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Delay-dependent robust resilient H_∞ control for uncertain singular time-delay system with Markovian jumping parameters

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Abstract
This paper is concerned with the delay-dependent robust resilient H_∞ control problem for uncertain singular time-delay system with Markovian jumping parameters. First, a delay-dependent bounded real lemma in terms of linear matrix inequalities is established, which guarantees the nominal Markovian jump singular system to be regular, impulse free and stochastically stable. Then, based on this condition, sufficient conditions in terms of LMIs are given to ensure the existence of the desired robust resilient H_∞ controllers. The uncertainties of the controllers are considered in two cases, that is the additive controller gain uncertainties and the multiplicative controller gain uncertainties. Finally, numerical examples illustrate the applicability of the results proposed in this paper.

Keywords Markovian jump systems · Singular systems · Linear matrix inequalities (LMIs) · Delay-dependent

1 Introduction

Singular systems contain three kinds of modes: finite dynamic modes, non-dynamic modes and impulsive modes. So, the singular systems can describe the actual system more appropriate when modeling in many practical systems, such as chemical systems, aerospace engineering systems, electrical networks, social economic systems, power systems, circuit systems [1–5]. In addition, when the physical systems appear abrupt variations, Markovian jump systems, as a special class of stochastic hybrid systems, can better describe the actual physical process. Many applications of such systems can be found in [6–9] and the references therein. Recently, many scholars dedicate to the study of the stability analysis and controller synthesis for Markovian jump systems and a lot of relevant conclusion are reported, please see [10–15]. On the other hand, time-delay is also a factor that can not be ignored in the actual process of modeling. It commonly encounters in various engineering systems and frequently leads to the instability and poor performance. In general, the results of time-delay system are divided into two categories, namely, delay-dependent conditions and delay-independent ones [16–21]. Since the stability of systems depends explicitly on the time-delay, a delay-independent condition is more conservative than the delay-dependent ones, especially for small delays. Very recently, much attention has been paid to the study on the singular time-delay systems with Markovian jumping parameters. Many important results on such systems have been reported, see [22–32] and the references therein. In spite of the recent developments on delay-dependent methods, only few results about the singular Markovian jump systems with time delay and the theory is far from being completed. Firstly, the above reports are all using memoryless feedback controllers. Memory state feedback controllers with the feedback provisions on both the current state and the past history of the state may lead to an improved performance. On the other hand, it is worth noting that an implicit assumption inherent in these above design techniques is that the controller is precise, and exactly implemented. But in practice, controllers do have a certain degree of errors due to finite word length in any digital systems, the imprecision inherent in analog systems and need for additional tuning

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of parameters in the final controller implementation [29,30]. These two cases inspire the present work.

In this paper, we deal with the delay-dependent robust resilient H_∞ control problem for uncertain singular Markovian jump time-delay systems. First, a new delay-dependent bounded real lemma (BRL) is provided to guarantee the considered system to be regular, impulse free and stochastically stable with H_∞ performance γ. Then based on this lemma, robust resilient H_∞ controllers are respectively designed to guarantee the resultant closed-loop system is delay-dependent stochastically admissible and satisfies a given H_∞ performance in terms of a set of strict LMIs. Finally numerical examples illustrate the effectiveness and less conservative of the results obtained in this note than the existing approaches.

Notations R^n denotes the n-dimensional Euclidean space; $R^{n\times m}$ is the set of all $n \times m$ real matrices; $X \geq 0 (X > 0)$ means that the symmetrical matrix X is positive semidefinite (positive definite); The superscript T stands for transpose of a matrix; $C_{n,d} = C([-d, 0], R^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-d, 0]$ into R^n. And $x_t := x(t+\theta), \theta \in [-d, 0]$ denotes the function family on $[-d, 0]$, which is generated by n-dimensional real valued continuous function $x(t), t \in [-d, +\infty)$. Obviously $x_t \in C_{n,d}$. The following norms will be used: $\|\cdot\|$ refers to Euclidean vector norm or spectral matrix norm. $\|\phi\|_c := \sup_{-d\leq t\leq 0} \|\phi(t)\|$ stands for the norm of a function $\phi \in C_{n,d}$. $\mathcal{L}_2[0,\infty)$ stands for the space of square integrable functions on $[0, \infty)$. $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space, Ω is the sample space, $\mathcal{F}$ is the algebra of events and $\mathcal{P}$ is the probability measure defined on $\mathcal{F}$. $\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$.

2 Problem formulation and preliminaries

Given a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where Ω is the sample space, $\mathcal{F}$ is the algebra of events and $\mathcal{P}$ is the probability measure defined on $\mathcal{F}$, the singular time-delay Markovian jump system considered in this paper is described by the following dynamics:

$$\begin{aligned} E\dot{x}(t) &= (A(r_t) + \triangle A(r_t))x(t) + (A_d(r_t) \\ &\quad + \triangle A_d(r_t))x(t-d) \\ &\quad + B(r_t)u(t) + (B_w(r_t) + \triangle B_w(r_t))w(t) \\ z(t) &= (C(r_t) + \triangle C(r_t))x(t) + (C_d(r_t) \\ &\quad + \triangle C_d(r_t))x(t-d) \\ &\quad + D(r_t)u(t) + (D_w(r_t) + \triangle D_w(r_t))w(t) \\ x(t) &= \phi(t), t \in [-\bar{d}, 0] \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control input, $w(t) \in R^p$ is the disturbance input and $w(t) \in \mathcal{L}_2[0,\infty]$. $z(t) \in R^q$ is the controlled output. d is an unknown constant delay and satisfies $0 < d \leq \bar{d}$. $\phi(t) \in C_{n,\bar{d}}$ is a compatible initial function. $\{r_t, t \geq 0\}$ is a homogeneous finite-state Markovian process with right continuous trajectories and taking values in a finite set $\mathcal{S} = \{1, \ldots, s\}$ with transition probability matrix $\Pi = \{\pi_{ij}\}$ given by

$$Pr\{r_{t+h} = j | r_t = i\} = \begin{cases} \pi_{ij}h + o(h), \; j \neq i \\ 1 + \pi_{ii}h + o(h), \; j = i \end{cases}$$

where $h > 0, lim_{h\to 0}\frac{o(h)}{h} = 0$ and $\pi_{ij} \geq 0$, for $j \neq i$, is the transition rate from the mode i at time t to the mode j at time $t+h$ and $\pi_{ii} = -\sum_{j=1, j\neq i}^{s} \pi_{ij}$. The matrix $E \in R^{n\times n}$ is singular and $0 < rankE = r \leq n$. $A(r_t), A_d(r_t), B(r_t), B_w(r_t), C(r_t), C_d(r_t), D(r_t)$ and $D_w(r_t)$ are known real constant matrices with appropriate dimensions. And it is assumed that the uncertainties $\triangle A(r_t), \triangle A_d(r_t), \triangle B_w(r_t), \triangle C(r_t), \triangle C_d(r_t)$ and $\triangle D_w(r_t)$ are norm-bounded and for each $i \in \mathcal{S}$ can be described as

$$\begin{aligned} &\begin{bmatrix} \triangle A_i(t) & \triangle A_{di}(t) & * & \triangle B_{wi}(t) \\ \triangle C_i(t) & \triangle C_{di}(t) & * & \triangle D_{wi}(t) \end{bmatrix} \\ &= \begin{bmatrix} M_{1i} \\ M_{2i} \end{bmatrix} F_{1i}(t) \begin{bmatrix} N_{1i} & N_{2i} & * & N_{3i} \end{bmatrix} \\ &F_{1i}^T(t)F_{1i}(t) \leq I, \forall i \in \mathcal{S} \end{aligned} \tag{2}$$

where $M_{1i}, M_{2i}, N_{1i}, N_{2i}$ and N_{3i} are known real constant matrices with appropriate dimensions for each $i \in \mathcal{S}$, and $F_{1i}(t)$ are unknown matrix functions.

The objective of this note is to develop resilient memory state feedback controllers:

$$\begin{aligned} u(t) &= (K(r_t) + \triangle K(r_t))x(t) \\ &\quad + (K_d(r_t) + \triangle K_d(r_t))x(t-d) \end{aligned} \tag{3}$$

such that the closed-loop system constructed by (1)–(3) is robustly and stochastically admissible with H_∞ performance γ for any constant time-delay d satisfying $0 \leq d \leq \bar{d}$. In this case, the controller (3) is called a robust resilient controller of the system (1).

For the controller gain uncertainties $\triangle K(r_t), \triangle K_d(r_t)$, for each possible $i \in \mathcal{S}$, the following two forms will be considered:

(a) Additive controller gain uncertainties:

$$\begin{aligned} &\begin{bmatrix} \triangle K_i(t) & \triangle K_{di}(t) \end{bmatrix} \\ &= M_{+i}F_{2i}(t)\begin{bmatrix} N_{+i} & N_{+di} \end{bmatrix}, \\ &F_{2i}^T(t)F_{2i}(t) \leq I, \forall i \in \mathcal{S} \end{aligned} \tag{4}$$

(b) Multiplicative controller gain uncertainties:

$$\begin{aligned}&\left[\triangle K_i(t)\ \triangle K_{di}(t)\right]\\&=M_{\times i}F_{3i}(t)\left[N_{\times i}K_i(t)\ N_{\times di}K_{di}(t)\right],\\&F_{3i}^T(t)F_{3i}(t)\le I,\forall i\in\mathcal{S}\end{aligned}\tag{5}$$

where M_{+i}, $M_{\times i}$, N_{+i}, $N_{\times i}$, N_{+di} and $N_{\times di}$ are known real constant matrices with appropriate dimensions for each $i\in\mathcal{S}$, and $F_{2i}(t)$, $F_{3i}(t)$ are unknown matrix functions.

$\triangle A(r_t)$, $\triangle A_d(r_t)$, $\triangle B_w(r_t)$, $\triangle C(r_t)$, $\triangle C_d(r_t)$, $\triangle D_w(r_t)$, $\triangle K(r_t)$ and $\triangle K_d(r_t)$ are said to be admissible if (2), (4) and (5) are satisfied.

For notational simplicity, in the sequel, for each possible $i\in\mathcal{S}$, matrices $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $B_w(r_t)$, $C(r_t)$, $C_d(r_t)$, $D(r_t)$, $D_w(r_t)$, $K(r_t)$ and $K_d(r_t)$ will be respectively denoted by A_i, A_{di}, B_i, B_{wi}, C_i, C_{di}, D_i, D_{wi}, K_i and K_{di}.

About the definition of the robustly and stochastically admissible with H_∞ performance, it can be referred to following definitions:

Definition 2.1 [7]

1. For a given scalar $\bar{d}>0$, the singular Markovain jump time-delay system

$$\begin{aligned}E\dot{x}(t)&=A_ix(t)+A_{di}x(t-d)\\x(t)&=\phi(t),t\in[-\bar{d},0]\end{aligned}\tag{6}$$

is said to be regular and impulse free for any constant d satisfying $0\le d\le\bar{d}$, if the pairs (E,A_i) and (E,A_i+A_{di}) are regular and impulse free for any $i\in S$;

2. The singular Markovian jump time-delay system (6) is said to be stochastically stable, if there exists a scalar $M(r_0,\phi(\cdot))$ such that

$$lim_{t\to\infty}\mathcal{E}\left\{\int_0^t\|x(s)\|^2ds|r_0,x(s)=\phi(s),s\in[-\bar{d},0]\right\}\le M(r_0,\phi(\cdot))$$

3. The singular Markovian jump time-delay system (6) is said to be stochastically admissible, if it is regular, impulse free and stochastically stable.

Definition 2.2 [7] The singular Markovian jump time-delay system (1) is said to be stochastically admissible with H_∞ performance γ, if the system with $F_i(t)=0$, $u(t)=0$ and $w(t)=0$ is stochastically admissible and under zero initial condition, the output vector $z(t)$ satisfies

$$\mathcal{E}\left\{\int_0^\infty z^T(t)z(t)dt\right\}\le\gamma^2\int_0^\infty w^T(t)w(t)dt$$

for any non-zero $w(t)\in\mathcal{L}_2[0,\infty]$.

3 Main results

In this section, we shall solve the delay-dependent robust resilient H_∞ control problem for the singular time-delay Markovian jump system (1) in terms of LMIs approach. Initially, we give a sufficient condition of stochastic admissibility for the singular Markovian jump time-delay system (1) with $u(t)\equiv 0$ and $F_i(t)=0$, that is

$$\begin{aligned}E\dot{x}(t)&=A_ix(t)+A_{di}x(t-d)+B_{wi}w(t)\\z(t)&=C_ix(t)+C_{di}x(t-d)+D_{wi}w(t)\\x(t)&=\phi(t),t\in[-\bar{d},0]\end{aligned}\tag{7}$$

which will play a key role in solving the problem.

Lemma 3.1 *For given a scalar $\bar{d}>0,\gamma>0$, the singular Markovian jump time-delay system (7) is regular, impulse free and stochastically stable with H_∞ performance under zero initial conditions for any d satisfying $0\le d\le\bar{d}$, if there exist matrices $Q_{2i}>0$, $Q_2>0$, $R_1>0$, $R_2>0$, $Q_{1i}=\begin{bmatrix}Q_{1i11}&Q_{1i12}\\\star&Q_{1i22}\end{bmatrix}>0$, $Q_1=\begin{bmatrix}Q_{111}&Q_{112}\\\star&Q_{122}\end{bmatrix}>0$, and matrices F_i,G_i,H_i,J_i and nonsingular matrices P_i such that for every $i\in\mathcal{S}$,*

$$E^TP_i=P_i^TE\ge 0\tag{8}$$

$$\bar{\Theta}_i=\begin{bmatrix}\Theta_{i11}&\Theta_{i12}&\Theta_{i13}&\Theta_{i14}&F_i^TB_{wi}&C_i^T\\\star&\Theta_{i22}&-H_i&\Theta_{i24}&G_i^TB_{wi}&0\\\star&\star&\Theta_{i33}&\Theta_{i34}&H_i^TB_{wi}&0\\\star&\star&\star&\Theta_{i44}&J_i^TB_{wi}&C_{di}^T\\\star&\star&\star&\star&-\gamma^2I&D_{wi}^T\\\star&\star&\star&\star&\star&-I\end{bmatrix}<0\tag{9}$$

$$Q_{1i}<Q_1,\ \ Q_{2i}<Q_2\tag{10}$$

where $\mu=max\{|\pi_{ii}|,i\in\mathcal{S}\}$ and

$$\begin{aligned}\Theta_{i11}&=\sum_{j=1}^{s}\pi_{ij}E^TP_j+Q_{1i11}\\&\quad+\frac{\bar{d}}{2}\mu(Q_{111}+Q_2)+F_i^TA_i+A_i^TF_i,\Theta_{i22}\\&=R_1+R_2-G_i^T-G_i,\\\Theta_{i33}&=Q_{1i22}-Q_{1i11}+\frac{\bar{d}}{2}\mu Q_{122}+Q_{2i},\Theta_{i13}\\&=Q_{1i12}+\frac{\bar{d}}{2}\mu Q_{112}+A_i^TH_i,\\\Theta_{i12}&=P_i^T-F_i^T+A_i^TG_i,\Theta_{i44}\\&=-Q_{1i22}-Q_{2i}+J_i^TA_{di}+A_{di}^TJ_i,\\\Theta_{i14}&=F_i^TA_{di}+A_i^TJ_i,\Theta_{i24}=G_i^TA_{di}-J_i,\Theta_{i34}\\&=-Q_{1i12}+H_i^TA_{di}.\end{aligned}$$

Proof Now we firstly prove the system (7) is stochastically admissible. From Definition 2.2, it is to prove that the system (6) is stochastically admissible. Considering the system (6) and from (9), we can easily deduce that for every $i \in \mathcal{S}$, $\mathcal{I}_1 \bar{\Theta}_i \mathcal{I}_1 < 0$. Then

$$\tilde{E}^T \tilde{P} = \tilde{P}^T \tilde{E} \geq 0 \tag{11}$$

$$\begin{bmatrix} \pi_{ii} \tilde{E}^T \tilde{P}_i + \tilde{Q}_1 + \tilde{P}_i^T \tilde{A}_i + \tilde{A}_i^T \tilde{P}_i & \tilde{Q}_2 + \tilde{P}_i^T \tilde{A}_{di} + \tilde{A}_i^T \tilde{J}_i \\ \star & \tilde{Q}_3 + \tilde{J}_i^T \tilde{A}_{di} + \tilde{A}_{di}^T \tilde{J}_i \end{bmatrix} < 0 \tag{12}$$

where

$$\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}, \tilde{A}_i = \begin{bmatrix} 0 & I_n \\ A_i & -I_n \end{bmatrix}, \tilde{A}_{di} = \begin{bmatrix} 0 & 0 \\ A_{di} & 0 \end{bmatrix},$$

$$\tilde{P}_i = \begin{bmatrix} P_i & 0 \\ F_i & G_i \end{bmatrix}, \bar{P}_i = \begin{bmatrix} P_i & 0 \\ F_i + J_i & G_i + H_i \end{bmatrix},$$

$$\tilde{Q}_1 = \begin{bmatrix} Q_{1i11} + \frac{\bar{d}}{2}\mu(Q_{111} + Q_2) & 0 \\ 0 & R_1 + R_2 \end{bmatrix},$$

$$\tilde{Q}_2 = \begin{bmatrix} 0 & Q_{1i12} + \frac{\bar{d}}{2}\mu Q_{112} \\ 0 & 0 \end{bmatrix}, \tilde{J}_i = \begin{bmatrix} 0 & 0 \\ J_i & H_i \end{bmatrix},$$

$$\tilde{Q}_3 = \begin{bmatrix} -Q_{1i22} - Q_{2i} & -Q_{1i12}^T \\ \star & Q_{1i22} - Q_{1i11} + \frac{\bar{d}}{2}\mu Q_{122} + Q_{2i} \end{bmatrix},$$

$$\tilde{R} = \begin{bmatrix} \frac{\bar{d}}{2}\mu Q_2 & 0 \\ 0 & R_1 + R_2 \end{bmatrix},$$

$$\mathcal{I}_1 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}.$$

Obviously, the following inequality is true:

$$\pi_{ii} \tilde{E}^T \tilde{P}_i + \tilde{P}_i^T \tilde{A}_i + \tilde{A}_i^T \tilde{P}_i < 0 \tag{13}$$

Since $rank\, \tilde{E} = rank\, E = r \leq n$, there exist nonsingular matrices $\bar{G}$ and $\bar{H}$, such that

$$\bar{E} = \bar{G} \tilde{E} \bar{H} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \tag{14}$$

Then, for every $i \in \mathcal{S}$, we denote: $\bar{A}_i \triangleq \bar{G} \tilde{A}_i \bar{H}$, $\bar{A}_{di} \triangleq \bar{G} \tilde{A}_{di} \bar{H}$, $\bar{P}_i \triangleq \bar{G}^{-T} \tilde{P}_i \bar{H}$

$$\bar{A}_i \triangleq \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}, \bar{A}_{di} \triangleq \begin{bmatrix} A_{di11} & A_{di12} \\ A_{di21} & A_{di22} \end{bmatrix},$$

$$\bar{P}_i \triangleq \begin{bmatrix} P_{i11} & P_{i12} \\ P_{i21} & P_{i22} \end{bmatrix}.$$

From (11), it is clear that $P_{i12} = 0$, for every $i \in \mathcal{S}$. Pre-multiplying and post-multiplying (13) by $\bar{H}^T$ and $\bar{H}$, respectively, we can deduce

$$A_{i22}^T P_{i22} + P_{i22}^T A_{i22} < 0$$

which implies that A_{i22} and P_{i22} are nonsingular for every $i \in \mathcal{S}$. This implies the pair $(\tilde{E}, \tilde{A}_i)$ is regular and impulse free and $\tilde{P}_i$ is nonsingular, that is G_i is nonsingular for every $i \in \mathcal{S}$. Note that $det(s\tilde{E} - \tilde{A}_i) = det(sE - A_i)$, we can easily get that the pair (E, A_i) is regular and impulse free for every $i \in \mathcal{S}$.

Now, pre-multiplying and post-multiplying (12) by $\begin{bmatrix} I_n & I_n & I_n & I_n \end{bmatrix}$ and $\begin{bmatrix} I_n & I_n & I_n & I_n \end{bmatrix}^T$, respectively, we can obtain

$$\begin{aligned} &\pi_{ii} E^T P_i + F_i^T A_i + A_i^T F_i + A_i^T G_i + G_i^T A_i \\ &+ P_i + P_i^T - F_i - F_i^T - G_i - G_i^T + F_i^T A_{di} \\ &+ A_{di}^T F_i + G_i^T A_{di} + A_{di}^T G_i + A_i^T J_i \\ &+ J_i^T A_i - J_i - J_i^T + J_i^T A_{di} + A_{di}^T J_i + A_i^T H_i \\ &+ H_i^T A_i - H_i - H_i^T + H_i^T A_{di} + A_{di}^T H_i + \frac{\bar{d}}{2}\mu(Q_{111} \\ &+ Q_{112} + Q_{112}^T + Q_{122} + Q_2) + R_1 + R_2 < 0 \end{aligned}$$

that is

$$\begin{aligned} &\begin{bmatrix} I_n & I_n \end{bmatrix} [\pi_{ii} \tilde{E}^T \bar{P}_i + \bar{P}_i^T (\tilde{A}_i + \tilde{A}_{di}) + (\tilde{A}_i \\ &+ \tilde{A}_{di})^T \bar{P}_i + \frac{\bar{d}}{2}\mu Q_1 + \tilde{R}] \begin{bmatrix} I_n & I_n \end{bmatrix}^T < 0 \end{aligned}$$

which implies that

$$\begin{aligned} &\pi_{ii} \tilde{E}^T \bar{P}_i + \bar{P}_i^T (\tilde{A}_i + \tilde{A}_{di}) + (\tilde{A}_i + \tilde{A}_{di})^T \bar{P}_i \\ &+ \frac{\bar{d}}{2}\mu Q_1 + \tilde{R} < 0. \end{aligned}$$

Note that $Q_1 > 0$, $\tilde{R} \geq 0$ and similar to the above analysis, it is true that the $pair(E, A_i + A_{di})$ is regular and impulse free for every $i \in \mathcal{S}$. So, according to Definition 2.1, the singular Markovian jump time-delay system (6) is regular and impulse free for any constant time delay d satisfying $0 \leq d \leq \bar{d}$.

Next we are in the position to prove that the system (6) is stochastically stable. Choose a Lyapunov–Krasovskii functional candidate as

$$V(x_t, r_t, t) = V_1(x_t, r_t, t) + V_2(x_t, r_t, t) + V_3(x_t, r_t, t) \tag{15}$$

where

$$V_1(x_t, r_t, t) = x^T(t)E^T P(r_t)x(t)$$

$$\begin{aligned} V_2(x_t, r_t, t) = & \int_{t-\frac{d}{2}}^{t} \xi^T(\alpha)Q_1(r_t)\xi(\alpha)d\alpha \\ & + \mu \int_{-\frac{d}{2}}^{0} \int_{t+\theta}^{t} \xi^T(\alpha)Q_1\xi(\alpha)d\alpha d\theta \\ & + \frac{2}{\bar{d}} \int_{-\frac{d}{2}}^{0} \int_{t+\theta}^{t} \dot{x}^T(\alpha)E^T R_1 E\dot{x}(\alpha)d\alpha d\theta \end{aligned}$$

$$\begin{aligned} V_3(x_t, r_t, t) = & \int_{t-d}^{t-\frac{d}{2}} x^T(\alpha)Q_2(r_t)x(\alpha)d\alpha \\ & + \mu \int_{-d}^{-\frac{d}{2}} \int_{t+\theta}^{t} x^T(\alpha)Q_2 x(\alpha)d\alpha d\theta \\ & + \frac{2}{\bar{d}} \int_{-d}^{-\frac{d}{2}} \int_{t+\theta}^{t} \dot{x}^T(\alpha)E^T R_2 E\dot{x}(\alpha)d\alpha d\theta \end{aligned}$$

here $\xi^T(t) = \left[x^T(t)\ x^T(t-\frac{d}{2}) \right]$. Let $\mathcal{L}[\cdot]$ be the weak infinitesimal operator of the stochastic process $\{x_t, r_t\}$, then for each $i \in \mathcal{S}$, we have

$$\begin{aligned} \mathcal{L}V(x_t, i, t) \le & \dot{x}^T(t)E^T P_i x(t) + x^T(t)E^T P_i\dot{x}(t) \\ & + x^T(t)\sum_{j=1}^{s} \pi_{ij} E^T P_j x(t) + x^T(t)Q_{1i11}x(t) \\ & + 2x^T(t)Q_{1i12}x\left(t-\frac{d}{2}\right) \\ & + x^T\left(t-\frac{d}{2}\right)Q_{1i22}x\left(t-\frac{d}{2}\right) \\ & - x^T\left(t-\frac{d}{2}\right)Q_{1i11}x\left(t-\frac{d}{2}\right) \\ & - 2x^T\left(t-\frac{d}{2}\right)Q_{1i12}x(t-d) \\ & - x^T(t-d)Q_{1i22}x(t-d) \\ & + 2\cdot\frac{\bar{d}}{2}\mu x^T(t)Q_{112}x\left(t-\frac{d}{2}\right) \\ & + \frac{\bar{d}}{2}\mu x^T(t)Q_{111}x(t) + \int_{t-\frac{d}{2}}^{t} \xi^T(\alpha)\sum_{j=1}^{s} \pi_{ij}Q_{1j}\xi(\alpha)d\alpha \\ & + \frac{\bar{d}}{2}\mu x^T\left(t-\frac{d}{2}\right)Q_{122}x\left(t-\frac{d}{2}\right) \\ & - \mu\int_{t-\frac{d}{2}}^{t} \xi^T(\alpha)Q_1\xi(\alpha)d\alpha + \dot{x}^T(t)E^T R_1 E\dot{x}(t) \\ & - \frac{2}{\bar{d}}\int_{t-\frac{d}{2}}^{t} \dot{x}^T(\alpha)E^T R_1 E\dot{x}(\alpha)d\alpha \\ & + x^T\left(t-\frac{d}{2}\right)Q_{2i}x\left(t-\frac{d}{2}\right) - x^T(t-d)Q_{2i}x(t-d) \\ & + \int_{t-d}^{t-\frac{d}{2}} x^T(\alpha)\sum_{j=1}^{s} \pi_{ij}Q_{2j}x(\alpha)d\alpha \\ & + \frac{\bar{d}}{2}\mu x^T(t)Q_2 x(t) \\ & - \mu\int_{t-d}^{t-\frac{d}{2}} x^T(\alpha)Q_2 x(\alpha)d\alpha + \dot{x}^T(t)E^T R_2 E\dot{x}(t) \\ & - \frac{2}{\bar{d}}\int_{t-d}^{t-\frac{d}{2}} \dot{x}^T(\alpha)E^T R_2 E\dot{x}(\alpha)d\alpha + [-E\dot{x}(t) \\ & + A_i x(t) + A_{di}x(t-d)]^T\Big[F_i x(t) + G_i E\dot{x}(t) \\ & + J_i x(t-d) + H_i x\left(t-\frac{d}{2}\right)\Big] \\ & + [F_i x(t) + G_i E\dot{x}(t) + J_i x(t-d) \\ & + H_i x\left(t-\frac{d}{2}\right)]^T[-E\dot{x}(t) + A_i x(t) + A_{di}x(t-d)] \end{aligned} \quad (16)$$

According to Jesen integral inequality, the following in equation is true

$$\begin{aligned} & -\frac{2}{\bar{d}}\int_{t-\frac{d}{2}}^{t} \dot{x}^T(\alpha)E^T R_1 E\dot{x}(\alpha)d\alpha \\ & \le -\int_{t-\frac{d}{2}}^{t} \frac{2}{\bar{d}}\dot{x}^T(\alpha)E^T d\alpha R_1 \int_{t-\frac{d}{2}}^{t} \frac{2}{\bar{d}}E\dot{x}(\alpha)d\alpha \\ & \quad -\frac{2}{\bar{d}}\int_{t-d}^{t-\frac{d}{2}} \dot{x}^T(\alpha)E^T R_2 E\dot{x}(\alpha)d\alpha \\ & \le -\int_{t-d}^{t-\frac{d}{2}} \frac{2}{\bar{d}}\dot{x}^T(\alpha)E^T d\alpha R_2 \int_{t-d}^{t-\frac{d}{2}} \frac{2}{\bar{d}}E\dot{x}(\alpha)d\alpha \end{aligned} \quad (17)$$

Noting $\pi_{ij} > 0$, for $i \ne j$ and $-\mu \le \pi_{ii} < 0$, and from (10), we can get

$$\begin{aligned} & \int_{t-\frac{d}{2}}^{t} \xi^T(\alpha)\sum_{j=1}^{s} \pi_{ij}Q_{1j}\xi(\alpha)d\alpha \le \int_{t-\frac{d}{2}}^{t} \xi^T(\alpha) \\ & \quad \sum_{j=1, j\ne i}^{s} \pi_{ij}Q_{1j}\xi(\alpha)d\alpha \le \mu\int_{t-\frac{d}{2}}^{t} \xi^T(\alpha)Q_1\xi(\alpha)d\alpha \end{aligned} \quad (18)$$

$$\begin{aligned} & \int_{t-d}^{t-\frac{d}{2}} x^T(\alpha)\sum_{j=1}^{s} \pi_{ij}Q_{2j}x(\alpha)d\alpha \le \int_{t-d}^{t-\frac{d}{2}} x^T(\alpha) \\ & \quad \sum_{j=1, j\ne i}^{s} \pi_{ij}Q_{2j}x(\alpha)d\alpha \le \mu\int_{t-d}^{t-\frac{d}{2}} x^T(\alpha)Q_2 x(\alpha)d\alpha \end{aligned} \quad (19)$$

Then from (16)–(19), we can obtain that, for every $i \in \mathcal{S}$,

$$\mathcal{L}V(x_t, i, t) \le \eta^T(t)\Theta_i\eta(t) \quad (20)$$

where

$$\eta(t)^T = \left[x^T(t)\ (E\dot{x}(t))^T\ x^T(t-\tfrac{d}{2})\ x^T(t-d)]^T \right],$$

$$\Theta_i = \begin{bmatrix} \Theta_{i11} & \Theta_{i12} & \Theta_{i13} & \Theta_{i14} \\ \star & \Theta_{i22} & -H_i & \Theta_{i24} \\ \star & \star & \Theta_{i33} & \Theta_{i34} \\ \star & \star & \star & \Theta_{i44} \end{bmatrix}$$

Note that $R_1 > 0$, $R_2 > 0$ and from (9), it is easy to get that for every $i \in \mathcal{S}$,

$$\mathcal{L}V(x_t, i, t) \le \lambda_{max}(\Theta_i) \parallel x(t) \parallel^2 .$$

Therefore, using Dynkin's formula, for any $t \ge \bar{d}$,

$$\mathcal{E}V(x_t, i, t) - \mathcal{E}V(x_{\bar{d}}, r_{\bar{d}}, \bar{d}) \le -\lambda_{max}(\Theta_i)\mathcal{E}\int_{\bar{d}}^{t} \parallel x(s) \parallel^2 ds.$$

So the following in equation is true

$$\mathcal{E}\int_{d}^{t} \parallel x(s) \parallel^2 ds \le \lambda_{max}^{-1}(\Theta_i)\mathcal{E}V(x_{\bar{d}}, r_{\bar{d}}, \bar{d}). \tag{21}$$

From the above analysis, we know that for every $i \in \mathcal{S}$, the $pair(E, A_i)$ is regular and impulse free. Then there exist nonsingular matrices $\bar{M}$, $\bar{N}$ such that (E, A_i) for every $i \in \mathcal{S}$ is r.s.e. to the Weierstrass standard form $\bar{E} = \bar{M}E\bar{N}$, $\bar{A}_i = \bar{M}A_i\bar{N}$,

$$\bar{E} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \bar{A}_i = \begin{bmatrix} A_{i1} & 0 \\ 0 & I_{n-r} \end{bmatrix}, \bar{A}_{di} = \begin{bmatrix} A_{di11} & A_{di12} \\ A_{di21} & A_{di22} \end{bmatrix}.$$

Then, for every $i \in \mathcal{S}$, system (6) can be equivalently transformed into

$$\begin{aligned} \dot{y}_1(t) &= A_{i1}y_1(t) + A_{di11}y_1(t-d) + A_{di12}y_2(t-d), \\ -y_2(t) &= A_{di21}y_1(t-d) + A_{di22}y_2(t-d), \\ \psi(t) &= \bar{N}^{-1}\phi(t), t \in [-\bar{d}, 0], \end{aligned} \tag{22}$$

where $y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \bar{N}^{-1}x(t)$, $y_1(t) \in R^r$, $y_2(t) \in R^{n-r}$. Then for any $t \in [0, \bar{d}]$,

$$\parallel y_1(t) \parallel \le \parallel e^{A_{i1}t}y_1(0)\parallel + \parallel \int_0^t e^{A_{i1}(t-s)}[A_{di11}y_1(s-d) + A_{di12}y_2(s-d)]ds \parallel \le k_1 \parallel \psi \parallel_{\bar{d}}$$

where $k_1 = \max_{i\in\mathcal{S}}\{[1+\bar{d}(\|A_{di11}\|+\|A_{di12}\|)] \max_{t\in[0,\bar{d}]} \|e^{A_{i1}t}\|\} \ge 0$.

Similarly, we can get

$$\parallel y_2(t) \parallel \le k_2 \parallel \psi \parallel_{\bar{d}}$$

where $k_2 = \max_{i\in\mathcal{S}}\{\parallel A_{di21} \parallel + \parallel A_{di22} \parallel\} \ge 0$.

It is clear that

$$\sup_{0\le s\le\bar{d}} \parallel y_1(s) \parallel^2 \le k_1^2 \parallel \psi \parallel_{\bar{d}}^2,\ \sup_{0\le s\le\bar{d}} \parallel y_2(s) \parallel^2 \le k_2^2 \parallel \psi \parallel_{\bar{d}}^2 . \tag{23}$$

Obviously, there exists a scalar $k_3 > 0$ such that

$$\sup_{0\le s\le\bar{d}} \parallel x(s) \parallel^2 \le k_3 \parallel \phi \parallel_{\bar{d}}^2 .$$

Noticing (15) and system (22), we can get that there exists a scalar $\sigma > 0$ such that

$$V(x_d, i, d) \le \sigma \parallel \phi \parallel_{\bar{d}}^2$$

From the above analysis, together with (21), there exist a scalar $\rho > 0$ such that

$$\mathcal{E}\int_0^t \parallel x(s) \parallel^2 ds \le \mathcal{E}\int_0^{\bar{d}} \parallel x(s) \parallel^2 ds + \mathcal{E}\int_{\bar{d}}^t \parallel x(s) \parallel^2 ds \le \rho \parallel \phi \parallel_{\bar{d}}^2 \tag{24}$$

Then from Definition 2.1, system (6) is stochastically stable for any constant time delay d satisfying $0 \le d \le \bar{d}$.

Secondly we prove that the output vector $z(t)$, under zero initial condition, satisfies

$$\mathcal{E}\left\{\int_0^\infty z^T(t)z(t)dt\right\} \le \gamma^2 \int_0^\infty w^T(t)w(t)dt \tag{25}$$

for any non-zero $w(t) \in \mathcal{L}_2[0, \infty]$.

Now we quote the following index:

$$J_{zw}(t) \triangleq \mathcal{E}\left\{\int_0^t [z^T(s)z(s) - \gamma^2 w^T(s)w(s)]ds\right\}$$

Under zero initial conditions, we can easily obtain that

$$J_{zw}(t) \le \mathcal{E}\left\{\int_0^t [z^T(s)z(s) - \gamma^2 w^T(s)w(s) + \mathcal{L}V(x_s, i, s)]ds\right\} \le \mathcal{E}\left\{\int_0^t \zeta^T(s)(\Psi_i + \Xi_i^T\Xi_i)\zeta(s)ds\right\}$$

where

$$\Psi_i = \begin{bmatrix} \Theta_{i11} & \Theta_{i12} & \Theta_{i13} & \Theta_{i14} & F_i^T B_{wi} \\ \star & \Theta_{i22} & -H_i & \Theta_{i24} & G_i^T B_{wi} \\ \star & \star & \Theta_{i33} & \Theta_{i34} & H_i^T B_{wi} \\ \star & \star & \star & \Theta_{i44} & J_i^T B_{wi} \\ \star & \star & \star & \star & -\gamma^2 I \end{bmatrix},$$

$$\zeta^T(t) = \left[x^T(t)\ (E\dot{x}(t))^T\ x^T(t-\tfrac{d}{2})\ x^T(t-d)\ w^T(t) \right],$$

$$\Xi_i = \left[C_i\ 0\ 0\ C_{di}\ D_{wi} \right].$$

By using the Schur's complement lemma, it is easy from (9) to get that for any $t > 0$,

$$J_{zw}(t) < 0$$

To this end, we get (25) for any non-zero $w(t) \in \mathcal{L}_2[0, \infty]$. This completes the proof. □

Remark 3.1 This result is similar to the conclusion of the author's earlier work in [22]. But here in this note, when dealing with the Lypunov function $\mathcal{L}V(x_t, i, t)$, we quote new matrices $H_i, i \in \mathcal{S}$ in order to take the time-delay state $x\left(t - \frac{d}{2}\right)$ into account in the conditions. This makes the results here more freedom.

Now, we will try to talk about the sufficient conditions for the existence of the robust resilient controllers of the systems (1). Applying the memory state feedback controllers (3) to the systems (1), the closed-loop system can be obtained as the following

$$\begin{aligned} E\dot{x}(t) &= (A_{ki} + \triangle A_{ki})x(t) + (A_{kdi} + \triangle A_{kdi})x(t-d) \\ &\quad + (B_{wi} + \triangle B_{wi})w(t) \\ z(t) &= (C_{ki} + \triangle C_{ki})x(t) + (C_{kdi} + \triangle C_{kdi})x(t-d) \\ &\quad + (D_{wi} + \triangle D_{wi})w(t) \\ x(t) &= \phi(t), t \in [-\bar{d}, 0] \end{aligned} \tag{26}$$

where $A_{ki} = A_i + B_i K_i, \triangle A_{ki} = \triangle A_i + B_i \triangle K_i, A_{kdi} = A_{di} + B_i K_{di}, \triangle A_{kdi} = \triangle A_{di} + B_i \triangle K_{di}, C_{ki} = C_i + D_i K_i, \triangle C_{ki} = \triangle C_i + D_i \triangle K_i, C_{kdi} = C_{di} + D_i K_{di}, \triangle C_{kdi} = \triangle C_{di} + D_i \triangle K_{di}$.

From Lemma 3.1, we know that for given scalars $\bar{d} > 0, \gamma > 0$, the sufficient conditions to guarantee that the closed systems (26) is regular, impulse free and stochastically stable with H_∞ performance under zero initial conditions for any d satisfying $0 \le d \le \bar{d}$ are the following: there exist matrices $Q_{2i} > 0, Q_2 > 0, R_1 > 0, R_2 > 0$, $Q_{1i} = \begin{bmatrix} Q_{1i11} & Q_{1i12} \\ \star & Q_{1i22} \end{bmatrix} > 0$, $Q_1 = \begin{bmatrix} Q_{111} & Q_{112} \\ \star & Q_{122} \end{bmatrix} > 0$, and matrices F_i, G_i, H_i, J_i and nonsingular matrices P_i satisfying (8), (10) for every $i \in \mathcal{S}$ and

$$\bar{\Theta}_{ki} = \begin{bmatrix} \Theta_{ki11} & \Theta_{ki12} & \Theta_{ki13} & \Theta_{ki14} & F_i^T(B_{wi} + \triangle B_{wi}) & (C_{ki} + \triangle C_{ki})^T \\ \star & \Theta_{i22} & -H_i & \Theta_{ki24} & G_i^T(B_{wi} + \triangle B_{wi}) & 0 \\ \star & \star & \Theta_{i33} & \Theta_{ki34} & H_i^T(B_{wi} + \triangle B_{wi}) & 0 \\ \star & \star & \star & \Theta_{ki44} & J_i^T(B_{wi} + \triangle B_{wi}) & (C_{kdi} + \triangle C_{kdi})^T \\ \star & \star & \star & \star & -\gamma^2 I & (D_{wi} + \triangle D_{wi})^T \\ \star & \star & \star & \star & \star & -I \end{bmatrix} < 0 \tag{27}$$

where

$$\begin{aligned} \Theta_{ki11} &= \sum_{j=1}^{s} \pi_{ij} E^T P_j + Q_{1i11} + \frac{\bar{d}}{2}\mu(Q_{111} + Q_2) \\ &\quad + F_i^T(A_{ki} + \triangle A_{ki}) + (A_{ki} + \triangle A_{ki})^T F_i, \\ \Theta_{ki44} &= -Q_{1i22} - Q_{2i} + J_i^T(A_{kdi} + \triangle A_{kdi}) + (A_{kdi} \\ &\quad + \triangle A_{kdi})^T J_i, \Theta_{ki24} = G_i^T(A_{kdi} + \triangle A_{kdi}) - J_i, \\ \Theta_{ki12} &= P_i^T - F_i^T + (A_{ki} + \triangle A_{ki})^T G_i, \Theta_{ki13} = Q_{1i12} \\ &\quad + \frac{\bar{d}}{2}\mu Q_{112} + (A_{ki} + \triangle A_{ki})^T H_i, \\ \Theta_{ki14} &= F_i^T(A_{kdi} + \triangle A_{kdi}) + (A_{ki} + \triangle A_{ki})^T J_i, \Theta_{ki34} \\ &= -Q_{1i12} + H_i^T(A_{kdi} + \triangle A_{kdi}). \end{aligned}$$

Now pre-multiplying and post-multiplying $\bar{\Theta}_{ki}$ by $\mathcal{I}_2$ and $\mathcal{I}_2^T$, respectively. The following in equation can be obtained

$$\mathcal{I}_2 \bar{\Theta}_{ki} \mathcal{I}_2^T = \Gamma_i^T(\Omega_i + \Delta\Omega_i) + (\Omega_i + \Delta\Omega_i)^T \Gamma_i + \Sigma_i < 0 \tag{28}$$

where

$$\mathcal{I}_2 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}, \Gamma_i = \begin{bmatrix} P_i & 0 & 0 & 0 & 0 & 0 \\ 0 & P_i & 0 & 0 & 0 & 0 \\ F_i & J_i & G_i & H_i & 0 & 0 \\ 0 & 0 & 0 & P_i & 0 & 0 \\ 0 & 0 & 0 & 0 & I_n & 0 \\ 0 & 0 & 0 & 0 & 0 & I_n \end{bmatrix}, \Omega_i = \begin{bmatrix} 0 & 0 & I_n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ A_{ki} & A_{kdi} & -I_n & 0 & B_{wi} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ C_{ki} & C_{kdi} & 0 & 0 & D_{wi} & 0 \end{bmatrix} \Delta\Omega_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta A_{ki} & \Delta A_{kdi} & 0 & 0 & \Delta B_{wi} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta C_{ki} & \Delta C_{kdi} & 0 & 0 & \Delta D_{wi} & 0 \end{bmatrix},$$

$$\Sigma_i = \begin{bmatrix} \sum_{j=1}^{s} \pi_{ij} E^T P_j + Q_{1i11} + \frac{\bar{d}}{2}\mu(Q_{111} + Q_2) & 0 & 0 & Q_{1i12} + \frac{\bar{d}}{2}\mu Q_{112} & 0 & 0 \\ 0 & -Q_{1i22} - Q_{2i} & 0 & -Q_{1i12}^T & 0 & 0 \\ 0 & 0 & R_1 + R_2 & 0 & 0 & 0 \\ Q_{1i12}^T + \frac{\bar{d}}{2}\mu Q_{112}^T & -Q_{1i12} & 0 & \Theta_{i33} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\gamma^2 I_n & 0 \\ 0 & 0 & 0 & 0 & 0 & -I_n \end{bmatrix}$$

Now considering the controllers with the additive gain uncertainties (4), we have

$$\mathcal{I}_2\bar{\Theta}_{ki}\mathcal{I}_2^T = \Gamma_i^T\Omega_i + \Omega_i^T\Gamma_i + \Sigma_i + \Gamma_i^T \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ M_{1i} & B_iM_{+i} \\ 0 & 0 \\ 0 & 0 \\ M_{2i} & D_iM_{+i} \end{bmatrix} \begin{bmatrix} F_{1i} & 0 \\ 0 & F_{2i} \end{bmatrix} \begin{bmatrix} (N_{1i}L_i)^T & (N_{+i}L_i)^T \\ (N_{2i}L_i)^T & (N_{+di}L_i)^T \\ 0 & 0 \\ 0 & 0 \\ N_{3i}^T & 0 \\ 0 & 0 \end{bmatrix}^T + \begin{bmatrix} (N_{1i}L_i)^T & (N_{+i}L_i)^T \\ (N_{2i}L_i)^T & (N_{+di}L_i)^T \\ 0 & 0 \\ 0 & 0 \\ N_{3i}^T & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_{1i}^T & 0 \\ 0 & F_{2i}^T \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ M_{1i} & B_iM_{+i} \\ 0 & 0 \\ 0 & 0 \\ M_{2i} & D_iM_{+i} \end{bmatrix}^T \Gamma_i < 0 \tag{29}$$

From Lemma 3.1, we know that P_i, G_i are nonsingular for every $i \in \mathcal{S}$. Then we define

$$\Gamma_i^{-1} = \begin{bmatrix} P_i & 0 & 0 & 0 & 0 & 0 \\ 0 & P_i & 0 & 0 & 0 & 0 \\ F_i & J_i & G_i & H_i & 0 & 0 \\ 0 & 0 & 0 & P_i & 0 & 0 \\ 0 & 0 & 0 & 0 & I_n & 0 \\ 0 & 0 & 0 & 0 & 0 & I_n \end{bmatrix}^{-1} \triangleq \begin{bmatrix} L_i & 0 & 0 & 0 & 0 & 0 \\ 0 & L_i & 0 & 0 & 0 & 0 \\ M_i & T_i & N_i & W_i & 0 & 0 \\ 0 & 0 & 0 & L_i & 0 & 0 \\ 0 & 0 & 0 & 0 & I_n & 0 \\ 0 & 0 & 0 & 0 & 0 & I_n \end{bmatrix}.$$

And set

$$\bar{Q}_{1i} = diag\{L_i^T, L_i^T\} \cdot Q_{1i} \cdot diag\{L_i, L_i\} = \begin{bmatrix} \bar{Q}_{1i11} & \bar{Q}_{1i12} \\ \star & \bar{Q}_{1i22} \end{bmatrix},$$

$$\bar{Q}_1 = diag\{L_i^T, L_i^T\} \cdot Q_1 \cdot diag\{L_i, L_i\} = \begin{bmatrix} \bar{Q}_{111} & \bar{Q}_{112} \\ \star & \bar{Q}_{122} \end{bmatrix},$$

$$\bar{Q}_{2i} = L_i^T Q_{2i} L_i,\ \bar{Q}_2 = L_i^T Q_2 L_i,\ S_i = K_i L_i,\ S_{di} = K_{di} L_i.$$

Then pre-multiplying and post-multiplying (8) by L_i^T and L_i, respectively, we can have

$$L_i^T E^T = EL_i \geq 0 \tag{30}$$

Pre-multiplying and post-multiplying $Q_{1i} < Q_1$ by $diag\{L_i^T, L_i^T\}$ and $diag\{L_i, L_i\}$, respectively, and pre-multiplying and post-multiplying $Q_{2i} < Q_2$ by L_i^T and L_i, respectively, following in equations can be obtained

$$\bar{Q}_{1i} < \bar{Q}_1, \quad \bar{Q}_{2i} < \bar{Q}_2 \tag{31}$$

Pre-multiply and post-multiply (29) by Γ_i^{-T} and Γ_i^{-1}. It is worthy to notice that there appears the term $\sum_{j=1, j\neq i}^{s} \pi_{ij} L_i^T E^T L_j^{-1} L_i$. Now without loss of generality, in the following we assume that $E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$. From Lemma 3.1, $L_i = \begin{bmatrix} L_{i11} & 0 \\ L_{i21} & L_{i22} \end{bmatrix}$ and $L_{i11} > 0$. It is easy to get that

$$\sum_{j=1, j\neq i}^{s} \pi_{ij} L_i^T E^T L_j^{-1} L_i = \sum_{j=1, j\neq i}^{s} \pi_{ij} \begin{bmatrix} L_{i11} \\ 0 \end{bmatrix} L_{j11}^{-1} \begin{bmatrix} L_{i11} & 0 \end{bmatrix}$$

Then by Schur complement lemma, the following condition can be obtained

$$\begin{bmatrix} \Upsilon_{i11} & T_i & \Upsilon_{i13} & \Upsilon_{i14} & 0 & \Upsilon_{i16} & M_i^T & M_i^T & X_i & 0 & 0 & \bar{N}_1 & \Lambda_+ \\ \star & \Upsilon_{i22} & \Upsilon_{i23} & -\bar{Q}_{1i12}^T & 0 & \Upsilon_{i26} & T_i^T & T_i^T & 0 & 0 & 0 & \bar{N}_2 & \Lambda_{+d} \\ \star & \star & \Upsilon_{i33} & -W_i & B_{wi} & 0 & N_i^T & N_i^T & 0 & \alpha_i M_{1i} & \alpha_i B_i M_{+i} & 0 & 0 \\ \star & \star & \star & \Upsilon_{i44} & 0 & 0 & W_i^T & W_i^T & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & -\gamma^2 I & D_{wi}^T & 0 & 0 & 0 & 0 & 0 & N_{3i}^T & 0 \\ \star & \star & \star & \star & \star & -I & 0 & 0 & 0 & \alpha_i M_{2i} & \alpha_i D_i M_{+i} & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -R_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & -R_2^{-1} & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & -Z_i & 0 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & -\alpha_i I & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\alpha_i I & 0 & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\alpha_i I & 0 \\ \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\alpha_i I \end{bmatrix} < 0 \tag{32}$$

where

$$\Lambda_{+} = (N_{+i}L_i)^T, \Lambda_{+d} = (N_{+di}L_i)^T, \bar{N}_1 = (N_{1i}L_i)^T, \bar{N}_2 = (N_{2i}L_i)^T,$$

$$\Upsilon_{i11} = \pi_{ii}L_i^T E^T + M_i + M_i^T + \bar{Q}_{1i11} + \frac{\bar{d}}{2}\mu(\bar{Q}_{111} + \bar{Q}_2),$$

$$\Upsilon_{i22} = -\bar{Q}_{1i22} - \bar{Q}_{2i},$$

$$\Upsilon_{i16} = L_i^T C_i^T + S_i^T D_i^T, \Upsilon_{i26} = L_i^T C_{di}^T + S_{di}^T D_i^T,$$

$$\Upsilon_{i14} = \bar{Q}_{1i12} + \frac{\bar{d}}{2}\mu\bar{Q}_{112} + W_i$$

$$\Upsilon_{i13} = N_i - M_i^T + L_i^T A_i^T + S_i^T B_i^T, \Upsilon_{i23} = L_i^T A_{di}^T + S_{di}^T B_i^T - T_i^T, \Upsilon_{i33} = -N_i - N_i^T$$

$$\Upsilon_{i44} = \bar{Q}_{1i22} - \bar{Q}_{1i11} + \frac{\bar{d}}{2}\mu\bar{Q}_{122} + \bar{Q}_{2i}, Z_i = diag\{L_{111}, \ldots, L_{(i-1)11}, L_{(i+1)11}, \ldots, L_{s11}\}$$

$$X_i = \left[\sqrt{\pi_{i1}}\begin{bmatrix} L_{i11} \\ 0 \end{bmatrix} \cdots \sqrt{\pi_{i(i-1)}}\begin{bmatrix} L_{i11} \\ 0 \end{bmatrix} \sqrt{\pi_{i(i+1)}}\begin{bmatrix} L_{i11} \\ 0 \end{bmatrix} \cdots \sqrt{\pi_{is}}\begin{bmatrix} L_{i11} \\ 0 \end{bmatrix}\right]$$

$$\begin{bmatrix}
\Upsilon_{i11} & T_i & \Upsilon_{i13} & \Upsilon_{i14} & 0 & \Upsilon_{i16} & M_i^T & M_i^T & X_i & 0 & 0 & \bar{N}_1 & \Lambda_{\times} \\
\star & \Upsilon_{i22} & \Upsilon_{i23} & -\bar{Q}_{1i12}^T & 0 & \Upsilon_{i26} & T_i^T & T_i^T & 0 & 0 & 0 & \bar{N}_2 & \Lambda_{\times d} \\
\star & \star & \Upsilon_{i33} & -W_i & B_{wi} & 0 & N_i^T & N_i^T & 0 & \alpha_i M_{1i} & \alpha_i B_i M_{\times i} & 0 & 0 \\
\star & \star & \star & \Upsilon_{i44} & 0 & 0 & W_i^T & W_i^T & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & -\gamma^2 I & D_{wi}^T & 0 & 0 & 0 & 0 & 0 & N_{3i}^T & 0 \\
\star & \star & \star & \star & \star & -I & 0 & 0 & 0 & \alpha_i M_{2i} & \alpha_i D_i M_{\times i} & 0 & 0 \\
\star & \star & \star & \star & \star & \star & -R_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & -R_2^{-1} & 0 & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & -Z_i & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & -\alpha_i I & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\alpha_i I & 0 & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\alpha_i I & 0 \\
\star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & \star & -\alpha_i I
\end{bmatrix} < 0 \quad (33)$$

Now, from the above analysis, we can get the following results.

Theorem 3.1 *Consider the Markovian jump singular time-delay system (1). For given scalars $\bar{d} > 0$ and $\gamma > 0$, if there exist matrices $\bar{Q}_{1i} > 0, \bar{Q}_{2i} > 0, \bar{Q}_1 > 0, \bar{Q}_2 > 0, R_1 > 0, R_2 > 0$, and matrices L_i, M_i, T_i, N_i, W_i, S_i, S_{di} and a scalar $\alpha_i > 0$, for every $i \in \mathcal{S}$ satisfying (30), (31) and (32) then there exist controllers with the additive uncertainties $u(t) = (S_i L_i^{-1} + M_{+i}F_{2i}N_{+i})x(t) + (S_{di}L_i^{-1} + M_{+i}F_{2i}N_{+di})x(t-d), i \in \mathcal{S}$ such that the closed-loop systems (26) is stochastically admissible with H_∞ performance γ under zero initial conditions for any constant time delay d satisfying $0 \le d \le \bar{d}$.*

As to the case of the controllers with the multiplicative gain uncertainties, similar to the above analysis we can get the following theorem.

Theorem 3.2 *Consider the Markovian jump singular time-delay system (1). For given scalars $\bar{d} > 0$ and $\gamma > 0$, if there exist matrices $\bar{Q}_{1i} > 0, \bar{Q}_{2i} > 0, \bar{Q}_1 > 0, \bar{Q}_2 > 0, R_1 > 0, R_2 > 0$, and matrices L_i, M_i, T_i, N_i, W_i, S_i, S_{di} and a scalar $\alpha_i > 0$, for every $i \in \mathcal{S}$ satisfying (30), (31) and (33) then there exist controllers with the multiplicative uncertainties $u(t) = (I + M_{\times i}F_{3i}N_{\times i})S_i L_i^{-1}x(t) + (I + M_{\times i}F_{3i}N_{\times di})S_{di}L_i^{-1}x(t-d), i \in \mathcal{S}$ such that the closed-loop systems (26) is stochastically admissible with H_∞ performance γ under zero initial conditions for any constant time delay d satisfying $0 \le d \le \bar{d}$.*

where

$$\Lambda_{\times} = (N_{\times i}S_i)^T, \Lambda_{\times d} = (N_{\times di}S_{di})^T.$$

Remark 3.2 Theorems 3.1 and 3.2 provide the delay-dependent sufficient conditions for the design of the robust resilient H_∞ controllers of the Markovian jump singular time-delay systems (1) in terms of LMIs. The controllers considered in these results are $u(t) = (K(r_t) + \triangle K(r_t))x(t) + (K_d(r_t) + \triangle K_d(r_t))x(t-d)$.

The state feedback controllers $u(t)$ cover not only the current state $x(t)$ but also the state of $t \in [t-d, t]$. This leads to that the results are more general. To the contrary, if we take the $K_d(r_t) + \triangle K_d(r_t) = 0$ in Theorems 3.1 and 3.2, the results can be deduced into the well-known LMI conditions for memoryless controller synthesis similarly to references [14–17].

Remark 3.3 The gains the controllers include two parts: $K(r_t)$, $K_d(r_t)$ and $\triangle K(r_t)$, $\triangle K_d(r_t)$. The existence of the controller gain perturbations ensures that the controllers themselves have certain robustness. This means that the controllers needn't to be precise and exactly implemented. And the results allow the controllers gains have a certain degree of perturbations, which is more advantageous to the actual situation of the realization of the controllers.

4 Numerical examples

Example 4.1 Consider the singular Markovian jump time-delay system (7) with two modes, that is $\mathcal{S} = \{1, 2\}$. The mode switching is governed by the rate matrix $\begin{bmatrix} -0.5 & 0.5 \\ 0.3 & -0.3 \end{bmatrix}$. And the system parameters are as follows:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} -2 & 1 \\ -1 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} -2 & 0.5 \\ 0.5 & -1 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} -1 & 0.5 \\ 2 & 0.3 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.2 & 0.1 \\ 0.5 & 0.2 \end{bmatrix},$$
$$B_{w1} = \begin{bmatrix} 0.4 \\ -0.5 \end{bmatrix}, B_{w2} = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix},$$
$$D_{w1} = 0.2, D_{w2} = 0.1, C_1 = \begin{bmatrix} -0.1 & 0.3 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} -0.1 & 0.3 \end{bmatrix},$$
$$C_{d1} = \begin{bmatrix} -1 & 0.2 \end{bmatrix}, C_{d2} = \begin{bmatrix} -2 & 0.2 \end{bmatrix}.$$

Tables 1 and 2 provide the comparison results, respectively via the method provided in [26] and Lemma 3.1 in this paper.

Table 1 shows that for a given H_∞ performance level γ, the conditions in Lemma 3.1 allow a bigger delay constant d than the results gotten in reference [26]. So the conditions of Lemma 3.1 for singular Markovian jump time-delay systems is less conservative than those obtained in [26].

Table 2 illustrates that for a given delay constant d, the conditions in Lemma 3.1 allow a smaller H_∞ performance γ. This means that the results here can allow greater energy external perturbation than the results obtained in [26].

Example 4.2 Consider singular Markovian jumping system (1) with two modes and the system parameters are described as follows:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 2 & -1.5 & 1 \\ -1.2 & 1 & 2 \\ 1 & 2 & -1.5 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1.5 & -1.5 & 0 \\ -1 & -2 & 0.5 \\ 1 & 1.2 & -1 \end{bmatrix}, A_{d1} = \begin{bmatrix} 0.5 & 1.5 & 0 \\ -1 & -2 & 0.5 \\ 1 & 1.2 & -1 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} 1 & 1.2 & 0.5 \\ 1.5 & -0.2 & 1 \\ -1 & 2 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 1.5 & -2 \\ 1 & -1 \\ 1 & 2 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.5 & 1 \\ 1 & 0.5 \\ 1 & 1.5 \end{bmatrix}, B_{w2} = \begin{bmatrix} 0.5 \\ -1 \\ 0.1 \end{bmatrix}, B_{w1} = \begin{bmatrix} -0.02 \\ -0.1 \\ 0.2 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -0.1 & -0.3 & 0.1 \end{bmatrix}, C_2 = \begin{bmatrix} 0.2 & -0.2 & -0.01 \end{bmatrix},$$
$$C_{d1} = \begin{bmatrix} -1 & 0.5 & 0.1 \end{bmatrix},$$
$$C_{d2} = \begin{bmatrix} -2 & 0.2 & 0.1 \end{bmatrix}, D_1 = \begin{bmatrix} -0.2 & 0.2 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} -0.3 & 0.1 \end{bmatrix}, D_{w1} = 0, D_{w2} = 0,$$
$$N_{11} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}, N_{12} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix},$$
$$N_{21} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}, N_{22} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix},$$
$$N_{31} = 0.1, N_{32} = 0.1, M_{21} = 0.1, N_{+1} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix},$$
$$M_{22} = 0.1, N_{+2} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix},$$
$$N_{+d1} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}, N_{+d2} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix},$$
$$N_{\times 1} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},$$
$$M_{11} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}, M_{12} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix},$$
$$M_{+1} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, M_{+2} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$
$$M_{\times 1} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, M_{\times 2} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},$$
$$N_{\times 2} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, N_{\times d1} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},$$
$$N_{\times d2} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix},$$

$F_{11}(t) = F_{12}(t) = F_{21}(t) = F_{22}(t) = F_{31}(t) = F_{32}(t) = sin(t)$. The transition probability matrix is given by

$$\pi = \begin{bmatrix} -0.3 & 0.3 \\ 1 & -1 \end{bmatrix}$$

When $d = 0.5$ and $\gamma = 1.0$, using Theorems 3.1 and 3.2, we can find feasible solutions for a set of LMIs (30)–(32) and LMIs (30)–(33) respectively by using MATLAB LMI Control Toolbox. Then we can get the robust resilient H_∞ controllers. The controllers with the additive uncertainties

$$u(t) = (K_i + M_{+i} sin(t) N_{+i}) x(t) + (K_{di} + M_{+i} sin(t) N_{+di}) x(t-d), i \in \mathcal{S}$$

Table 1 Comparison results of maximum allowed time-delay d for different γ

γ	1.8	1.6	1.4	1.2	1.0	0.8
[26]	4.0413	3.8917	3.7005	3.4504	3.1141	2.6495
Lemma 3.1	4.5749	4.4817	4.3386	4.1107	3.7235	3.0515

Table 2 Comparison results of minimum allowed γ for different $\bar{d}$

$\bar{d}$	4.5	4.0	3.5	3.0	2.5	2.0
[26]	2.9473	1.7400	1.2354	0.9443	0.7481	0.6204
Lemma 3.1	1.6328	1.1308	0.9207	0.7884	0.6915	0.6169

which has the following gains:

$$K_1 = \begin{bmatrix} 30.4737 & -37.7166 & -2.7148 \\ 30.7386 & -23.6282 & -0.8215 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.3769 & -0.4568 & 0.2871 \\ -902.5297 & 258.6746 & -4.7776 \end{bmatrix}$$
$$K_{d1} = \begin{bmatrix} -0.2264 & -0.4239 & 0.2648 \\ -0.1221 & -0.1459 & 0.2952 \end{bmatrix},$$
$$K_{d2} = \begin{bmatrix} -10.2761 & 2.6463 & -0.5260 \\ 7.4781 & -3.0840 & -0.3604 \end{bmatrix}.$$

The controllers with the multiplicative uncertainties

$$u(t) = (I + M_{\times i} sin(t) N_{\times i}) K_i x(t) + (I + M_{\times i} sin(t) N_{\times di}) K_{di} x(t - d), i \in \mathcal{S}$$

which has the following gains:

$$K_1 = \begin{bmatrix} 34.4338 & -40.4507 & -2.7959 \\ 40.0444 & -27.6446 & -0.8795 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 188.4577 & -56.1672 & -1.7445 \\ -806.2580 & 231.4064 & -8.4865 \end{bmatrix}$$
$$K_{d1} = \begin{bmatrix} -0.4225 & -0.4238 & 0.2914 \\ -0.0369 & -0.2756 & 0.2931 \end{bmatrix},$$
$$K_{d2} = \begin{bmatrix} -7.3369 & 1.5352 & -0.3566 \\ 5.4625s & -2.3023 & -0.4821 \end{bmatrix}.$$

These controllers can guarantee that the closed-loop systems are stochastically stable with H_∞ performance γ, for all admissible uncertainties. It is worth noting both the additive controller gain variance and the multiplicative controller gain variance appear the function $sin(t)$, which means that the controllers needn't be precise and exactly implemented. That is the controller gain changes in a certain range, the corresponding closed systems can all be regular, impulse free and stochastically stable. In addition the corresponding given H_∞ performance can be satisfied. So the controller algorithms presented here are more conducive to the realization.

5 Conclusions

The problem of delay-dependent robust resilient H_∞ control for singular Markovian jump time-delay systems has been discussed. In terms of the Lyapunov technique and linear matrix inequalities, a new delay-dependent BRL is established such that the system is stochastically admissible with a given H_∞ performance γ. Then, delay-dependent design algorithms for the desired state feedback robust resilient controllers are proposed in terms of a set of strict linear matrix inequalities (LMIs) to guarantee that the closed-loop systems are not only regular, impulse free and stochastically stable, but also satisfy a prescribed H_∞ performance level. Finally, numerical examples illustrate that the results proposed in this paper is valid.

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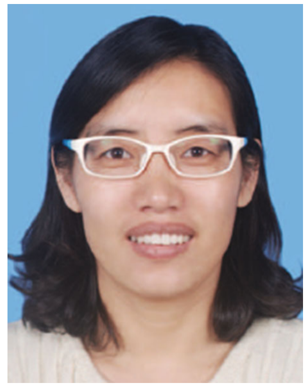

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