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Research on modal parameters identification of bridge structure based on adaptive signal de-noising method

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Abstract

For signal de-noising approach based on singular value decomposition (SVD), the method of determining the row number (p) of Hankel matrix and the effective rank (r) both are key problems. In this paper, an adaptive signal de-noising approach which based on genetic algorithm (GA) and SVD was proposed. Choosing signal to noise ratio (SNR) as fitness function, GA was introduced to automatically optimize the parameter of p and r. Then inverse SVD was conducted to achieve the de-noised signal. In order to demonstrate the validity of the approach, two numerical simulation signals with different frequency components are employed. The results show that p can be $N/4$ or $N/3$ (N is the length of data), r is twice as the number of dominating frequency. As for measured signal, the complication of the frequency components might be taken into consideration. And in order not to miss the true frequency components when dealing with measured signals, r should be more than twice as the number of dominating frequency, but p can still be $N/4$ or $N/3$.

Keywords Singular value decomposition · Genetic algorithm · Signal de-noising · Modal parameter identification

1 Introduction

The ambient excitation method to identify large-scale structure modal parameters is widely used for structural health monitoring (SHM). Different from traditional methods, modal parameters (e.g., natural frequency, damping ratio, mode shape) can be identified from output-only data in ambient excitation method. As affected by the violent background vibration or the strong environmental electromagnetic disturbances, the signals collected from field are often inevitably contaminated by noise. This situation often leads to serious difficulties in many applications as they require high quality measured signal data.

In order to reduce the noise residents in the measured signals, many methods had been tried, such as time series analysis [1, 2], wavelet analysis [3–5], parameter estimation algorithms [6–8], special numerical filters [9–13] and so on. The singular value decomposition (SVD) technique is a very useful tool in linear matrix theory, due to its simple calculation method, now it has been widely used in many fields in recent years, such as acoustics [14], smart control [15, 16], electronics [17, 18], signal processing [19, 20], mathematics [21, 22] and so on. However, compared to its achievements in above-mentioned fields, the research in noise reduction field has not been done sufficiently, especially for the two key problems: determining the row number (p) of Hankel matrix and the effective rank (r). Earlier works done by the authors concentrated on the determination of row number of Hankel matrix by utilizing "cut-and-try" method [23], singularity spectrum [24, 25], Singular Entropy [26], and other methods [27–29]. For the effective rank, structural risk minimization [30], dynamic clustering [31] and principal component analysis [32] are applied. The methods mentioned above were illustrated using simulated test cases where additive and multiplicative types of noise were added to otherwise clean data. It was found that the methods were fairly successful. However, the long-term field practices proved that these methods have their own limitations more or less [33–36]. This current paper is an extension of the works mentioned above and presents a new method for the elimination of noise from measured signal data. As signal to noise ratio (SNR) was chosen as object function, GA is introduced to

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find out the best row number of Hankel matrix and effective rank so that we can get the most "cleanest" signal.

2 Theorem of noise reduction applied SVD

2.1 Statement of the theorem

Every real matrix A of dimensions $p \times q$ can be factorized as $A = U \sum V^T$, where U of dimensions $p \times p$ and V of dimensions $q \times q$ are the orthogonal matrix, $\sum$ is $p \times q$ diagonal matrix with its only nonzero elements on the diagonal. These elements are called the singular values and ordered as $\sigma_1 \geq \sigma_2 \geq \cdots \sigma_R$, where $R = \min(p, q)$. The columns of U and V are the left, respectively, right singular vectors of A. More details, including algorithms, on the SVD can be found in [37].

Then there exists an $p \times q$ matrix $\hat{A}$ of rank $r \leq R$ which minimizes the sum of the squared error between the elements of A and the corresponding elements of $\widehat{A}$ when $\widehat{A} = U \overline{\sum} V^T$, where $\overline{\sum}$ is obtained by setting to zero all but its r largest singular values. Note that $[]^T$ indicates complex conjugate transpose in the case of the matrix A being complex.

2.2 Employed in noise reduction

The SVD technique is also employed in noise reduction. Jensen et al. [38] considered a noisy signal vector $\{x_k\} = \{x_1, x_2, \cdots x_N\}$ of N samples and assumed that the noise is additive and correlated with the signal, i.e.

$$\{x_k\} = \{s_k\} + \{n_k\} \tag{1}$$

where $\{s_k\}$ represents the signal component and the $\{n_k\}$ represents the noise. As in other applications, it is proposed in Jensen et al. [38] that Hankel matrix of dimensions $p \times q$ can be constructed using the signal vector $\{x_k\}$ as

$$A_{p \times q} = \begin{bmatrix} x(1) & x(2) & \cdots & x(q) \\ x(2) & x(3) & \cdots & x(q+q) \\ \vdots & \vdots & \cdots & \vdots \\ x(p) & x(p+1) & \cdots & x(p+q-1) \end{bmatrix} \tag{2}$$

where $p + q - 1 = N$ and $p \geq q$. Again, the signal component of the noisy signal vector is estimated in accordance with [37]. Equation (2) could be written as follows:

$$
A_{p \times q} = \begin{bmatrix} s(1) & s(2) & \cdots & s(q) \\ s(2) & s(3) & \cdots & s(q+1) \\ \vdots & \vdots & \cdots & \vdots \\ s(p) & s(p+1) & \cdots & s(p+q-1) \end{bmatrix}
$$
$$
+ \begin{bmatrix} n(1) & n(2) & \cdots & n(q) \\ n(2) & n(3) & \cdots & n(q+1) \\ \vdots & \vdots & \cdots & \vdots \\ n(p) & n(p+1) & \cdots & n(p+q-1) \end{bmatrix}
$$
$$
= A_s + A_n = [U_r \quad U_0] \begin{bmatrix} \sum_r & 0 \\ 0 & \sum_0 \end{bmatrix} \begin{bmatrix} V_r^T \\ V_0^T \end{bmatrix} \tag{3}
$$

where A_s represents the signal component and the A_n represents the noise.

Then SVD technique is utilized for the estimation of the rank of the matrix A in the process of noise elimination. $\sum_r$, which represents uncontaminated data space, contains significant singular values (σ_i, $i = 1, \ldots, r$). $\sum_0$, which represents the noise space, contains small singular values below a threshold ($\sigma_i, i = r + 1, \ldots, R$).

Essentially, the rank is an indicator of the number of independent characteristics in the data. However, the main problem in practice is the estimation of the noise threshold, i.e., the effective rank of the matrix $A_{p \times q}$. The ways to estimate the effective rank were described in [23–32]. It should be noted that the elimination of small singular values from $A_{p \times q}$ results in a matrix $\widehat{A}_{p \times q}$ which is of non-Hankel form. Consequently, the content of the matrix $\widehat{A}_{p \times q}$ needs to be transformed back to a vector form so as to obtain the uncontaminated data. This is achieved by arithmetic averaging along the anti-diagonals of $\widehat{A}_{p \times q}$. If the signal vector $\widehat{s}$ of length N is used to construct the $\widehat{A}_{p \times q}$ matrix, the ith element of the noise-free signal vector $\widehat{s}$ can be reconstructed as:

$$\widehat{s}(i) = \frac{1}{k - l + 1} \sum_{j=1}^{k} \widehat{A}_{p \times q}(i + j - 1) \tag{4}$$

where $l = \max(1, i - p + 1)$ and $k = \min(q, i)$.

Once the noise-free signal vector $\widehat{s}$ is established, SNR can also be estimated as

$$SNR = 10 \times \lg \left[\frac{P_S}{P_N} \right] = 10 \times \lg \left[\frac{\sum_{i}^{N} \widehat{s}(i)^2}{\sum_{i}^{N} (x(i) - \widehat{s}(i))} \right] \text{(dB)} \tag{5}$$

where Ps and P_N represent the effective power of noise-free signal and noise respectively, $x(i)$ and $\widehat{s}(i)$ represent the signal vector of raw signal data and noise-free signal data.

The procedure is illustrated in Fig. 1.

Fig. 1 Procedure of SVD-based de-nosing approach
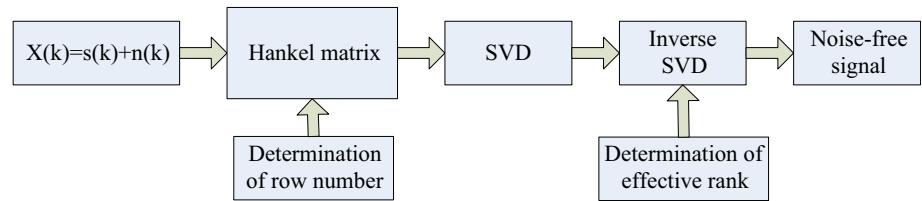
3 Determination of key parameters

For noise-free signal, the Hankel matrix of A_s is singular, where $k < \min(p, q)$. But the Hankel matrix ξ_i is column full rank for Gaussian white noise signal, where $k = \min(p, q)$. To ease the comprehension of this problem, the following simulated signal is considered

$$x(t) = x_0(t) + \alpha e(t) \tag{6}$$

where $x_0(t) = \sin(2\pi f_1 t) + \cos(2\pi f_2 t)$, $f_1 = 5$ Hz, $f_2 = 10$ Hz, $e(t)$ represents the Gaussian white noise, α indicates the noise level. In this paper, the length N of the simulated signal above is chosen as 2048.

3.1 Row number of Hankel matrix

For the first step of SVD-based de-noising approach, the signal should be transferred form time series format $x(t)$ to Hankel matrix format $A_{p \times q}$. The size of the Hankel matrix deserves special attention. For a given data with a sample number of N; a square or a nearly square Hankel matrix can be constructed. However, it is neither necessary nor desirable to construct a square or nearly square Hankel matrix for practical applications of the method. Instead, a rectangular matrix with appropriate selection of the smaller dimension can be used effectively, provided that the smaller dimension, i.e. the maximum possible rank, is large enough to represent the system behavior including the effect of noise. The reason for this is that nearly square Hankel matrix may require unnecessary computations while inadequate setting of the maximum possible rank of the system may cause loss of performance. It is expected that if the dimensions of the Hankel matrix are set properly small singular values will approach an asymptote, otherwise not.

From the results of [23–29], the row number p of Hankel matrix could be chosen in the range of [N/10, N/2]. In this section, the simulated signal of Eq. (6) was utilized for the investigation of distribution characteristic of singular values when different row number was chosen. The noise level is 10%, SNR = 10 dB, the length N of the simulated signal is 2048. The row number was selected as 100, 300, 600, 900, 1024, which is in the range of [N/10, N/2]. Results are illustrated in Fig. 2. It is seen that for all situations, the distribution of singular values shows an

asymptote after the 4th singular values (twice the number of modes), the rest of which represent noise. So the row number is reasonable in the range of [N/10, N/2], but the trend is more obvious when row number was selected as 1024 (N/2). It means that a square or nearly square Hankel matrix is preferred if the computation cost is not an issue.

3.2 Effective rank

Then SVD technique is utilized for the estimation of the rank of the matrix A in the process of noise elimination. In fact, it has been found that the rank, i.e. the number of singular values is expected to be twice the number of frequency components included in the signal. But sometimes, the distribution of singular values is not straightforward enough for the signal with high noise level, especially for field measurements. Some real frequency components would be lost if the rank is set to be too low, otherwise the noise residual is too much to identify the real frequency components.

In [26], Yang proposed a method named Singular Entropy as follows:

$$E_k = \sum_{i=1}^{k} \Delta E_i \quad (k \leq p) \tag{7}$$

where k is effective rank, ΔE_i is the increment in kth singular value and it can be obtained through Eq. (8).

$$\Delta E_i = -\left(\lambda_i \sum_{j=1}^{p} \lambda_j \right) \log \left(\lambda_i \sum_{j=1}^{p} \lambda_j \right) \tag{8}$$

where λ_i is ith singular value.

The simulated signal of Eq. (6) with different additive noise levels (10, 20, 50 and 100%) were used in this section again. It is seen that the distribution of Singular Entropy increment is straightforward when the noise level is low and it shows an asymptote after the 4th singular values. But the asymptote is not obvious enough to determine the rank of the system in the situation of 100% additive noise. In order not to miss the real frequency components, usually we must determine a higher rank, which need more computation. In fact, sometimes the power of noise is even greater than the power of useful signal for field measurements of civil structures such as bridge and tall building. So Singular Entropy is not suitable for field measurements.
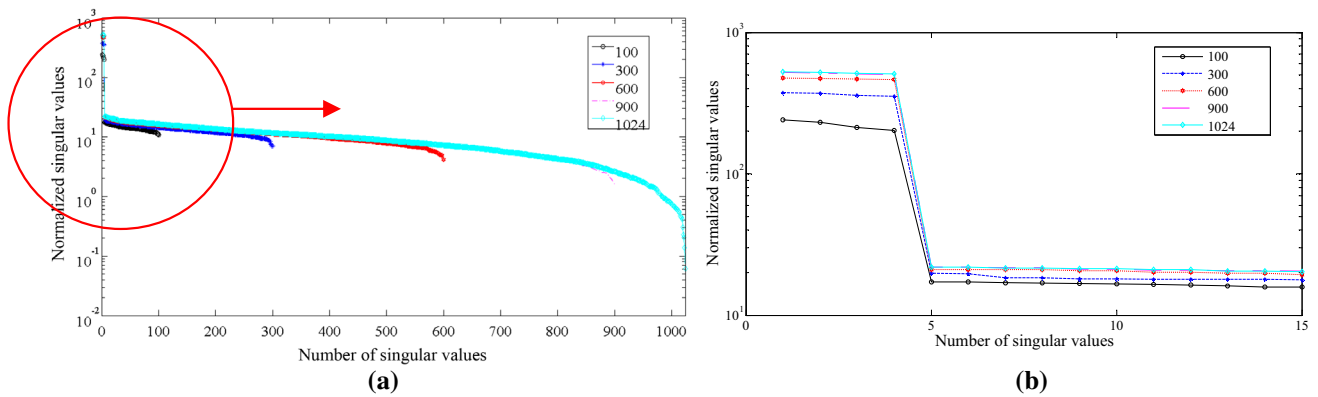
Fig. 2 Distribution of normalized singular values to a simulated signal with 10% additive noise. **a** Whole, **b** detail

4 Adaptive de-nosing approach based on GA–SVD

4.1 Overview

Genetic algorithm (GA), which is known as a random optimization method, was invented by John Holland in the 1970s [39]. GA is based on Darwin's theory of gradual evolution. For developing the solutions of an optimization problem, the algorithm uses the same principles that nature implement on the evolution of gene symbols. A common method used to implement the GA is as the following:

- Defining the individual for the GA
  The population is a set of individuals, and each individual is a potential solution to the problem. Therefore, once the target parameter is provided, each individual has a unique variable (gen). For SVD-based de-noising approach, the target parameters are the row number p and effective rank r. The range of p can be defined to $[N/10, N/2]$, and the range of r could be determined by distribution of Singular Entropy.
  A set of random solutions, which are called populations, are generated.
- Calculating fitness
  The fitness function indicates the accuracy of a solution by enabling determination of which of two solutions is superior. Thus, it is essential to define the cost function correctly if the algorithm is to be effective. In the proposed approach, SNR is selected as the fitness function to evaluate the effect of noise reduction.

$$Fit(p,k) = \begin{cases} \min\left(\dfrac{1}{SNR}\right) \\ \displaystyle\sum_{i=1}^{n} \dfrac{1}{SNR} \Big/ n \end{cases} \tag{9}$$

where n is population quantity.
In each iteration, all solutions are evaluated using the

fitness function. Then, some of the best solutions are selected using a probability function and constitute a new population.

- Selection, crossing and mutation
  Some of these selected solutions are used without changing and others using genetic operators such as Crossover and Mutation are used to generate offspring. The parameters were selected as follows [39]: selection percentage: 80%, crossing probability: 7%, mutation probability: 5%.
  The process is continued to find the optimal solution.
- Stopping criterion
  The algorithm terminates when the number of iterations reaches a maximum or the change in the average fitness value is less than a given constant value.

The GA flowchart is shown in Fig. 3.

4.2 Numerical study: case I

As illustrated before, the simulated signal of Eq. (6) contains two frequency components: $f_1 = 5$ Hz, $f_2 = 10$ Hz. Different additive noise levels (10, 20, 50 and 100%) were used and the distribution of Singular Entropy can been found in Fig. 4. It is seen that the abrupt change of Singular Entropy at the 15th singular for the situation of additive
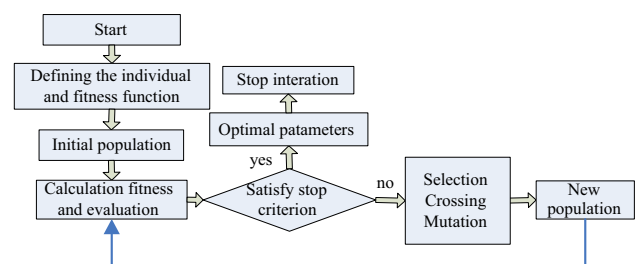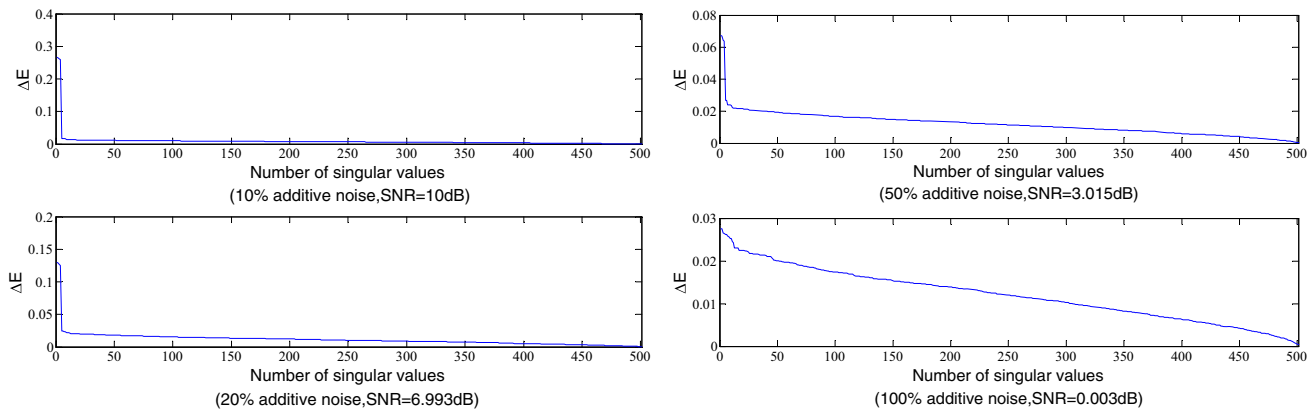

Fig. 3 Flowchart of GA

Fig. 4 Distribution of Singular Entropy to a simulated signal with different additive noise level

noise levels: 100%. Using the GA–SVD de-noising approach, the parameters are defined as follow:

Sampling frequency: 100 Hz;
Length N of data: 1024
Range of p is defined to $[N/10, N/2]$;
Range of r is defined to $[1, 15]$;
Number of population: 20;
Number of iterations: 30
It take 16 min for computation and the procedure of iteration is illustrated in Fig. 5.

The effect of de-noising is illustrated in Table 1. The increment of SNR for the four situations is about 15–20 dB. It is seen that the row number p is almost half of the length of data, which means that the Hankel matrix is nearly a square matrix. The effective rank r is 4, which is twice the number of frequency components.

For numerical case I, the length data is 1024, which is suitable for numerical study. But for real measurements of civil structures, it usually need to do data collection for more than 20 min. In order to get better effect of de-noising, for this huge amounts of data, is it necessary to construct a square Hankel matrix? It might take us a couple of days for computing. The second situation, which is disturbed by 50% additive noise level, is under investigated here. The effective rank r is set to be 4. Row number p is set to be from 50 to 900, with a increment step of 50. Figure 6 shows that in the range of 200–800 ($p \approx [N/5, 2N/3]$), we get very close results, which means that SNRs of de-noised signal are almost about 23 dB. So, for improving the calculating efficiency, a rectangular matrix with appropriate selection of row number p is large enough to represent the system behavior including the effect of noise.

4.3 Numerical study: case II

In case I, the simulated signal with two frequency components was disturbed by Gaussian white noise. As affected by the violent background vibration or the strong environmental electromagnetic disturbances, the signals collected from field are often inevitably contaminated by not only Gaussian white noise, but also non-Gaussian colored noise. Additionally, for vibration-based SHM, we usually care about the first few frequencies [39, 40]. In case II, the following simulated signal is considered

$$x(t) = \sin(3\pi t) + \sin(5\pi t) + \sin(10\pi t) + \sin(1\pi t) \\ + \cos(20\pi t) + \alpha_1 e_1(t) + e_2(t) + e_3(t) \tag{10}$$

where $f_1 = 0.5$ Hz, $f_2 = 1.5$ Hz, $f_3 = 2.5$ Hz, $f_4 = 5$ Hz, $f_5 = 10$ Hz; $e_1(t)$ represents the Gaussian white noise, α indicates 50% noise level; $e_2(t)$ represents colored noise with bandwidth of [0.2–20] Hz; $e_3(t)$ represents impulse noise; the length N of the simulated signal above is chosen as 1024.

SNR of the mixed signal is -5.5742 dB, which indicates that the power of noise is greater than the power of pure signal. Distribution of Singular Entropy is illustrated in Fig. 7, which shows an asymptote after the 20th singular values. Using GA–SVD approach, the range of effective rank r is set to be in the range of [4, 20], the range of row number p is set to be in the range of $[N/10, N/2]$. Procedure of iteration is shown in Fig. 8. SNR of the de-noised signal is 6.1388 dB, which illustrates that the noise has been weakened greatly. The row number $p = 413$ (about $N/3$), effective rank $r = 10$ (twice the number of frequency components). Figure 9 shows a pure signal in red color, which is so clear that the waveform can be distinguished from noise. The five frequency components can be identified accurately by peak picking method.
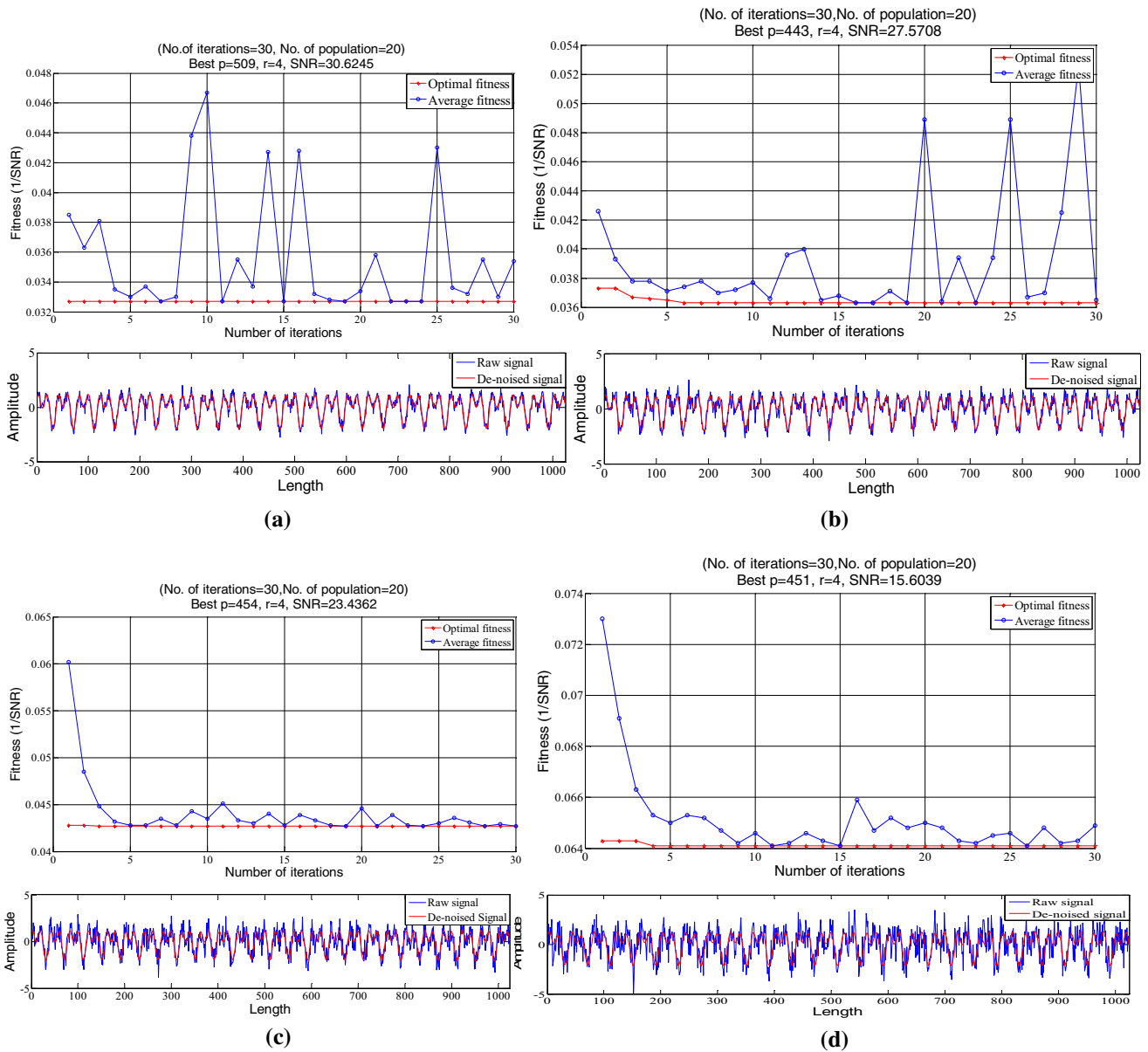
Fig. 5 Procedure of iterations and effect of de-noising. **a** additive noise levels: 10%, **b** additive noise levels: 20%, **c** additive noise levels: 50%, **d** additive noise levels: 100%

Table 1 Effect of de-noising using GA–SVD

| Noise level (%)	Raw signal (SNR: dB)	De-noised signal (SNR: dB)	Row number p	Rank r
10	10.0000	30.6245	509 ($\approx N/2$)	4
20	6.9930	27.5708	443 ($\approx 2N/5$)	4
50	3.0136	23.4362	454 ($\approx 2N/5$)	4
100	0.0033	15.6093	451 ($\approx 2N/5$)	4

It is worth noted that, from the results of case I and II: (1) effective rank can be defined as twice the number of frequency components; (2) row number p can be defined in the range of $[N/5, 2\,N/3]$, which is $< N/2$. For real measurements of civil structures, it usually need to do data collection for > 20 min. So we can defined row number p as $N/4$ or $N/3$, which construct a rectangular matrix and less computations are needed.
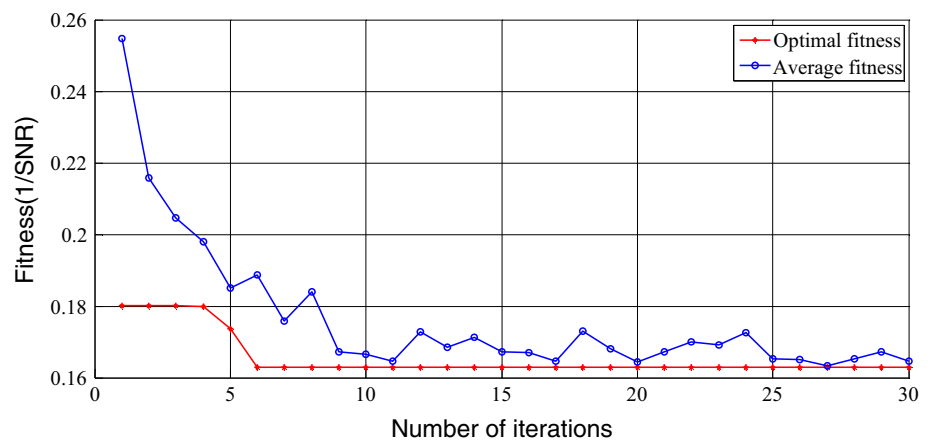
Fig. 6 Effect of de-noising under the situation: $k = 4$, $p = 200$–800
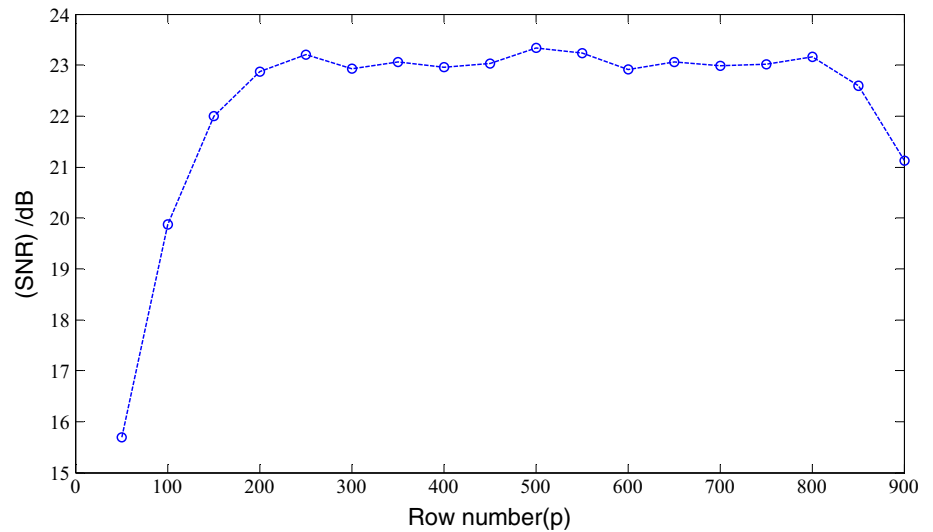
Fig. 7 Distribution of Singular Entropy
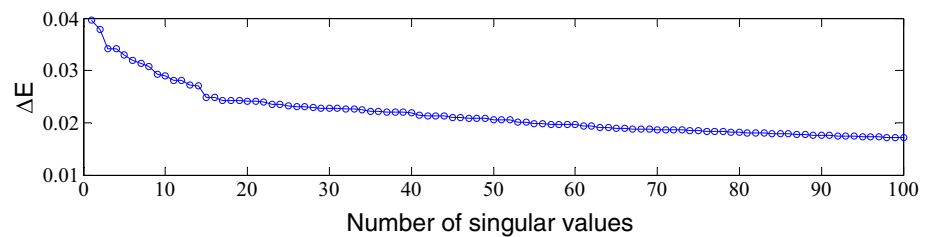

Fig. 8 Procedure of iterations

4.4 Experimental study

An operational modal test was conducted in 2011, for a long span cable-stayed bridge, whose span combination is 165 m + 338 m + 165 m, Fig. 10. For vertical vibration modal test, the sampling frequency was set to 160 Hz, each group of data lasts for about 20 min. At first, we must confirm the modes that we care about. From the analysis of Finite Element Model (FEM), it is known that the first 15 modes are < 5 Hz, who are usually used for SHM. Secondary, the data was resampled at 12 Hz. The data of the first 170 s was chosen for analyzing, whose length is 2048.

FFT was conducted to the raw acceleration data, the spectrogram illustrated in Fig. 11 that: (1) the first three natural frequencies are obviously to identified by the peaks in the range of [0, 1] Hz; (2) in the range of [1, 2] Hz, the peaks are not so obvious. The identification accuracy is determined by frequency resolution; (3) in the range of [2, 6] Hz, the peaks are too messy to identified.

Using GA–SVD, row number p was set as 512 ($N/4$) to construct Hankel matrix. Distribution of Singular Entropy was illustrated in Fig. 12. From the analysis of FEM, it is known that there are 15 modes in the range of [0, 5] Hz. According to the conclusion of numerical studies above,
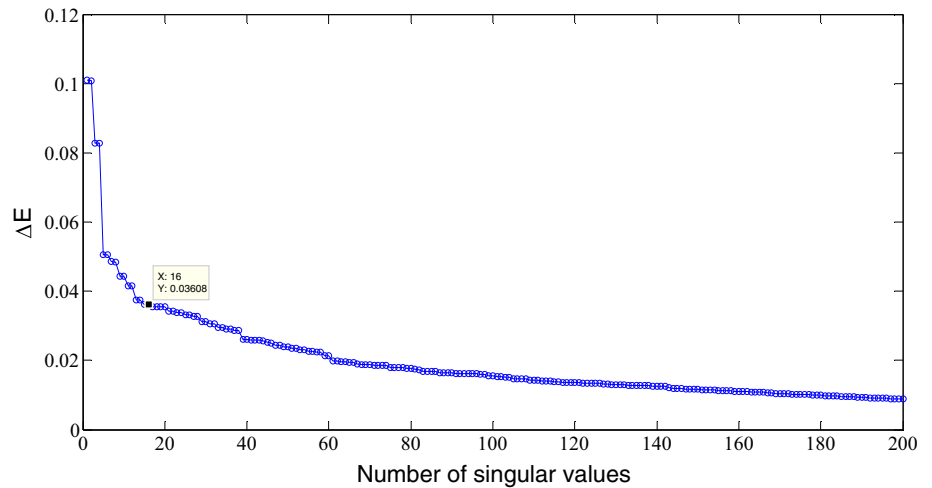
Fig. 9 Effect of de-noising
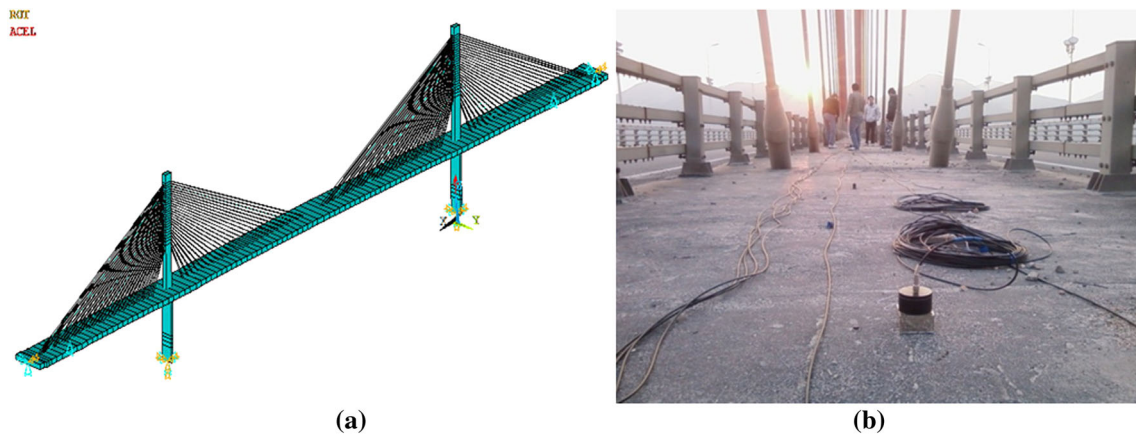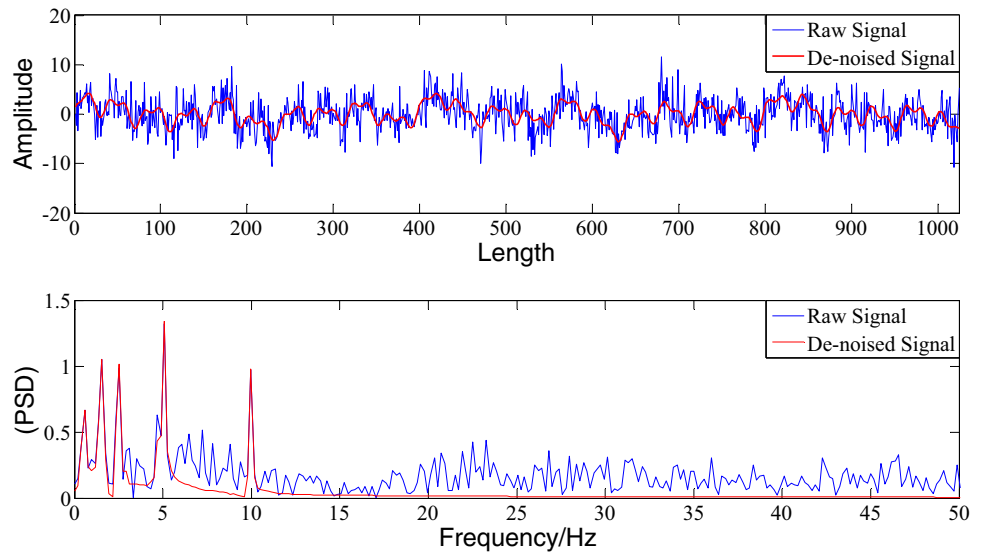
Fig. 10 Experimental study: Yamen bridge. **a** Finite element mode, **b** modal test of vertical vibration

Fig. 11 Raw acceleration data and spectrogram
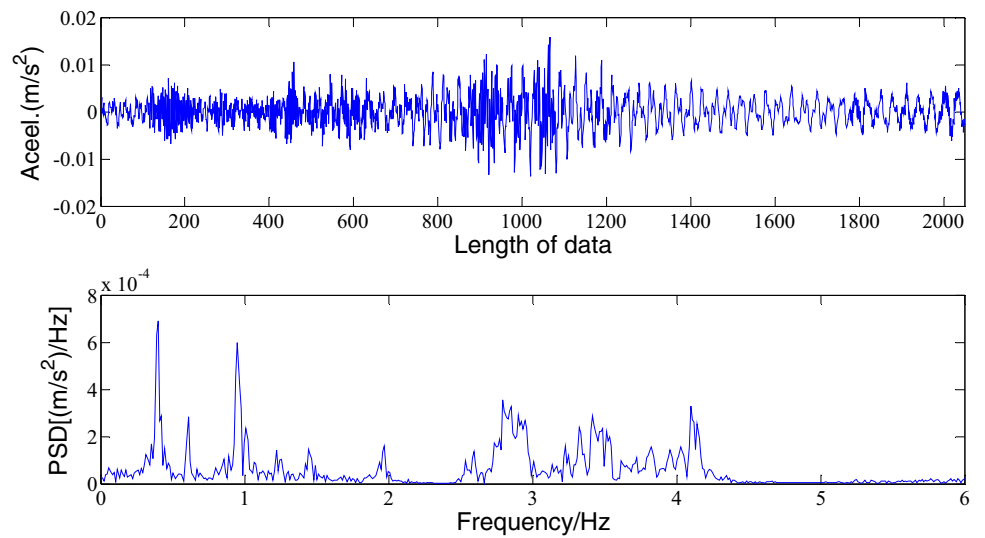
Fig. 12 Distribution of Singular Entropy

the effective rank r should be as twice the number of frequency components. But for this field measurement data, some real modal frequency will be filtered if effective rank r is set as 30. When row number p was set as 512 and effective rank r is set as 40, it is obvious that better result will be gotten. So when dealing with real measurements, effective rank r should be greater than twice the number of frequency components.

Eigensystem Realization Algorithm was applied to identify modal parameters based on the de-noised signal.

Table 2 Result of modal parameters identification

Modal parameters	Mode order					
	1	2	3	4	5	6
Frequency (Hz)	0.399	0.611	0.957	1.247	1.468	1.961
Damping ratio (%)	2.89	1.45	1.04	0.92	0.88	0.85

As shown in Table 2 and Fig. 13, the first six modes identified are in great agreement with [40], which can reflect the real dynamic characteristic of the structure, though there are closely spaced modes in the range of [0, 2] Hz.

5 Conclusion

An adaptive approach for noise reduction is proposed which based on GA and SVD. Both the row number of Hankel matrix p and effective rank r are difficult to determine for noise reduction based on SVD. For the adaptive approach, GA is applied to determine the two key parameters adaptively. In order to demonstrate the validity of the approach, two numerical simulation signals with different frequency components are employed. The results show that p can be $N/4$ or $N/3$ (N is the length of data), r is twice as the number of dominating frequency. As for

Fig. 13 Spectrogram of raw signal and de-noised signal
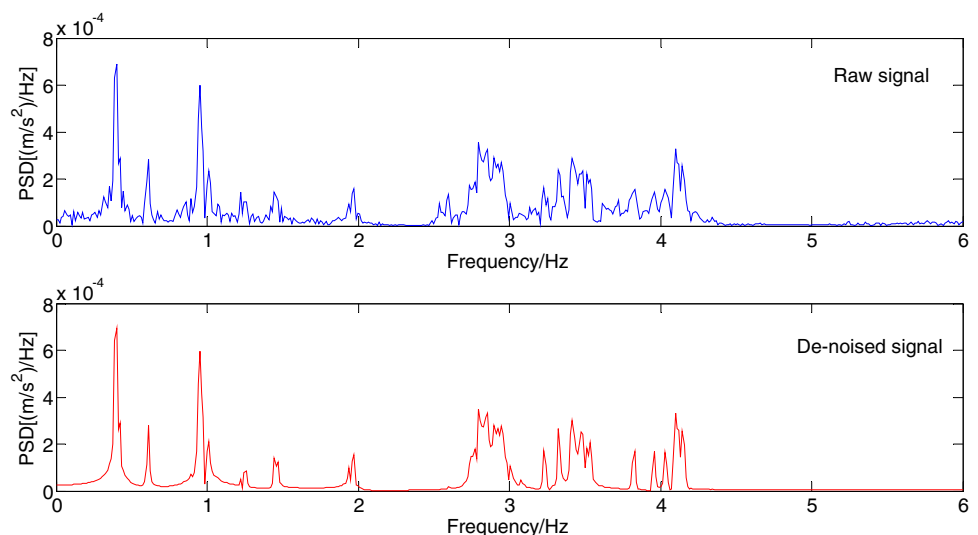
measured signal, the complication of the frequency components might be taken into consideration. And in order not to miss the real frequency components when dealing with measured signals, r should be more than twice as the number of dominating frequency, but p can still be $N/4$ so that no additional computation is needed.

Compliance with ethical standards

Conflict of interest The authors confirm that this article content has no conflict of interest.
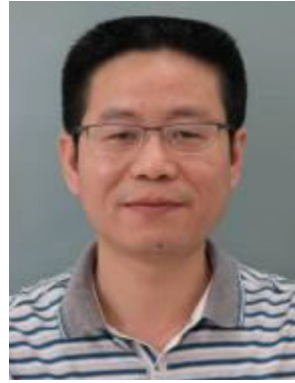
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