exceeding counterparts across various benchmark functions.

Table 11 Comparison table of HSO with other metaheuristics on IEEE CEC2015 benchmark test functions

Function	Parameters	HSO	MPA	TSA	WOA	GWO
CEC-1	Avg	4.39E+04	1.32E+06	3.47E+05	1.25E+05	5.05E+05
	STD	7.15E+05	2.19E+06	4.91E+05	2.67E+06	5.51E+05
CEC-2	Avg	7.46E+03	4.45E+06	8.52E+03	1.85E+04	1.22E+04
	STD	1.82E+03	6.52E+06	1.27E+04	1.51E+06	1.22E+04
CEC-3	Avg	3.04E+02	3.29E+02	3.12E+02	3.21E+02	3.13E+02
	STD	1.30E+00	6.69E−02	8.56E−02	1.02E−01	3.27E−02
CEC-4	Avg	7.59E+02	4.15E+02	4.07E+02	4.25E+02	4.20E+02
	STD	2.92E+02	1.22E+01	4.25E+00	1.27E+01	1.04E+01
CEC-5	Avg	5.00E+02	9.21E+02	6.74E+02	1.59E+03	1.27E+03
	STD	2.11E−01	1.85E+02	2.27E+02	3.61E+02	3.18E+02
CEC-6	Avg	6.00E+02	1.15E+04	1.76E+03	7.14E+03	3.62E+03
	STD	7.40E−02	2.62E+04	2.01E+03	4.82E+03	2.47E+03
CEC-7	Avg	7.00E+02	7.01E+02	7.01E+02	7.02E+02	7.04E+02
	STD	2.88E−01	7.67E−01	7.42E−01	1.19E+00	1.11E+00
CEC-8	Avg	8.02E+02	3.38E+03	3.49E+03	9.82E+03	2.36E+03
	STD	2.16E+00	2.19E+03	2.04E+03	9.51E+03	1.48E+03
CEC-9	Avg	9.03E+02	1.01E+03	1.00E+03	1.01E+03	1.07E+03
	STD	2.25E−01	1.32E−01	6.84E−02	2.31E−01	5.51E−02
CEC-10	Avg	3.55E+03	4.82E+03	3.15E+03	8.27E+03	2.31E+03
	STD	1.76E+03	3.29E+03	2.01E+03	1.22E+04	1.56E+03
CEC-11	Avg	1.11E+03	1.38E+03	1.24E+03	1.26E+03	1.26E+03
	STD	1.58E+01	5.27E+01	1.72E+02	9.65E+01	4.78E+01
CEC-12	Avg	1.24E+03	1.47E+03	1.32E+03	1.32E+03	1.31E+03
	STD	6.91E+01	7.25E−01	6.52E−01	1.41E+00	6.61E−01
CEC-13	Avg	1.58E+03	1.38E+03	1.32E+03	1.33E+03	1.39E+03
	STD	6.69E+00	2.12E−03	4.76E−03	1.82E+00	1.56E−03
CEC-14	Avg	2.14E+03	7.38E+03	7.12E+03	1.91E+03	7.12E+03
	STD	2.52E+02	2.62E+03	3.12E+03	1.93E−03	2.61E+03
CEC-15	Avg	1.90E+03	1.52E+03	1.55E+03	7.59E+03	1.52E+03
	STD	5.23E−01	5.37E+00	2.82E+07	1.22E+01	1.78E+02

Function	Parameters	HSO	TLBO	GSA	GA	PSO
CEC-1	Avg	4.39E+04	6.45E+06	1.42E+06	1.47E+06	3.12E+07
	STD	7.15E+05	3.27E+06	1.29E+06	1.32E+06	8.81E+06
CEC-2	Avg	7.46E+03	7.21E+08	6.61E+06	6.61E+06	3.47E+04
	STD	1.82E+03	2.45E+08	1.36E+08	1.36E+08	1.19E+04
CEC-3	Avg	3.04E+02	3.15E+02	3.18E+02	3.19E+02	3.17E+02
	STD	1.30E+00	8.21E−02	1.68E−02	1.27E−02	1.22E−02
CEC-4	Avg	7.59E+02	4.29E+02	4.43E+02	4.12E+02	4.28E+02
	STD	2.92E+02	7.38E+00	6.26E+01	6.16E+01	6.87E+00
CEC-5	Avg	5.00E+02	1.65E+03	9.72E+02	9.71E+02	1.64E+03
	STD	2.11E−01	2.42E+02	2.16E+02	2.16E+02	3.06E+02
CEC-6	Avg	6.00E+02	2.21E+04	4.14E+03	4.04E+03	2.82E+06
	STD	7.40E−02	2.54E+04	1.25E+04	1.75E+04	1.77E+06
CEC-7	Avg	7.00E+02	7.05E+02	7.01E+02	7.02E+02	7.05E+02
	STD	2.88E−01	9.87E−01	6.14E−01	6.04E−01	1.34E+00
CEC-8	Avg	8.02E+02	6.62E+03	1.36E+03	1.66E+03	5.06E+05
	STD	2.16E+00	3.62E+03	2.46E+03	2.46E+03	5.18E+05
CEC-9	Avg	9.03E+02	1.00E+03	1.01E+03	1.00E+03	1.01E+03

Table 11 (continued)

Function	Parameters	HSO	TLBO	GSA	GA	PSO
	STD	2.25E−01	1.37E+00	1.71E+01	1.58E+01	5.67E+00
CEC-10	Avg	3.55E+03	8.81E+03	1.23E+03	1.12E+03	2.31E+05
	STD	1.76E+03	7.62E+03	2.65E+04	2.65E+04	1.82E+05
CEC-11	Avg	1.11E+03	1.24E+03	1.24E+03	1.24E+03	1.31E+03
	STD	1.58E+01	1.71E+02	1.43E+01	1.54E+01	6.47E+01
CEC-12	Avg	1.24E+03	1.35E+03	1.31E+03	1.39E+03	1.23E+03
	STD	6.91E+01	1.58E+00	7.14E+00	5.15E+00	2.25E+00
CEC-13	Avg	1.58E+03	1.30E+03	1.33E+03	1.34E+03	1.38E+03
	STD	6.69E+00	3.15E−03	7.06E−03	7.06E−03	5.16E+01
CEC-14	Avg	2.14E+03	6.42E+03	6.11E+03	6.12E+03	9.23E+03
	STD	2.52E+02	1.59E+03	2.23E+03	2.28E+03	4.34E+02
CEC-15	Avg	1.90E+03	1.51E+03	1.56E+03	1.54E+03	1.52E+03
	STD	5.23E−01	4.01E+00	6.25E+01	6.25E+01	1.67E+01

5.2.2 Discussion of the results of the proposed algorithm (HSO) using Friedman ranking tests across 100 dimension

The proposed HSO ranking algorithm is evaluated across benchmark functions (F1 to F13) in 100 dimensions. Table 20 results show that HSO consistently outperforms, securing top ranks. In F1, it stands out, while the Genetic Algorithm (GA) lags at 14. In F2, HSO leads at 1, surpassing GA and Firefly Algorithm (FA). This trend persists, emphasizing HSO's superiority. GA averages 11th, while FA and BAT Algorithm rank 8 and 16. Overall, HSO maintains dominance with an average rank of 1, affirming its efficacy. In conclusion, HSO demonstrates promising results, positioning it as a robust choice for optimization tasks compared to other algorithms.

5.2.3 Discussion of the results of the proposed algorithm (HSO) using Friedman ranking tests across 500 dimension

The HSO algorithm excels across benchmark functions (F1 to F13) in 500 dimensions, outperforming fifteen well-known optimization algorithms, as represented in Table 21. In F1, HSO secures the top rank, surpassing GA, FA, BAT, and others. This trend persists across functions; for example, in F5, HSO ranks 4, showcasing its effectiveness. With an average rank of 1.69, HSO demonstrates superior overall performance. Conversely, GA averages the 11th rank, while FA and BAT average ranks of 8 and 15. Overall, HSO emerges as a robust and competitive optimization approach, consistently surpassing or matching other algorithms. The average ranks affirm HSO's efficacy, making it a promising choice for diverse optimization tasks.

5.2.4 Discussion of the results of the proposed algorithm (HSO) using Friedman ranking tests across 1000 dimension

The HSO algorithm excels when compared against fifteen different optimization algorithms across functions F1 to F13 in 1000 dimensions, as illustrated in Table 22. For instance, in F1, HSO claims the top position with a rank of 1, surpassing PSO (rank 2) and leaving GA and WOA behind with ranks 15 and 16. This trend repeats across functions, emphasizing HSO's consistent superiority. In F5, HSO secures a competitive rank of 5, showcasing its adaptability, even though PSO claims the top spot. Overall, HSO maintains robust performance, exemplified by its rank of 3 in F6, outshining GA and WOA. HSO's lead in F7 and F8, with ranks 1, emphasizes its adaptability and consistent excellence. Function F10 sees HSO with a rank of 2, highlighting its strong performance, while in F11, HSO secures the top position, showcasing versatility. Even in F13, HSO maintains its robustness, securing the third position. The average ranks affirm HSO's dominance with an average of 1.85, surpassing GA and FA with average ranks of 9.77 and 11.69. This concise analysis highlights HSO's exceptional optimization capabilities across a diverse set of functions, making it a promising choice for various optimization tasks.

5.2.5 Discussion of the results of the proposed algorithm (HSO) using Friedman ranking tests across fixed dimension

In the proposed algorithm, HSO is ranked against fifteen optimization algorithms across different benchmark functions (F1 to F13) on fixed dimensions. The performance is

Table 12 HSO results on the IEEE CEC-2017 benchmark suite. (dimensions = 10)

Functions	Parameters	HSO	AVOA	TLBO	TSA	WSO	MVO	RSA	WOA	GWO	MPA	GSA
C17-F1	Mean	1.29E+02	3.85E+03	1.47E+08	1.74E+09	5.63E+09	7.53E+03	1.02E+10	6.46E+06	8.83E+07	3.53E+07	7.47E+02
	STD	2.17E+00	6.00E+03	1.52E+08	1.66E+09	1.21E+09	3.21E+03	1.64E+09	1.75E+06	1.69E+08	6.77E+07	7.96E+02
C17-F3	Mean	3.00E+02	3.02E+02	7.27E+02	1.12E+04	8.54E+03	3.00E+02	9.65E+03	1.73E+03	3.07E+03	1.41E+03	1.03E+04
	STD	0.00E+00	2.39E+00	2.01E+02	5.35E+03	3.38E+03	5.34E−02	3.84E+03	1.39E+03	2.19E+03	8.76E+02	3.36E+03
C17-F4	Mean	4.29E+02	4.05E+02	4.09E+02	5.77E+02	9.34E+02	4.03E+02	1.35E+03	4.25E+02	4.12E+02	4.07E+02	4.05E+02
	STD	2.19E+00	2.71E+00	5.98E−01	1.14E+02	2.25E+02	1.87E+00	4.66E+02	3.53E+01	1.21E+01	4.80E+00	1.26E+00
C17-F5	Mean	5.09E+02	5.45E+02	5.34E+02	5.65E+02	5.65E+02	5.24E+02	5.74E+02	5.41E+02	5.13E+02	5.13E+02	5.54E+02
	STD	1.97E+00	2.08E+01	4.36E+00	2.59E+01	1.20E+01	1.27E+01	1.81E+01	2.75E+01	5.60E+00	5.59E+00	8.74E+00
C17-F6	Mean	6.00E+02	6.18E+02	6.07E+02	6.25E+02	6.33E+02	6.02E+02	6.41E+02	6.24E+02	6.01E+02	6.01E+02	6.17E+02
	STD	0.00E+00	1.88E+00	2.71E+00	1.21E+01	3.75E+00	1.91E+00	3.71E+00	1.75E+01	5.13E−01	8.89E−01	1.70E+01
C17-F7	Mean	7.89E+02	7.66E+02	7.53E+02	8.30E+02	8.05E+02	7.31E+02	8.06E+02	7.63E+02	7.26E+02	7.25E+02	7.17E+02
	STD	3.19E+00	2.51E+01	6.26E+00	3.91E+01	1.72E+01	1.53E+01	1.34E+01	2.17E+01	1.32E+01	4.02E+00	2.89E+00
C17-F8	Mean	8.09E+02	8.32E+02	8.38E+02	8.49E+02	8.49E+02	8.12E+02	8.55E+02	8.37E+02	8.16E+02	8.13E+02	8.20E+02
	STD	1.19E+00	1.24E+01	8.42E+00	1.75E+01	8.28E+00	4.18E+00	8.40E+00	1.42E+01	4.77E+00	3.04E+00	7.35E+00
C17-F9	Mean	9.00E+02	1.19E+03	9.12E+02	1.39E+03	1.43E+03	9.01E+02	1.48E+03	1.38E+03	9.12E+02	9.05E+02	9.00E+02
	STD	0.00E+00	3.62E+02	6.20E+00	2.40E+02	1.41E+02	1.70E+00	1.10E+02	2.71E+02	1.69E+01	6.47E+00	0.00E+00
C17-F10	Mean	1.12E+03	1.78E+03	2.18E+03	2.04E+03	2.31E+03	1.79E+03	2.59E+03	2.03E+03	1.73E+03	1.52E+03	2.29E+03
	STD	1.97E+00	4.78E+02	3.16E+02	3.05E+02	2.21E+02	4.38E+02	2.71E+02	5.82E+02	2.11E+02	1.03E+02	2.05E+02
C17-F11	Mean	2.29E+03	1.15E+03	1.15E+03	5.48E+03	3.87E+03	1.13E+03	4.00E+03	1.15E+03	1.16E+03	1.13E+03	1.14E+03
	STD	1.92E+02	4.08E+01	1.63E+01	1.12E+02	1.20E+03	2.37E+01	2.47E+03	3.04E+01	5.44E+01	2.35E+01	2.29E+01
C17-F12	Mean	2.29E+06	1.11E+06	5.09E+06	1.05E+06	3.56E+08	1.04E+06	7.11E+08	2.37E+06	1.43E+06	5.72E+05	1.03E+06
	STD	1.59E+06	8.41E+05	4.41E+06	3.81E+05	2.98E+08	1.63E+06	5.97E+08	1.90E+06	1.05E+06	4.19E+05	5.81E+05
C17-F13	Mean	4.49E+03	1.85E+04	1.68E+04	1.28E+04	1.73E+07	6.77E+03	3.46E+07	7.63E+03	1.04E+04	5.47E+03	1.01E+04
	STD	2.29E+02	1.63E+04	1.68E+03	5.96E+03	2.92E+07	6.24E+03	5.84E+07	5.93E+03	3.54E+03	1.53E+03	4.23E+03
C17-F14	Mean	1.47E+03	2.03E+03	1.59E+03	3.40E+03	4.02E+03	1.57E+03	5.38E+03	1.52E+03	2.35E+03	1.94E+03	5.60E+03
	STD	1.71E+00	5.94E+02	5.49E+01	2.39E+03	1.16E+03	3.09E+02	1.14E+03	4.32E+01	1.91E+03	7.56E+02	1.52E+03
C17-F15	Mean	1.69E+03	5.33E+03	1.71E+03	7.05E+03	1.04E+04	1.54E+03	1.40E+04	6.26E+03	5.85E+03	4.00E+03	2.41E+04
	STD	2.98E+01	5.40E+03	1.16E+02	4.82E+03	6.88E+03	1.34E+01	1.32E+04	5.47E+03	1.68E+03	7.60E+02	1.29E+04
C17-F16	Mean	1.97E+03	1.81E+03	1.68E+03	2.05E+03	2.02E+03	1.82E+03	2.02E+03	1.95E+03	1.73E+03	1.68E+03	2.08E+03
	STD	1.31E+01	1.31E+02	4.10E+01	1.84E+02	1.15E+02	7.03E+01	2.18E+02	1.64E+02	9.49E+01	3.45E+01	1.60E+02
C17-F17	Mean	1.79E+03	1.75E+03	1.76E+03	1.80E+03	1.82E+03	1.84E+03	1.82E+03	1.84E+03	1.77E+03	1.74E+03	1.85E+03
	STD	1.03E+01	3.23E+01	1.09E+01	1.23E+01	7.03E+00	8.94E+01	1.28E+01	5.52E+01	7.58E+01	2.87E+01	1.26E+02
C17-F18	Mean	3.59E+04	1.19E+04	2.97E+04	1.21E+04	2.87E+06	2.11E+04	5.73E+06	2.34E+04	2.00E+04	1.11E+04	9.76E+03
	STD	6.79E+03	5.28E+03	6.50E+03	4.02E+03	4.12E+06	1.29E+04	8.25E+06	1.59E+04	1.51E+04	6.15E+03	2.55E+03
C17-F19	Mean	2.19E+03	6.74E+03	4.71E+03	1.26E+05	3.99E+05	1.91E+03	7.08E+05	3.50E+04	5.41E+03	5.62E+03	4.07E+04
	STD	5.17E+01	5.89E+03	5.69E+03	1.56E+05	3.84E+05	7.70E+00	7.24E+05	2.52E+04	6.21E+03	3.96E+03	2.33E+04

Table 12 (continued)

Functions	Parameters	HSO	AVOA	TLBO	TSA	WSO	MVO	RSA	WOA	GWO	MPA	GSA
C17-F20	Mean	2.03E+03	2.17E+03	2.07E+03	2.21E+03	2.22E+03	2.14E+03	2.22E+03	2.21E+03	2.17E+03	2.09E+03	2.26E+03
	STD	5.01E+00	1.30E+02	9.84E+00	9.93E+01	5.74E+01	9.01E+01	6.14E+01	9.92E+01	5.68E+01	2.35E+01	8.47E+01
C17-F21	Mean	2.13E+03	2.21E+03	2.30E+03	2.33E+03	2.29E+03	2.25E+03	2.27E+03	2.31E+03	2.31E+03	2.26E+03	2.37E+03
	STD	7.89E+00	1.85E+01	7.06E+01	7.72E+01	3.75E+01	6.72E+01	3.28E+01	6.76E+01	4.13E+00	2.33E+00	1.59E+01
C17-F22	Mean	2.25E+03	2.31E+03	2.32E+03	2.72E+03	2.74E+03	2.29E+03	2.92E+03	2.32E+03	2.31E+03	2.31E+03	2.30E+03
	STD	2.94E+00	3.42E+00	9.03E+00	2.31E+02	1.34E+02	4.12E+01	1.67E+02	6.03E+00	1.07E+01	3.89E+00	9.74E−03
C17-F23	Mean	2.79E+03	2.64E+03	2.64E+03	2.72E+03	2.70E+03	2.62E+03	2.70E+03	2.65E+03	2.61E+03	2.61E+03	2.79E+03
	STD	5.81E+00	1.52E+01	9.82E+00	6.62E+01	3.39E+01	1.18E+01	3.57E+01	2.25E+01	7.17E+00	2.68E+00	1.05E+02
C17-F24	Mean	2.59E+03	2.77E+03	2.76E+03	2.67E+03	2.78E+03	2.68E+03	2.85E+03	2.76E+03	2.75E+03	2.63E+03	2.75E+03
	STD	9.29E+01	2.79E+01	1.07E+01	1.68E+02	7.18E+01	1.33E+02	4.48E+01	2.51E+01	1.92E+01	9.60E+00	1.86E+02
C17-F25	Mean	2.98E+03	2.91E+03	2.93E+03	3.14E+03	3.16E+03	2.92E+03	3.28E+03	2.91E+03	2.94E+03	2.92E+03	2.92E+03
	STD	3.81E+01	2.60E+01	2.20E+01	3.88E+02	1.52E+02	2.68E+01	6.58E+01	1.05E+02	1.28E+01	4.45E+00	2.49E+01
C17-F26	Mean	3.51E+03	2.98E+03	3.21E+03	3.63E+03	3.61E+03	2.90E+03	3.76E+03	3.19E+03	3.27E+03	3.01E+03	3.87E+03
	STD	3.39E+02	2.19E+02	4.93E+02	6.04E+02	3.11E+02	3.93E−02	3.13E+02	3.20E+02	4.74E+02	2.07E+02	7.84E+02
C17-F27	Mean	3.01E+03	3.12E+03	3.12E+03	3.18E+03	3.21E+03	3.09E+03	3.23E+03	3.20E+03	3.12E+03	3.10E+03	3.23E+03
	STD	8.23E+01	4.47E+01	4.11E+01	5.94E+01	5.70E+01	2.71E+00	1.44E+02	1.27E+01	4.44E+01	2.15E+01	1.65E+01
C17-F28	Mean	3.27E+03	3.24E+03	3.33E+03	3.59E+03	3.63E+03	3.24E+03	3.78E+03	3.29E+03	3.35E+03	3.22E+03	3.45E+03
	STD	1.92E+01	1.41E+02	9.24E+01	2.18E+02	4.21E+01	1.76E+02	7.21E+01	1.34E+02	1.11E+02	3.88E+01	1.61E+01
C17-F29	Mean	3.27E+03	3.29E+03	3.21E+03	3.24E+03	3.33E+03	3.20E+03	3.38E+03	3.35E+03	3.27E+03	3.20E+03	3.35E+03
	STD	4.89E+01	8.76E+01	3.59E+01	6.30E+01	2.02E+01	6.69E+01	7.84E+01	1.20E+02	9.90E+01	3.80E+01	2.12E+02
C17-F30	Mean	4.89E+04	2.96E+05	6.09E+04	6.17E+05	2.23E+06	3.04E+05	3.69E+06	9.97E+05	9.40E+05	4.17E+05	7.86E+05
	STD	2.98E+03	3.46E+05	3.87E+04	5.51E+05	9.00E+05	6.21E+05	2.28E+06	2.01E+06	6.78E+05	2.96E+05	1.81E+05

Table 13 HSO results on the IEEE CEC-2017 benchmark suite (Dimensions = 30)

Functions	Parameters	HSO	AVOA	TLBO	TSA	WSO	MVO	RSA	WOA	GWO	MPA	GSA
C17-F1	Mean	2.29E+03	3.11E+03	6.17E+09	1.79E+10	2.63E+10	5.38E+05	4.11E+10	1.70E+09	1.67E+09	2.68E+04	1.05E+07
	STD	1.97E+02	3.76E+03	2.41E+09	6.69E+09	5.20E+09	1.43E+05	6.97E+09	4.29E+08	2.45E+09	1.49E+04	1.92E+07
C17-F3	Mean	3.00E+02	4.47E+04	3.47E+04	4.72E+04	9.73E+04	1.78E+03	7.36E+04	2.32E+05	4.17E+04	1.10E+03	9.58E+04
	STD	0.00E+00	1.56E+04	3.94E+03	2.73E+03	9.65E+03	5.04E+02	1.21E+04	3.37E+04	4.52E+03	2.47E+02	1.13E+04
C17-F4	Mean	4.57E+02	5.15E+02	9.08E+02	4.55E+03	6.45E+03	4.97E+02	9.82E+03	8.57E+02	5.72E+02	4.93E+02	5.95E+02
	STD	1.19E+01	1.85E+01	2.96E+02	2.99E+03	2.30E+03	1.03E+01	3.36E+03	7.26E+01	4.16E+01	1.61E+01	2.08E+01
C17-F5	Mean	5.09E+02	7.26E+02	7.71E+02	7.95E+02	8.46E+02	6.20E+02	8.85E+02	8.24E+02	6.22E+02	5.83E+02	7.23E+02
	STD	1.67E+01	4.72E+01	2.58E+01	3.20E+01	1.88E+01	2.57E+01	3.14E+01	2.13E+01	3.73E+01	2.08E+01	2.22E+01
C17-F6	Mean	6.00E+02	6.47E+02	6.43E+02	6.77E+02	6.80E+02	6.24E+02	6.83E+02	6.76E+02	6.12E+02	6.03E+02	6.56E+02
	STD	0.00E+00	2.39E+00	8.94E+00	1.24E+01	1.18E+00	1.26E+01	6.01E+00	8.09E+00	6.39E+00	1.26E+00	8.31E−01
C17-F7	Mean	8.10E+02	1.15E+03	1.08E+03	1.22E+03	1.30E+03	8.54E+02	1.34E+03	1.31E+03	8.86E+02	8.47E+02	9.71E+02
	STD	2.59E+01	1.34E+02	9.38E+01	1.40E+02	3.98E+01	5.94E+01	1.79E+01	6.29E+01	5.27E+01	3.98E+01	5.63E+01
C17-F8	Mean	8.12E+02	9.50E+02	1.02E+03	1.06E+03	1.09E+03	8.94E+02	1.12E+03	1.03E+03	8.93E+02	8.91E+02	9.62E+02
	STD	3.91E+00	2.57E+01	2.48E+01	7.63E+01	1.78E+01	2.89E+01	2.64E+01	4.58E+01	7.20E+00	6.70E+00	2.46E+01
C17-F9	Mean	9.00E+02	4.82E+03	5.77E+03	1.15E+04	1.10E+04	5.45E+03	1.06E+04	1.10E+04	2.08E+03	1.08E+03	4.08E+03
	STD	0.00E+00	9.41E+02	2.24E+03	3.83E+03	1.40E+03	2.10E+03	1.93E+02	2.58E+03	7.00E+02	1.55E+02	6.55E+02
C17-F10	Mean	5.16E+03	5.46E+03	7.94E+03	6.57E+03	7.23E+03	4.66E+03	7.92E+03	6.51E+03	4.80E+03	4.00E+03	4.86E+03
	STD	3.79E+02	6.47E+02	2.70E+02	1.02E+03	4.34E+02	3.55E+02	6.80E+02	1.08E+03	3.98E+02	4.07E+02	3.40E+02
C17-F11	Mean	1.29E+03	1.26E+03	1.99E+03	5.14E+03	7.52E+03	1.31E+03	8.82E+03	7.83E+03	2.20E+03	1.17E+03	2.90E+03
	STD	4.13E+01	5.97E+01	5.48E+02	2.01E+03	1.16E+03	5.24E+01	1.37E+03	2.83E+03	1.56E+03	3.83E+01	6.82E+02
C17-F12	Mean	3.59E+04	2.09E+07	3.03E+08	5.09E+09	7.05E+09	1.13E+07	1.09E+10	2.48E+08	5.27E+07	2.17E+04	2.00E+08
	STD	6.69E+03	2.31E+07	1.64E+08	1.90E+09	1.45E+09	1.16E+07	2.08E+09	2.17E+08	5.00E+07	5.65E+03	3.18E+08
C17-F13	Mean	1.65E+03	1.50E+05	8.84E+07	1.47E+09	5.73E+09	9.12E+04	1.06E+10	9.07E+05	7.57E+05	1.89E+03	3.66E+04
	STD	8.91E+02	6.97E+04	3.25E+07	2.65E+09	2.36E+09	7.50E+04	3.70E+09	5.18E+05	1.17E+06	4.01E+02	1.24E+04
C17-F14	Mean	1.41E+03	2.72E+05	1.40E+05	1.18E+06	1.90E+06	2.04E+04	2.20E+06	2.23E+06	5.34E+05	1.44E+03	1.15E+06
	STD	5.12E+00	2.84E+05	4.26E+04	4.09E+05	6.28E+05	1.39E+04	1.14E+06	3.38E+06	6.13E+05	4.07E+00	5.05E+05
C17-F15	Mean	1.59E+03	3.75E+04	5.14E+06	1.44E+07	3.04E+08	4.28E+04	5.98E+08	5.05E+06	1.58E+07	1.62E+03	1.61E+04
	STD	1.91E+01	2.29E+04	3.79E+06	1.40E+07	3.98E+07	2.17E+04	7.69E+07	8.34E+06	3.15E+07	2.75E+01	4.73E+03
C17-F16	Mean	2.39E+03	3.00E+03	3.47E+03	3.27E+03	4.32E+03	2.59E+03	4.98E+03	4.25E+03	2.55E+03	2.03E+03	3.67E+03
	STD	1.39E+02	4.33E+02	2.07E+02	3.26E+02	3.05E+02	2.39E+02	8.65E+02	7.24E+02	1.52E+02	2.70E+02	1.73E+02
C17-F17	Mean	2.97E+03	2.48E+03	2.20E+03	3.28E+03	3.41E+03	2.08E+03	3.72E+03	2.85E+03	1.94E+03	1.87E+03	2.53E+03
	STD	7.81E+01	1.25E+02	2.43E+02	2.01E+03	6.26E+02	1.10E+02	5.22E+02	3.76E+02	1.45E+02	8.30E+01	1.34E+02
C17-F18	Mean	1.87E+03	2.65E+06	1.67E+06	3.64E+07	2.84E+07	6.41E+05	3.27E+07	5.91E+06	4.20E+05	1.90E+03	5.15E+05
	STD	2.91E+01	2.55E+06	6.61E+05	4.08E+07	2.26E+07	7.98E+05	2.48E+07	4.77E+06	5.12E+05	1.76E+01	3.59E+05
C17-F19	Mean	1.97E+03	6.78E+04	5.76E+06	2.95E+08	5.81E+08	9.40E+05	9.80E+08	1.43E+07	4.04E+06	1.92E+03	8.18E+04
	STD	8.45E+00	6.47E+04	2.78E+06	4.08E+08	1.76E+08	1.11E+06	3.75E+08	1.14E+07	6.56E+06	3.69E+00	2.98E+04

Table 13 (continued)

Functions	Parameters	HSO	AVOA	TLBO	TSA	WSO	MVO	RSA	WOA	GWO	MPA	GSA
C17-F20	Mean	3.48E+03	2.64E+03	2.80E+03	2.86E+03	2.91E+03	2.61E+03	2.96E+03	2.84E+03	2.38E+03	2.18E+03	3.02E+03
	STD	5.69E+01	1.89E+02	1.09E+02	1.35E+02	9.42E+01	3.06E+02	1.38E+02	1.82E+02	1.54E+02	9.33E+01	3.92E+02
C17-F21	Mean	2.98E+03	2.44E+03	2.50E+03	2.54E+03	2.63E+03	2.41E+03	2.68E+03	2.61E+03	2.39E+03	2.37E+03	2.57E+03
	STD	3.96E+01	1.76E+02	1.21E+01	1.75E+02	8.09E+01	3.01E+01	8.19E+01	7.75E+01	2.63E+01	1.24E+01	2.67E+01
C17-F22	Mean	3.14E+03	5.80E+03	5.72E+03	8.83E+03	8.04E+03	3.97E+03	7.79E+03	7.45E+03	2.71E+03	2.30E+03	6.36E+03
	STD	4.82E+00	2.54E+03	3.73E+03	1.76E+02	4.08E+02	2.12E+03	9.75E+02	8.26E+02	1.98E+02	1.35E+00	1.71E+03
C17-F23	Mean	2.47E+03	2.93E+03	2.91E+03	3.20E+03	3.20E+03	2.74E+03	3.26E+03	3.05E+03	2.75E+03	2.65E+03	3.78E+03
	STD	1.32E+00	1.36E+02	3.77E+01	1.54E+02	8.70E+01	3.47E+01	6.39E+01	1.36E+02	1.92E+01	1.29E+02	1.26E+02
C17-F24	Mean	2.91E+03	3.18E+03	3.05E+03	3.29E+03	3.32E+03	2.91E+03	3.42E+03	3.12E+03	2.92E+03	2.88E+03	3.37E+03
	STD	8.76E+00	1.43E+02	2.83E+01	8.30E+01	5.87E+01	3.60E+01	1.25E+02	4.76E+01	9.76E+00	1.17E+01	3.66E+01
C17-F25	Mean	2.81E+03	2.91E+03	3.08E+03	3.48E+03	3.96E+03	2.91E+03	4.59E+03	3.08E+03	2.99E+03	2.89E+03	3.00E+03
	STD	5.34E+00	2.88E+01	1.36E+02	4.16E+02	3.04E+02	4.74E+01	6.49E+02	2.89E+01	5.58E+01	6.46E+00	1.10E+01
C17-F26	Mean	3.97E+03	7.38E+03	5.96E+03	8.80E+03	9.25E+03	4.80E+03	9.85E+03	8.44E+03	4.58E+03	2.94E+03	7.53E+03
	STD	5.61E+00	9.92E+02	1.36E+03	5.11E+02	6.15E+02	5.02E+02	1.20E+03	7.20E+02	4.74E+02	1.69E+00	8.23E+02
C17-F27	Mean	4.91E+03	3.36E+03	3.32E+03	3.48E+03	3.62E+03	3.23E+03	3.77E+03	3.43E+03	3.25E+03	3.21E+03	5.00E+03
	STD	4.69E+01	9.45E+01	6.84E+01	1.89E+02	7.90E+01	2.10E+01	2.69E+02	1.40E+02	1.14E+01	1.74E+01	4.22E+02
C17-F28	Mean	2.97E+03	3.27E+03	3.68E+03	4.18E+03	4.81E+03	3.26E+03	5.73E+03	3.44E+03	3.61E+03	3.22E+03	3.53E+03
	STD	1.32E+01	2.85E+01	2.57E+02	5.78E+02	2.33E+02	3.15E+01	3.35E+02	5.60E+01	3.62E+02	2.32E+01	1.12E+02
C17-F29	Mean	4.38E+03	4.35E+03	4.53E+03	5.27E+03	5.43E+03	3.85E+03	5.66E+03	5.12E+03	3.80E+03	3.66E+03	5.09E+03
	STD	2.03E+02	2.78E+02	3.84E+02	7.41E+02	4.96E+02	1.16E+02	8.16E+02	2.10E+02	1.01E+02	1.43E+02	3.15E+02
C17-F30	Mean	5.89E+04	1.44E+06	3.81E+07	3.87E+07	1.44E+09	3.11E+06	2.84E+09	3.95E+07	6.42E+06	7.70E+03	2.28E+06
	STD	5.07E+03	9.26E+05	3.05E+07	3.81E+07	2.76E+08	1.89E+06	5.82E+08	2.51E+07	7.99E+06	1.98E+03	3.52E+05

Table 14 HSO results on the IEEE CEC-2017 benchmark suite (Dimensions = 50)

Functions	Parameters	HSO	AVOA	TLBO	TSA	WSO	MVO	RSA	WOA	GWO	MPA	GSA
C17-F1	Mean	4.97E+05	9.22E+06	2.07E+10	3.80E+10	5.97E+10	4.06E+06	9.35E+10	7.68E+09	9.33E+09	5.62E+06	1.71E+10
	STD	7.91E+04	1.13E+07	7.31E+09	2.66E+09	5.09E+09	9.61E+05	9.68E+09	3.58E+09	2.74E+09	6.30E+06	3.03E+09
C17-F3	Mean	3.00E+00	1.46E+05	9.80E+04	1.09E+05	1.58E+05	4.61E+04	1.57E+05	2.33E+05	1.30E+05	1.79E+04	1.77E+05
	STD	0.00E+00	3.22E+04	1.87E+04	1.03E+04	2.12E+04	9.43E+03	1.39E+04	9.22E+04	1.41E+04	2.76E+03	2.12E+04
C17-F4	Mean	4.96E+02	6.97E+02	2.79E+03	8.29E+03	1.47E+04	5.63E+02	2.37E+04	1.94E+03	1.43E+03	5.31E+02	3.06E+03
	STD	1.97E+01	2.19E+01	1.53E+03	1.87E+03	2.59E+03	5.83E+01	6.29E+03	5.33E+02	3.37E+02	4.44E+01	3.67E+02
C17-F5	Mean	6.72E+02	8.56E+02	9.97E+02	1.14E+03	1.10E+03	7.38E+02	1.13E+03	9.54E+02	7.24E+02	7.35E+02	8.05E+02
	STD	1.09E+01	3.36E+01	3.36E+01	1.35E+02	3.81E+01	9.04E+01	1.60E+01	3.17E+01	3.23E+01	6.61E+01	4.55E+01
C17-F6	Mean	6.00E+00	6.60E+02	6.64E+02	6.89E+02	6.94E+02	6.37E+02	6.96E+02	6.97E+02	6.23E+02	6.11E+02	6.57E+02
	STD	0.00E+00	5.12E+00	9.88E+00	1.78E+01	3.97E+00	1.76E+01	2.63E+00	6.37E+00	7.53E+00	2.99E+00	3.72E+00
C17-F7	Mean	9.23E+02	1.66E+03	1.47E+03	1.68E+03	1.79E+03	1.05E+03	1.89E+03	1.70E+03	1.06E+03	1.03E+03	1.41E+03
	STD	1.75E+01	6.32E+01	9.30E+01	1.52E+02	2.57E+01	3.13E+01	8.56E+01	7.55E+01	2.13E+01	5.50E+01	1.45E+02
C17-F8	Mean	1.26E+03	1.12E+03	1.32E+03	1.43E+03	1.42E+03	1.02E+03	1.44E+03	1.32E+03	1.03E+03	1.01E+03	1.14E+03
	STD	2.61E+01	5.75E+01	5.03E+01	1.09E+02	4.92E+01	5.28E+01	2.45E+01	1.09E+02	3.53E+01	3.51E+01	1.11E+01
C17-F9	Mean	9.00E+00	1.32E+04	2.39E+04	3.76E+04	3.59E+04	1.95E+04	3.61E+04	3.28E+04	6.83E+03	3.31E+03	1.06E+04
	STD	0.00E+00	6.96E+02	4.38E+03	3.42E+03	2.44E+03	7.86E+03	2.04E+03	4.05E+03	1.04E+03	1.23E+03	8.09E+02
C17-F10	Mean	7.89E+03	8.32E+03	1.40E+04	1.17E+04	1.30E+04	7.66E+03	1.42E+04	1.18E+04	8.66E+03	6.54E+03	8.59E+03
	STD	5.03E+02	4.91E+02	7.27E+02	9.48E+02	6.53E+02	1.12E+03	3.33E+02	1.14E+03	3.77E+03	7.98E+02	8.79E+02
C17-F11	Mean	1.19E+03	1.60E+03	5.10E+03	1.29E+04	1.53E+04	1.57E+03	2.09E+04	5.08E+03	6.11E+03	1.25E+03	1.41E+04
	STD	2.95E+01	1.37E+02	4.48E+02	2.06E+03	9.49E+02	1.44E+02	1.85E+03	9.41E+02	3.49E+03	3.83E+01	1.38E+03
C17-F12	Mean	2.79E+05	7.32E+07	5.04E+09	2.58E+10	4.35E+10	7.91E+07	7.10E+10	1.32E+09	9.56E+08	1.44E+07	2.17E+09
	STD	3.59E+03	4.79E+07	3.61E+09	1.46E+10	7.68E+09	3.81E+07	2.28E+10	3.53E+08	8.82E+08	7.66E+05	1.43E+09
C17-F13	Mean	1.53E+03	1.49E+05	5.84E+08	1.01E+10	2.45E+10	2.41E+05	4.30E+10	9.47E+07	3.57E+08	1.63E+04	1.85E+07
	STD	2.59E+01	1.37E+05	1.58E+08	4.75E+09	9.23E+09	1.04E+05	1.83E+10	1.75E+07	3.92E+08	5.57E+03	3.23E+07
C17-F14	Mean	4.78E+04	1.22E+06	8.65E+05	2.68E+06	2.59E+07	1.91E+05	4.83E+07	4.76E+06	1.15E+06	1.56E+03	1.51E+07
	STD	2.79E+02	1.25E+06	1.62E+05	1.60E+06	1.94E+07	1.31E+05	3.84E+07	6.78E+05	9.46E+05	1.88E+01	1.06E+07
C17-F15	Mean	1.97E+03	3.80E+04	7.05E+07	1.70E+09	2.60E+09	1.21E+05	4.18E+09	9.91E+06	5.94E+06	2.26E+03	1.97E+08
	STD	9.23E+01	2.34E+04	2.29E+07	1.58E+09	8.02E+08	6.32E+04	8.14E+08	8.41E+06	7.41E+06	1.61E+02	4.12E+08
C17-F16	Mean	2.43E+03	4.35E+03	4.54E+03	4.64E+03	6.28E+03	3.33E+03	7.58E+03	5.49E+03	3.32E+03	2.75E+03	3.95E+03
	STD	1.91E+02	3.83E+02	3.30E+02	4.24E+02	1.32E+03	2.17E+02	2.81E+03	7.97E+02	5.21E+02	2.18E+02	3.90E+02
C17-F17	Mean	3.76E+03	3.56E+03	4.15E+03	3.96E+03	7.62E+03	3.07E+03	1.10E+04	4.53E+03	2.97E+03	2.56E+03	3.82E+03
	STD	6.09E+01	5.05E+02	4.89E+02	5.89E+02	1.59E+03	4.70E+02	2.82E+03	3.52E+02	2.05E+02	5.63E+01	3.60E+02
C17-F18	Mean	3.09E+03	2.41E+06	8.20E+06	3.50E+07	7.57E+07	2.64E+06	1.12E+08	4.52E+07	5.72E+06	2.62E+04	8.41E+06
	STD	2.37E+02	2.26E+06	2.65E+06	4.85E+07	1.35E+07	1.33E+06	5.63E+07	3.74E+07	5.86E+06	1.69E+04	5.82E+06
C17-F19	Mean	2.39E+03	2.60E+05	5.07E+07	2.67E+09	2.72E+09	5.12E+06	3.84E+09	6.84E+06	1.16E+06	2.08E+03	4.52E+05
	STD	6.79E+01	2.10E+05	1.03E+07	3.80E+09	1.49E+09	1.09E+06	1.04E+09	7.06E+06	5.54E+05	4.55E+01	3.91E+05

Table 14 (continued)

Functions	Parameters	HSO	AVOA	TLBO	TSA	WSO	MVO	RSA	WOA	GWO	MPA	GSA
C17-F20	Mean	2.34E+03	3.27E+03	3.78E+03	3.43E+03	3.83E+03	3.28E+03	4.10E+03	3.75E+03	2.62E+03	2.66E+03	4.04E+03
	STD	7.59E+01	5.29E+02	1.40E+02	3.34E+02	2.55E+02	3.52E+02	2.13E+02	4.41E+02	2.33E+02	2.60E+02	2.55E+02
C17-F21	Mean	2.59E+03	2.76E+03	2.82E+03	2.96E+03	2.99E+03	2.57E+03	3.03E+03	2.95E+03	2.52E+03	2.45E+03	2.84E+03
	STD	3.15E+01	1.46E+02	3.51E+01	1.30E+02	3.88E+01	4.16E+01	9.96E+01	9.84E+01	4.53E+01	2.51E+01	5.45E+01
C17-F22	Mean	6.79E+03	1.12E+04	1.57E+04	1.38E+04	1.50E+04	9.01E+03	1.63E+04	1.37E+04	8.89E+03	5.36E+03	1.15E+04
	STD	3.59E+02	2.07E+03	6.06E+02	4.87E+02	2.17E+02	1.56E+03	2.49E+02	4.60E+02	7.69E+02	3.70E+03	2.93E+02
C17-F23	Mean	3.85E+03	3.30E+03	3.29E+03	3.75E+03	3.83E+03	2.99E+03	3.91E+03	3.76E+03	3.02E+03	2.89E+03	4.78E+03
	STD	2.97E+02	8.65E+01	7.21E+01	2.88E+02	8.51E+01	6.04E+01	4.13E+01	1.40E+02	1.06E+02	1.57E+01	1.66E+02
C17-F24	Mean	3.49E+03	3.52E+03	3.46E+03	4.02E+03	4.23E+03	3.14E+03	4.51E+03	3.84E+03	3.20E+03	3.07E+03	4.40E+03
	STD	4.09E+01	1.42E+02	7.03E+01	1.19E+02	4.24E+02	3.54E+01	8.88E+02	8.43E+01	1.06E+02	3.37E+01	4.55E+01
C17-F25	Mean	5.95E+03	3.18E+03	4.39E+03	6.04E+03	8.66E+03	3.06E+03	1.20E+04	4.17E+03	4.04E+03	3.07E+03	4.29E+03
	STD	3.98E+02	3.54E+01	5.99E+02	1.04E+03	1.21E+03	2.70E+01	1.96E+03	3.36E+02	2.30E+02	1.78E+01	5.22E+02
C17-F26	Mean	3.27E+03	1.11E+04	9.80E+03	1.27E+04	1.42E+04	5.82E+03	1.52E+04	1.39E+04	6.55E+03	3.32E+03	1.16E+04
	STD	1.97E+02	4.62E+02	7.23E+02	1.62E+03	2.18E+02	3.78E+02	7.52E+02	1.30E+03	4.38E+02	2.46E+02	3.46E+02
C17-F27	Mean	3.29E+03	3.86E+03	3.84E+03	4.73E+03	4.82E+03	3.36E+03	5.01E+03	4.47E+03	3.65E+03	3.39E+03	8.15E+03
	STD	4.63E+01	5.61E+01	1.77E+02	5.79E+02	2.59E+02	5.51E+01	3.39E+02	5.98E+02	5.63E+01	9.44E+01	3.28E+02
C17-F28	Mean	4.15E+03	3.60E+03	5.27E+03	7.30E+03	8.79E+03	3.29E+03	1.13E+04	4.84E+03	4.42E+03	3.35E+03	5.08E+03
	STD	3.71E+01	9.38E+01	5.29E+02	1.56E+03	1.60E+03	2.21E+01	2.50E+03	4.49E+02	3.14E+02	4.43E+01	9.05E+01
C17-F29	Mean	4.89E+03	5.53E+03	6.58E+03	6.95E+03	1.38E+04	4.86E+03	1.97E+04	9.12E+03	4.90E+03	4.11E+03	8.24E+03
	STD	2.69E+02	1.30E+02	9.90E+02	4.46E+02	4.93E+03	4.84E+02	1.01E+04	2.61E+03	2.55E+02	3.00E+02	1.98E+03
C17-F30	Mean	5.79E+05	2.19E+07	3.01E+08	1.66E+09	3.28E+09	7.05E+07	5.50E+09	1.59E+08	1.40E+08	1.66E+06	1.85E+08
	STD	4.69E+03	8.88E+06	7.80E+07	1.77E+09	9.10E+08	8.21E+06	2.46E+09	6.10E+07	7.67E+07	7.62E+05	4.59E+07

Table 15 HSO results on the IEEE CEC-2017 benchmark suite (Dimensions = 100)

Functions	Parameters	HSO	AVOA	TLBO	TSA	WSO	MVO	RSA	WOA	GWO	MPA	GSA
C17-F1	Mean	5.97E+05	3.82E+09	9.11E+10	1.26E+11	1.67E+11	6.58E+07	2.33E+11	6.27E+10	5.71E+10	5.19E+08	1.36E+11
	STD	2.59E+04	1.63E+09	6.87E+09	1.35E+10	3.71E+09	1.18E+07	2.92E+09	5.49E+09	7.81E+09	1.38E+08	9.44E+09
C17-F3	Mean	3.00E+00	3.26E+05	2.96E+05	3.63E+05	4.28E+05	4.66E+05	3.22E+05	7.88E+05	3.68E+05	1.58E+05	3.43E+05
	STD	0.00E+00	6.86E+03	1.58E+04	5.70E+04	2.93E+04	9.45E+04	9.22E+03	1.05E+05	3.85E+04	3.31E+04	2.62E+04
C17-F4	Mean	6.03E+02	1.55E+03	1.07E+04	1.60E+04	4.45E+04	7.60E+02	7.50E+04	1.09E+04	4.46E+03	1.02E+03	3.41E+04
	STD	4.03E+01	2.01E+02	7.13E+02	4.85E+03	3.66E+03	5.24E+01	7.03E+03	1.24E+03	1.64E+03	1.20E+02	6.02E+03
C17-F5	Mean	8.34E+02	1.29E+03	1.84E+03	2.10E+03	1.95E+03	1.21E+03	1.92E+03	1.80E+03	1.16E+03	1.20E+03	1.31E+03
	STD	6.54E+00	9.55E+00	2.48E+01	2.69E+01	1.43E+01	8.64E+01	3.80E+01	1.16E+02	4.93E+01	1.10E+02	3.65E+01
C17-F6	Mean	6.00E+02	6.60E+02	6.79E+02	7.08E+02	7.03E+02	6.72E+02	7.02E+02	7.01E+02	6.39E+02	6.36E+02	6.62E+02
	STD	0.00E+00	3.64E+00	7.16E+00	1.07E+01	2.51E+00	5.94E+00	3.68E+00	1.29E+01	5.21E+00	5.18E+00	3.19E+00
C17-F7	Mean	1.79E+02	3.01E+03	3.03E+03	3.36E+03	3.53E+03	1.97E+03	3.64E+03	3.50E+03	1.98E+03	1.81E+03	3.05E+03
	STD	3.81E+01	1.56E+02	1.16E+02	1.67E+02	8.29E+01	1.26E+02	7.71E+01	1.45E+02	1.47E+02	6.43E+01	1.62E+02
C17-F8	Mean	2.39E+03	1.71E+03	2.21E+03	2.35E+03	2.37E+03	1.43E+03	2.42E+03	2.27E+03	1.49E+03	1.41E+03	1.79E+03
	STD	2.61E+01	4.11E+01	5.39E+01	7.97E+01	5.07E+01	1.47E+02	1.86E+01	1.93E+02	1.18E+02	1.29E+02	1.01E+02
C17-F9	Mean	9.00E+00	2.56E+04	7.18E+04	1.16E+05	8.69E+04	5.71E+04	7.45E+04	7.41E+04	3.47E+04	2.16E+04	2.27E+04
	STD	0.00E+00	3.40E+03	2.31E+03	2.27E+04	1.07E+04	7.53E+03	2.14E+03	1.94E+04	1.37E+04	1.10E+03	1.27E+03
C17-F10	Mean	4.96E+03	1.58E+04	3.11E+04	2.89E+04	2.98E+04	1.68E+04	3.11E+04	2.79E+04	1.51E+04	1.38E+04	1.70E+04
	STD	1.97E+01	2.29E+03	1.02E+03	7.83E+02	2.66E+02	5.50E+02	6.69E+02	1.09E+03	8.13E+02	6.87E+02	1.38E+03
C17-F11	Mean	3.79E+03	6.28E+04	7.03E+04	6.40E+04	1.61E+05	4.52E+03	2.02E+05	2.04E+05	8.54E+04	4.72E+03	1.69E+05
	STD	5.12E+01	9.37E+03	1.45E+04	2.81E+04	2.92E+04	4.28E+02	6.54E+04	1.06E+05	1.18E+04	8.97E+02	2.53E+04
C17-F12	Mean	6.78E+06	6.45E+08	2.15E+10	5.57E+10	1.03E+11	3.26E+08	1.68E+11	1.29E+10	1.12E+10	2.55E+08	6.55E+10
	STD	4.03E+04	3.23E+08	6.33E+09	2.90E+10	2.18E+10	1.46E+08	3.47E+10	1.96E+09	2.62E+09	8.25E+07	9.16E+09
C17-F13	Mean	5.69E+03	9.63E+04	2.76E+09	2.09E+10	2.73E+10	3.47E+05	4.18E+10	5.12E+08	9.28E+08	9.50E+04	8.56E+09
	STD	2.98E+02	2.92E+04	7.10E+08	4.70E+09	3.69E+09	4.71E+04	7.57E+09	1.84E+08	1.19E+09	1.03E+05	2.61E+09
C17-F14	Mean	3.92E+04	6.55E+06	1.37E+07	8.74E+06	4.46E+07	2.98E+06	7.82E+07	1.43E+07	9.44E+06	8.92E+04	1.13E+07
	STD	1.59E+02	3.31E+06	4.14E+06	6.27E+06	5.94E+06	1.55E+06	7.47E+06	5.06E+06	3.90E+06	8.00E+04	4.14E+06
C17-F15	Mean	2.59E+03	8.28E+04	1.17E+09	1.18E+10	1.51E+10	1.24E+05	2.31E+10	6.88E+07	4.92E+08	5.50E+04	1.22E+09
	STD	2.07E+01	1.89E+04	1.01E+09	1.04E+10	1.43E+09	4.69E+04	6.64E+09	4.67E+07	7.26E+08	3.11E+04	5.39E+08
C17-F16	Mean	3.51E+02	7.06E+03	1.13E+04	1.44E+04	1.87E+04	6.53E+03	2.23E+04	1.60E+04	6.05E+03	5.48E+03	1.09E+04
	STD	2.19E+02	9.12E+02	8.27E+02	2.39E+03	9.64E+02	5.92E+02	3.68E+03	2.28E+03	7.09E+02	1.11E+02	1.55E+03
C17-F17	Mean	2.97E+03	5.79E+03	8.64E+03	2.15E+05	4.15E+06	4.93E+03	8.16E+06	1.68E+04	5.45E+03	4.61E+03	4.57E+04
	STD	1.91E+02	3.40E+02	1.51E+02	2.67E+05	4.22E+06	4.09E+02	8.48E+06	8.84E+03	1.27E+03	2.31E+02	2.13E+04
C17-F18	Mean	4.92E+04	2.77E+06	1.59E+07	1.47E+07	5.74E+07	4.83E+06	1.01E+08	1.18E+07	1.08E+07	2.28E+05	1.16E+07
	STD	2.97E+03	1.48E+06	5.04E+06	1.20E+07	3.62E+07	2.39E+06	6.69E+07	2.58E+06	6.28E+06	1.33E+05	1.04E+07
C17-F19	Mean	7.52E+04	2.83E+06	6.58E+08	4.97E+09	1.25E+10	1.64E+07	2.20E+10	1.32E+08	3.55E+08	2.75E+05	1.55E+09
	STD	5.98E+03	1.90E+06	6.28E+08	3.69E+09	1.81E+09	8.85E+06	5.08E+09	8.56E+07	5.41E+08	1.85E+05	1.44E+09

Table 15 (continued)

Functions	Parameters	HSO	AVOA	TLBO	TSA	WSO	MVO	RSA	WOA	GWO	MPA	GSA
C17-F20	Mean	2.39E+03	6.14E+03	7.20E+03	6.98E+03	7.24E+03	5.78E+03	7.49E+03	7.00E+03	6.04E+03	4.48E+03	6.29E+03
	STD	3.61E+01	3.37E+02	5.82E+02	6.40E+02	1.61E+02	4.12E+02	9.79E+01	3.94E+02	1.16E+03	5.86E+01	3.59E+02
C17-F21	Mean	2.56E+03	3.64E+03	3.68E+03	4.09E+03	4.25E+03	3.22E+03	4.37E+03	4.19E+03	2.97E+03	2.82E+03	4.67E+03
	STD	2.93E+01	1.62E+02	1.58E+02	1.28E+02	6.05E+01	9.68E+01	6.28E+01	2.57E+02	6.24E+01	3.65E+01	4.50E+02
C17-F22	Mean	2.97E+03	2.01E+04	3.30E+04	3.04E+04	3.14E+04	1.72E+04	3.31E+04	2.88E+04	2.32E+04	1.86E+04	2.10E+04
	STD	9.78E+01	1.58E+03	7.19E+02	9.96E+02	6.32E+02	9.31E+02	4.97E+02	1.29E+03	8.16E+03	1.39E+03	5.41E+02
C17-F23	Mean	2.78E+03	4.10E+03	4.20E+03	5.43E+03	5.31E+03	3.48E+03	5.32E+03	5.13E+03	3.61E+03	3.29E+03	7.85E+03
	STD	5.09E+01	8.52E+01	6.10E+01	8.71E+02	2.66E+02	9.69E+01	2.15E+02	1.50E+02	3.90E+01	2.30E+01	5.01E+02
C17-F24	Mean	3.97E+03	5.37E+03	4.75E+03	6.66E+03	8.51E+03	3.97E+03	1.05E+04	6.37E+03	4.29E+03	3.71E+03	1.08E+04
	STD	1.69E+02	1.95E+02	2.09E+02	3.79E+02	1.64E+03	9.28E+01	3.04E+03	5.05E+02	2.56E+02	5.92E+01	1.25E+03
C17-F25	Mean	4.01E+03	4.14E+03	8.89E+03	1.05E+04	1.52E+04	3.41E+03	2.13E+04	7.29E+03	6.42E+03	3.68E+03	1.10E+04
	STD	4.79E+02	3.32E+02	1.43E+03	5.32E+02	1.29E+03	6.15E+01	2.57E+03	4.95E+02	2.96E+02	1.37E+02	1.15E+03
C17-F26	Mean	4.98E+03	2.46E+04	2.38E+04	3.32E+04	3.94E+04	1.18E+04	4.53E+04	3.38E+04	1.68E+04	1.16E+04	3.37E+04
	STD	3.78E+02	2.69E+03	4.45E+03	9.62E+02	4.77E+02	1.79E+03	2.16E+03	3.48E+03	1.60E+03	7.94E+02	1.47E+03
C17-F27	Mean	3.92E+03	4.16E+03	4.33E+03	6.61E+03	9.33E+03	3.62E+03	1.23E+04	6.01E+03	4.08E+03	3.53E+03	1.41E+04
	STD	2.48E+02	2.17E+02	3.59E+02	3.21E+02	1.76E+03	7.22E+01	3.70E+03	8.74E+02	1.63E+02	3.36E+01	3.04E+02
C17-F28	Mean	4.02E+03	4.71E+03	1.12E+04	1.57E+04	2.09E+04	3.46E+03	2.82E+04	1.04E+04	9.27E+03	3.77E+03	1.88E+04
	STD	2.09E+01	2.43E+02	2.34E+03	3.11E+03	2.04E+03	7.50E+01	3.03E+03	1.16E+03	1.59E+03	9.77E+01	2.06E+03
C17-F29	Mean	4.89E+03	9.61E+03	1.24E+04	1.84E+04	1.83E+05	8.67E+03	3.49E+05	1.65E+04	8.29E+03	6.87E+03	2.49E+04
	STD	1.59E+01	9.67E+02	6.77E+02	4.19E+03	6.75E+04	7.56E+02	1.38E+05	2.76E+03	2.41E+02	7.09E+02	6.15E+03
C17-F30	Mean	4.59E+03	2.76E+07	3.77E+09	1.33E+10	2.31E+10	1.03E+08	3.76E+10	1.49E+09	1.83E+09	4.68E+06	7.32E+09
	STD	2.49E+02	1.60E+07	3.04E+09	4.00E+09	2.22E+09	3.05E+07	2.59E+09	3.92E+08	8.03E+08	2.79E+06	1.67E+09

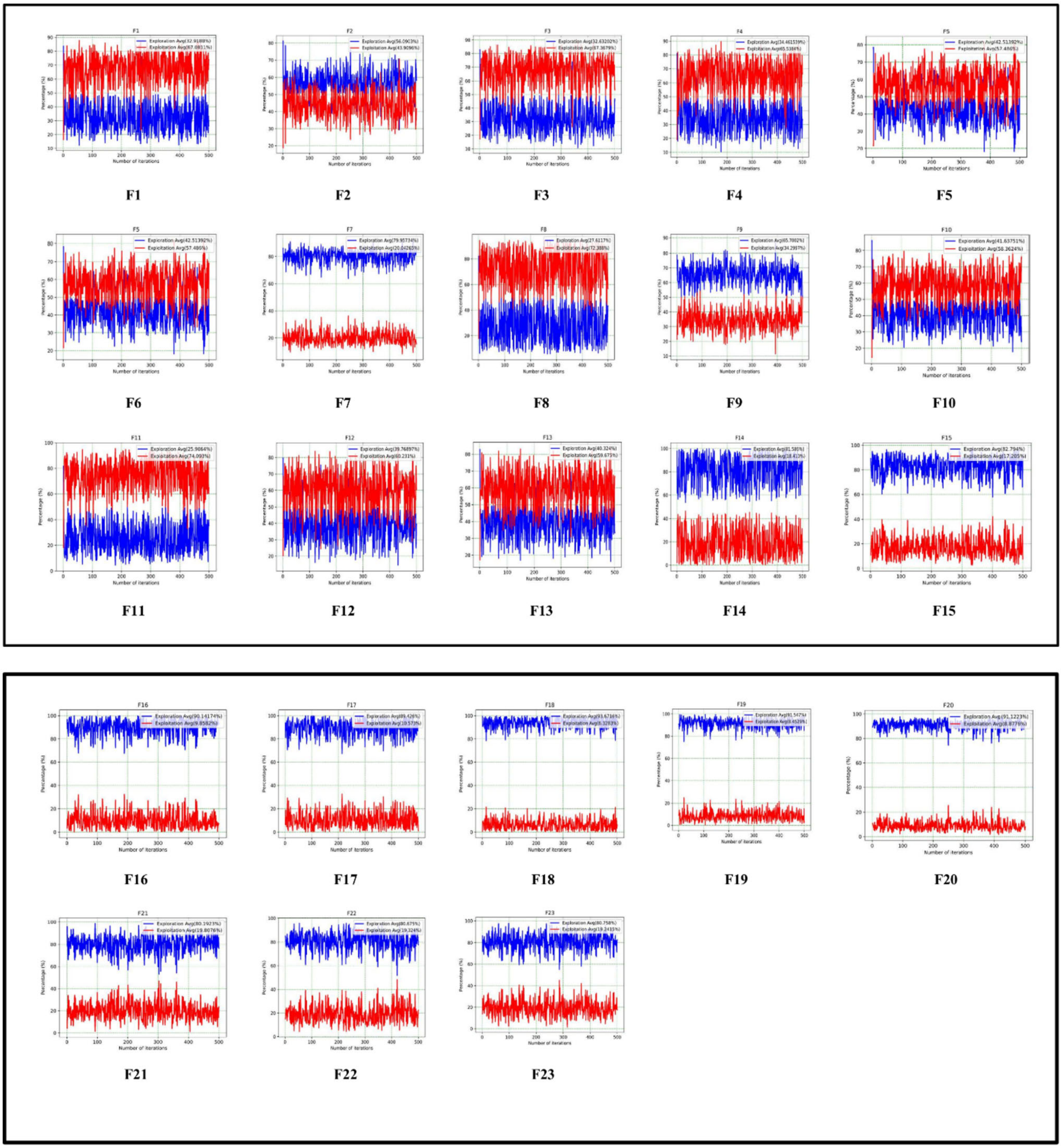


Fig. 3 Diversity plots of 23 standard benchmark functions

assessed based on the average ranks of each algorithm, as shown in Table 23. For Function F14, HSO secures the top position with an average rank of 4.10, outperforming the rest of the algorithms. Similarly, in F15, HSO maintains its dominance with an average rank of 3, indicating its superior performance compared to the others. In F16, HSO again emerges as the top-ranked algorithm with an average rank of 2. However, in F17, HSO experiences a slight decline in performance, securing the 7th position with an average rank of 7.40. This trend continues in F18, where HSO is outranked by GA, securing the 2nd position with an average rank of 15. In F19, HSO bounces back, regaining its top position with an average rank of 1. HSO consistently performs well in F20, F21, and F22, securing the 1st, 1st,

Table 16 Sensitivity analysis of the HSO for the number of population members (N = 20, 30, 50, 100)

Functions	Parameters	N = 20	N = 30	N = 50	N = 100
F1	Mean	0	0.00E+00	0	0
	STD	0	0.00E+00	0	0
F2	Mean	0	0.00E+00	0	0
	STD	0	0.00E+00	0	0
F3	Mean	0	0.00E+00	0	0
	STD	0	0.00E+00	0	0
F4	Mean	0	0.00E+00	0	0
	STD	0	0.00E+00	0	0
F5	Mean	27.96684	2.70E+01	28.91451	28.42622
	STD	5.179709	5.82E+00	0.140361	2.3149421
F6	Mean	2.096954971	1.56E+00	1.241605115	1.6121026
	STD	1.792380961	1.48E+00	1.443642005	1.8299623
F7	Mean	6.55E−05	2.90E−05	2.41E−05	7.71E−06
	STD	5.77E−05	2.47E−05	2.42E−05	7.14E−06
F8	Mean	− 12,555.49382	− 1.26E+04	− 12,569.47545	− 12,569.48145
	STD	75.2057782	1.84E−02	0.021568536	0.007481758
F9	Mean	0	0.00E+00	0	0
	STD	0	0.00E+00	0	0
F10	Mean	4.44E−16	4.44E−16	4.44E−16	4.44E−16
	STD	0	0.00E+00	0	0
F11	Mean	0	0.00E+00	0	0
	STD	0	0.00E+00	0	0
F12	Mean	14.20287	1.05E−01	8.146909	6.9549762
	STD	10.49694	1.94E−01	6.470876	4.7206544
F13	Mean	0.787118	7.32E−01	0.391034	0.4388542
	STD	0.901895	9.54E−01	0.270972	0.4646127

and 4th positions, respectively. In F23, HSO maintains a competitive edge, securing the 3rd position with an average rank of 5.90. The overall average ranks highlight HSO's strong performance, securing the 1st position with an average rank of 1. In summary, the proposed algorithm HSO demonstrates robust performance across various benchmark functions, consistently outperforming other optimization algorithms in terms of average ranks.

6 Computational time complexities

The algorithm initiates a search optimization procedure utilizing a population of search agents within an $m \times n-$ dimensional domain. In the initialization phase, operating in $O(m \times n)$ time complexity, random values are assigned to the population in the $m \times n-$ dimensional space. Simultaneously, the evaluation of the target objective function during this phase incurs a time complexity of $O(m)$, performed for each of the m search agents. Moving forward, the primary iteration loop demonstrates a time complexity of $(iter \times m \times n)$, reflecting its dependence on '$iter$' iterations. In each iteration, the loop traverses through each of the 'm' search agents and each of the 'n' dimensions. Within the exploration phase, the nested loops responsible for updating the population contribute to a time complexity of $O(m \times n)$. Additionally, the evaluation of the target objective function in this phase incurs a time complexity of$O(m)$. Similarly, in the exploitation phase, the nested loops for updating the population and the evaluation of the target objective function both carry a time complexity of $O(m \times n)$ and $O(m)$, respectively. The conditional checks within both the exploration and exploitation phases minimally impact the overall time complexity, involving fundamental arithmetic operations and comparisons. In conclusion, the aggregate time complexity of the algorithm is succinctly expressed as$O(iter \times m \times n)$, where '$iter$' represents the number of iterations, 'm' denotes the quantity of search agents, and 'n' signifies the number of dimensions within the search space.

Table 17 Sensitivity analysis of the TDO for the maximum number of iterations (T = 100, 500, 1000, 2000)

Functions	Parameters	T = 100	T = 500	T = 1000	T = 2000
F1	Mean	0	0.00E+00	0	0
	STD	0	0.00E+00	0	0
F2	Mean	0	0.00E+00	0	0
	STD	0	0.00E+00	0	0
F3	Mean	0	0.00E+00	0	0
	STD	0	0.00E+00	0	0
F4	Mean	0	0.00E+00	0	0
	STD	0	0.00E+00	0	0
F5	Mean	28.08269099	2.70E+01	27.78966	28.838428
	STD	4.72440981	5.82E+00	5.199464	0.2390185
F6	Mean	2.852133	1.56E+00	2.143618	1.4845312
	STD	1.791617	1.48E+00	2.602609	1.9577613
F7	Mean	0.000156	2.90E−05	1.18E−05	1.08E−05
	STD	0.000169	2.47E−05	1.39E−05	9.11E−06
F8	Mean	− 12,568.9	− 1.26E+04	− 12,555.5	− 12,569.49
	STD	1.180006	1.84E−02	75.2105	0.0014336
F9	Mean	0	0.00E+00	0	0
	STD	0	0.00E+00	0	0
F10	Mean	4.44E−16	4.44E−16	4.44E−16	4.44E−16
	STD	0	0.00E+00	0	0
F11	Mean	0.059506604	0.00E+00	0	0
	STD	0.179282108	0.00E+00	0	0
F12	Mean	21.87216	1.05E−01	5.694502	5.1943021
	STD	18.76721	1.94E−01	5.837189	9.6091305
F13	Mean	1.224024	7.32E−01	0.516487	0.3855146
	STD	1.076604	9.54E−01	0.698257	0.4971515

7 Classical real-world engineering design problems

7.1 Constrained optimization based on IEEE−CEC-20 benchmarks suits

7.1.1 Pressure vessel design

The primary purpose of this pressure vessel design [120] is to keep the overall cost to a minimum. This total cost includes the material, forming, and welding of a cylindrical vessel. The vessel is capped on both ends, and the head is hemispherical in shape. The four variables in this problem are the thickness of the shell (T_s), the thickness of the head (T_h), the inner radius (R), and the length of the cylindrical section without considering the head (L).

There are four limits on the subject of this problem. This problem's mathematical model is as follows:

$$\text{Consider}\,\overrightarrow{X} = [X_1, X_2, X_3, X_4] = \left[T_s, T_h, R, L\right],$$

$$f\left(\overrightarrow{X}\right) = 0.6224X_1X_3X_4 + 1.7781X_2X_3^2 + 3.1661X_1^2X_4 + 19.84X_1^2X_3,$$

$$g_1\left(\overrightarrow{X}\right) = -X_1 + 0.019X_3 \leq 0,$$

$$g_2\left(\overrightarrow{X}\right) = -X_3 + 0.00954X_3 \leq 0,$$

$$g_3\left(\overrightarrow{X}\right) = -\pi X_3^2X_4 - \frac{4}{3}\pi X_3^3 + 1296000 \leq 0,$$

$$g_4\left(\overrightarrow{X}\right) = X_4 - 240 \leq 0,$$

Variable range

$0 \leq X_1 \leq 99,$

$0 \leq X_2 \leq 99,$

$10 \leq X_3 \leq 200,$

$10 \leq X_4 \leq 200,$

This problem is very common for all optimizers. Researchers and Scientists solve this problem using conventional optimization algorithms and meta-heuristic

Table 18 Student T tests for the proposed algorithm (HSO) across 30 dimensions

Functions	VS GA	VS FA	VS BAT	VS DE	VS TSA	VS GSA	VS PSO	VS WOA	VS GWO
F1	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)
F2	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)
F3	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)
F4	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)
F5	5.54E−15 (+)	4.30E−14 (+)	2.09E−10 (+)	1.60E−10 (+)	2.17E−19 (+)	5.06E−03 (+)	1.53E−14 (+)	3.29E−11 (+)	3.12E−12 (+)
F6	5.87E−13 (+)	4.91E−13 (+)	3.68E−11 (+)	3.17E−11 (+)	2.09E−13 (+)	9.15E−10 (+)	2.30E−13 (+)	2.87E−18 (+)	2.09E−13 (+)
F7	8.51E−14 (+)	3.25E−15 (+)	1.11E−10 (+)	5.60E−11 (+)	7.91E−12 (+)	6.72E−11 (+)	1.19E−13 (+)	2.19E−01 (−)	3.01E−03 (+)
F8	3.05E−10 (+)	6.59E−12 (+)	2.69E−11 (+)	4.60E−12 (+)	3.29E−04 (+)	6.61E−07 (+)	3.95E−11 (+)	2.31E−14 (+)	8.71E−14 (+)
F9	8.48E−13 (+)	6.63E−13 (+)	5.90E−11 (+)	2.84E−13 (+)	6.17E−08 (+)	3.17E−08 (+)	3.16E−10 (+)	0.323931 (−)	2.23E−14 (+)
F10	7.74E−15 (+)	8.64E−10 (+)	4.96E−12 (+)	5.75E−14 (+)	3.96E−16 (+)	2.73E−11 (+)	6.14E−13 (+)	3.82E−13 (+)	3.06E−15 (+)
F11	6.64E−17 (+)	6.58E−12 (+)	1.47E−10 (+)	1.69E−12 (+)	2.37E−04 (+)	3.20E−07 (+)	3.41E−12 (+)	2.13E−01 (−)	3.79E−05 (+)
F12	5.39E−10 (+)	9.17E−13 (+)	6.88E−09 (+)	4.50E−11 (+)	6.79E−09 (+)	7.34E−06 (+)	4.30E−13 (+)	2.19E−12 (+)	2.95E−09 (+)
F13	3.15E−07 (+)	8.65E−09 (+)	5.35E−05 (+)	5.00E−07 (+)	4.81E−13 (+)	2.12E−11 (+)	6.03E−11 (+)	5.19E−19 (+)	3.98E−17 (+)
F14	2.03E−10 (+)	8.00E−10 (+)	1.04E−10 (+)	6.59E−09 (+)	6.21E−04 (+)	6.42E−12 (+)	3.34E−07 (+)	2.17E−08 (+)	2.19E−04 (+)
F15	4.56E−11 (+)	6.12E−13 (+)	2.34E−11 (+)	6.66E−10 (+)	5.26E−05 (+)	7.65E−13 (+)	5.60E−09 (+)	1.92E−04 (+)	8.39E−01 (−)
F16	2.63E−12 (+)	4.75E−14 (+)	4.79E−07 (+)	3.98E−11 (+)	3.27E−14 (+)	9.25E−09 (+)	2.65E−10 (+)	3.49E−15 (+)	3.49E−15 (+)
F17	8.27E−03 (+)	5.74E−15 (+)	4.04E−13 (+)	1.46E−06 (+)	1.27E−12 (+)	5.77E−07 (+)	3.33E−07 (+)	3.49E−15 (+)	2.97E−15 (+)
F18	5.07E−11 (+)	8.97E−11 (+)	9.72E−12 (+)	3.72E−06 (+)	2.31E−04 (+)	6.97E−05 (+)	3.31E−03 (+)	3.49E−15 (+)	3.29E−13 (+)
F19	5.59E−04 (+)	9.09E−16 (+)	5.40E−09 (+)	7.20E−05 (+)	1.29E−14 (+)	8.88E−07 (+)	1.32E−02 (+)	8.13E−14 (+)	2.79E−13 (+)
F20	3.38E−04 (+)	4.71E−11 (+)	2.56E−08 (+)	5.19E−04 (+)	4.15E−09 (+)	7.75E−10 (+)	5.19E−05 (+)	2.17E−12 (+)	3.86E−13 (+)
F21	1.53E−06 (+)	2.99E−04 (+)	6.48E−10 (+)	1.09E−02 (+)	2.87E−18 (+)	2.05E−02 (+)	7.08E−07 (+)	8.13E−01 (−)	8.53E−09 (+)
F22	3.01E−03 (+)	5.83E−03 (+)	4.74E−07 (+)	9.47E−02 (+)	2.18E−14 (+)	3.63E−02 (+)	1.11E−09 (+)	3.14E−03 (+)	2.17E−09 (+)
F23	2.57E−02 (+)	5.47E−03 (+)	6.85E−01 (−)	2.92E−03 (+)	4.79E−13 (+)	6.51E−02 (+)	3.26E−02 (+)	2.19E−03 (+)	3.64E−03 (+)

Functions	VS CS	VS BBO	VS SCA	VS MFO	VS FPA	VS AOA
F1	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)
F2	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	N/A (=)
F3	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)
F4	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)
F5	3.29E−11 (+)	2.39E−14 (+)	2.39E−14 (+)	2.39E−14 (+)	2.56E−11 (+)	2.39E−12 (+)
F6	2.17E−08 (+)	4.79E−13 (+)	2.87E−18 (+)	2.87E−18 (+)	7.27E−10 (+)	4.79E−13 (+)

Table 18 (continued)

Functions	VS CS	VS BBO	VS SCA	VS MFO	VS FPA	VS AOA
F7	6.71E−01 (−)	3.23E−15 (+)	2.19E−16 (+)	2.87E−18 (+)	4.79E−13 (+)	3.89E−15 (+)
F8	5.29E−14 (+)	5.29E−14 (+)	4.45E−10 (+)	5.29E−14 (+)	5.29E−14 (+)	5.29E−14 (+)
F9	N/A (=)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	4.49E−14 (+)	N/A (=)
F10	7.39E−16 (+)	1.92E−14 (+)	3.49E−15 (+)	3.49E−15 (+)	2.39E−14 (+)	N/A (=)
F11	N/A (=)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)
F12	2.87E−18 (+)	2.39E−14 (+)	1.19E−17 (+)	1.19E−17 (+)	4.79E−13 (+)	2.27E−04 (+)
F13	7.81E−03 (−)	4.49E−14 (+)	2.87E−18 (+)	2.87E−18 (+)	1.27E−16 (+)	4.81E−13 (+)
F14	4.29E−14 (+)	4.29E−14 (+)	4.52E−08 (+)	2.39E−14 (+)	1.48E−07 (+)	6.71E−05 (+)
F15	6.21E−05 (+)	2.39E−14 (+)	3.89E−15 (+)	2.45E−11 (+)	1.48E−11 (+)	7.89E−09 (+)
F16	3.49E−15 (+)	3.49E−15 (+)	1.43E−11 (+)	3.49E−15 (+)	3.49E−15 (+)	3.49E−15 (+)
F17	3.49E−15 (+)	3.49E−15 (+)	6.72E−10 (+)	3.49E−15 (+)	3.49E−15 (+)	2.49E−04 (+)
F18	3.49E−15 (+)	N/A (=)	2.42E−10 (+)	3.49E−15 (+)	3.49E−15 (+)	6.79E−08 (+)
F19	3.49E−15 (+)	3.49E−15 (+)	7.37E−11 (+)	3.49E−15 (+)	3.49E−15 (+)	3.21E−13 (+)
F20	2.19E−13 (+)	2.39E−14 (+)	2.49E−02 (+)	6.39E−11 (+)	2.39E−14 (+)	6.79E−06 (+)
F21	2.71E−08 (+)	2.85E−02 (−)	4.79E−13 (+)	2.96E−02 (+)	2.59E−02 (+)	2.87E−18 (+)
F22	2.17E−08 (+)	2.24E−02 (+)	2.87E−18 (+)	5.37E−03 (+)	7.44E−03 (+)	2.87E−18 (+)
F23	9.45E−04 (+)	6.06E−03 (+)	2.87E−18 (+)	1.96E−02 (+)	4.65E−02 (+)	2.87E−18 (+)

Table 19 Friedman ranking tests of the proposed algorithm with other competitor metaheuristics for 30 dimensions

F.no	HSO	GA	FA	BAT	DE	TSA	GSA	PSO	WOA	GWO	CS	BBO	SCA	MFO	FPA	AOA
F1	1	14	8	15	7	4	13	12	2	3	6	11	10	9	16	5
F2	1	14	9	16	7	5	12	15	3	4	10	6	8	11	13	1
F3	1	11	8	15	12	3	16	13	14	2	5	10	9	7	6	4
F4	1	11	5	15	7	6	16	12	13	2	4	10	9	14	8	3
F5	6	12	8	16	2	1	9	15	3	5	4	10	13	11	14	7
F6	6	11	3	16	1	7	14	15	4	5	2	10	9	12	13	8
F7	1	10	8	16	7	4	11	15	5	3	9	6	12	14	13	2
F8	2	1	10	15	8	11	14	12	4	6	16	3	13	5	7	9
F9	1	6	8	13	12	14	9	16	5	4	7	1	11	10	15	1
F10	2	12	7	16	6	8	10	15	1	4	5	9	13	14	11	3
F11	1	12	5	16	6	3	9	15	7	4	2	11	10	14	13	8
F12	6	11	3	16	2	10	8	15	5	4	1	9	14	12	13	7
F13	6	11	1	16	2	9	10	15	5	4	3	7	13	14	12	8
Avg	2.69	10.46	6.38	15.46	6.08	6.54	11.62	14.23	5.46	3.85	5.69	7.92	11.08	11.31	11.85	5.08
Avg. ranks	1	10	7	16	6	8	13	15	4	2	5	9	11	12	14	3

Nature Inspired algorithms. The comparison table of HSO with some of Nature Inspired Algorithm are as follows:

Pressure Vessel Design is one of the most popular constrained optimization problems due to the complexity of its constraint search spaces (Tables 24 and 25), and it is included in the CEC 2020 real-world constrained optimization benchmarks. Using metaheuristics, eight competitive algorithms are used to solve this problem to ensure fair comparisons. Due to its optimal cost value, HSO ranked first among all algorithms, while TDO and TSA ranked second and third, respectively, in the experimental results. According to Tables 26 and 27, the HSO

Table 20 Friedman ranking tests of the proposed algorithm with other competitor metaheuristics for 100 dimensions

F. No	HSO	GA	FA	BAT	DE	TSA	GSA	PSO	WOA	GWO	CS	BBO	SCA	MFO	FPA	AOA
F1	1	14	6	16	15	4	8	12	2	3	7	9	10	13	11	5
F2	1	14	9	16	12	5	6	15	3	4	7	10	8	13	11	2
F3	1	6	12	13	11	5	10	15	16	4	3	14	8	9	7	2
F4	1	13	6	15	16	9	3	8	12	5	4	11	14	10	7	2
F5	5	11	7	14	12	1	8	16	4	3	6	9	15	13	10	2
F6	7	13	2	16	10	6	1	15	4	5	3	12	11	14	9	8
F7	1	12	7	14	13	5	6	16	3	4	9	8	11	15	10	2
F8	2	3	6	14	15	11	7	13	5	8	1	4	16	9	10	12
F9	1	10	8	14	7	15	5	16	3	6	9	4	11	12	13	1
F10	2	12	8	14	11	4	6	14	1	3	7	16	13	10	9	5
F11	1	13	6	16	10	5	7	15	1	3	4	8	9	12	11	14
F12	3	13	7	16	15	8	6	14	2	4	1	9	11	12	10	5
F13	4	14	7	16	12	6	8	13	1	2	3	9	11	15	10	5
Avg	2.31	11.38	7.00	14.92	12.23	6.46	6.23	14.00	4.38	4.15	4.92	9.46	11.38	12.08	9.85	5.00
Avg. ranks	1	11	8	16	14	7	6	15	3	2	4	9	11	13	10	5

Table 21 Friedman ranking tests of the proposed algorithm with other competitor metaheuristics for 500 dimensions

F.No	HSO	GA	FA	BAT	DE	TSA	GSA	PSO	WOA	GWO	CS	BBO	SCA	MFO	FPA	AOA
F1	1	12	7	16	13	4	14	11	2	3	6	8	14	10	9	5
F2	1	12	11	16	13	5	6	15	2	3	7	10	8	14	9	4
F3	1	9	10	13	14	11	4	6	12	8	3	7	16	15	5	2
F4	1	13	5	11	12	15	4	9	10	6	3	8	16	14	7	2
F5	4	13	9	11	16	7	6	12	2	1	5	10	15	14	8	3
F6	2	11	7	14	13	5	8	12	1	4	3	10	9	16	15	6
F7	1	11	10	15	12	5	6	13	3	4	7	9	14	16	8	2
F8	3	6	7	14	10	9	2	13	4	8	1	5	12	15	16	11
F9	1	9	10	14	15	13	11	6	1	4	5	7	8	16	12	3
F10	2	12	10	13	14	5	8	16	1	3	6	9	11	15	7	4
F11	1	15	8	12	10	3	9	13	1	4	5	7	14	11	6	16
F12	2	12	9	15	6	10	7	16	1	4	3	11	13	14	8	5
F13	2	12	8	16	15	7	6	11	1	3	4	10	13	14	9	5
Avg	1.69	11.31	8.54	13.85	12.54	7.62	7.00	11.77	3.15	4.23	4.46	8.54	12.54	14.15	9.15	5.23
Avg. ranks	1	11	8	15	13	7	6	12	2	3	4	8	13	16	10	5

results are superior. Figure 4 depicts the convergence curve of HSO after 500 iterations. Each run consists of 500 iterations, and 30 runs are performed. All parameter charts for algorithms should be represented in Figs. 5 and 6.

7.1.2 Welded beam design

The major goal of this topic is to reduce the time it takes to fabricate a welded beam [121]. In this problem, there are five restrictions to consider: In the beam, shear stress (τ), bending stress (θ), the buckling load on the bar (P_c), the beam's end deflection (δ), and the side limitations.

The thickness of the weld (h), the length of the attached part of the bar (l), the height of the bar (t), and the thickness of the bar (t) are the four variables in this problem (b). The following is the mathematical model for this problem:

Consider $\overrightarrow{X} = [X_1, X_2, X_3, X_4] = [h, l, t, b]$

Minimize

$$f\left(\overrightarrow{X}\right) = 1.10471X_1^2X_2 + 0.04811X_3X_4(14.0 + X_2),$$

Table 22 Friedman ranking tests of the proposed algorithm with other competitor metaheuristics for 1000 dimensions

F.No	HSO	GA	FA	BAT	DE	TSA	GSA	PSO	WOA	GWO	CS	BBO	SCA	MFO	FPA	AOA
F1	1	15	14	16	13	5	7	11	2	3	6	10	9	12	8	4
F2	1	11	15	15	13	4	7	12	2	5	6	9	8	14	10	3
F3	1	14	12	15	13	7	4	11	9	6	3	8	10	16	5	2
F4	1	11	7	12	10	14	4	8	9	6	3	13	14	16	5	2
F5	5	12	9	16	15	7	3	11	2	1	6	10	13	14	8	4
F6	3	11	14	16	13	1	7	12	2	4	5	10	9	15	8	6
F7	1	10	11	14	16	5	8	12	3	4	7	9	13	15	6	2
F8	1	3	16	15	10	8	7	14	12	6	4	2	13	5	9	11
F9	1	7	13	14	15	8	16	10	1	4	6	5	12	11	9	3
F10	2	12	11	15	8	5	16	9	1	4	6	10	13	14	7	3
F11	1	14	13	15	10	5	6	11	2	4	3	9	8	12	7	16
F12	3	6	9	15	16	12	7	11	2	4	1	10	13	14	8	5
F13	3	1	8	13	16	10	7	12	2	5	6	11	14	15	9	4
Avg	1.85	9.77	11.69	14.69	12.92	7.00	7.62	11.08	3.77	4.31	4.77	8.92	11.46	13.31	7.62	5.00
Avg. ranks	1	10	13	16	14	6	7	11	2	3	4	9	12	15	7	5

$$g_1\left(\overrightarrow{X}\right) = \tau\left(\overrightarrow{X}\right) - \tau_{max} \leq 0,$$

$$g_2\left(\overrightarrow{X}\right) = \sigma\left(\overrightarrow{X}\right) - \sigma_{max} \leq 0,$$

$$g_3\left(\overrightarrow{X}\right) = \delta\left(\overrightarrow{X}\right) - \delta_{max} \leq 0,$$

$$g_4\left(\overrightarrow{X}\right) = X_1 - X_4 \leq 0,$$

$$g_5\left(\overrightarrow{X}\right) = P - P_c(\overrightarrow{X}) \leq 0,$$

$$g_6\left(\overrightarrow{X}\right) = 0.125 - X_1 \leq 0,$$

$$g_7\left(\overrightarrow{X}\right) = 1.10471X_1^2 + 0.04811X_3X_4(14.0 + X_2) - 5.0 \leq 0,$$

Variable range

$0.1 \leq X_1 \leq 2,$

$0.1 \leq X_2 \leq 10,$

$0.1 \leq X_3 \leq 10,$

$0.1 \leq X_4 \leq 2$

where $\tau\left(\overrightarrow{X}\right) = \sqrt{(\tau\prime)^2 + 2\tau\prime\tau''\frac{X_2}{2R} + (\tau'')^2},$

$$\tau\prime = \frac{P}{\sqrt{2}X_1X_2}, \tau'' = \frac{MR}{J}, M = P\left(L + \frac{X_2}{2}\right),$$

$$R = \sqrt{\frac{X_2^2}{4} + \left(\frac{X_1 + X_2}{2}\right)^2},$$

$$J = 2\left\{\sqrt{2}X_1X_2\left[\frac{X_2^2}{4} + \left(\frac{X_1 + X_2}{2}\right)^2\right]\right\},$$

$$\sigma\left(\overrightarrow{X}\right) = \frac{6PL}{X_4X_3^2}, \delta\left(\overrightarrow{X}\right) = \frac{6PL^3}{EX_3^2X_4}$$

$$P_c\left(\overrightarrow{X}\right) = \frac{4.013E\sqrt{\frac{X_3^2X_4^6}{36}}}{L^2}\left(1 - \frac{X_3}{2L}\sqrt{\frac{E}{4G}}\right),$$

$$P = 6000lb, L = 14in.\delta_{max} = 0.25in., E = 30 \times 1^6psi, G = 12 \times 10^6psi, \tau_{max} = 13600psi,$$

$\sigma_{max} = 30000psi.$

The following are the comparison findings of welded beam design of HSO with several Nature Inspired Optimization algorithms are as follows:

The experimental results for the welded beam design problem are displayed in Table 28. To ensure fairness, HSO is compared to seven other competitor algorithms. HSO outperforms other algorithms when determining optimal beam design costs. HSO was ranked first for this problem, while TDO and TSA were ranked second and third, respectively (Figs. 7 and 8). The CEC 2020 benchmark suite for real-world constraint optimization includes welded beam design. The statistical comparison between HSO and other algorithms for this problem is displayed in Table 29. Figures 7 and 9 depict, respectively, a statistical graph and a convergence analysis of HSO over 500 iterations. Figure 8 is a comparison of the HSO parameters graph to other algorithms.

Table 23 Friedman ranking tests of the proposed algorithm with other competitor metaheuristics for fixed dimensions

F.No	HSO	GA	FA	BAT	DE	TSA	GSA	PSO	WOA	GWO	CS	BBO	SCA	MFO	FPA	AOA
F14	6	1	12	14	8	16	1	9	10	13	15	1	7	11	1	1
F15	3	15	7	14	4	16	8	10	6	12	2	13	9	11	5	1
F16	2	16	3	15	3	3	10	3	10	10	3	14	1	10	3	3
F17	7	15	8	4	8	8	3	14	8	5	8	16	2	1	8	6
F18	1	1	1	15	1	16	11	12	1	10	1	1	13	1	1	13
F19	1	16	2	14	2	2	11	2	9	11	2	10	15	2	2	11
F20	13	15	3	8	4	5	10	16	7	9	1	14	12	6	2	10
F21	1	10	8	14	3	11	4	15	7	6	13	5	16	9	12	2
F22	4	12	6	13	5	10	2	11	9	2	15	7	16	8	14	1
F23	3	14	6	16	4	7	1	13	7	2	12	10	15	9	11	4
Avg	4.10	11.50	5.60	12.70	4.20	9.40	6.10	10.50	7.40	8.00	7.20	9.10	10.60	6.80	5.90	5.20
Avg. ranks	1	15	4	16	2	12	6	13	9	10	8	11	14	7	5	3

7.1.3 Tension compression spring design

The purpose of this Tension Compression Spring Design [122] challenge is to make a tension/compression spring that is as light as feasible. Shear stress, surge frequency, and minimum deflection must all be considered during the minimization process. This optimization is influenced by wire diameter (d), mean coil diameter (D), and the number of active coils (N) (Figs. 9 and 10).

The following is the mathematical model for this problem:

$$\vec{X} = [X_1, X_2, X_3] = [d, D, N],$$

$$\text{Minimize} f(\overrightarrow{X}) = (X_3 + 2)X_2X_1^2,$$

$$\text{Subject to } g_1\left(\vec{X}\right) = 1 - \frac{X_2^3X_3}{7185X_1^4} \le 0,$$

$$g_2\left(\vec{X}\right) = \frac{4X_2^2 - X_1X_2}{12566(X_2X_1^3 - X_1^4)} + \frac{1}{5108X_1^2} \le 0,$$

$$g_3\left(\vec{X}\right) = 1 - \frac{140.45X_1}{X_2^2X_3} \le 0,$$

$$g_4\left(\vec{X}\right) = \frac{X_1 + X_2}{1.5} - 1 \le 0,$$

Variable range

$$0.05 \le X_1 \le 2.00,$$

$$0.25 \le X_2 \le 1.30,$$

$$2.00 \le X_3 \le 15.0$$

This optimization problem is solved by both mathematical and heuristic approach. To solve this problem by HSO, various Nature Inspired algorithms and their comparison results are reported in comparison table:

Table 24 Balance analysis between exploration and exploitation of the proposed algorithm (HSO)

Function name	Exploration rate (%)	Exploitation rate (%)
F1	32.92	67.08
F2	56.09	43.91
F3	32.63	67.37
F4	34.46	65.54
F5	42.51	57.49
F6	32.29	67.71
F7	79.96	20.04
F8	27.61	72.39
F9	65.70	34.30
F10	41.64	58.36
F11	25.91	74.09
F12	39.77	60.23
F13	40.32	59.68
F14	81.59	18.41
F15	82.79	17.21
F16	90.14	9.86
F17	89.43	10.57
F18	93.67	6.33
F19	91.55	8.45
F20	91.12	8.88
F21	80.19	19.81
F22	80.68	19.32
F23	80.76	19.24

Tension compression spring design is an additional CEC 2020 real-world engineering problem benchmark suite. This problem is recognized due to its complex mathematical model and complex constraint search spaces. The experimental results for HSO for this problem are shown in

Table 25 Advantages and disadvantages of various metaheuristic techniques

Algorithms	Advantages	Disadvantages
Grey Wolf Optimizer [13]	Efficient exploration and exploitation Simplicity of implementation Inspiration from social behavior of grey wolves Effectiveforvarioustypesofoptimizationproblems Goodbalancebetweenglobalandlocalsearch. it′sagoodchoiceforlargescaleoptimizationproblems.	Sensitivitytoparametersettingsandlackofstandardizedguidelines. Limitedtheoreticalunderstandingandconvergenceanalysis. Suboptimalperformanceinhighdimensionalorcomplexoptimizationproblems. Relativelysmallerresearchbaseandbenchmarkproblemscomparedtoother Popularalgorithms. Pronenesstoprematureconvergenceorstagnationinsuboptimalsolutions.
Marinepredatrors algorithm [17]	Utilizesthehuntingstrategiesofmarinepredatorsfor Efficientoptimization. Potentialforfastconvergenceandaccuratesolutions. Abilitytohandlevarioustypesofoptimizationproblems. Adaptabilitytodynamicenvironments. Balancebetweenexplorationandexploitationforrobustsearch.	Limitedtheoreticalfoundationandconvergenceanalysis. Compleximplementationandparametertuning. Relativelysmallerresearchbaseandbenchmarkproblems. Sensitivitytoproblemcharacteristics, affectingperformance. Potentialcomputationaloverheadduetotheemulationofcomplexpredator behaviors.
Archimedes optimization algorithm(AOA) [55]	UtilizestheprincipleofArchimedes′leverforefficient Optimization. Strongtheoreticalfoundationandmathematicalbasis. Adaptabilitytovariousoptimizationproblemsanddomains. Potentialforfastconvergenceandaccuratesolutions. Balancesexplorationandexploitationforrobustsearch capabilities.	Sensitivetoinitialconditionsandparametersettings. Slowconvergenceratecomparedtosomeotheroptimizationalgorithms. Maygettrappedinlocaloptimaandstrugglewithmultimodaloptimization Problems. Requiresderivativeinformation, limitingitsapplicabilitytodifferentiable functions. Maysufferfromhighmemoryusageforlargescaleoptimizationproblems.
Newton metaheuristic algorithm(NMA) [112]	Efficientlyexploitsthelocallandscapebyutilizingsecond −orderderivativeinformation. Fastconvergenceratecomparedtosomeotheroptimization Algorithms. Canhandlebothconvexandnonconvexoptimizationproblems. Providesaccurateestimationoftheoptimumandgradient Direction. Suitableforhighdimensionaloptimizationproblemsdueto Itseffectiveexplorationexploitationbalance.	Sensitivitytoinitialconditionsandlocaloptima. Dependencyontheavailabilityofsecondorderderivativeinformation. Limitedapplicabilitytonondifferentiableordiscontinuousfunctions. Highcomputationalandmemoryrequirementsforlargescaleproblems. Susceptibilitytonumericalinstabilityandconvergenceissuesinillconditioned problems. Notsuitablefordiscreteproblems. Lackofglobalexplorationcapabilitymayresultinsuboptimalsolutionsfor multimodaloptimizationproblems.

Table 25 (continued)

Algorithms	Advantages	Disadvantages
Rainoptimization algorithm(ROA) [113]	Simulatesthenaturalbehaviorofrainfall, allowingfor Efficientexplorationandexploitationofthesearchspace. Offersfastconvergenceandrobustnessinfindingoptimal solutions. Canhandlebothcontinuousanddiscreteoptimizationproblems. Providesflexibilityinincorporatingdifferentobjectivefunctions andconstraints. Suitableforsolvingcomplexoptimizationproblemsinvarious fields, includingengineeringandfinance.	Sensitivitytoparametersettingsandinitialconditions, affectingalgorithm performance. Limitedscalabilityforlargescaleoptimizationproblemsduetohigh computationalrequirements. Maystrugglewithmultimodaloptimizationproblemsandgetstuckinlocal optima. Lackoftheoreticalguaranteesorconvergencepropertiesincertaincases. Requirescarefultuningandexpertiseforoptimalresults, makingitlessuser friendlyforinexperiencedusers.
Heapbased optimizer (HBO) [114]	Efficientlymanagesmemoryallocationanddeallocationfor improvedcomputationalperformance. Providesconstanttimeaccesstothesmallestorlargestelement intheheap. Supportsdynamicinsertionandremovalofelements, allowingfor realtimeupdates. Facilitatesprioritybasedoptimizationbyeasilyextractingthe optimalelementfromtheheap. Enablesefficientimplementationofvariousalgorithmslike Dijkstra′salgorithmandPrim′salgorithm.	Limitedflexibilityinhandlingcomplexoptimizationproblemsbeyond prioritybasedscenarios. Maysufferfrominefficiencywhenfrequentupdatesormodificationsto theheaparerequired Requiresadditionalmemoryoverheadforstoringtheheapstructure. Lackofinherentsupportforparallelprocessing, limitingscalabilityin certainscenarios. Complexityandpotentialforerrorsinimplementingandmaintaining theheapstructureandoperations.
Equilibrium optimizer(EO) [51]	Mimicstheconceptofequilibriuminnature, leadingtorobust andefficientoptimization. Providesfastconvergenceandglobaloptimizationcapabilities. Supportsbothcontinuousanddiscreteoptimizationproblems. Exhibitsgoodexplorationexploitationbalanceforeffective searchincomplexsearchspaces. Offersflexibilityinincorporatingdiverseobjectivefunctions andconstraints.	Sensitivitytoparametersettings, requiringcarefultuningforoptimal performance. Limitedscalabilityforlargescaleoptimizationproblemsduetocomputational complexity. Maystrugglewithmultimodaloptimizationproblemsandgettingtrappedinlocal optima. Lackoftheoreticalguaranteesorconvergencepropertiesincertaincases. Dependencyonderivativeinformationrestrictsitsapplicabilitytonon differentiablefunctions.

Table 25 (continued)

Algorithms	Advantages	Disadvantages
Photonsearch algorithm(PSA) [115]	Exploitstheprinciplesofphotonbehaviorforefficientand parallelexplorationofsearchspace. Providesfastconvergenceandrobustnessinfindingoptimal solutions. Supportsbothcontinuousanddiscreteoptimizationproblems. Offerspotentialforparallelizationandscalabilityincomputing environments. Suitableforsolvingcomplexoptimizationproblemsinvarious fields, includingphotonicsandcomputationalphysics.	Sensitivitytoparametersettingsandinitializationconditions, affectingalgorithm performance. Limitedapplicabilitytooptimizationproblemsoutsidethedomainofphotonicsor physics. Requiresspecializedknowledgeandexpertisetoeffectivelyimplementandtune thealgorithm. Limitedtheoreticalunderstandingandlackofestablishedconvergenceguarantees incertaincases. Highercomputationalrequirementscomparedtosimpleroptimizationalgorithms.
Transientsearch optimization(TSO) [116]	Efficientlyexplorestransientstatesfordynamicoptimization problems. Providesfastconvergenceandadaptabilitytochangingproblem conditions. Supportsrealtimedecisionmakingandcontrolindynamic environments. Offersflexibilityinincorporatingtimevaryingconstraints andobjectives.Enablesoptimizationofcomplexsystemswith timedependentvariablesanddynamics.	Sensitivitytoparametersettingsandtuning, impactingalgorithmperformance. Limitedapplicabilitytostaticoptimizationproblemsorsystemswithout transientbehavior. Highercomputationalrequirementscomparedtostaticoptimizationalgorithms. Lackoftheoreticalguaranteesorconvergencepropertiesincertaincases. Challengesinmodelingandcapturingcomplexdynamicsandtransient behaviorsaccurately.
Artificialelectric fieldalgorithm (AEFA) [117]	Efficientlyexploresthesearchspaceusingartificialelectric fieldinspiredmechanisms. Providesfastconvergenceandglobaloptimizationcapabilities. Supportsbothcontinuousanddiscreteoptimizationproblems. Exhibitsgoodbalancebetweenexplorationandexploitationfor effectivesearch. Offersflexibilityinincorporatingdiverseobjectivefunctions andconstraints.	Sensitivetoparametersettings, requiringcarefultuningforoptimalperformance. Limitedscalabilityforlargescaleoptimizationproblemsduetocomputational complexity. Maystrugglewithmultimodaloptimizationproblemsandgettingtrappedin localoptima. Lackoftheoreticalguaranteesorconvergencepropertiesincertaincases. Limitedabilitytohandlenondifferentiableordiscontinuousobjectivefunctions.
Volcanoeruption algorithm(VEA) [118]	Simulatesthedynamicsofvolcanoeruptionstoefficiently explorethesearchspace. Providesfastconvergenceandrobustnessinfindingoptimal solutions. Supportsbothcontinuousanddiscreteoptimizationproblems. Allowsforadaptiveanddynamicexplorationexploitationbalance. Suitableforsolvingcomplexoptimizationproblemsinvarious fields, includinggeophysicsandnaturalresourcemanagement.	Sensitivetoparametersettingsanderuptionsimulationdynamics, impacting algorithmperformance. Limitedscalabilityforlargeoptimizationproblemsduetocomputational complexity. Potentialforprematureconvergenceorgettingtrappedinlocaloptima. Lackoftheoreticalguaranteesorconvergencepropertiesincertaincases. Limitedapplicabilitytooptimizationproblemsoutsidethedomainofvolcano dynamicsorgeophysicalsystems.

Table 25 (continued)

Algorithms	Advantages	Disadvantages
Groupteaching optimization algorithm(GTOA) [119]	Utilizesteachingstrategiesinspiredbygrouplearning dynamicsforefficientoptimization. Offersfastconvergenceandrobustnessinfindingoptimal solutions. Supportsbothcontinuousanddiscreteoptimizationproblems. Provideseffectiveexplorationexploitationbalancethrough collaborationandknowledgesharing. Enablesadaptivelearningandknowledgetransferwithinthe groupforimprovedoptimizationperformance.	Sensitivitytoparametersettingsandgroupdynamics, affectingalgorithm performance. Limitedscalabilityforlargescaleoptimizationproblemsdueto computationalcomplexity. Potentialforsuboptimalsolutionsifgroupdynamicsarenoteffectively managed. Lackoftheoreticalguaranteesorconvergencepropertiesincertaincases. Requirescarefulcoordinationandcommunicationamonggroupmembers, increasingcomplexityandpotentialforinefficiencies.
HyperbolicSineOptimizer(HSO)	Utilizepreciseglobalminimaforthemajorityoftheproblems. Earlyiterationsofhyperbolicfunctionssinhconvergetoglobal optimalsolutions. Duetoitsrobustmathematicalmodel, itcanhandlecomplicated optimizationissues. Thesolutionsareaccurateandhaveahighprecisionfactor. Capableofbalancingtheexplorationandexploitationstages. Goodperformance. Easyimplementation	Parametertuningcomplexity. Difficultyinhandlingdynamicorchangingproblemconditions. Performanceheavilyinfluencedbyrandomization. Sensitivitytoproblemrepresentation.

Table 26 Comparison table of HSO for pressure vessel design

Algorithm	Optimum variables				Optimal cost
	T_s	T_h	R	L	
HSO	1.1828	1.259E−06	61.2858	28.186	3097.68
TSA	0.83037	0.41621	42.7513	169.345	6048.78
GSA	1.0858	0.94961	49.3452	169.487	11,550.3
GA	1.09952	0.90658	44.4564	179.659	6550.02
TDO	0.77805	0.38604	40.3136	199.984	5887.18
GWO	0.84572	0.41856	43.8163	156.382	6011.52
PSO	0.75236	0.39954	40.4525	198.003	5890.33
MPA	0.77904	0.38466	40.3278	199.65	5889.37
TLBO	0.81758	0.41793	41.7494	183.573	6137.37
WOA	0.77896	0.38468	40.3209	200	5891.39

Table 27 Statistical analysis of HSO for pressure vessel design

Algorithm	Best	Mean	Worst	SD	Median
HSO	3097.68	3625.88	4234.56	237.95	3650.602
TSA	6048.78	6052.62	6071.25	2.893	6050.23
GSA	11,550.3	23,342.3	33,226.3	5790.63	24,010
GA	6550.02	6643.99	8005.44	657.523	7586.01
TDO	5887.18	5890.02	5892.2	1.0215	5888.91
GWO	6011.52	6477.31	7250.92	327.007	6397.48
PSO	5890.33	6264.01	7005.75	496.128	6112.69
MPA	5889.37	5891.53	5894.62	13.91	5890.65
TLBO	6137.37	6326.76	6512.35	126.609	6318.32
WOA	5891.39	6531.5	7394.59	534.119	6416.11

Tables 30 and 31 respectively. Then, compare it to seven additional metaheuristics algorithms. The results indicate that HSO performed exceptionally well and placed first, while TDO and TSA placed second and third, respectively, due to their optimal cost. Statistical and parameter graphs are depicted in Figs. 11 and 12, respectively. The convergence curve of HSO after 500 iterations is depicted in Fig. 10.

7.1.4 Speed reducer design

The Speed reducer Design Problem [123], which is a discrete problem, has four design constraints: the bending stress of the gear teeth, the covering stress, the transverse deflections of the shafts, and the stresses in the shafts. The

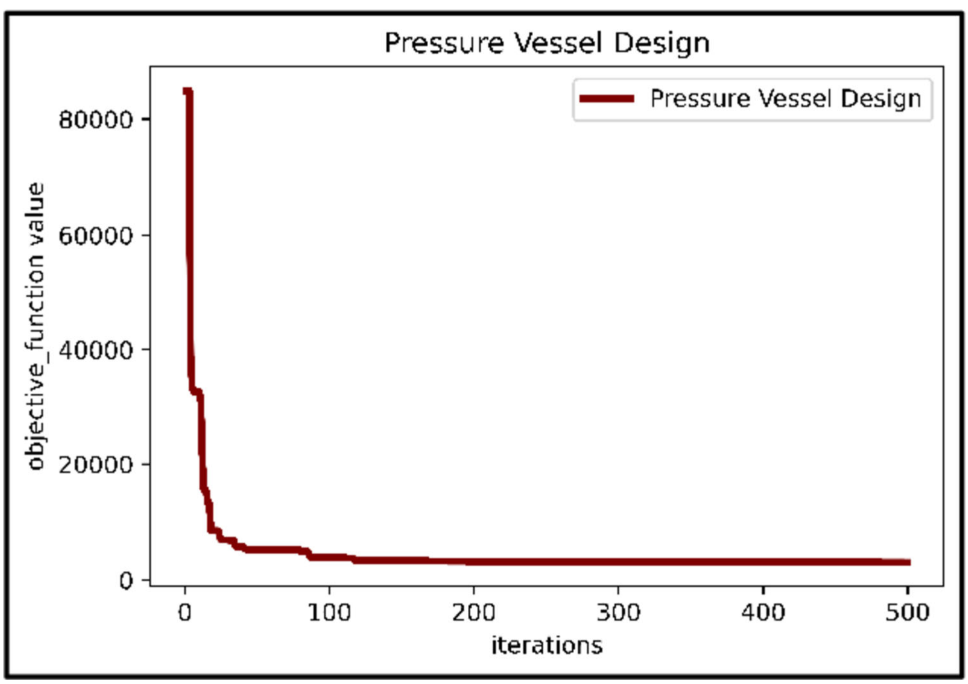


Fig. 4 Convergence curve of HSO for Pressure Vessel Design

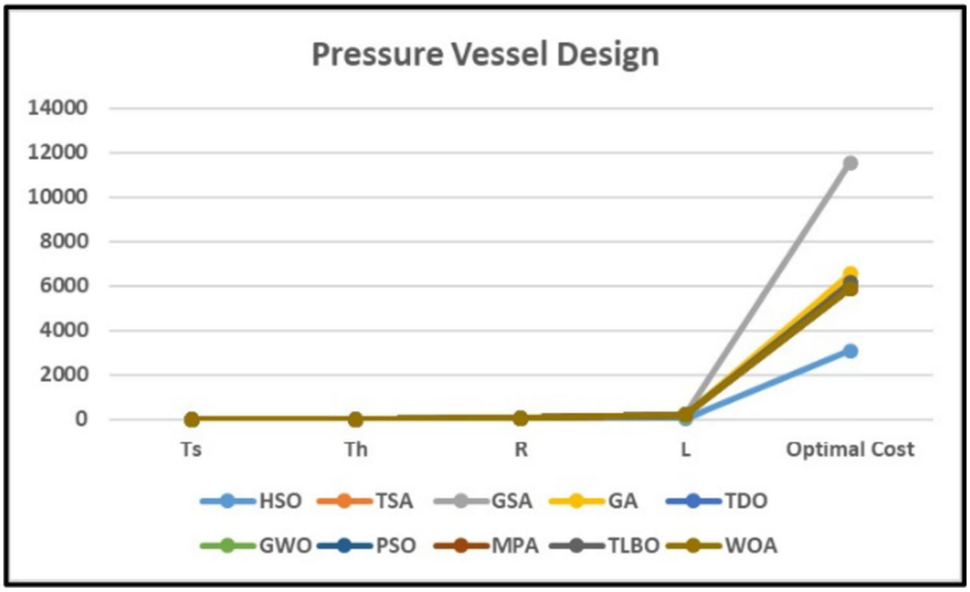


Fig. 5 Parameters graph of HSO for Pressure Vessel Design

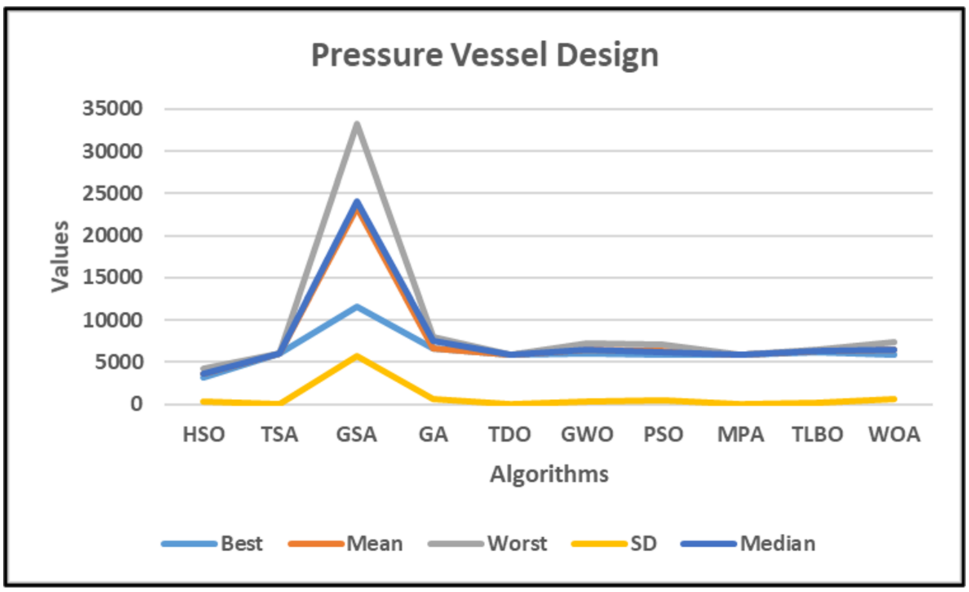


Fig. 6 Statistical graph of HSO for pressure vessel design

main goal of the problem is to determine the minimum weight of the speed reducer to satisfy these constraints. As a result, six continuous variables and one discrete variable are detected. In this instance, x_1 represents the face width, x_2 the module of teeth, and x_3 is a discrete design variable that displays the pinion's teeth. Similar to this, x_4 represents the distance between the first and second shafts' bearings, and x_5 represents that distance. The diameters of the first and second shafts, x_6 and x_7, respectively, are the sixth and seventh design variables.

The mathematical model of this problem are as follows: Consider: $X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]$

$X = [b, m, p, l_1, l_2, d_1, d_2]$

Minimize: $f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$

Subject to: $g_1(x) = \frac{27}{x_1x_2^2x_3} - 1 \leq 0,$

$$g_2(x) = \frac{397.5}{x_1x_2^2x_3} - 1 \leq 0,$$

$$g_3(x) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \leq 0,$$

$$g_4(x) = \frac{1.93x_5^3}{x_2 x_3 x_7^4} - 1 \le 0,$$

$$g_5(x) = \frac{1}{110x_6^3}\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9.10^6} - 1 \le 0,$$

$$g_6(x) = \frac{1}{85x_7^3}\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2 + 157.5.10^6} - 1 \le 0,$$

$$g_7(x) = \frac{x_2x_3}{40} - 1 \le 0,$$

$$g_8(x) = \frac{5x_2}{x_1} - 1 \le 0,$$

$$g_9(x) = \frac{x_1}{12x_2} - 1 \le 0,$$

$$g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \le 0,$$

$$g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0$$

$$2.6 \le x_1 \le 3.6, 0.7 \le x_2 \le 0.8, 17 \le x_3 \le 28, 7.3 \le x_4 \le 8.3, 7.8 \le x_5 \le 8.3, 2.9 \le x_6 \le 3.9, 5 \le x_7 \le 5.5$$

Speed Reducer Design is yet another constrained real-world engineering design problem (Fig. 13). This problem falls under the CEC 2020 benchmarking suit. This

Table 28 Comparison table of HSO for welded beam design

Algorithms	Optimum variables				
	H	l	T	b	Optimum cost
HSO	0.125	0.1085	0.359	0.127	0.03293
GSA	0.1471	5.49074	10	0.21773	2.17286
MPA	0.20568	3.4754	9.03696	0.20623	1.727
GA	0.20649	3.63587	10	0.20325	1.83625
TDO	0.20573	3.47052	9.0366	0.20573	1.7249
WOA	0.19741	3.31506	10	0.2014	1.8204
PSO	0.16417	4.03254	10	0.22365	1.87397
GWO	0.20561	3.4721	9.04093	0.20571	1.72547
TSA	0.20556	3.47485	9.0358	0.20581	1.72566
TLBO	0.2047	3.53629	9.00429	0.21003	1.75917

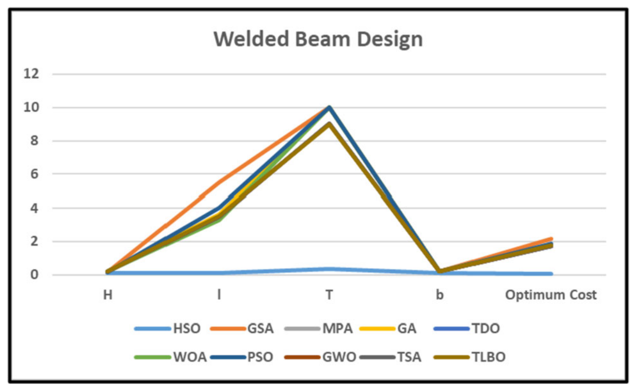
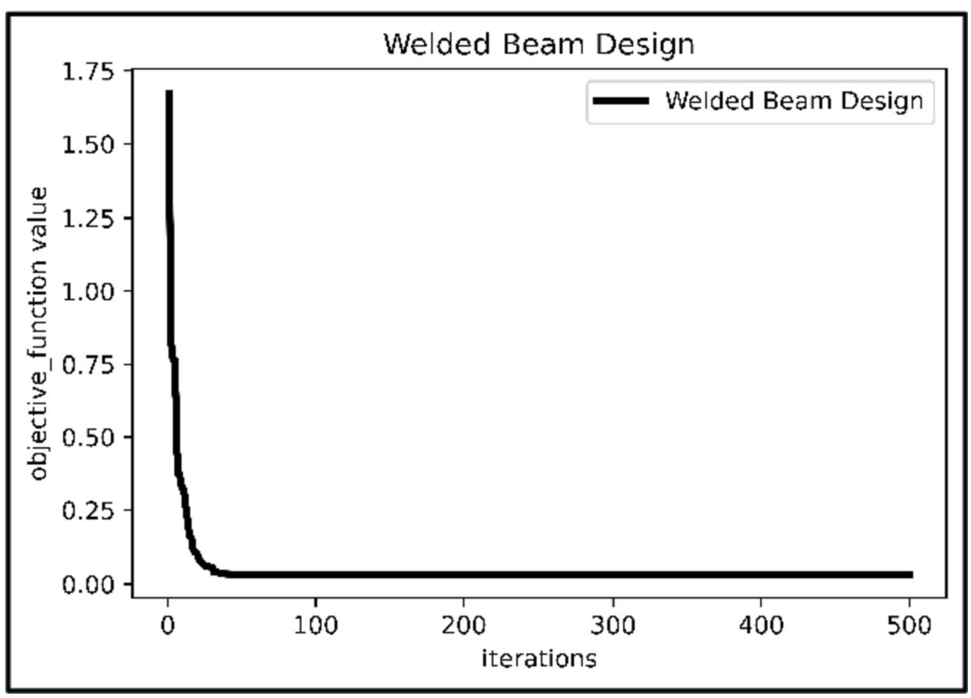


Fig. 8 Parameters graph of HSO for Welded beam design

Fig. 7 Convergence curve of HSO for Welded beam design



problem is recognized due to its complex nature, complicated search space, and high dimensionality with constraints. This problem is solved using HSO, and the experimental results are shown in Tables 32 and 33 respectively. The experimental results show that HSO performs marginally well in this problem. TDO and TSA outperforms and were ranked first and second, respectively. The statistical and parameters graph of HSO with other algorithms are shown in Figs. 14 and 15, respectively, while the convergence curve of HSO is shown in Fig. 13.

7.1.5 3 Bar truss design

The goal of this engineering design problem is to build a truss [121] with three bars to save weight. The search space for this problem is extremely constrained.

Table 29 Statistical analysis of HSO for welded beam design

Algorithms	Best	Mean	Worst	SD	Median
HSO	0.0329	0.03463	0.0386	0.0015	0.034083
GSA	2.17286	2.54424	3.00366	0.25586	2.49511
MPA	1.727	1.72713	1.72756	0.00116	1.72709
GA	1.83625	1.36353	2.03525	0.13949	1.93575
TDO	1.7249	1.72521	1.7254	1.00E−05	1.72518
WOA	1.8204	2.23031	3.04823	0.32453	2.24466
PSO	1.87397	2.11924	2.32013	0.03482	2.09705
GWO	1.72547	1.72968	1.74165	0.00487	1.72742
TSA	1.72566	1.72583	1.72606	0.00029	1.72579
TLBO	1.75917	1.81766	1.87341	0.02754	1.82013

The mathematical model of this problem are as follows:
Consider: $[x_1, x_2]$,
Minimize: $f(x) = \left(2\sqrt{2}x_1 + x_2\right) \times L$,
Subject to:

$$g_1 = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2}P - \sigma \leq 0,$$

$$g_2 = \frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2}P - \sigma \leq 0,$$

$$g_3 = \frac{1}{x_1 + \sqrt{2}x_2}P - \sigma \leq 0,$$

$$where 0 \leq x_1 \leq 1. The\ constants\ are\ L = 100cm, P = \frac{2KN}{cm^2} and \sigma = \frac{2KN}{cm^2}.$$

The comparison table of HSO with various algorithms are as follows:

3 Bar truss is another CEC 2020 real-world constrained optimization problem benchmarking suit. Table 34 shows the comparison results of HSO for the three Bar truss problems. HSO slightly outperforms other competing algorithms in this case. DEDS and SSA performed similarly and were ranked first, with MBA and PSO-DE ranking second and third. In this category, HSO was ranked sixth, with a slight advantage over the others in determining the optimal weight. Figures 16 and 17 depicts a convergence and parameters graph of the threE−bar truss design problem respectively.

7.1.6 Optimal gas production facilities

Since the consumption of gas products is so great today in the actual world, it is imperative to upgrade the facilities

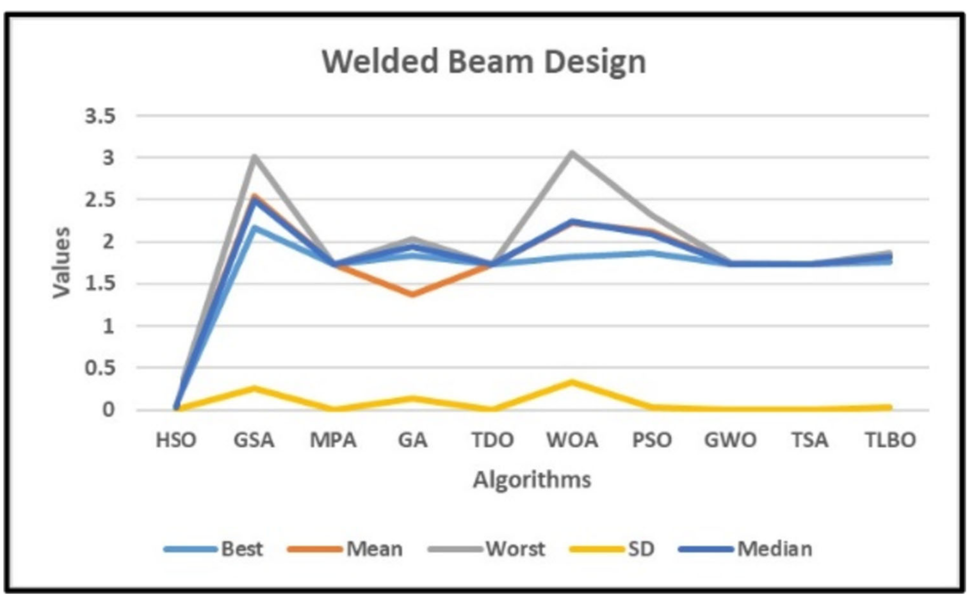


Fig. 9 Statistical graph of HSO for Welded beam design

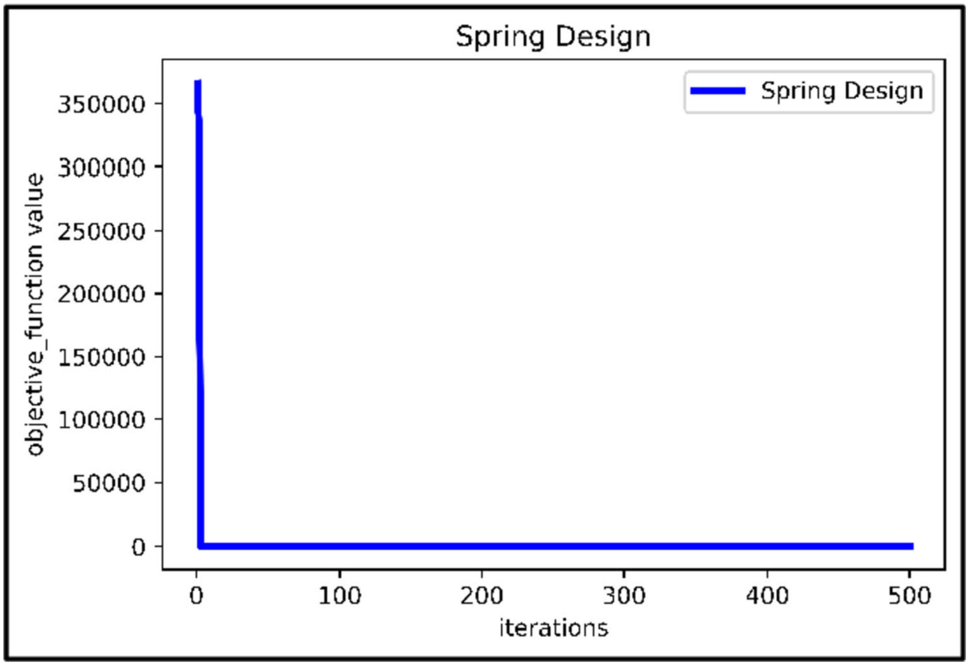


Fig. 10 Convergence curve of HSO for tension compression spring design

Table 30 Comparison table of HSO for tension compression spring design

Algorithm	Optimum variables			Optimum cost
	d	D	P	
HSO	0.0535	0.40118	9.17479	0.01286
WOA	0.05	0.31041	15	0.01319
PSO	0.0501	0.31011	14	0.01304
TDO	0.0518	0.35938	11.1509	0.01267
GA	0.05025	0.31635	15.2396	0.01278
TLBO	0.05078	0.33478	12.7227	0.01271
TSA	0.05114	0.34375	12.0955	0.01267
GSA	0.05	0.31731	14.2287	0.01287
MPA	0.05018	0.34154	12.0735	0.01268
GWO	0.05	0.31596	14.2262	0.01282

Table 31 Statistical analysis of HSO for tension compression spring design

Algorithm	Best	Mean	Worst	SD	Median
HSO	0.01286	0.01531	0.01858	0.00142	0.015196
WOA	0.01319	0.01482	0.01786	0.00227	0.01319
PSO	0.01304	0.01404	0.01625	0.00207	0.013
TDO	0.01267	0.01268	0.0127	2.00E−05	0.01268
GA	0.01278	0.01307	0.01521	0.00038	0.01295
TLBO	0.01271	0.01284	0.013	7.80E−05	0.01285
TSA	0.01267	0.01268	0.01272	2.70E−05	0.01269
GSA	0.01287	0.01344	0.01421	0.00029	0.01337
MPA	0.01268	0.0127	0.01272	4.10E−05	0.0127
GWO	0.01282	0.01446	0.01784	0.00162	0.01402

for the production of gas. It is quite challenging to supply the most facilities required for gas production [124] everywhere because there are many locations where they cannot be. Therefore, determining the ideal production facility capacity that joins to form an oxygen producing and storing system is an optimization problem.

This optimization problem can be modelled mathematically as:

Consider: $X = [X_1, X_2]$

Minimize: $F(x) = 61.8 + 5.72x_1 + 0.2623 \quad [(40 - x_1)\, In\left(\frac{x_2}{200}\right)]^{-0.85} + 0.087(40 - x_1)\, In\left(\frac{x_2}{200}\right) + 700.23{x_2}^{-0.75}$

Subject to: $x_1 \geq 17.5, x_2 \geq 300$

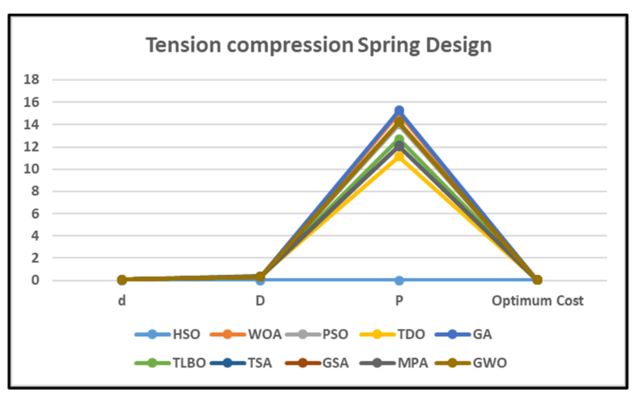


Fig. 11 Parameters graph of HSO for tension compression spring design

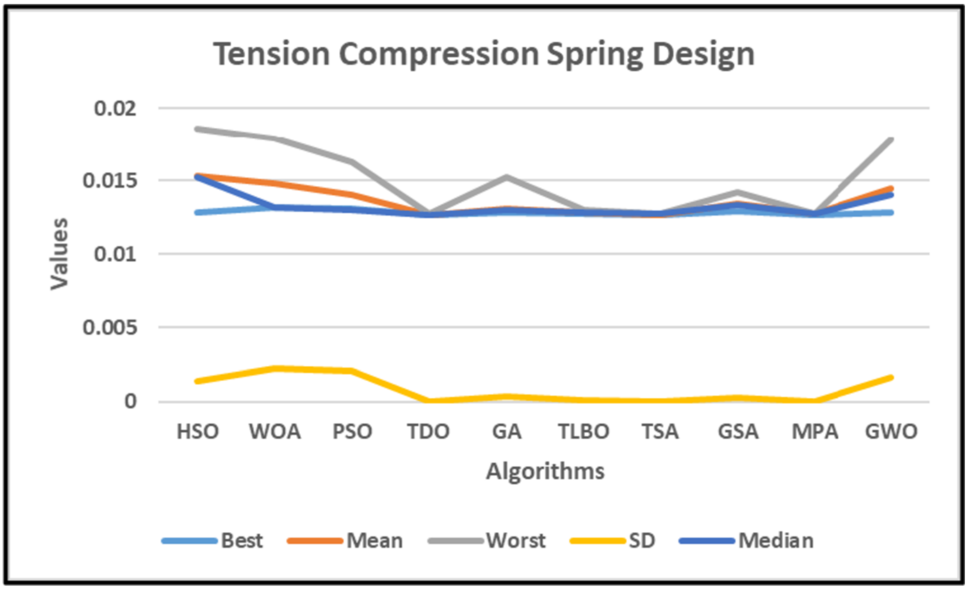


Fig. 12 Statistical graph of HSO for tension compression spring design

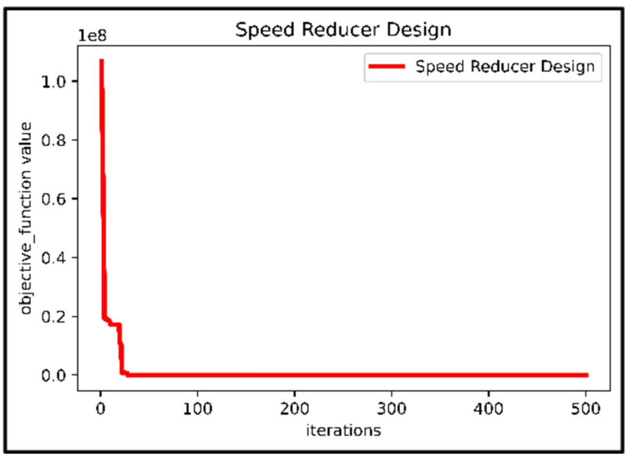


Fig. 13 Convergence curve of HSO for speed reducer design

Table 33 Statistical analysis of speed reducer design

Algorithms	Best	Mean	Worst	SD	Median
HSO	3099.01	3247.87	3556.49	98.76	3217.182
MPA	3001.29	3005.85	3008.75	5.83794	3004.52
GSA	3051.12	3170.33	3363.87	92.5726	3156.75
TDO	2996.35	2997.81	2999.61	1.1642	2997.02
PSO	3067.56	3186.52	3313.2	17.1186	3198.19
GWO	3002.93	3028.84	3060.96	13.0186	3027.03
GA	3029	3295.33	3619.47	57.0235	3288.66
TSA	2998.55	2999.64	3003.89	1.93193	2999.19
TLBO	3030.56	3065.92	3104.78	18.0742	3065.61
WOA	3005.76	3105.25	3211.17	79.6381	3105.25

Table 32 Comparison table of HSO for Speed reducer design

Algorithm	Optimum variables							Optimum cost
	b	m	p	l_1	l_2	d_1	d_2	
HSO	3.53317	0.706	17.5	7.7821	8.082	3.4016	5.288	3099.01
MPA	3.50669	0.7	17	7.38093	7.81573	3.35785	5.28677	3001.29
GSA	3.6	0.7	17	8.3	7.8	3.36966	5.28922	3051.12
APSO	3.50131	0.7	18	8.12781	8.04212	3.35245	5.28708	3187.63
TDO	3.5	0.7	17	7.3	7.8	3.35021	5.28668	2996.35
PSO	3.51025	0.7	17	8.35	7.8	3.3622	5.28772	3067.56
GWO	3.5085	0.7	17	7.39284	7.81603	3.35807	5.28678	3002.93
GA	3.52012	0.7	17	8.37	7.8	3.36697	5.28872	3029
TSA	3.50159	0.7	17	7.3	7.8	3.35127	5.28874	2998.55
TLBO	3.50876	0.7	17	7.3	7.8	3.46102	5.28921	3030.56
WOA	3.50002	0.7	17	8.3	7.8	3.35241	5.28672	3005.76

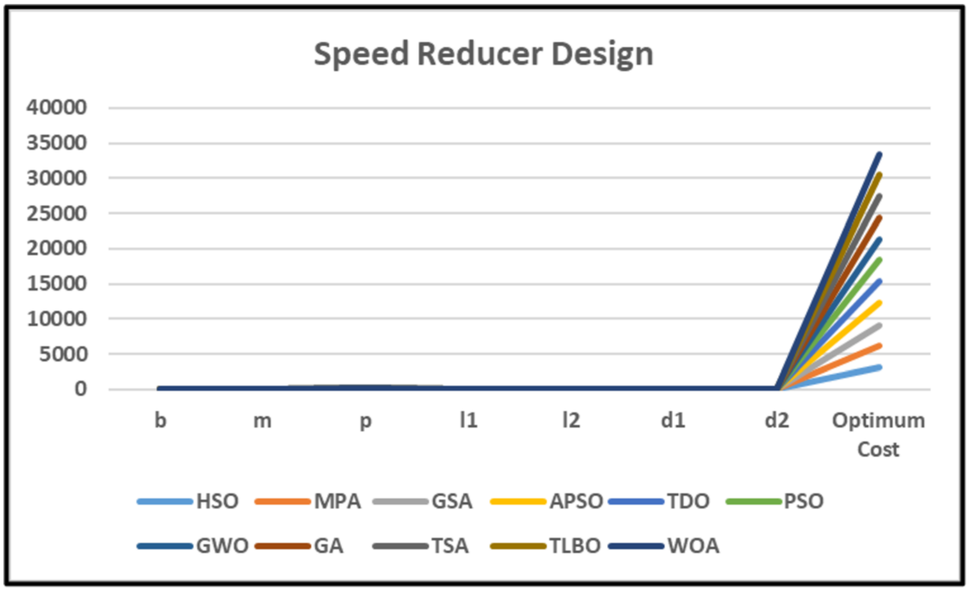


Fig. 14 Parameters graph of HSO for speed reducer design

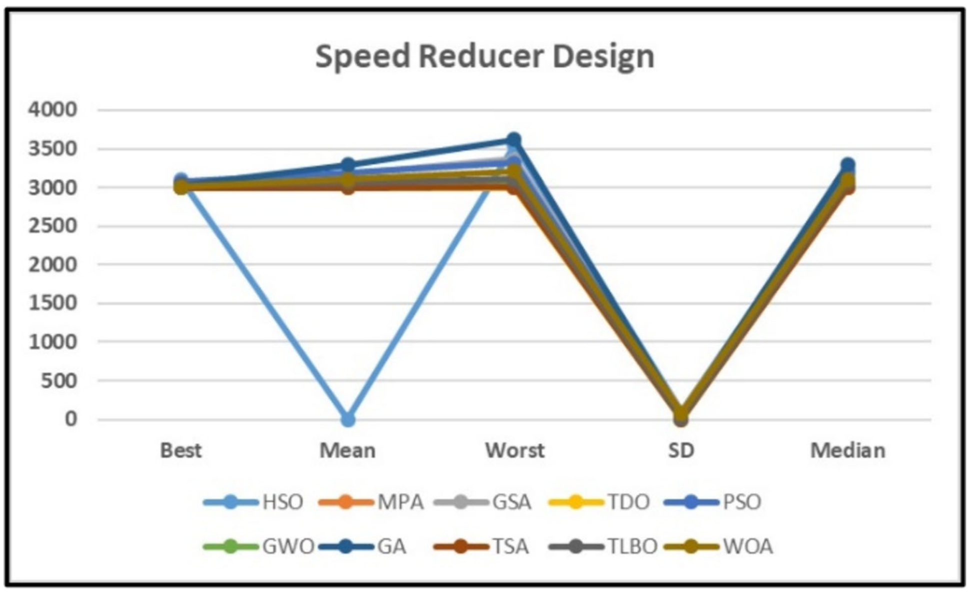


Fig. 15 Statistical graph of HSO for speed reducer design

Bounds: $17.5 \leq x_1 \leq 40, 300 \leq x_2 \leq 600$

Determining the optimal capacity of gas production facilities is another IEEE CEC-2020-based constrained optimization problem. The HSO has been evaluated for this issue, and it outperformed four other algorithms in terms of performance (Fig. 18). With only 120 evaluations, HSO received the highest score and ranked first, followed by ALO and DE, in that order. The outcomes of a comparison between HSO and other algorithms for this problem, as well as the parameters graph, are displayed in Table 35 and Fig. 19, respectively. The convergence curve of this problem is illustrated in Fig. 18.

Table 34 Comparison table of HSO for 3 Bar truss design

Algorithm	Optimal value for variables		Optimal weight
	X_1	X_2	
HSO	0.797023	0.38515	263.915
DEDS	0.788675	0.40824828	263.8958
CS	0.78867	0.40902	263.9716
PSO-DE	0.788675	0.4082482	263.8958
AOA	0.79369	0.39426	263.9154
Ray and Sain	0.795	0.395	264.3
SSA	0.788665	0.408275784	263.8958
TSA	0.788	0.408	263.68
MBA	0.788565	0.4085597	263.8959

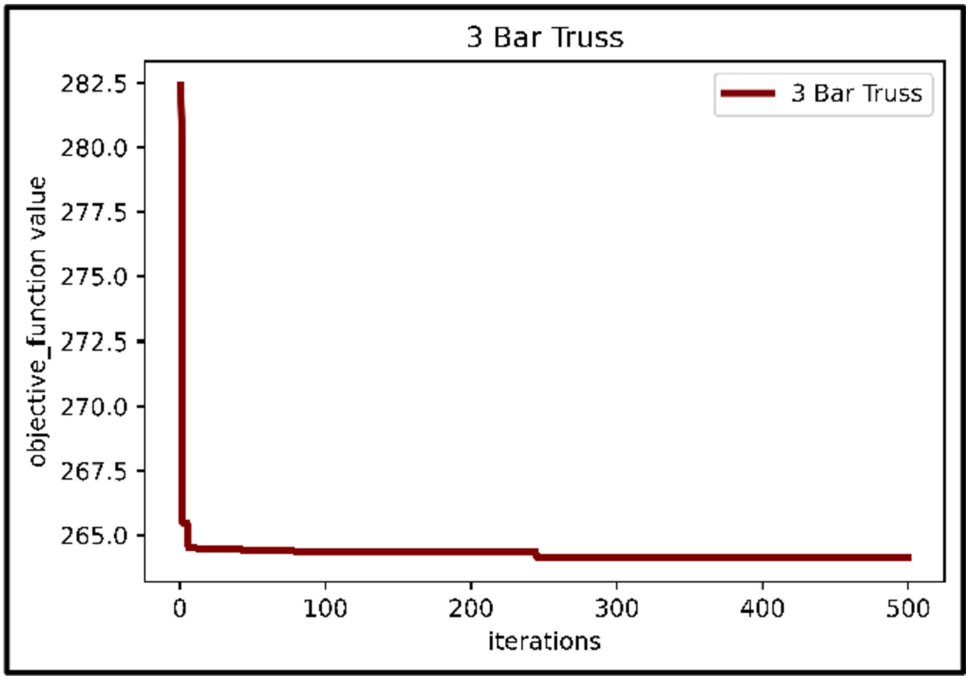


Fig. 16 Convergence curve of HSO for 3 bar truss design

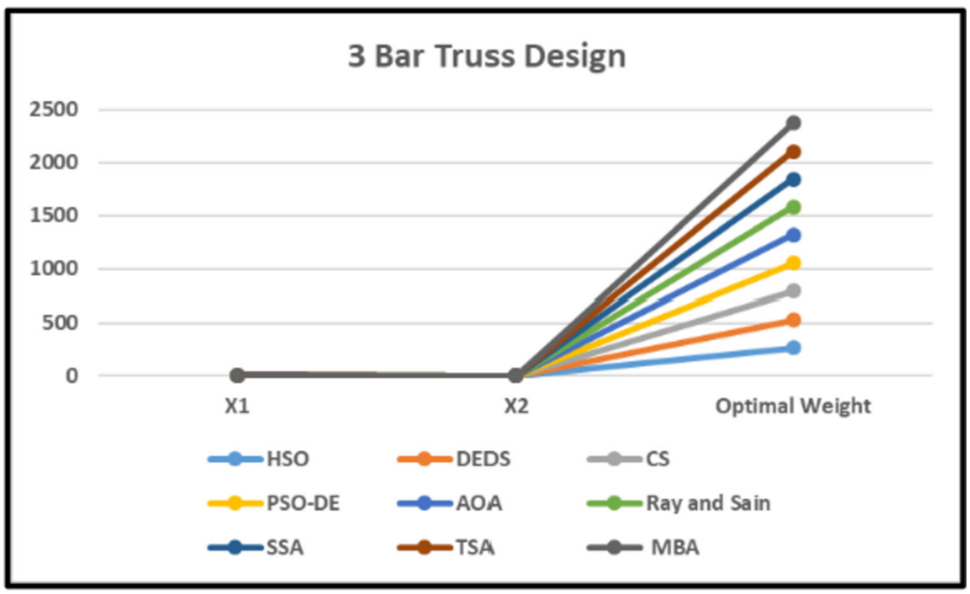


Fig. 17 Parameters graph of HSO for 3 bar truss design

7.2 Unconstrained real-world engineering design problems

7.2.1 Gear train design problem

Cost minimization of the gear ratio of a complex gear train [125] is the goal of this unconstrained discrete optimization gear train design problem. The definition of the gear ratio is:

$$Gear_{ratio} = \frac{angular\ velocity\ of\ the\ output\ shaft}{angular\ velocity\ of\ the\ input\ shaft}$$

Four gears make up the gear design problem, and there should be less of a discrepancy between the necessary gear ratio (1/6.931) and the actual gear ratio. The total number of gearwheel teeth is the number of variables. Therefore, the goal is to determine the four-gear train's best tooth configuration to reduce the gear ratio.

The mathematical model of the problem is:

Consider:$X = [X_1, X_2, X_3, X_4] = [n_1, n_2, n_3, n_4]$,

Minimize: $f(X) = \left(\frac{1}{6.931} - \frac{X_3 X_2}{X_1 X_4}\right)^2$,

Subject to: $12 \le X_1, X_2, X_3, X_4 \le 60$,

The gear train design problem is an unconstrained real-world engineering design problem (Fig. 20). Table 36 shows the results of testing HSO to solve this problem. For this problem, HSO is compared to eight other competing algorithms. HSO excelled in this category, finishing first out of 500 total evaluations. ALO and CS were ranked second and third, respectively. Figure 21 shows a parameters graph of HSO with other algorithms while Fig. 20 represents convergence curve of HSO of this problem.

7.2.2 Parameter estimation for frequency–modulated (FM) sound waves.

Parameter estimation for Frequency-Modulated (FM) [126] sound waves one of the most challenging problem. The most challenging factors of today's modern music systems are the FM sound wave synthesis. Its play's a significant role for sound systems. To optimize the FM synthesizer parameter, the major issue is that it contains six dimensions. The six parameters vector of X are as follows:

$X = [a_1, \omega_1, a_2, \omega_2, a_3, \omega_3]$. The following equation is used to optimize a sound wave using this vector. This is one of the most difficult problems since it involves a multimodal problem with a strong mathematical concept. This problem is optimized by various Nature Inspired optimization algorithms. This problem contains the lowest value $f\left(\overrightarrow{X_{sol}}\right) = 0$. The following is the mathematical model for this problem:

$$y(t) = a_1.\sin(\omega_1.t.\theta + a_2.\sin(\omega_2.t.\theta + a_3.\sin(\omega_3.t.\theta)))$$

$$y_0(t) = (1.0).\sin((5.0).t.\theta - (1.5).\sin((4.8).t.\theta + (2.0).\sin((4.9).t.\theta)))$$

Here $\theta = 2\pi/100$ and the parameters are defined in the range of $[-6.4, 6.35]$ and the cost functions is calculated by using the sum of the square errors between the estimate wave and the target wave as follows:

$$f\left(\overrightarrow{X}\right) = \sum_{i=0}^{100} (y(t) - y_0(t))^2$$

Frequency modulation sound waves represent an unrestricted engineering design problem in the real world. Using HSO to resolve this issue would yield marginal results at best. HSO achieved satisfactory results and ranked fifth in the minimization problem, whereas GWO and MFO achieved outstanding results and ranked first and second, respectively. The results of all algorithms discovered for this problem are summarised in Table 37. Figures 22 and 23 depict the parameters graph and convergence curve for this problem, respectively.

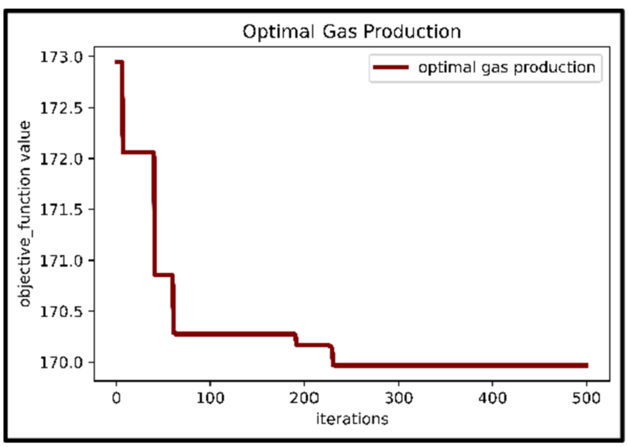


Fig. 18 Convergence curve of HSO for optimal gas production design

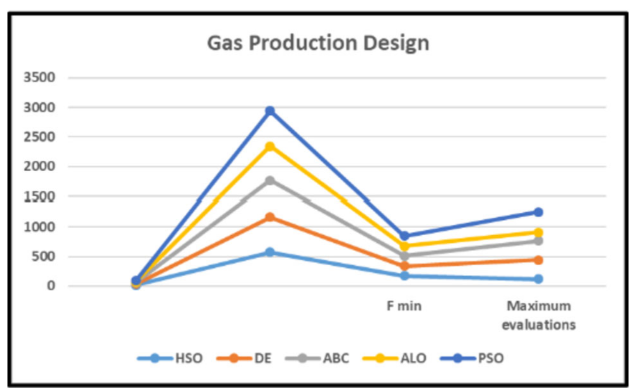


Fig. 19 Parameters graph of HSO for optimal gas production design

Table 35 Comparison table of HSO for Gas production design

Algorithm	Optimal values of variables		F min	Maximum evaluations
HSO	17.549	573.086	169.82	120
DE	17.5	593	169.996	324
ABC	17.5	600	169.012	319
ALO	17.5	575.22	169.947	150
PSO	17.5	600	169.844	342

8 Effect of the proposed algorithm (HSO) on training multilayer perceptron

The proposed MSCA is put to use for the purposes of multilayer perceptron training within this section (MLP). The term "MLP" refers to the neural networks that only have a single hidden layer and connections that only go in one direction between their neurons. Additionally, MLPs only have one hidden layer. In these MLPs, the layer that comes first is known as the input layer, and the layer that comes last is known as the output layer. After the weights, inputs, and biases have been provided, the following steps need to be taken in order to calculate the outputs of MLP:

1. The weighted sum of inputs is first calculated as follows:

$$s_j = \sum_{i=1}^{n} W_{i,j}X_i - \theta_j, j = 1,2,\ldots.,h$$

$W_{i,j}$ represents the connection weights from the i^{th} input node to the j^{th} node in the hidden layer; x_i is the i^{th} input; and θ_j is the threshold or bias of the j^{th} node in the hidden layer.

2. Each node's output in the hidden layer is calculated as follows:

$$S_j = \frac{1}{1+e^{-s_j}} = sigmoid(s_j), j = 1,2,..,h$$

3. Ultimately, the final output can be calculated using the outputs of the hidden layer nodes.

$$o_k = \sum_{j=1}^{h} W_{j,k}S_j - \theta_k\prime, k = 1,2,\ldots,m$$

$$o_k = \frac{1}{1+e^{-o_k}} = sigmoid(o_k), k = 1,2,\ldots,m$$

where $W_{j,k}$ indicate the connection weight from the j^{th} hidden node to the k^{th} output node, $\theta_k\prime$ is the threshold or bias of the k^{th} output node. It is evident that the biases and weights used in an MLP model are what ultimately decides what the model produces in response to a set of inputs. Since training MLP's goal is to achieve highest classification accuracy for both training and testing samples. A common metric, which

Table 36 Comparison table of HSO for Gear Train design problem

Algorithms	Optimal values for variables				f_{min}	Max. eval
	η_1	η_2	η_3	η_4		
HSO	41.28	18.55	17.13	53.36	0	500
MBA	43	16	19	49	2.70E−12	10,000
GA	N/A	N/A	N/A	N/A	2.33E−07	10,000
DE	33	14	17	50	1.36E−09	N/A
CS	43	16	19	49	2.70E−12	5000
ISA	N/A	N/A	N/A	N/A	2.70E−12	200
ALM	33	15	13	41	2.15E−08	N/A
ALO	49	19	16	43	2.70E−12	120
ABC	19	16	44	49	2.78E−11	40,000

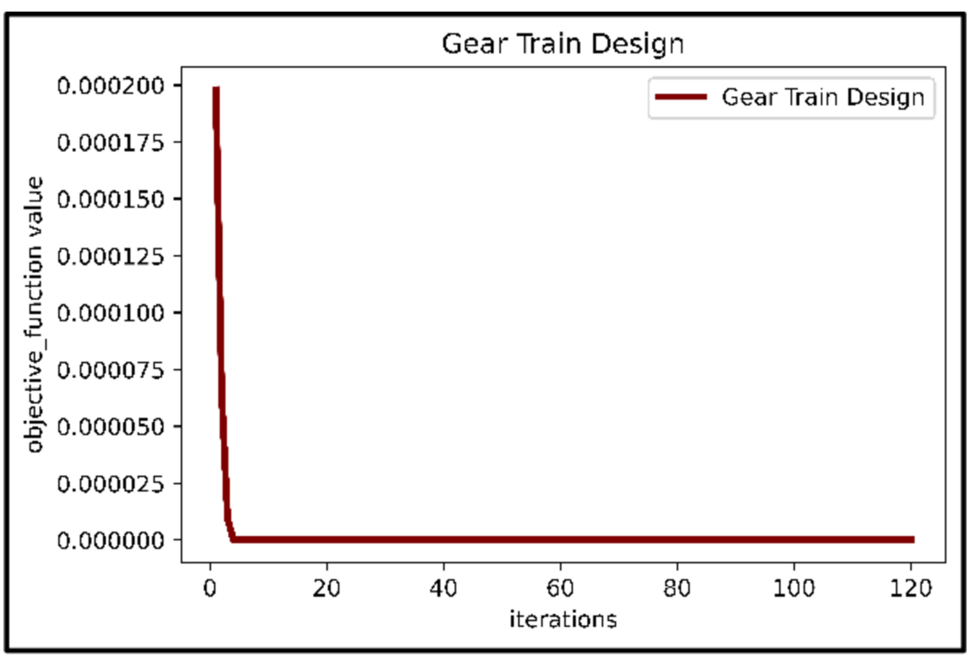


Fig. 20 Convergence curve of HSO for Gear train design problem

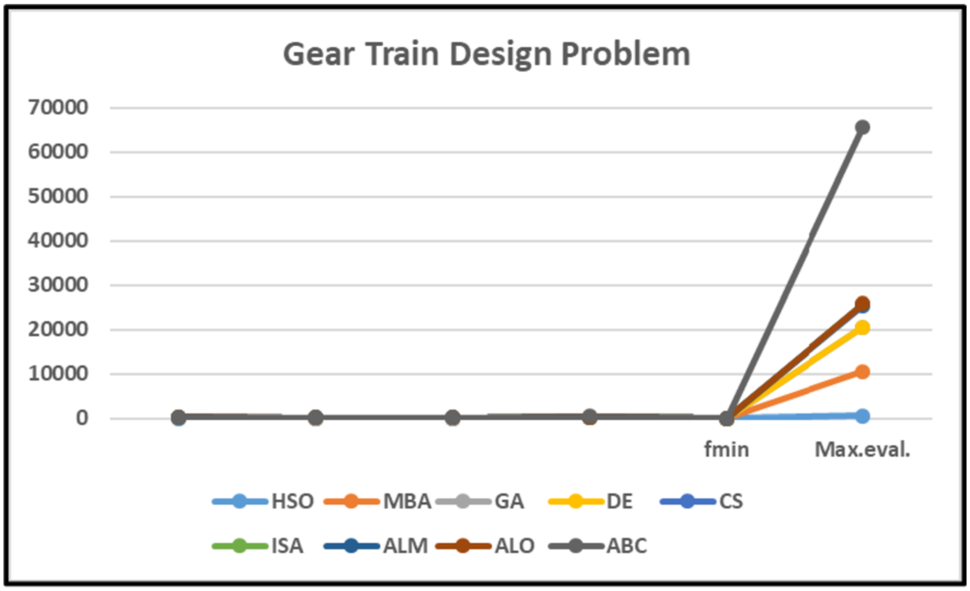


Fig. 21 Parameters graph of HSO for Gear train design problem

is used to evaluate the MLP is Mean Square Error (MSE) and is defined as follows:

$$MSE = \sum_{i=1}^{m} (o_i^k - d_i^k)^2$$

The desired output for the i^{th} input is denoted byd_i^k, and the actual output, o_i^k, is given when the i^{th} training sample appears in the input (where m is the total number of outputs). The MLP's efficacy can be tested by ensuring it optimises itself to produce the smallest mean squared error (MSE) across all training samples. Consider the following definition for the mean squared error,F:

$$F = \frac{\sum_{k=1}^{N} \sum_{i=1}^{m} (o_i^k - d_i^k)^2}{N}$$

Here N denotes the number of training samples. Thus, during the training of MLP, the task is to minimize the objective function F given above with the decision variables called weights and biases.

The number of samples used for training, denoted here by. Training an MLP entails, therefore, minimizing the objective function F with the decision variables weights and biases (Fig. 24).

In this section, the effectiveness of the proposed HSO for training MLP is evaluated through the utilization of four different classification data sets, namely XOR, balloon, iris, and breast cancer. These data sets were obtained from the Machine Learning Repository at the University of California, Irvine (UCI). The experimental

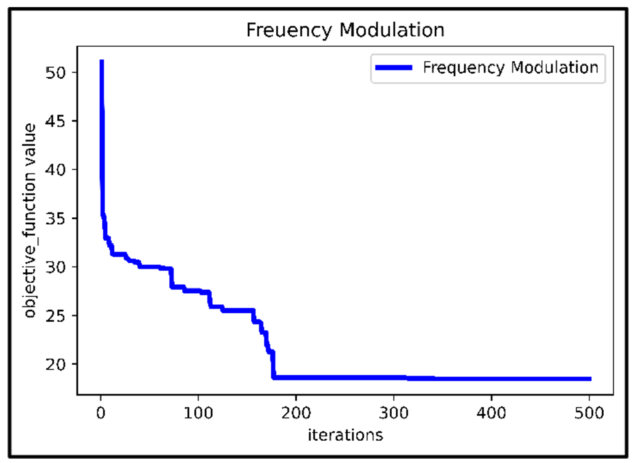


Fig. 22 Convergence curve of HSO for frequency modulation sound waves

results are compared with eight popular competitor metaheuristics algorithm such as Sine Cosine Algorithm (SCA) [52], Equilibrium Optimizer (EO) [51], Whale Optimization Algorithm (WOA) [14], Grey Wolf Optimizer (GWO) [13], Moth Flame Optimization (MFO) [107], Arithmetic Optimization Algorithm (AOA) [68], Particle Swarm Optimization (PSO) [127], Multiverse Optimizer (MVO) [128]. In this scenario, the decision variables are constrained to a range of [10, 10]. For the XOR and Balloon dataset, the population size of all algorithms is set to 50, while for all other datasets, it is set to 200. Each algorithm has a hard limit of 250 iterations. Table 38 provides information about the datasets. The number of hidden nodes is assumed to be 2N+1 in this work, where N is the

Table 37 Comparison table of HSO for parameter estimation of frequency modulation sound waves

Algorithms	MFO	PSO	GWO	TSA	HSO
a_1	0.6141	− 0.5886	− 0.6654	0.3415	0.8116
ω_1	0.0432	5.0145	− 0.1684	4.7881	− 0.2362
a_2	− 4.3251	− 3.2779	1.5173	1.4309	− 1.3609
ω_2	4.7923	− 4.9324	− 0.1287	0.1158	− 2.5174
a_3	0.8339	− 0.8562	− 4.1335	0.0975	0.7694
ω_3	0.1278	− 0.1476	− 4.8997	0.548	2.818901
Minimum Cost	11.8969	13.1807	8.4725	25.1052	10.07061

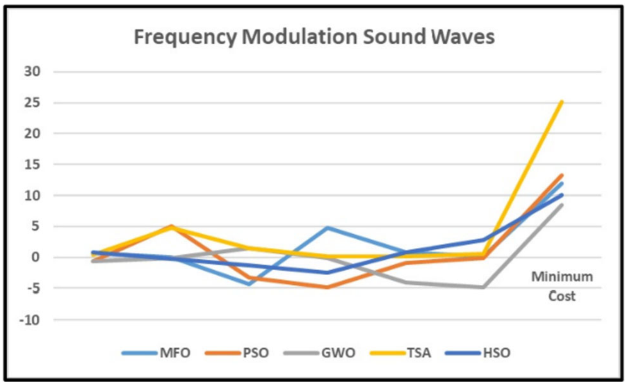


Fig. 23 Parameters graph of HSO for frequency modulation sound waves

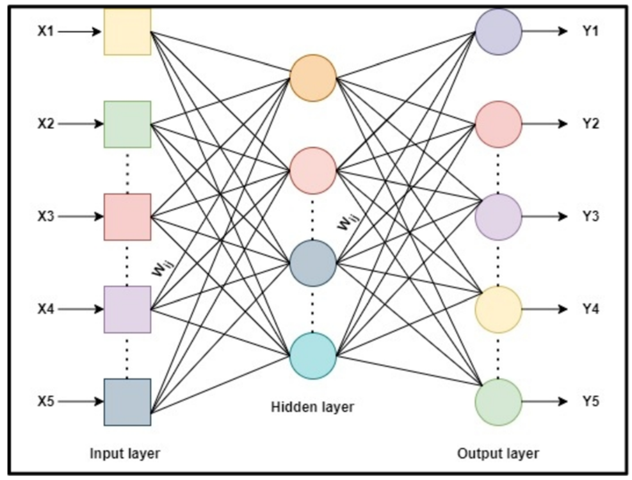


Fig. 24 Training of multilayer perceptron

total number of inputs to the dataset. Table 39 displays the mean and standard deviation value of the objective function F (average of MSE) based on 10 replicate runs of each algorithm on each dataset. Classification success rates for all used algorithms are summarized in the same table. The proposed HSO outperforms the other classical meta heuristics algorithms in terms of MSE, standard deviation, classification rate, and statistical outcomes, suggesting that it is an effective MLP trainer. The HSO's enhanced capability to avoid the local optima during the search procedure is the root cause of the reduced MSE. Thus, the proposed HSO is preferable to other classical metaheuristics for use in practical optimization tasks like MLP training, as shown by the comparative study.

9 Conclusion and future scope

In summary, this research presents the Hyperbolic Sine Optimizer (HSO), an innovative meta-heuristic algorithm developed to address scientific optimization issues. What distinguishes HSO from traditional approaches is its emphasis on mathematical principles, particularly the investigation of algebraic and hyperbolic *sinh* function in the context of population dynamics. This study stands out by uniquely focusing on the behavioral patterns linked to hyperbolic function convergence in both exploration and exploitation phases, thereby promoting the active involvement of population members and improving overall efficiency.

The conducted experiments, spanning 23 standard and 42 varied benchmark functions, including prominent IEEE CEC-2015 and CEC-2017 benchmarks, demonstrate the superior performance of HSO. Notably, HSO outperforms other metaheuristics for unimodal functions in 30 dimensions and achieves exact global optima for specific functions across various dimensions. The stability and predictability of HSO are further affirmed in results for 100, 500, and 1000 dimensions. In addressing multimodal functions, where local stagnation avoidance is crucial,

Table 38 Details of the datasets

Datasets	No. of attributes	No. of training samples	No. of test samples	Number of classes
3-bit-XOR	3	8	8	2
Balloon	4	16	16	2
Iris	4	150	150	3
Cancer	9	599	100	2

Table 39 Experimental results for different datasets

Dataset	Algorithm	MSE		Classification rate
		Mean	STD	
Iris	HSO	1.54E−01	1.67E−02	98.3
	SCA	1.79E−01	2.97E−02	60
	EO	2.31E−02	7.27E−03	91.31
	WOA	1.38E−02	3.19E−03	91.47
	GWO	2.39E−02	3.17E−03	91.21
	MFO	9.85E−02	2.69E−02	42.58
	AOA	2.15E−02	1.69E−03	91.41
	PSO	1.73E−02	3.59E−03	96
	MVO	1.49E−01	3.52E−02	61.21
3-bit XOR	HSO	1.12E−05	2.79E−04	100
	SCA	1.69E−04	3.43E−04	97.6
	EO	4.73E−02	3.39E−02	98.7
	WOA	7.62E−06	3.21E−05	96.3
	GWO	1.29E−02	3.81E−02	92
	MFO	5.72E−02	2.49E−02	75
	AOA	7.81E−05	3.39E−05	98.3
	PSO	1.03E−04	9.81E−05	96
	MVO	5.15E−02	3.47E−02	78
Cancer	HSO	3.42E−05	2.71E−04	99.3
	SCA	9.81E−03	4.84E−03	95
	EO	1.03E−02	3.62E−03	97.8
	WOA	1.19E−03	1.27E−04	96.5
	GWO	1.32E−02	5.47E−03	99.2
	MFO	1.29E−03	1.47E−04	96
	AOA	1.69E−03	3.64E−04	98.7
	PSO	1.39E−03	6.92E−05	96.3
	MVO	6.83E−03	1.59E−03	92.4
Baboon	HSO	2.76E−09	3.79E−08	100
	SCA	3.41E−05	5.89E−05	100
	EO	3.27E−02	8.68E−03	100
	WOA	1.79E−27	6.23E−27	100
	GWO	3.72E−06	3.84E−06	100
	MFO	1.71E−04	3.85E−04	100
	AOA	1.71E−08	5.62E−08	100
	PSO	1.53E−05	2.18E−05	100
	MVO	7.26E−15	2.19E−14	100

HSO exhibits remarkable performance, surpassing various metaheuristics for specific functions and providing exceptional outcomes for others. The evaluation extends to 42 supplementary functions, showcasing HSO's ability to attain precise global optima across diverse categories. Extensive analysis and qualitative assessment, including trajectory plots, average fitness values, contour plots, and diversity graphs, reinforce the optimization prowess of HSO. Real-world engineering design problems, both constrained and unconstrained, further validate HSO's superiority in solution quality and computational efficiency over well-known optimization algorithms. Moreover, our exploration extends to the application of HSO in training multilayer perceptron's, revealing its superiority over alternative metaheuristics.

As we look towards the future, this study identifies key research directions to enhance optimization strategies and problem-solving capabilities. These directions include combining metaheuristics, developing real-time adaptive metaheuristics for dynamic environments, scaling for large problems using parallel and distributed computing, and improving explain-ability and interpretability for critical decision-making. As a future extension of this research, incorporating arithmetic and evolutionary operators, along with integrating complementary methods such as Levy flight, disruption, mutation, and opposition-based learning, is planned. This aims to propose new and enhanced versions of HSO tailored for optimization problems involving binary, discrete, or multiple objectives. Boosting the efficiency of HSO will involve further integration with stochastic elements like local search or global search strategies. Exploring the diverse applications of HSO in various fields signifies a significant advancement and opens avenues for continued innovation in meta-heuristic optimization.

Appendix

See Tables 40, 41, 42, 43, 44.

Table 40 Benchmark functions for optimization

S.no	Function	Search domain	Dimensions	X_{min}
$F1$	$F_1(X) = \sum_{i=1}^{D} x_i^2$	[− 100,100]	[30, 100, 500, 1000]	$F_1(X_{min}) = 0$
$F2$	$F_2(X) = \sum_{i=1}^{D} \|x\| + \prod_{i=1}^{D} x_i$	[− 10,10]	[30, 100, 500, 1000]	$F_2(X_{min}) = 0$
$F3$	$F_3(X) = \sum_{i=1}^{D} (\sum_{j=1}^{i} x_j)^2$	[− 100,100]	[30, 100, 500, 1000]	$F_3(X_{min}) = 0$
$F4$	$F_4(X) = max\|x_i\|, 1 \le i \le D$	[− 100,100]	[30, 100, 500, 1000]	$F_4(X_{min}) = 0$
$F5$	$F_5(X) = \sum_{i}^{D-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	[− 30,30]	[30, 100, 500, 1000]	$F_5(X_{min}) = 0$
$F6$	$F_6(X) = \sum_{i}^{D} ([x_i + 0.5])^2$	[− 100,100]	[30, 100, 500, 1000]	$F_6(X_{min}) = 0$
$F7$	$F_7(X) = \left(\sum_{i}^{D} i.x_i^4\right) + rand[0, 1]$	[− 1.28,1.28]	[30, 100, 500, 1000]	$F_7(X_{min}) = 0$
$F8$	$F_8(X) = \sum_{i}^{D} -x_i.sin(\sqrt{\|x_i\|})$	[− 500,500]	[30, 100, 500, 1000]	$F_8(X_{min}) = -418.9829*D$
$F9$	$F_9(X) = \sum_{i}^{D} [x_i^2 - 10cos(2\pi x_i) + 10]$	[− 5.12,5.12]	[30, 100, 500, 1000]	$F_9(X_{min}) = 0$
$F10$	$F_{10}(X) = -20 \exp\left(-\sqrt[0.2]{\frac{1}{D}\sum_{i}^{D} x_i^2}\right) - \exp\left(\frac{1}{D}\sum_{i}^{D} \cos 2\pi x_i\right) + 20 + e$	[− 32,32]	[30, 100, 500, 1000]	$F_{10}(X_{min}) = 0$
$F11$	$F_{11}(X) = \frac{1}{4000}\sum_{i}^{D} x_i^2 - \prod_{1}^{D} cos(\frac{x_i}{\sqrt{i}}) + 1$	[− 600,600]	[30, 100, 500, 1000]	$F_{11}(X_{min}) = 0$
$F12$	$F_{12}(X) = \frac{\pi}{D}\left\{10sin^2(\pi y_1) + \sum_{i=1}^{D} (x_i - 1)^2.\left[1 + sin^2(xy_{i+1})\right] + (y_D - 1)^2\right\} + \sum_{i}^{D} u(x_{i,} 10, 100, 4)$	[− 50,50]	[30, 100, 500, 1000]	$F_{12}(X_{min}) = 0$
$F13$	$F_{13}(X) = 0.1\left\{sin^2(3\pi x_1) + \sum_{i=1}^{D} (x_i - 1)^2.\left[1 + sin^2(3\pi x_{i+1})\right] + (x_D - 1)^2\right\} + \sum_{i}^{D} u(x_{i,} 5,100,4)$	[− 50,50]	[30, 100, 500, 1000]	$F_{13}(X_{min}) = 0$
$F14$	$F_{14}(X) = \left(\frac{1}{500} + \sum_{j=1}^{25} (j + 1 + \sum_{i=0}^{1}(x_i - a_{ij})^6\right)^{-1}$	[− 65.54,65.54]	2	$F_{14}(X_{min}) = 0.998$
$F15$	$F_{15}(X) = \sum_{i=0}^{10}\left(a_i - \frac{x_0(b_i^2 + b_i x_1)}{b_i^2 + b_i x_2 + x_3}\right)^2$	[− 5,5]	4	$F_{15}(X_{min}) = 0.0003075$
$F16$	$F_{16}(X) = 4x_0^2 - 2.1x_0^4 + \frac{1}{3}x_0^6 + x_0 x_1$	[− 5,5]	2	$F_{16}(X_{min}) = -1.0316$
$F17$	$F_{17}(X) = (x_1 - \frac{5.1}{4\pi^2}x_0^2 + \frac{5}{\pi}x_0 - 6)^2 + 10(1 - \frac{1}{8\pi})\cos(x_0) + 10$	[− 5,5]	2	$F_{17}(X_{min}) = 0.398$
$F18$	$F_{18}(X) = \{1 + (x_0 + x_1 + 1)^2(19 - 14x_0 + 3x_0^2 - 14x_1 - 6x_0x_1 + 3x_1^2)\}\{30 + (2x_0 - 3x_1)^2(18 - 32x_0 + 12x_0^2 + 48x_1 - 36x_0x_1 + 27x_1^2)\}$	[− 2,2]	2	$F_{18}(X_{min}) = 3$
$F19$	$F_{19}(X) = -\sum_{i=1}^{4} c_i exp(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2)$	[1,3]	3	$F_{19}(X_{min}) = -3.86$
$F20$	$F_{20}(X) = -\sum_{i=1}^{4} c_i exp(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2)$	[0,1]	6	$F_{20}(X_{min}) = -3.32$
$F21$	$f_{19}(X) = -\sum_{i=1}^{5}((X - a_i)^T(X - a_i) + c_i)^{-1}$	[0,10]	4	$F_{21}(X_{min}) = -10.1532$
$F22$	$F_{20}(X) = -\sum_{i=1}^{7}((X - a_i)^T(X - a_i) + c_i)^{-1}$	[0,10]	4	$F_{22}(X_{min}) = -10.4029$
$F23$	$F_{21}(X) = -\sum_{i=1}^{10}((X - a_i)^T(X - a_i) + c_i)^{-1}$	[0,10]	4	$F_{23}(X_{min}) = -10.5364$

Table 41 Forty-two extra functions based on various modalities

Category	Func-no	Function-name	Expression	Domain	dim	Global minima
Many Local Minima	G1	Ackley	$f(x) = -20e^{-b\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i{}^2}} - e^{\frac{1}{d}\sum_{i=1}^{d}\cos(cx_i)} + 20 + e$	$x_i \in [-32.768, 32.768]$	D = 30	0
	G2	Bukin Function N.6	$f(x) = 100\sqrt{\lvert x_2 - 0.01x_1^2\rvert} + 0.01\lvert x_1 + 10\rvert$	$x_1 \in [-15, -5], x_2 \in [-3, 3]$	2	0
	G3	Cross-in-tray-function	$f(x) = -0.0001\left(\left\lvert\sin(x_1)\sin(x_2)\exp\left(\left\lvert 100 - \frac{\sqrt{x_1^2+x_2^2}}{\pi}\right\rvert\right)\right\rvert + 1\right)^{0.1}$	$x_i \in [-10, 10]$	2	− 2.06261
	G4	Drop Wave function	$f(x) = -\frac{1+\cos\left(12\sqrt{x_1^2+x_2^2}\right)}{0.5\left(x_1^2+x_2^2\right)+2}$	$x_i \in [-5.12, 5.12]$	2	− 1
	G5	Egg holder Function	$f(x) = -(x_2 + 47)\sin\left(\sqrt{\left\lvert x_2 + \frac{x_1}{2} + 47\right\rvert}\right) - x_1\sin\left(\sqrt{\lvert x_1 - (x_2 + 47)\rvert}\right)$	$x_i \in [-512, 512]$	2	− 959.6407
	G6	Griewank function	$f(x) = \sum_{i=1}^{d}\frac{x_i^2}{4000} - \prod_{i=1}^{d}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$x_i \in [-600, 600]$	D = 30	0
	G7	Holder Table Function	$f(x) = -\left\lvert\sin(x_1)\cos(x_2)\exp\left(\left\lvert 1 - \frac{\sqrt{x_1^2+x_2^2}}{\pi}\right\rvert\right)\right\rvert$	$x_i \in [-10, 10]$	2	− 19.2085
	G8	Langerman Function	$f(x) = \sum_{i=1}^{m} c_i\exp\left(-\frac{1}{\pi}\sum_{j=1}^{d}\left(x_j - A_{ij}\right)^2\right)\cos\left(\pi\sum_{j=1}^{d}\left(x_j - A_{ij}\right)^2\right)$	$x_i \in [0, 10]$	D = 2	− 1.4
	G9	Levy function	$f(x) = sin^2(\pi w_1) + \sum_{i=1}^{d-1}(w_1 - 1)^2[1 + 10sin^2(\pi w_1 + 1)] + (w_d - 1)^2\left[1 + sin^2(2\pi w_d)\right]$	$x_i \in [-10, 10]$	D = 30	0
	G10	Levy function N.13	$f(x) = sin^2(3\pi x_1) + (x_1 - 1)^2\left[1 + sin^2(3\pi x_2)\right] + (x_2 - 1)^2\left[1 + sin^2(2\pi x_2)\right]$	$x_i \in [-10, 10]$	2	0
	G11	Rastrigin Function	$f(x) = 10d + \sum_{i=1}^{d}\left[x_i^2 - 10\cos(2\pi x_i)\right]$	$x_i \in [-5.12, 5.12]$	D = 30	0
	G12	Schaffer Function N.2	$f(x) = 0.5 + \frac{sin^2\left(x_1^2-x_2^2\right)-0.5}{\left[1+0.001\left(x_1^2+x_2^2\right)\right]^2}$	$x_i \in [-100, 100]$	2	0
	G13	Schaffer Function N.4	$f(x) = 0.5 + \frac{cos^2\left(\sin\left(\lvert x_1^2-x_2^2\rvert\right)\right)-0.5}{\left[1+0.001\left(x_1^2+x_2^2\right)\right]^2}$	$x_i \in [-100, 100]$	2	0.29257
	G14	Schwefel function	$f(x) = 418.9829d - \sum_{i=1}^{d} x_i\sin\left(\sqrt{\lvert x_i\rvert}\right)$	$x_i \in [-500, 500]$	D = 30	0
	G15	Shubert function	$f(x) = \left(\sum_{i=1}^{5} icos((i+1)x_1 + i)\right)\left(\sum_{i=1}^{5} icos((i+1)x_2 + i)\right)$	$x_i \in [-5.12, 5.12]$	2	− 186.7309
Bowl Shaped	G16	Bohachevsky Functions	$f_1(x) = x_1^2 + 2x_2^2 - 0.3\cos(3\pi x_1) - 0.4\cos(4\pi x_2) + 0.7$	$x_i \in [-100, 100]$	2	0
	G17	Perm Function	$f(x) = \sum_{i=1}^{d}\left(\sum_{j=1}^{d}(j + \beta)\left(x_j^i - \frac{1}{j^i}\right)\right)^2$	$x_i \in [-d, d]$	D = 30	0
	G18	Rotated Hyper − Ellipsoid Function	$f(x) = \sum_{i=1}^{d}\sum_{j=1}^{i} x_j^2$	$x_i \in [-65.536, 65.536]$	D	0
	G19	Sphere Function	$f(x) = \sum_{i=1}^{d} x_i^2$	$x_i \in [-5.12, 5.12]$	D = 30	0
	G20	Sum of Different Powers Function	$f(x) = \sum_{i=1}^{d}\lvert x_i\rvert^{i+1}$	$x_i \in [-1, 1]$	D = 30	0
	G21	Sum Squares Function	$f(x) = \sum_{i=1}^{d} ix_i^2$	$x_i \in [-10, 10]$	D = 30	0

Table 41 continued

Category	Func-no	Function-name	Expression	Domain	dim	Global minima
	G22	Trid Function	$f(x)=\sum_{i=1}^{d}(x_i-1)^2-\sum_{i=2}^{d}x_ix_{i-1}$	$x_i\in[-d^2,d^2]$	D = 30	− d(d+4)(d − 1)/6
Plate Shaped	G23	Booth Function	$f(x)=(x_1+2x_2-7)^2+(2x_1+x_2-5)^2$	$x_i\in[-10,10]$	2	0
	G24	Matyas Function	$f(x)=0.26(x_1^2+x_2^2)-0.48x_1x_2$	$x_i\in[-10,10]$	2	0
	G25	McCormick function	$f(x)=\sin(x_1+x_2)+(x_1-x_2)^2-1.5x_1+2.5x_2+1$	$x_1\in[-1.5,4],x_2\in[-3,4]$	2	− 1.9133
	G26	Power Sum function	$f(x)=\sum_{i=1}^{d}\left[\left(\sum_{j=1}^{d}x_j^i\right)-b_i\right]^2$	$x_i\in[0,d]$	D = 30	0
	G27	Zakharov Function	$f(x)=\sum_{i=1}^{d}x_i^2+\left(\sum_{i=1}^{d}0.5ix_i\right)^2+\left(\sum_{i=1}^{d}0.5ix_i\right)^4$	$x_i\in[-5,10]$	D = 30	0
Valley Based	G28	ThreE−Hump Camel Function	$f(x)=2x_1^2-1.05x_1^4+\frac{x_1^6}{6}+x_1x_2+x_2^2$	$x_i\in[-5,5]$	2	0
	G29	Six-Hump Camel Function	$f(x)=\left(4-2.1x_1^2+\frac{x_1^4}{3}\right)x_1^2+x_1x_2+(-4+4x_2^2)x_2^2$	$x_1\in[-3,3],x_2\in[-2,2]$	2	− 1.0316
	G30	Dixon-Price Function	$f(x)=(x_1-1)^2+\sum_{i=2}^{d}i(2x_i^2-x_{i-1})^2$	$x_i\in[-10,10]$	D = 30	0
	G31	Rosenbrock Function	$f(x)=\sum_{i=1}^{d-1}\left[100(x_{i+1}-x_i^2)^2+(x_i-1)^2\right]$	$x_i\in[-5,10]$	D = 30	0
Steep Ridges/ Drops	G32	De Jong Function N.5	$f(x)=\left(0.002+\sum_{i=1}^{25}\frac{1}{i+(x_1-a_{1i})^6+(x_2-a_{2i})^6}\right)^{-1}$	$x_i\in[-65.536,65.536]$	2	1
	G33	Easom Function	$f(x)=-\cos(x_1)\cos(x_2)\exp\left(-(x_1-\pi)^2-(x_2-\pi)^2\right)$	$x_i\in[0,\pi]$	2	− 1
	G34	Michalewicz Function	$f(x)=-\sum_{i=1}^{d}\sin(x_1)sin^{2m}\left(\frac{ix_i^2}{\pi}\right)$	$x_i\in[-4.5,4.5]$	10	− 9.66015
	G35	Beale Function	$f(x)=(1.5-x_1+x_1x_2)^2+(2.25-x_1+x_1x_2^2)^2+(2.625-x_1+x_1x_2^3)^2$	$x_i\in[-4.5,4.5]$	2	0
Others	G36	Branin Function	$f(x)=\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos(x_1)+10$	$x_1\in[-5,10],x_2\in[0,15]$	2	0.397887
	G37	Colville Function	$f(x)=100(x_1^2-x_2)^2+(x_1-1)^2+(x_3-1)^2+90(x_3^2-x_4)^2$ $+10.1\left((x_2-1)^2+(x_4-1)^2\right)+19.8(x_2-1)(x_4-1)$	$x_i\in[-10,10]$	4	0
	G38	Goldstein Price Function	$f(x)=\left[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)\right]$ $\times[30+(2x_1-3x_2)^2(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)]$	$x_i\in[-2,2]$	2	3
	G39	Hartmann 3D Function	$(x)=-\sum_{i=1}^{4}\alpha_i\exp\left(-\sum_{j=1}^{3}A_{ij}(x_j-P_{ij})^2\right)$	$x_i\in[0,1]$	3	− 3.86278
	G40	Hartmann 6D Function	$f(x)=-\sum_{i=1}^{4}\alpha_i\exp\left(-\sum_{j=1}^{6}A_{ij}(x_j-P_{ij})^2\right)$	$x_i\in[0,1]$	6	− 3.32237
	G41	Powell Function	$f(x)=\sum_{i=1}^{\frac{d}{4}}[(x_{4i-3}+10x_{4i-2})^2+5(x_{4i-1}-x_{4i})^2$ $+(x_{4i-2}-2x_{4i-1})^4+10(x_{4i-3}-x_{4i})^4]$	$x_i\in[-4,5]$	D = 30	0
	G42	Shekel Function	$f(x)=-\sum_{i=1}^{m}\left(\sum_{j=1}^{4}(x_j-C_{ji})^2+\beta_i\right)^{-1}$	$x_i\in[0,10]$	4	− 10.1532

Table 42 IEEE CEC-2015 benchmark test functions

S.No	Functions	Modality	Dim	F_{min}
CEC – 1	Rotatedbentcigarfunction	Bentcigarfunction	10	100
CEC – 2	Rotateddiscussfunction	Discussfunction	10	200
CEC – 3	Shiftedandrotatedweierstrassfunction	Weierstrassfunction	10	300
CEC – 4	Shiftedandrotatedschwefel′sfunction	Schwefel′sfunction	10	400
CEC – 5	Rotatedkatsuurafunction	Katsuurafunction	10	500
CEC – 6	Shiftedandrotatedhappycatfunction	Happycatfunction	10	600
CEC – 7	Shiftedandrotatedhgbatfunction	Hgbatfunction	10	700
CEC – 8	Shiftedandrotatedexpandedgriewank′s plusrosenbrock′sfunction	Griewank′sfunction Rosenbrock′sfunction	10	800
CEC – 9	Shiftedandrotated expandedscaffer′sfunction	Expandedscaffer′s function	10	900
CEC – 10	Hybridfunction1(N = 3)	Schwefel′sfunction Rastrigin′sfunction Highconditionedelliptic function	10	1000
CEC – 11	Hybridfunction2(N = 4)	Griewank′sfunction Weierstrassfunction Rosenbrock′sfunction scaffer′sfunction	10	1100
CEC – 12	Hybridfunction3(N = 5)	Katsuurafunction Happycatfunction ExpandedGriewank′splus Rosenbrock′sfunction Schwefel′sfunction Ackley′sfunction	10	1200
CEC – 13	Compositionfunction1(N = 5)	Rosenbrock′sfunction Highconditionedelliptic functionBentcigarfunction Discussfunction Highconditionedelliptic function	10	1300
CEC – 14	Compositionfunction2(N = 3)	Schwefel′sfunction Rastrigin′sfunction Highconditionedelliptic function	10	1400
CEC – 15	Compositionfunction2(N = 5)	Hgbatfunction Rastrigin′sfunction Schwefel′sfunction Weierstrassfunction Highconditionedelliptic function	10	1500

Table 43 IEEE CEC-2017 benchmark test functions

Notations	Functions	Modality	Dim	F_{min}
CEC − 1	Bentcigarfunction	ShiftedandRotated	30	100
CEC − 2	Sumofdifferentpowerfunction	ShiftedandRotated	30	200
CEC − 3	Zakharovfunction	ShiftedandRotated	30	300
CEC − 4	Rosenbrock′sfunction	ShiftedandRotated	30	400
CEC − 5	Rastrigin′sfunction	ShiftedandRotated	30	500
CEC − 6	ExpandedScaffer′sfunction	ShiftedandRotated	30	600
CEC − 7	Lunacekbirastriginfunction	ShiftedandRotated	30	700
CEC − 8	Noncontinuousrastrigin′sfunction	ShiftedandRotated	30	800
CEC − 9	Levyfunction	ShiftedandRotated	30	900
CEC − 10	Schwefel′sfunction	ShiftedandRotated	30	1000
CEC − 11	function1(N = 3)	Hybrid	30	1200
CEC − 12	function2(N = 3)	Hybrid	30	1300
CEC − 13	function3(N = 3)	Hybrid	30	1400
CEC − 14	function4(N = 4)	Hybrid	30	1500
CEC − 15	function5(N = 4)	Hybrid	30	1600
CEC − 16	function6(N = 4)	Hybrid	30	1700
CEC − 17	function6(N = 5)	Hybrid	30	1800
CEC − 18	function6(N = 5)	Hybrid	30	1900
CEC − 19	function6(N = 5)	Hybrid	30	2000
CEC − 20	function6(N = 6)	Hybrid	30	2100
CEC − 21	function1(N = 3)	Composite	30	2200
CEC − 22	function2(N = 3)	Composite	30	2300
CEC − 23	function3(N = 4)	Composite	30	2400
CEC − 24	function4(N = 4)	Composite	30	2500
CEC − 25	function5(N = 5)	Composite	30	2600
CEC − 26	function6(N = 5)	Composite	30	2700
CEC − 27	function7(N = 6)	Composite	30	2800
CEC − 28	function8(N = 6)	Composite	30	2900
CEC − 29	function9(N = 3)	Composite	30	3000
CEC − 30	function10(N = 3)	Composite	30	

Table 44 Abbreviations

ACO	Ant colony optimization
GWO	Grey wolf optimization
PSO	Particle swarm optimization
GTO	Gorilla troops optimizer
SCA	Sine cosine algorithm
CSA	Cuckoo search algorithm
TSA	Tree seed optimizer
MVO	Multi verse optimizer
WOA	Whale optimization algorithm
MFO	Moth flame optimization
FP	Flower pollination
PGO	Plant growth optimization
SFO	Sun flower optimization
GA	Genetics algorithm
PBIL	Probability based incremental learning
BBO	Biogeography based optimizer
FSO	Fish swarm optimization
CSO	Cat swarm optimization
BCO	Big crunch optimization
ASO	Atom search optimization
RO	Ray optimization
CFO	Central force optimization
SA	Simulated annealing
GLSA	Gravitational search algorithm
GSA	Gravitational search algorithm
CSS	Charged system search
ACROA	Artificial chemical reaction optimization algorithm
BHSO	Black hole optimization algorithm
SWOA	Small World Optimization Algorithm
GBSA	Galaxy based search algorithm
CSO	Curved space optimization
SGA	Fashion search group algorithm
SLC	Soccer league competition
GTO	Group teaching optimization
KIA	Kidney inspired algorithm
TLBO	Teaching learning based optimizer
QPSO	Quantum-behaved PSO
SPSO	Simplified PSO
BBPSO	BarE−bones PSO
CPSO	Chaotic PSO
FPSO	Fuzzy PSO
PSOTVAC	PSO with TVAC
OPSO	Opposition-based PSO

Acknowledgements The first author wishes to express his gratitude to Doon University in Uttarakhand, India, for providing all of the essential resources for this study.

Author contributions Shivankur Thapliyal, and Narender Kumar wrote the main manuscript text . All authors reviewed the manuscript

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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