

Synchronization study of Hindmarsh–Rose neuron coupled system based on numerical simulation of time delay

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Abstract This paper studied the synchronization of Hindmarsh–Rose neuron coupled system based on numerical simulation of time delay through theoretical derivation and numerical simulation. Researches found that coupling strength can influence the discharge activity of coupled neuron system and the synchronization of the coupled neurons. Adjusting coupling strength between coupled neuron systems can change two neuron systems from non-synchronization to synchronization. Meanwhile, the researches also found that proper time delay can change two Hindmarsh–Rose neuron systems coupled by electrical synapse from synchronization to non-synchronization, and then destroy the system’s synchronization.

Keywords Electrical synaptic coupling · Hindmarsh–Rose neuron · Synchronization · Time delay

1 Introduction

As an extreme complicated system which is made up of billions of interconnected neurons, human nervous system is an important regulating mechanism in human body. It can regulate and control the organs in human body directly

or indirectly, and make the body full unity and keep balance of internal and external environment, thus maintain the stability and coordination of various functional activities in the body. As the basic structure unit of neural system structure and basic work unit of brain, neurons—the key components of neural system, bear the important responsibility of conveying information in the neural system, and have the function of experiencing stimulation and conducting excitation. Activities of neurons mainly consist of the formation, change and spread of biological electrical signals. Biological nervous system, made up of a large number of interconnected nerve cells, is a fairly complex multi-level information network of the nervous system. Thus there is a complex nonlinear dynamics behavior in neuron’s participation in discharging activity and information coding process. Neural dynamics is the combination of biophysiology and non-linear dynamics, and research on discharging form of neurons is meaningful in both biology and dynamics. With the continuous development of biophysiology and non-linear dynamics, different data is produced in different experiments, and then different biological neuron models are built based on these data. We can calculate and analyze the neuron modal, find out the phenomenons which are difficult to observe in the neurophysiology experiments, guide the actual physiological experiments, and thus offer certain theoretical basis and promote the rapid development of medicine and neural science.

The process of neural information in brain is conducted through interworking of neural populations. Thus neural population’s pattern is quite important to information conveying. Synchronization occurred in the neural system’s information process is an important component in realizing activities like association and memory. As the basic behavior of neuron, synchronization is a kind of expression of neuron discharge, and widely exists in the neural system. Many activities of

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the neurons like selective attention, cognizing, memory, studying and certain diseases are all associated with synchronization [1]. This phenomenon has been observed in many physiological experiments and simulations. For example, scientists have observed synchronization in the visual cortex [2,3], and they have found the synchronization of γ frequency motion in the anesthetic cat's visual cortex [4] and synchronization in the awake monkey [5]. However, many physiological experiments about synchronization show that too much synchronization will do harm to the organism. For example, disease like hands trembling, Parkinson's disease and Epilepsy have some relationships with synchronization. Thus, it is quite important to explore the disease caused by synchronization with nonlinear dynamics [6–8].

With the non-linear system and chaos gradually known to us all, the concept of synchronization also becomes a hot content in the non-linear area and is being improved and enhanced. The study in coupled neuron system synchronization dynamics has achieved great achievements. For example, Yoshioka studied the dynamics behaviors of the neuron network coupled through chemical synapses, which illustrates the stability of any neuron and the neuron network cluster of any interaction strength [9]. Dhamala et al, through researching Hindmarsh–Rose neuron coupled by electrical synapse, found the synchronous transition from bursting synchronization to peak synchronization, and the bursting synchronization is always prior to the peak synchronization [10]. Belykh et al, through theoretical analysis and numerical simulation of coupled network, found that the stability of full synchronization depends on the amount of signal received by neurons and has nothing to do with the network topology [11]. Postnova et al. found the transfer of neuron synchronization from discharging to bursting, and they have also researched different kinds of synchronization phenomena ever happened, like cophase synchronization, anti-phase synchronization and phase synchronization in chaotic [12]. Shi Xia et al, researched and analyzed the synchronization pattern of electrical coupled Hindmarsh–Rose neurons which has a ring structure, and got one of the bases to judge whether the coupled neuron system achieved stable or not through the stability theory of differential equation [13]. Sun Xiaojuan et al, taking one dimension round coupled FitzHugh–Nagumo neuron synchronization pattern as the object, studied the influence of heterogeneous neurons under different arrangements on its frequency synchronization. The result shows that the arrangement of heterogeneous neurons is different, and the corresponding critical coupling strength needed for the neuron network to achieve the frequency synchronization is also not the same [14]. Wu Wangsheng et al, based on the dynamic model of Hindmarsh–Rose neurons, studied the two coupled Hindmarsh–Rose neurons' synchronization under the

feedback coupling through calculating the coupled neuron system's synchronization discrepancy and the changing of inter-spike interval with coupling strength, and the result shows that the initial chaotic Hindmarsh–Rose neurons have complex synchronization behavior in the coupling process [15].

In the organism, the information is transferred between neurons through a special structure—synapse, a transitive point of neural information which can realize the transfer of not only the electrical signals but also the chemical signals. Electrical synaptic coupling, widely existed in the neural system, is the simplest way to transfer information between neurons, and plays an important role in the information transfer. Thus, it has a great meaning for the research of electrical synaptic coupling neuron synchronization [16].

Time delay widely exists in most neuron coupled systems, and is the non-negligible part of the system. The early scientific workers didn't take time delay into consideration, however, they found in their researches that the message conveying is not in real time, and there was some delay both in the message conveying and the neuron communication. The result illustrates that time delay is real and inevitable in coupled neuron systems. The existence of time delay changes the dynamics system from finite dimension into infinite dimension, and then introduces the non-linear dynamics characteristics which were more complicated, thus time delay attracts wide attention owing to its influence on the dynamics behavior of the coupled neuron system. With the study going further, time delay Hopfield neural network [17], time delay bidirectional associative memory neural network [18], delay cell neural network (DCNN) [19], time delay Cohen–Grossberg [20] etc, have been put forward one by one. The coupling neuron system with the function of time delay also attracts a lot of experts' and scholars' attention and they have made impressive progress through research and exploration. For example, Reddy et al, explored two limit cycle powers which have time delay coupled system and found that time delay can make the limit cycle of two oscillators with the same oscillation frequency disappeared [21]; Park et al, studied the effect of time delay on the dynamic model of the phase oscillation and found that time delay can induce a rich nonlinear phenomenon, like clustering behavior, that is, the system can be divided into some phase locking, synchronization and multi stability, which indicates that time delay plays an important role in the information processing of the nervous system [22]. Cao Shuhong et al, studied the effects of time delay on the synchronized discharge mode of coupled Hindmarsh–Rose neurons through numerical simulation, and the research results showed that the delay makes the discharge pattern of neurons change, and at the same time, the increase of time delay can induce the decrease or disappearance of the peak in the neuron clusters [23].

This paper establishes mathematical model which contains two Hindmarsh–Rose neurons coupled by electrical synapse, studies the basic phenomenon of neuron coupled by electrical synapse, observes the variation of system discharging and synchronization, and also analyzes the influence of time delay on the coupling synchronization.

2 HR neuron model

HR neuron model is a data expression of the snail’s neuron cell which is gained according to the voltage clamp experiment by two biologists, and this neuron model doesn’t take the role of each ion channel on each neuron’s cytomembrane into consideration, but regard each neuron cell as a whole, a three-dimension dynamics mathematical model. HR neurons can well explain some dynamic characteristics like the discharge pattern and the non-periodic model of neuron clusters. Since this model is simple and relatively easy to study, therefore, it is used by many experts and scholars as an ideal model to study the discharge of realistic neurons. The HR model used in this paper is a neuron model similar to the HR model [24], which is constructed by Tsaneva-Atanasov Krasimira. Since the structure of this model is modified on the basis of HR model, and it has the same dynamic behavior as the latter, it is also called HR model. The formula is showed in formula (1):

$$\begin{cases} \frac{dx}{dt} = -s(-ax^3 + x^2) - y - bz \\ \frac{dy}{dt} = \varphi(x^2 - y), \\ \frac{dz}{dt} = \varepsilon(sa_1x_1 + b_1 - kz) \end{cases} \quad (1)$$

In the above formula, x is the membrane voltage, and y is the recovery variable relative to the current inside the neuron cell, and z is the slowly varying adaptive current. a, b, a_1, b_1, k, s are the parameters of the system, and φ & ε are the parameters that control the time scale. The values of these parameters are: $\varphi=1, a = 0.5, b = 1, a_1 = -0.1, k = 0.2$, and please refer to Reference [24] for the detailed information of these parameters. When $b_1 = -0.045, \varepsilon = 0.02$, and $s = -1.61$, the neuron is in chaotic discharging pattern, and its time response chart and phase plane chart are showed in Fig. 1 [25].

3 Synchronization of HR neuron coupled by electrical synapse

In order to verify the synchronization of HR neural system, this paper builds a mathematical modal of two HR Neurons coupled by electrical synapse, and the kinetic equation can be described by the following calculus equation:

$$\begin{cases} \frac{dx_1}{dt} = -s(-ax_1^3 + x_1^2) - y_1 - bz_1 + D(x_2 - x_1) \\ \frac{dy_1}{dt} = \phi(x_1^2 - y_1) \\ \frac{dz_1}{dt} = \varepsilon(sa_1x_1 + b_1 - kz_1) \\ \frac{dx_2}{dt} = -s(-ax_2^3 + x_2^2) - y_2 - bz_2 + D(x_1 - x_2) \\ \frac{dy_2}{dt} = \phi(x_2^2 - y_2) \\ \frac{dz_2}{dt} = \varepsilon(sa_1x_2 + b_1 - kz_2) \end{cases} \quad (2)$$

In the equation, D is coupling strength, $\varphi = 1, a = 0.5, b = 1, a_1 = -0.1, k = 0.2, \varepsilon = 0.02$, and $s = -1.61$. We adopt mode decomposition method in Reference [26] to calculate the coupling strength, and here, we suppose $X = (x, y, z)$, then formula (1) can be changed to the following formula:

$$\begin{cases} f_1 = -s(-ax^3 + x^2) - y - bz \\ f_2 = \varphi(x^2 - y), \\ f_3 = \varepsilon(sa_1x + b_1 - kz) \end{cases} \quad (3)$$

And

$$DF(X) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} \end{bmatrix} = \begin{bmatrix} 3asx^2 - 2sx & -1 & -b \\ 2\varphi x & -\varphi & 0 \\ \varepsilon sa_1 & 0 & -\varepsilon k \end{bmatrix} \quad (4)$$

$$A(X) = DF(X) + (DF(X))^T = \begin{bmatrix} 6asx^2 - 4sx & 2\varphi x - 1 & \varepsilon sa_1 - b \\ 2\varphi x - 1 & -2\varphi & 0 \\ \varepsilon sa_1 - b & 0 & -2\varepsilon k \end{bmatrix} \quad (5)$$

Let’s make $B(X) = A(x) - 2D$, and based on the principle that the same characteristic values have similar matrix, we just need to confirm that the entire characteristic values of $B(X)$ are all negative real part.

$$B(X) = \begin{bmatrix} 6asx^2 - 4sx - 2D & 2\varphi x - 1 & \varepsilon sa_1 - b \\ 2\varphi x - 1 & -2\varphi & 0 \\ \varepsilon sa_1 - b & 0 & -2\varepsilon k \end{bmatrix},$$

its ordered-main sub-determinants are:

$$\begin{cases} B_1 = 6asx^2 - 4sx - 2D \\ B_2 = -2\varphi(6asx^2 - 4sx - 2D) - (2\varphi x - 1)^2 \\ B_3 = \det(B(x)) = 4\varepsilon\varphi k(6asx^2 - 4sx - 2D) + 2\varepsilon k(2\varphi x - 1)^2 + 2\varphi(\varepsilon sa_1 - b)^2 \end{cases} \quad (6)$$

If $B_1 < 0$, $B_2 > 0$, $B_3 < 0$, the characteristic values of $B(X)$ are all negative real part. Therefore, put the parameters into formula (6), and we can get:

$$D > -2.415x_0^2 + 3.22x_0 + (2x_0 - 1)^2/8 \quad (7)$$

x_0 is the upper bound of action potential x in the chaotic solution of formula (6). We can make $x_0=1.05$ according to the numerical modeling in Fig. 1a, then we can get $D > 0.869$.

According to the result we get in formula (7), we make $D = 0$ and $D = 1$ to do numerical simulation. In order to have a better coupling study of the neuron synchronization, this paper introduces such synchronization discrepancy of coupled neuron as $e_0 = x_2 - x_1$, $e_1 = y_2 - y_1$, $e_2 = z_2 - z_1$, $e_3 = \frac{1}{3}(|e_0| + |e_1| + |e_2|)$ to analyze the change of neuron coupling synchronization.

Firstly, this paper takes the discharge activity of two HR modals without any coupling strength, that is $D = 0$, into consideration. When coupling strength $D = 0$, we can get the phase plane chart of two HR neurons coupled by electrical synapse in (x_1, x_2) , as showed in Fig. 2a. From the diagram we can see that phase plane presents random movement, which means at the moment, the system is in non-synchronization state. We can see from Fig. 2b time history chart of x_1 and x_2 over time that the time history diagram of the system presents an irregular chaotic discharge activity, and the values of x_1 and x_2 at the same time are different, which indicates that the system is in a chaotic state. From the synchronization discrepancy of the coupled neurons over time, we can find that with the change of time, the synchronization difference always exists, which also indicates that the two neuron systems are not in synchronization state. Through the above analysis, we can get that when coupling strength $D = 0$, the two neuron systems are in chaotic discharge and asynchronization state.

Then, this paper analyzed the electrical coupling synchronization of two neurons when coupling strength $D=1$. In order to get that, this paper also gets the phase plane chart of the two neurons in (x_1, x_2) when $D=1$, as showed in Fig. 3a, from which we can see that the phase plane chart is a straight line with slope equal to 1, and x_1 and x_2 are equal, which means that the system is in synchronization state at the moment. We can see from the time history Fig. 3b of x_1 and x_2 over time that the system presents chaotic discharging activity, which means the system now is in chaotic state, and x_1 and x_2 are equal in value at the same time, thus the two neurons are in synchronization state. In order to have a better study of the synchronization of the two neurons, we also bring in the synchronization discrepancy showed in Fig. 3c, and from this figure we can clearly find out that with the time going on, the synchronization discrepancy is always equal to 0, which shows that the two neural coupled systems are in synchronization state at the moment. Through the anal-

ysis above, a conclusion can be drawn that when coupling strength $D = 1$, the system is in chaotic discharge and non-synchronization state.

Through the above analysis, we can draw the conclusion that coupling strength can influence discharging activity of the coupled neural system and synchronization state between two coupled neurons. We can change the two neural systems from non-synchronization to synchronization by adjusting the coupling strength between coupling neural systems. It is necessary for us to study the coupling synchronization of neurons and show the abundant discharging activity. Meanwhile, it also can help to promote the further development of the neurophysiology experiments.

4 The function of time delay to the synchronization of neurons coupled by electrical synapse

With the existence of synaptic cleft and the limitation of message spreading speed, time delay widely exists in most physics and ecosystem and coupled system. As an important factor in the neuron system to influence the electricity activity, time delay can cause kinds of complicated actions in neuron synchronization and has an important impact on the system's dynamic behavior. Thus we can understand the dynamic mechanics arising in the different kinds of neuron synchronization patterns with the help of complicated synchronization introduced by the time delay.

This paper establishes two synchronization behaviors of HR neuron system coupled by electrical synapse with time delay, and their dynamic behaviors are described by the delay differential equations set as follows:

$$\begin{cases} \frac{dx_1}{dt} = -s(-ax_1^3 + x_1^2) - y_1 - bz_1 + D(x_2(t - \tau) - x_1) \\ \frac{dy_1}{dt} = \phi(x_1^2 - y_1) \\ \frac{dz_1}{dt} = \varepsilon(sa_1x_1 + b_1 - kz_1) \\ \frac{dx_2}{dt} = -s(-ax_2^3 + x_2^2) - y_2 - bz_2 + D(x_1(t - \tau) - x_2) \\ \frac{dy_2}{dt} = \phi(x_2^2 - y_2) \\ \frac{dz_2}{dt} = \varepsilon(sa_1x_2 + b_1 - kz_2) \end{cases} \quad (8)$$

In the equation, D is the coupling strength and τ is the time delay in HR neuron message spreading, while other variables keep the same meaning and value.

Firstly, this paper studies the synchronization of neuron system without any time delay. Make coupling strength $D = 0.02$, and time delay $\tau = 0$, we can get the phase plane chart of the two electrical coupled neurons in (x_1, x_2) , as shown in Fig. 4a, through solving the differential equation. We can see from the diagram that the phase plane is a straight line with slope equal to 1, which means x_1 and x_2 are equal, and the system is in synchronization. Then this

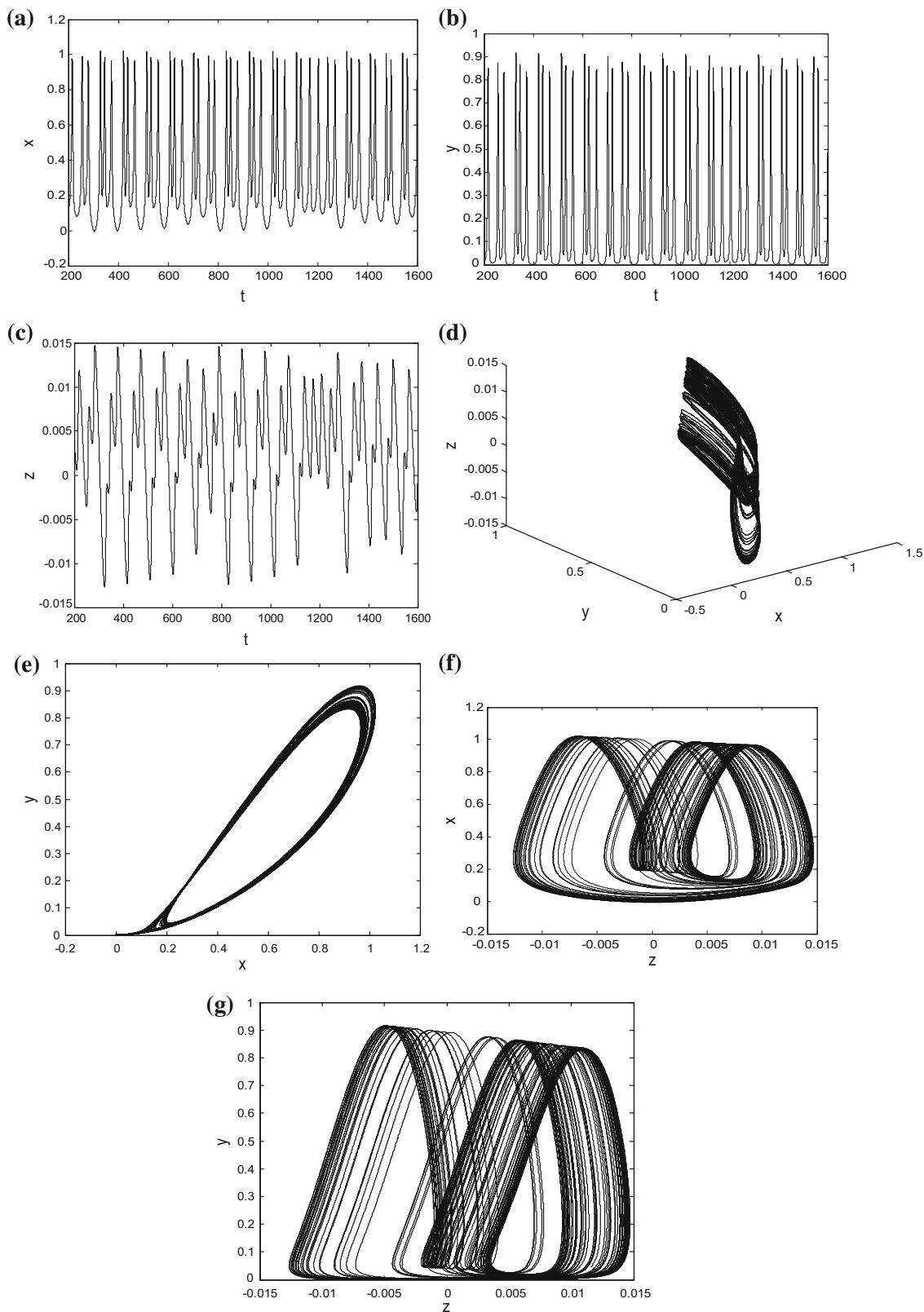


Fig. 1 The time response diagram and phase plane diagram of HR neuron when $b_1 = -0.045$, $\varepsilon = 0.02$, $s = -1.61$. **a** Time response diagram of Hindmarsh–Rose Neuron x . **b** Time response diagram of Hindmarsh–Rose Neuron y . **c** Time response diagram of Hindmarsh–Rose Neuron

z . **d** Phase plane diagram of HR neuron in the plane (x,y,z) . **e** Phase plane diagram of HR Neuron in the plane (x,y) **f** Phase plane chart of HR neuron in the plane (x,z) **g** Phase plane chart of HR Neuron in the plane (y,z)

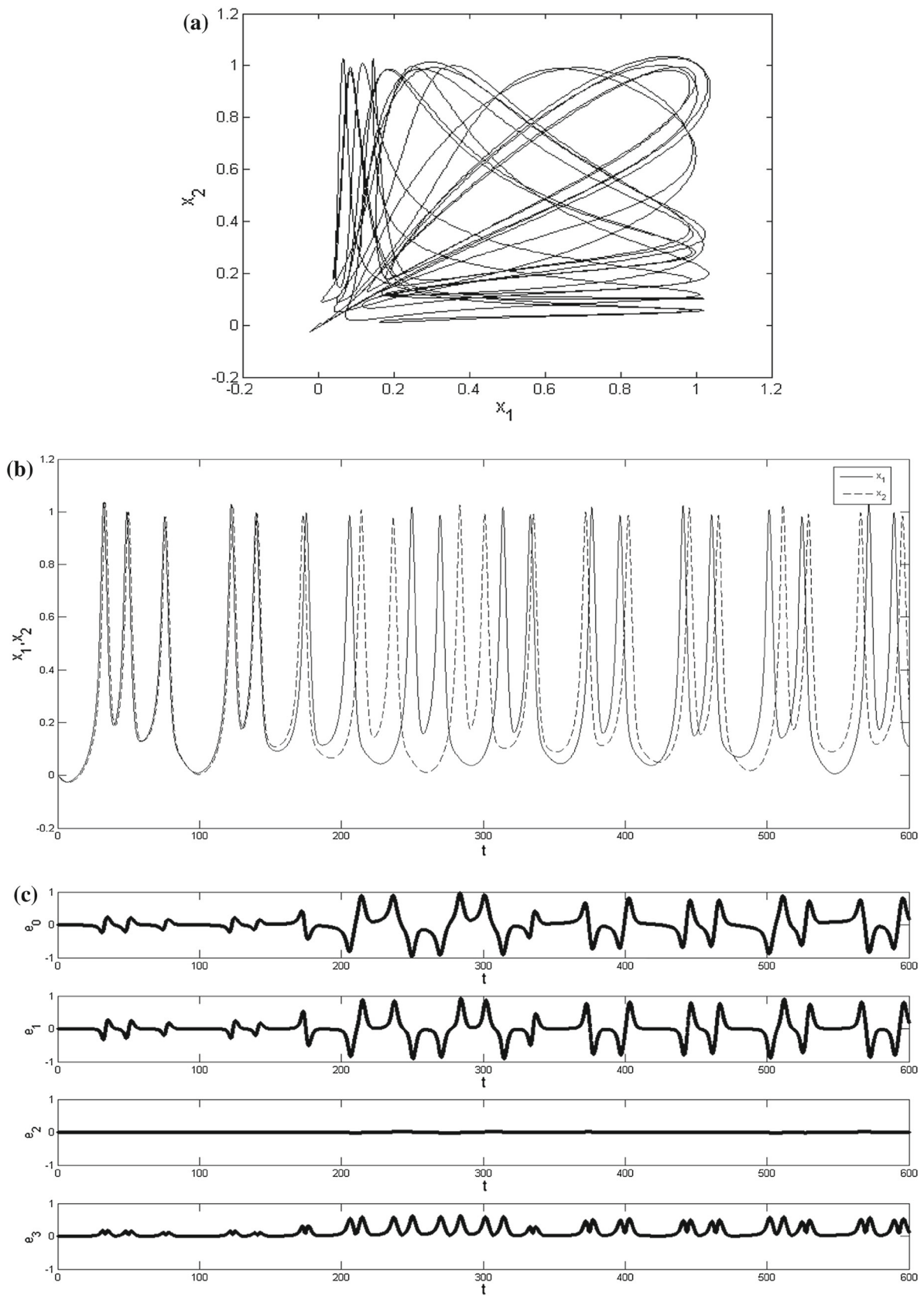


Fig. 2 Two HR neural systems coupled by electrical synapse when $D = 0$. **a** Phase plane diagram when $D = 0$. **b** Time history diagram when $D = 0$. **c** Synchronization discrepancy diagram when $D = 0$

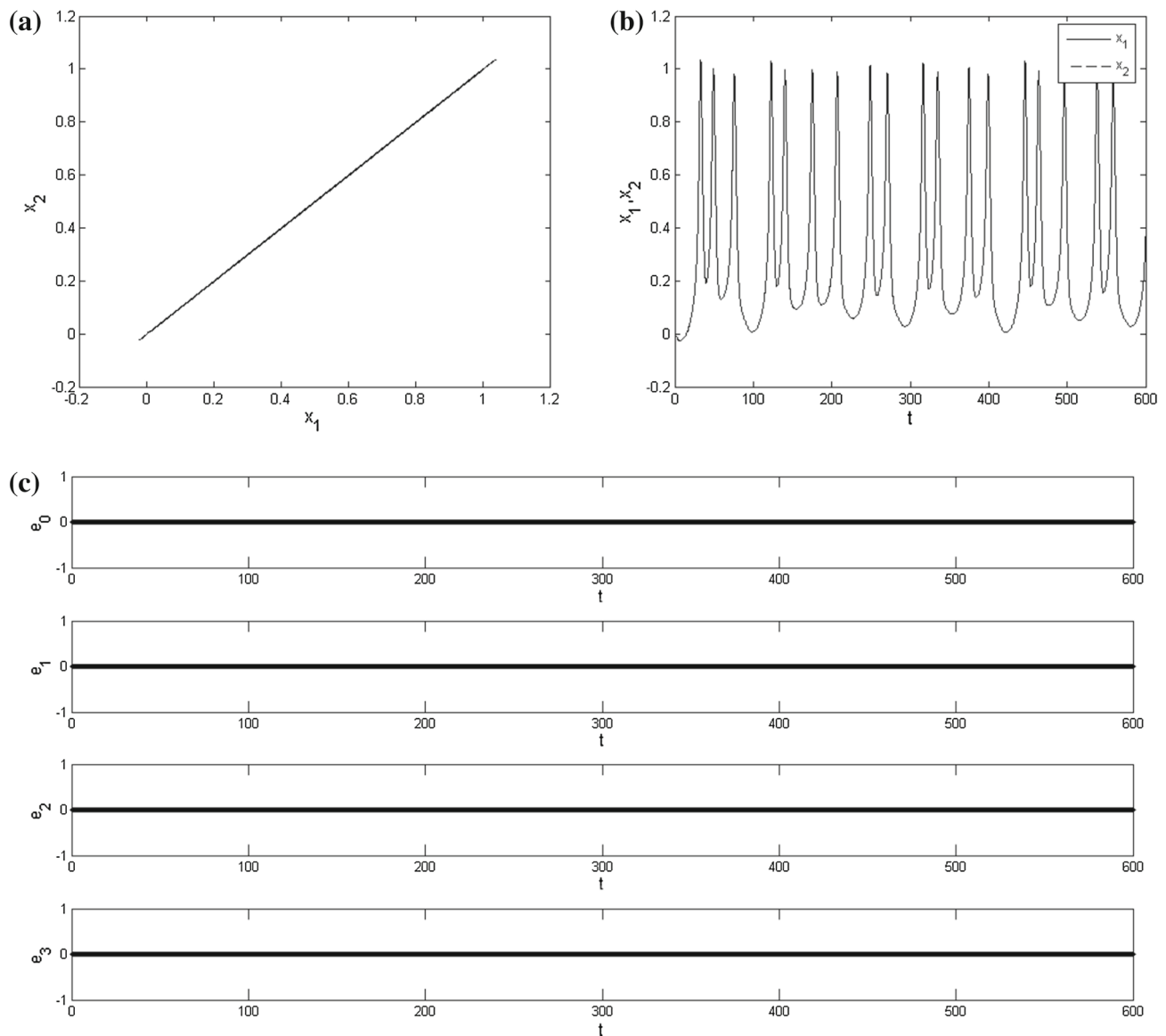


Fig. 3 Two HR neural systems coupled by electrical synapse when $D = 1$. **a** Phase plane diagram $D = 1$. **b** Time history diagram when $D = 1$. **c** Synchronization discrepancy diagram when $D = 1$

paper gets the time history diagram of x_1 and x_2 over time change, as is showed in Fig. 4b, in which the picture is in curvilinear motion. However, the value curves of x_1 and x_2 are overlapped, which demonstrates that x_1 and x_2 are the same in value. Also it can be clearly observed in Fig. 4c that the synchronization discrepancy is always 0, which means that the two neuron coupled systems reach synchronization. Through the above analysis we can conclude that when coupling strength $D = 0.02$ and time delay $\tau = 0$, the HR neuron systems coupled by electrical synapse are in synchronization.

Then, this paper takes the influence of time delay on the HR neuron model coupled by electrical synapse into consideration, by making time delay $\tau=3$, we can get the phase

plane chart of HR neuron coupled by electrical synapse in (x_1, x_2) , as shown in Fig. 5a. We can see from this diagram that the phase plane is in random movement, which illustrates that the system is in non-synchronization state. Then, this paper gets the time history diagram of x_1 and x_2 over time change, as is showed in Fig. 5b, in which we can see that x_1 and x_2 present chaotic discharge activities, which means the two HR neuron systems are chaotically discharging. However, we can also see from the diagram that when x_1 and x_2 are not equal, which means the two neurons do not achieve synchronization. In order to better observe whether the two nerves are in synchronization or not, this paper also introduces neuron coupling synchronization discrepancy chart, as shown in Fig. 5c. It can be clearly observed that with time

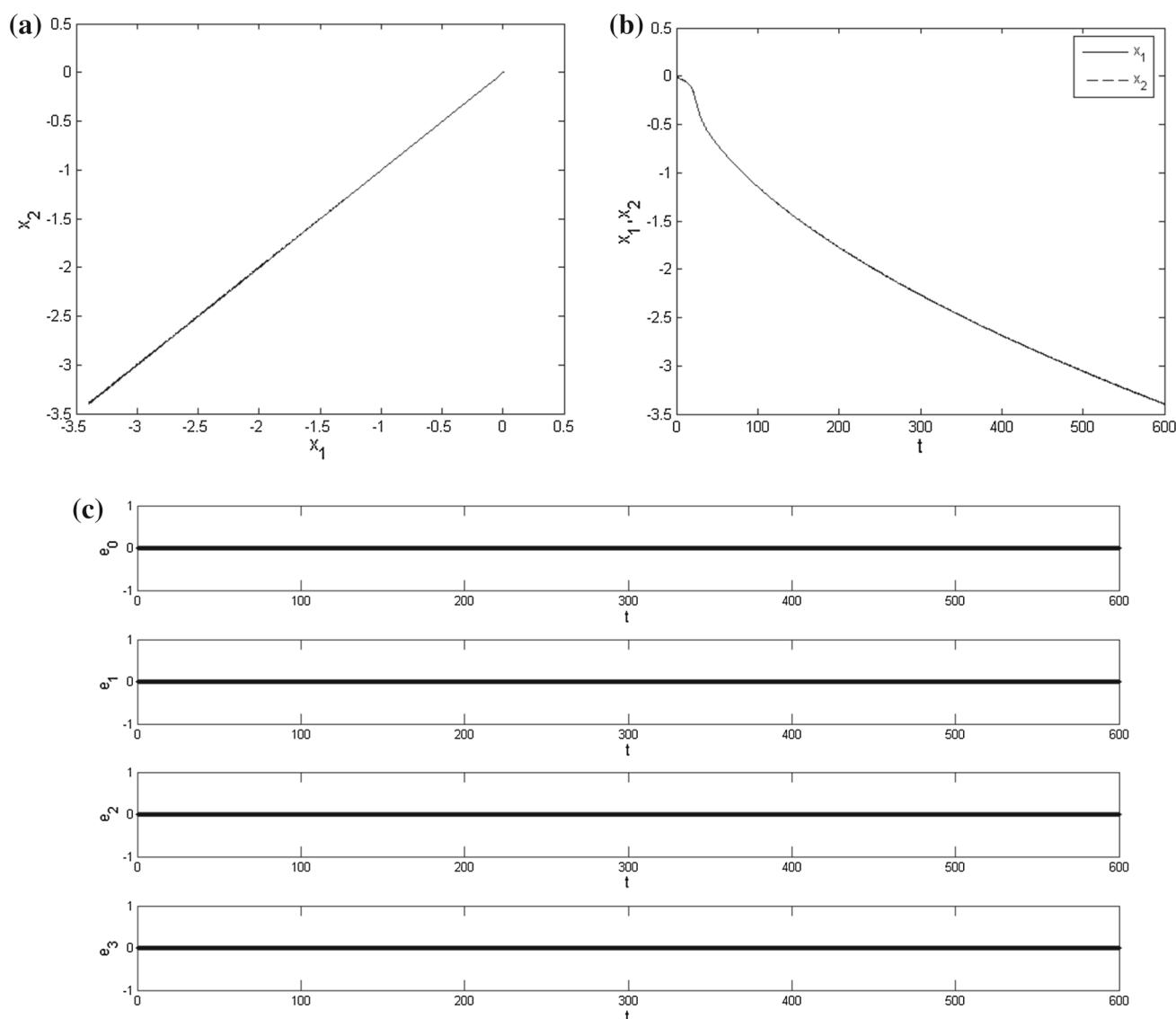


Fig. 4 Two HR systems coupled by electrical synapse when $D = 0.02$ and $\tau = 0$. **a** Phase plane diagram when $D = 0.02$ and $\tau = 0$. **b** Time history diagram when $D = 0.02$ and $\tau = 0$. **c** Synchronization discrepancy diagram when $D = 0.02$ and $\tau = 0$

going by, the discrepancy always exists and is changing all the time, which means the system is in non-synchronization state. Through the above analysis we can conclude that when coupling strength $D = 0.02$ and time delay, $\tau = 3$ the HR neuron systems coupled by electrical synapse are in chaotic discharge and non-synchronization state.

According to the above analysis we can get that proper time delay can change two HR neuron systems coupled by electrical synapse from synchronization to non-synchronization, and then destroy the system's synchronization. Modern medicine shows that there are certain relationships between the rising of such diseases as Parkinson's disease, hand shaking, epilepsy and neuron synchronization. Thus it is quite important for us to use coupled neuron mathematical model to explore the neuron synchronization discharging

activity in the physiological experiments and study how to suppress or eliminate the disease caused by neuron synchronization.

5 Conclusions

Taking Hindmarsh–Rose neuron model as the object of study, this paper has explored the synchronization of HR neuron systems coupled by electrical synapse and time delay's influence on coupled neuron synchronization, and therefore conclusions are shown as follows:

- (1) The two neurons which are in chaotic discharge cannot reach synchronization under uncoupled situation, how-

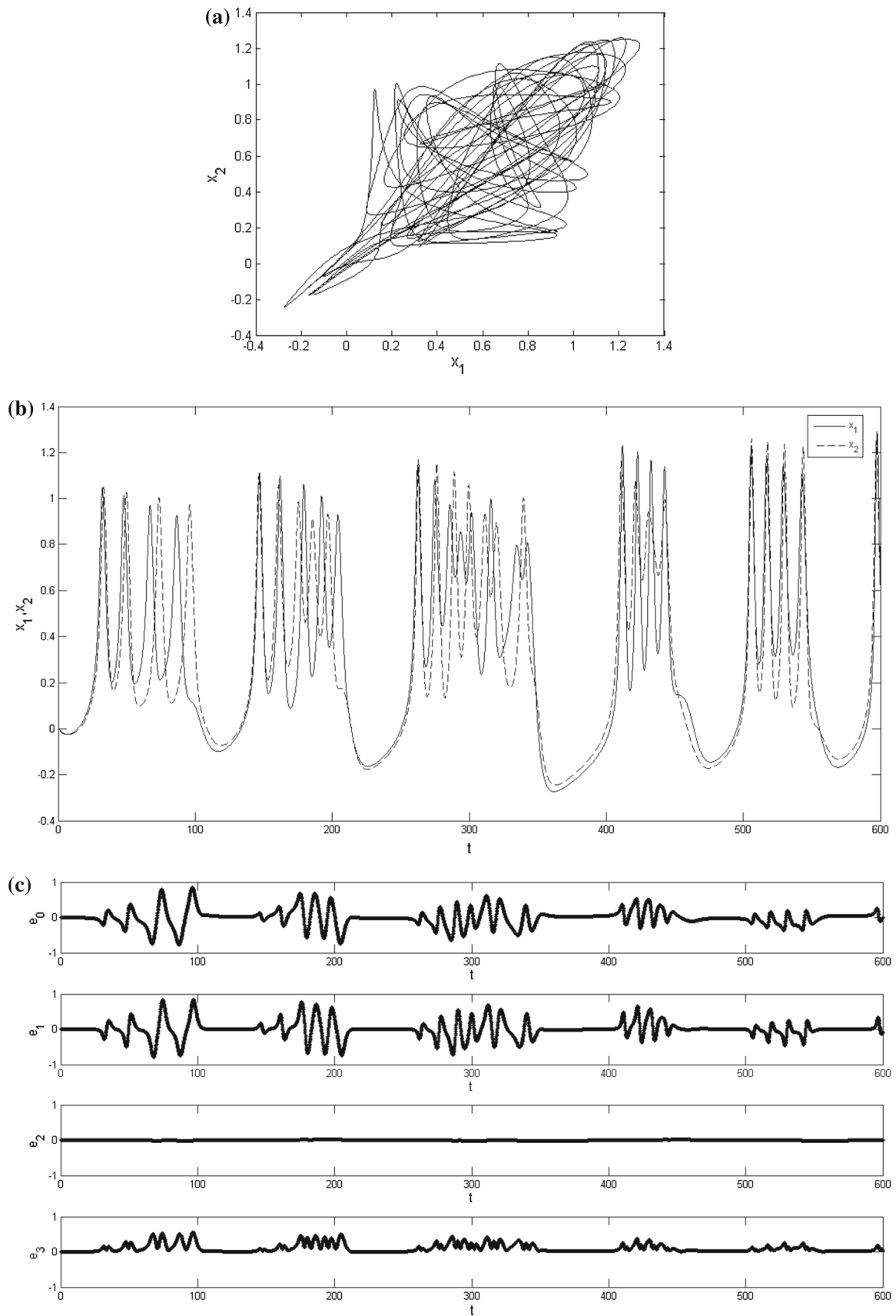


Fig. 5 Two HR neural systems coupled by electrical synapse when $D = 0.02$ and $\tau = 3$. **a** Phase plane diagram when $D = 0.02$ and $\tau = 3$, **b** time history diagram when $D = 0.02$ and $\tau = 3$, **c** synchronization discrepancy diagram when $D = 0.02$ and $\tau = 3$

ever, when the coupling strength reaches certain degree, such as $D = 1.0$, the system can achieve coupling synchronization.

- (2) Time delay can suppress the synchronization of neuron coupling, and proper time delay can make the two HR neuron systems coupled by electrical synapse change from synchronization into non-synchronization, and then destroy the synchronization of the system. For example, when $D = 0.02$, $\tau = 0$, the system is in synchronization, while when $D = 0.02$, $\tau = 3$, the system changes from synchronization into non-synchronization.

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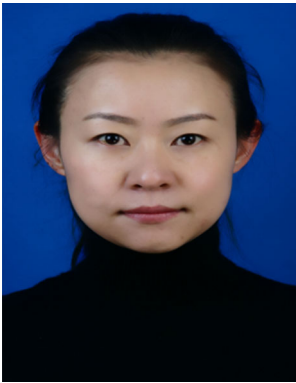
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