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Optimal path for sustainable development under the dual constraints based on endogenous growth algorithm

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Abstract Constructing an economic growth model comprising dual resource and environmental constraints by introducing both environmental quality and non-renewable resources as endogenous factors also introducing the production and utility functions. This was used to systematically analyze the endogenous mechanism through factors such as non-renewable resource consumption, environmental pollution externalities, the accumulation of physical capital, human capital development, and endogenous technological advancement could influence long-term economic growth. The basic conclusion of the model suggests that under both resource and environmental constraints, it is investment in both human capital and research & innovation that is the main driving and determining factor for long-term sustainable economic growth. The optimal development strategy of economic sustainability can be achieved through supporting human capital accumulation and technological innovation activities, promoting the advancement of clean production technologies, and formulating stringent environmental standards, as well as strengthening the society's awareness of the environment and sustainable development.

Keywords Dual constraints · Environmental quality · Endogenous growth · Sustainable development

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1 Introduction and review

With economic development and population growth, humankind is facing increasing pressure due to the depletion of resources and the environmental degradation. In 1987, the World Commission on Environment and Development (WCED) published a report titled *Our Common Future*. The concept of sustainable development was officially proposed in the report and defined as: "Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet". The emphasis was on the relationship between resources, environment and economic growth, as well as on how this relationship constituted an important component of research on sustainable development.

In reality, economists had begun to introduce the issues of energy, natural resources, and environmental pollution to neoclassical growth theories as early as the 1970s. This was in response to the oil crisis and pessimism perpetuated by the global think tank, Club of Rome. Specifically, Dasgupta and Heal (1974, 1979), Stiglitz [1] and other economists used the neoclassical growth model to analyze optimal paths for the exploitation and use of non-renewable resources. This led to the proposal of possible conditions under which economic growth could be sustained. Their models treated technological progress as exogenous and certain. The impact of environmental pollution was considered neither, despite environmental issues having become the most important topic in relation to sustainable development.

Subsequently, the endogenous growth theory came into prominence, with the main proponents being Romer [2,3] and Lucas [4], among others. Economists then introduced environmental quality into utility functions by incorporating the environment or resources into production functions. This led to extensive discussions on the issues of resource

depletion, deterioration of ecological environments and sustainable development under the framework of the endogenous growth model. Some examples include: Bovenberg and Smulders [5], who introduced the environmental factor into the production function, based on the model by Romer (1986). Scholz and Ziemes [6], who used the model by Romer (1990) as the basis to study issues of environmental externalities and sustained economic growth. Ligthart, Van der Ploeg [7] and Stokey [8], who also examined the two issues mentioned above, by expanding upon the "AK" model proposed by Barro [9]. Schou [10], Scholz et al. (1999) and Barbier [11], who introduced depleting resources as a critical factor for production, into the model by Romer (1990), to analyze the problem of sustained economic growth. Using natural resources as the constraint, Aghion & Howitt [12], Grimaud & Rouge [13], Bretschger [14], Tsur & Zemel [15], who incorporated environmental pollution and limited non-renewable resources into the neo-Schumpeterian model, to observe the means by which the dual constraints of resources and the environment affect sustainable development. With the progress of endogenous technological, Bovenberg & Smulders [16] made an attempt to categorize product and knowledge into two different departments, production department and R&D department. Production department is responsible for producing final products, while R&D department is responsible for producing public knowledge on reducing pollution, so as to realize dynamic analysis of long-term variation of economy and pollution increase ratio. Maltsoglou [17] established endogenous economic growth model and investigated the relation between technical progress and economic growth under high and low economic growth conditions. On the basis of model proposed by Romer (1986), Prettner and Werner [18] divided the whole economy into final product department, intermediate product department, technical research department, and scientific research department, and then studied the influence of scientific advance on technical progress as well as the fundamental role of scientific advance in economic growth.

With the worsening of resource and pollution issue, scholars have made extensive researches on resource and pollution under sustainable development framework. With regard to study on new classic growth model, Keeler et al. (1971) designed an optimized growth model with consideration on pollution and non-renewable energy consumption. Forster (1973), Becker et al. [19] introduced pollution into economic model and discussed the relation between environment pollution and long-term economic growth. Regarding the study on endogenous growth model, Lyon & Lee [20] studied the exhaustible resource exploitation model with pollution as state variable, analyzed the optimal path and gave analytical solution. Bastianoni et al. [21] studied replacement of fossil energy with renewable energy sources, and pointed out

that increasing investment on renewable energy sources can improve energy consumption structure. Nguyen & Nguyen-Van [22] established endogenous economic growth model with consideration of technical progress, and believed that government can take certain policy or measure to change energy consumption. Acemoglu [23] regarded technical progress as endogenous variable, and pointed out that only the substitution elasticity between clean and non-clean product was larger than 1, can the sustainable economic development be realized. Silva [24] discussed the influence of substitution between renewable energy sources and fossil energy to economy and environment. Kalkuhl [25] studied the advantage and disadvantage in policies on replacement of fossil energy by renewable energy sources. Peng [26] introduced environmental quality into utility function and production function, finding that optimal sustainable economic growth can be maintained when there were enough human capital accumulation and higher R&D output efficiency in economy. Cairns [27], Berk & Yetkiner [28] et al. successively studied the interaction among resources, environment and economy as well as sustainable development issue.

The relationship between resources, environment and sustainable economic development is essentially an issue of tradeoffs. On the one hand, sustainable development aims to ensure continuous enhancement in the level of wellbeing per capita over time or at the very least, to maintain the current level without decline. On the other hand, natural resources and the environment, which are essential for the survival of humankind, need to be well-protected. However, sustainable development is not necessarily the optimal development strategy for societies. For example, growth in per capita income and reduction in consumption rates can be realized by reducing resource-intensive production activities that cause environmental pollution and by maintaining the ratio between the accumulation and consumption of capital, appropriately. However, doing so is clearly not optimal.

Admittedly, the optimal growth path is not necessarily sustainable. Hence, the main task that the planners of societies dealing with the dual constraints of resources and the environment are facing is to realize sustainable development along the optimal growth path, as well as to ensure optimal social welfare and inter-generational equity when it comes to the use of resources and the environment. This has to be done through designed policies to encourage reasonable investments in natural, material and human capitals, which guide people to weigh the alternatives and tradeoffs between their material consumptions and the ecological or environmental needs.

Based on existing literature, this paper introduced environmental quality and non-renewable resources as endogenous factors to the analytical framework of the optimal growth model. The authors drew upon the ideas of Stoky (1988),

Aghion and Howitt (1998) for using the endogenous growth theory to analyze the issue of sustainable development. This paper also used the endogenous growth model proposed by Romer (1990) as the foundation for this study. These led to the construction of an endogenous growth model that took into account various factors simultaneously, like endogenous resources, the environment, and knowledge production, development of human capital and technological advancements. Through the analysis of the optimal growth path generated by the theoretical model, this paper was able to explore conditions for sustainable economic development under the dual constraints of resources and the environment. This paper further examined the impact of various economic, resource and environmental parameters on steady economic growth rates.

2 Economic development model with dual resource and environmental constraints

Consider a benchmark economy comprised of four divisions and six input factors. The four production divisions are the Knowledge Production Division, Human Capital Development Division, Intermediate Goods Production Division, and Final Goods Production Division. Production requires six types of input factors, including physical capital K, human capital H_Y, labor L, natural resources E, knowledge A, and clean technology z, which reflects environmental quality. This paper assumes that numerous individuals exist in the entire economy and that each individual is homogeneous, i.e., as the producer also consumer. For ease of analysis, the authors also assumes that the overall scale of the population is 1, i.e., $L = 1$; as this paper has not taken population growth into account, the total and per capita values are equal. Assuming that the total amount of human capital is H, the amount of human capital invested into the Final Goods Production Division is H_Y, human capital used in the Knowledge Production Division is H_A, and human capital for the Human Capital Development Division is H_H, so it will get

$$H = H_H + H_Y + H_A.$$

The final output function of the economic model therefore be expressed as

$$Y = F\left(1, K, H, A, R, z\right).$$

Conditions of the various divisions and input factors can derived from the following analyses. The relationships between each departments as shown in Fig. 1:

2.1 Human capital development division

Lucas (1988) introduced the concept of human capital into the economic growth model, and explored the relationship between human capital accumulation and economic growth, as well as establishing the "production function" for human capital production and accumulation. Based on Lucas (1988), the concepts of human capital and labor differ, as labor cannot accumulated while human capital can be accumulated through "learning by doing", and be transferred between old and new products. Assuming that a dedicated Human Capital Development Division exists in the economy, its role is mainly to enhance the standard of competitive human capital invested in production.

According to the assumptions of Uzawa (1965) and Lucas (1988), the production function of Human Capital Development Division can be set as

$$\dot{H} = \sigma H_H = \sigma(H - H_Y - H_A), \tag{1}$$

in which, $\dot{H}$ is the increment of human capital, representing the output of the Human Capital Development Division, and σ is the productivity of the Human Capital Development Division. When $\sigma > 0$, it reflects the time increment of human capital production. H_H is the amount of human capital invested by the Human Capital Development Division, which is a part of the overall human capital, $H_H = H - H_Y - H_A$.

Fig. 1 The relationships between each departments
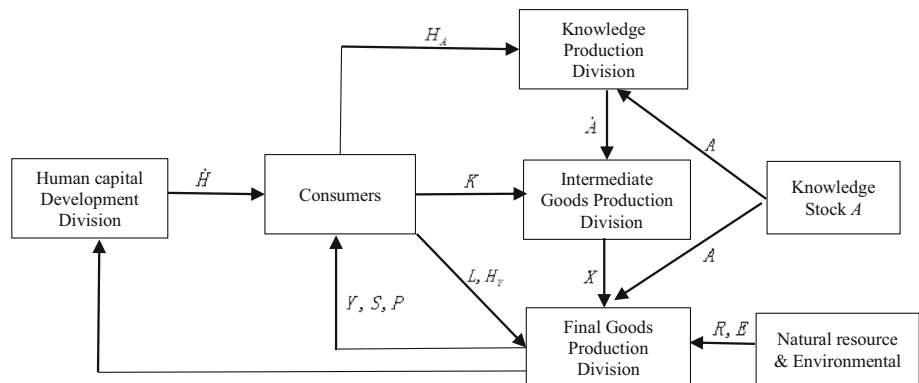
2.2 Knowledge production division

The Knowledge Production Division primarily uses human capital H_A and current knowledge stock A to produce new knowledge or the "design" for new intermediate products. Based on the studies of Jones (1995) and Romer (1990), the Knowledge Production Division can regarded as human capital intensive, and its production does not rely on physical products or labor. It is also assumed that knowledge is non-competitive, but firms that produce knowledge are competitive. Even if each firm is able to acquire the overall knowledge stock A at no cost and use A concurrently, and there is no competition in terms of knowledge usage among them, the human capital used in knowledge production is competitive. Based on Jones' (1995) improved version of Romer's (1990) model, the knowledge output of the jth knowledge producer at time t can be expressed as

$$\dot{A}_j = \pi H^j A^\xi,$$

in which $\dot{A}_j$ represents the knowledge output of the jth knowledge producer at time t; H^j represents the human capital stock used; π represents the knowledge productivity, i.e., the output rate of knowledge; A represents the initial knowledge stock or technological level; and ξ is the externality parameter of knowledge, $0 \leqslant \xi < 1$. $\xi = 0$ indicates that there is no relationship between technological advancement and the current knowledge level, and technological advancement is independent of the current knowledge level. $0 < \xi$ means that knowledge has positive external effects, i.e., the greater the existing stock of knowledge, the richer the research and development experience of the people, leading to higher productivity of the human capital invested by the Knowledge Production Division. $\xi < 1$ illustrates the congested nature of knowledge production, i.e., although knowledge is non-competitive and has positive externalities, technological innovation becomes relatively difficult with increasing knowledge levels, and the productivity of an initial piece of knowledge will show a diminishing trend.

Assuming that knowledge producers are homogeneous, i.e., they have the same knowledge production efficiency π. The total output of all knowledge producers at the time of t is

$$\dot{A} = \sum A_j = \pi H_A A^\xi, \tag{2}$$

in which, $\dot{A}$ represents the level of knowledge production within one time unit, which can also be termed the technological rate of change. π represents the production efficiency of knowledge. H_A represents the total human capital invested in the Knowledge Production Division, and is a part of the overall human capital H. Evidently, πH_A is equivalent to the rate of change of knowledge stock A given a certain human capital investment of H_A.

2.3 Intermediate goods production division

The Intermediate Goods Production Division mainly uses knowledge A and physical capital K for producing an intermediate good X, which is the input for the Final Goods Production Division. The paper assumes that physical capital can be broken down into numerous different types of intermediate goods x_i (to avoid integer constraints, the paper assumed that the intermediate products were continuous, i.e., i was a continuous and non-discrete variable). Our assumption differed slightly from that of Rivera-Batiz and Romer's (1991), as the quality of intermediate goods were simplified and excluded. Assuming that once a new technical proposal design has been created by the knowledge production division, the production of one unit of any type of intermediate good x_i would consume one unit of physical capital K, the total amount of per capita physical capital in the economy can be expressed as

$$K = \int_0^N x(i)di = AX, \tag{3}$$

in which K is the total amount of per capita physical capital; X is the per capita intermediate goods output; A is the current knowledge stock or technological level; and N is the total number of the different types of intermediate goods produced, $i \in [0, N]$. From the above equation, the production function of intermediate goods X can be derived as

$$X = \frac{K}{A}. \tag{4}$$

2.4 Final goods production division

Assuming that in the economy, the factors used the final goods production division comprise intermediate good input X, non-skilled labor L, competitive human capital H_Y, knowledge A, natural resource factor R, and clean technology z (reflecting environmental quality), then the output of production is Y. Based on earlier analyses, the natural resource factor is the fundamental input factor for economic growth. When $R = 0$, it is possible to have $Y = 0$; if $Y > 0$, then it is certain that $R > 0$. Y is the incremental function of R and fulfills the paddy condition of diminishing marginal productivity, i.e., $\lim_{R \to 0} F_R = \infty$, $\lim_{R \to \infty} F_R = 0$. Assuming that environmental pollution is an unavoidable consequence of the input of other factors, environment has a negative impact on output, and z represents the state of clean technologies adopted in production, $0 < z < 1$. If cleaner technologies are adopted, the intensity of pollution would decrease, and

economic output would increase as a result. However, as it is impossible to completely eliminate environmental pollution, $z < 1$. Assuming that the final production function adopts the Cobb-Douglas production function, assuming that labor population is a constant 1, the production function of per capital final production Y can be expressed as

$$
\begin{aligned}
Y &= A^{\alpha} H_Y^{\alpha} R^{\beta} z \left[\int_0^N x(i) di \right]^{\gamma} \\
&= A^{\alpha} H_Y^{\alpha} R^{\beta} (AX)^{\gamma} z = A^{\alpha} H_Y^{\alpha} R^{\beta} K^{\gamma} z,
\end{aligned} \tag{5}
$$

in which $0 < \alpha, \beta, \gamma < 1, \alpha + \beta + \gamma = 1$.

As seen from the above equation, when the level of knowledge and technology A remains constant, the final output Y is a function of human capital, natural resource input, physical capital input and clean technologies. Additionally, the level of knowledge and technology not only affects the final output directly, it is also a function of the human capital level, and influences the final output level by expanding the number of intermediate goods.

2.5 Natural resource constraint function

In terms of the natural resource constraint function, considering non-renewable resources as the main resources, the authors followed the model assumptions of Stiglitz (1974), and used $S(0) = S_0$ to represent the initial stock of non-renewable resources, $S_0 > 0$. At every point in time, the amount of resource developed and produced by the energy exploration division and sold to the Final Goods Production Division is R, $R > 0$. Excluding considerations of the cost of resource development, the stock equation of natural resources at each point in time can be expressed as

$$
S(t) = S(0) - \int_0^t R(V) dV, \quad S_0 > 0 \tag{6}
$$

Obtaining the derivatives for both sides of the above equation with respect to time, the differential equation of the natural resource stock with respect to changes in time can be derived, i.e., the dynamic equation for resource change is

$$
\dot{S} = -R, \quad R > 0 \tag{7}
$$

2.6 Environmental constraints function

Inevitably, economic activities will impact the environment, and the use of resources coupled with environmental pollution impacts the ecology. Assuming that this paper do not consider the pollution generated during consumption, then environmental pollution primarily generated through production. Changes in the stock of environmental pollutants during production mainly affected by two factors. First, is the discharge of pollutants, which can be assumed as the function of the output level and clean technologies. The higher the output level, the greater the amount of fossil fuels used, increasing emissions; the higher the standard of clean technologies, the more efficiently fossil fuels are used, decreasing emissions. When a specific clean technology z is adopted in production, the per capita emission in the environment can be expressed as $P = P(Y, z) = Y z^{\theta}$, in which θ represents the index of the degree of pollution. $\theta < 0$ indicates that as θ decreases, the emissions of the given clean technology are lower. θ can also be interpreted as the indicator for the strictness toward environmental pollution. The smaller θ, the more stringent the society's requirement toward the environment, and there would be less emissions correspondingly. The second factor is the self-purification capacity of the environmental system. The environment has an inherent self-purification capability toward pollutants. A contaminated environment has the capability to restore itself to its initial conditions through certain natural processes, as well as through physical, chemical, and biological intervention.

Similar to the thoughts of Aghion & Howitt (1998), this paper assumed that environmental quality had an upper limit, which could only be achieved by ceasing all productive activities. Therefore, changes in environmental quality at any point in time would be non-negative. The authors also assumed that there was a lower limit for environmental quality, i.e., emission had an upper limit of $P^{\max}$, and any emission exceeding this upper limit would mean the occurrence of irreversible devastation of the environment, such that it would be difficult for human production and living to continue. Considering the non-negative nature of emissions, a basic constraint condition of emission can derived as follows:

$$
0 \leq P < P^{\max}.
$$

Assuming that in a sustainable economy, the clean technology adopted in economic production is z; and assuming that pollution remediation and the self-purification effects of the environment could only reduce part of the emissions, the movement equation of environmental quality with changes in time can expressed as

$$
\dot{P} = A^{\alpha} H_Y^{\alpha} R^{\beta} K^{\gamma} z^{\theta+1} - \varpi P. \tag{8}
$$

In the above equation, $\dot{P}$ represents the rate of change of per capita pollutant in the environment, i.e., the amount of change of per capita pollutant stock in a unit of time; ϖ is the environmental system's self-purification coefficient toward pollutants, i.e., the environmental system's rate of regeneration, $0 < \varpi < 1$; $\overline{\varpi} P$ represents the amount of pollutants reduced through the environment's self-purification capability.

2.7 Physical capital accumulation function

Using the same assumptions as the neoclassical economic growth model, the net increment of per capita physical capital stock in an economy equals per capita total output Y less the various deductions such as per capita consumption and depreciations. Based on earlier analyses, the per capita total output of the Final Goods Production Division has three main uses: per capita consumption C, per capita depreciation δK, and per capita capital accumulation K. The dynamic change equation of the per capita physical capital accumulation can derived as

$$\dot{K} = Y - C - \delta K. \tag{9}$$

In the above equation, $\dot{K}$ represents the rate of change of per capita physical capital; δ represents the depreciation coefficient of physical capital.

2.8 Consumption utility function

In order to discuss sustainable development issues under resource and environmental constraints, the connotation of social welfare holistically were needed to considered . In a traditional economic growth model, social welfare is only a function of consumption, and the maximization of social welfare means the maximization of consumption by consumers, which evidently cannot cover the overall social welfare function. Drawing reference from the corrections made on the consumption utility function by Aghion & Howit (1998), Grimaud & Rouge (2003), this paper constructed the following separable instantaneous utility function for a representative consumer in an infinite time horizon that comprised consumption, resources, and environmental factors:

$$U(C, S, P) = \frac{C^{1-\varepsilon} - 1}{1 - \varepsilon} + \frac{S^{1-\omega} - 1}{1 - \omega} + \frac{[(-P)^{1+\phi} - 1]}{1 + \phi} \tag{10}$$

In the above equation, U represents the instantaneous utility function, C is the per capita goods consumption at time t, and P is the per capita pollutant stock at time t. As environmental pollutants have negative utility, $-P$ can be used to express the increment status of its consumption utility. Assuming the utilities generated by consumers when consuming C and $-P$ are substitutes, the marginal utilities generated by the above consumption activities are positive and diminishing, i.e., when $U_C, U_S, U_{-P} \to 0$ and when $U_C, U_S, U_{-P} \to \infty$. In the above equation, parameters ε and φ represent the risk aversion coefficients corresponding to consumption, resource and environmental pollution, and are the reciprocal of the intertemporal elasticity of substitution for consumption, resource, and environmental pollution,

respectively, $\varepsilon, \varphi > 0$. The larger the ε, φ, the faster the rate of diminishing marginal utilities caused by consumption, increased resource usage and environmental pollution, where reduction in current consumption in exchange for increased future consumption becomes uneconomical. As a result, consumers are more unwilling to reduce current spending and resource consumption, which in turn increases emissions.

Assuming that the optimal goal of the social planner is to maximize the present value of the social utility of the current and all future generations under the constraints of physical capital, human capital, non-renewable resources, and environment, a dynamic optimal planning function could therefore be constructed under the time continuity condition:

$$\max U(C, S, P) = \max \int_0^\infty U(C, S, P)e^{-\rho t}dt$$
$$= \max \int_0^\infty \left\{ \frac{C^{1-\varepsilon} - 1}{1 - \varepsilon} + \frac{S^{1-\omega} - 1}{1 - \omega} + \frac{[(-P)^{1+\phi} - 1]}{1 + \phi} \right\}$$
$$e^{-\rho t}dt.$$

In the above equation, ρ represents the consumer's subjective preference rate, which is also known as the intertemporal utility discount rate, and represents the discount rate when future consumptions are discounted as current consumption, $\rho > 0$. The larger the value of ρ, the greater the current consumptions and the less future spending arrangements by consumers.

3 Analysis of the optimal equilibrium of the society

Based on the above analyses, obtaining an economic growth model comprising four divisions and six input factors, in which human capital, knowledge, natural resources, and environment factors were also made endogenous, as illustrated below:

$$\max U(C, S, P) = \max \int_0^\infty U(C, S, P)e^{-\rho t}dt$$
$$= \max \int_0^\infty \left\{ \frac{C^{1-\varepsilon} - 1}{1 - \varepsilon} + \frac{S^{1-\omega} - 1}{1 - \omega} \right.$$
$$\left. + \frac{[(-P)^{1+\varphi} - 1]}{1 + \varphi} \right\} e^{-\rho t}dt$$

$$s.t. \begin{cases} Y = A^\alpha H_Y^\alpha R^\beta K^\gamma z \\ \dot{A} = \pi H_A A^\xi \\ \dot{H} = \begin{aligned} \sigma H_H = \sigma(H - H_Y - H_A) \\ \dot{K} = Y - C - \delta K \\ \dot{S} = -R \\ \dot{P} = A^\alpha H_Y^\alpha R^\beta K^\gamma z^{\theta+1} - \varpi P \end{aligned} \end{cases}$$

The above optimization problem is the maximization of integral functions within a continuous time, in which A, K, H, S,

and P are state variables, and C and z are control variables. This dynamic optimization problem can be processed using the Pontryagin maximum principle [120], and the present value of the Hamilton function is defined as

$$J = U(C, S, P) + \lambda_1(A^\alpha H_Y^\alpha R^\beta K^\gamma z - C - \delta K)$$
$$+ \lambda_2 \pi H_A A^\xi - \lambda_3 E + \lambda_4(A^\alpha H_Y^\alpha R^\beta K^\gamma z^{\theta+1} - \varpi P)$$
$$+ \lambda_5 \sigma(H - H_Y - H_A).$$

In the above equation, λ_1, λ_2, λ_3, λ_4 and λ_5 are the multipliers of Hamilton before discount, which represent the shadow price of physical capital, technology, natural resources, environment and human capital at time t, respectively.

3.1 Optimal first order conditions

From $\dfrac{\partial J}{\partial C} = 0$, obtained $\lambda_1 = C^{-\varepsilon}$. (11)

From $\dfrac{\partial J}{\partial H_Y} = 0$, obtained $\lambda_1 \dfrac{\partial Y}{\partial H_Y} + \lambda_4 z^\theta \dfrac{\partial Y}{\partial H_Y}$
$- \lambda_5 \sigma = 0.$

Reorganizing the above, obtained $\lambda_5 \sigma = \lambda_1 \alpha Y / H_Y$
$+ \lambda_4 z^\theta \alpha Y / H_Y.$ (12)

From $\dfrac{\partial J}{\partial H_A} = 0$, obtained $\lambda_2 \pi A^\xi - \lambda_5 \sigma = 0.$
Reorganizing the above, obtained $\lambda_2 \pi A^\xi = \lambda_5 \sigma.$ (13)

From $\dfrac{\partial J}{\partial E} = 0$, obtained $\lambda_1 \dfrac{\partial Y}{\partial E} - \lambda_3 + \lambda_4 z^\theta \dfrac{\partial Y}{\partial E} = 0.$
Reorganizing theabove, obtained $\lambda_3 = \lambda_1 \beta Y / E$
$+ \lambda_4 z^\theta \beta Y / E.$ (14)

From $\dfrac{\partial J}{\partial z} = 0$, obtained$\lambda_1 \dfrac{\partial Y}{\partial z} + \lambda_4 \dfrac{\partial (Y z^\theta)}{\partial z} = 0.$
Reorganizing the above, obtained $\lambda_1 Y / z$
$+ \lambda_4(\theta + 1) z^\theta Y / z = 0.$ (15)

3.2 Euler equations

According to the principle of dynamic optimization, the Euler equations of each common-mode variable can be obtained as follows:

$$\dot{\lambda}_1 = \rho\lambda_1 - \frac{\partial J}{\partial K}$$
$$= \rho\lambda_1 - \lambda_1(\frac{\partial Y}{\partial K} - \delta) - \lambda_4 z^\theta \frac{\partial Y}{\partial K}$$

$$= \rho\lambda_1 - \lambda_1(\gamma\frac{Y}{K} - \delta) - \lambda_4 z^\theta \gamma\frac{Y}{K}. \tag{16}$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \frac{\partial J}{\partial A} =$$
$$\rho\lambda_2 - \lambda_1 \frac{\partial Y}{\partial A} - \lambda_2 \pi\xi H_A A^{\xi-1} - \lambda_4 z^\theta \frac{\partial Y}{\partial A}$$
$$= \rho\lambda_2 - \lambda_1 \alpha\frac{Y}{A} - \lambda_2 \pi\xi H_A A^{\xi-1} - \lambda_4 z^\theta \alpha\frac{Y}{A}. \tag{17}$$

$$\dot{\lambda}_3 = \rho\lambda_3 - \frac{\partial J}{\partial S} = \rho\lambda_3 - S^{-\omega}. \tag{18}$$

$$\dot{\lambda}_4 = \rho\lambda_4 - \frac{\partial J}{\partial P} = \rho\lambda_4 - \left[-(-1)^{1+\varphi}P^\varphi - \lambda_4\overline{\omega}\right]$$
$$= \rho\lambda_4 + (-1)^{1+\varphi}P^\varphi + \lambda_4\overline{\omega}. \tag{19}$$

$$\dot{\lambda}_5 = \rho\lambda_5 - \frac{\partial J}{\partial H} = \rho\lambda_5 - \lambda_5\sigma. \tag{20}$$

3.3 Analysis of the stable growth rate

According to the definition of stable state, under stable growth conditions, the growth rate of each variable is constant, and the growth rates of per capita output, per capita consumption and per capita capital are equal, i.e., $g_Y = g_C = g_K = cons\tan t$. The authors defined the growth rate of the individual variable as $g_x\left(g_x = \frac{\dot{x}}{x}\right)$, and the growth rate of the variables can derived using the above equation as

$$g_H = (1 - \varepsilon)g_C + (\sigma - \rho) \tag{21}$$

$$g_A = \frac{1 - \varepsilon}{1 - \xi}g_C + \frac{\sigma - \rho}{1 - \xi} \tag{22}$$

$$g_S = g_R = \frac{1 - \varepsilon}{1 - \omega}g_C \tag{23}$$

$$g_P = \frac{1 - \varepsilon}{1 + \varphi}g_C \tag{24}$$

$$g_z = -\frac{\varepsilon + \varphi}{\theta(1 + \varphi)}g_C \tag{25}$$

$$g_Y = g_K = g_C = \alpha(\sigma - \rho)\left(1 + \frac{1}{1 - \xi}\right)\left[\alpha\frac{\varepsilon - 1}{1 - \xi}\right.$$
$$\left. + \alpha\varepsilon + \beta\frac{\varepsilon - \omega}{1 - \omega} + \frac{\varepsilon + \varphi}{\theta(1 + \varphi)}\right]^{-1} \tag{26}$$

4 Analysis of long-term sustainable development

4.1 The conditions of long-term sustainable development

The sustainable development theory suggests that the utilization of resources must comply with the minimum safety requirement. Therefore, in order to achieve sustainable economic development under the dual constraints of resource and environment, the following critical conditions must be fulfilled (Daly, 1989, 1994):

Firstly, the stable economic growth rate of sustainable development is a positive value, i.e., $g_Y = g_K = g_C > 0$;

Secondly, the irreversible nature of the environmental system determines that under the sustainable development condition, consumers with rational expectations will not damage the environment beyond its threshold, lest it would lead to irreparable damage on the environmental system. Therefore, sustainable development requires the growth rate of pollutants in stable growth to be a negative value, i.e., $g_P < 0$;

Thirdly, the growth rate of natural resource consumption (i.e., the input of natural resources in production) must not exceed the growth rate of natural resources production, i.e., $g_E \leq g_S$; and the growth rate of resource consumption must be lower than the economic growth rate, i.e., $g_E < g_Y = g_K = g_C$.

From the above restrictions, the critical condition for achieving stable growth in the economic system under the sustainable development state can derived as:

The productivity σ of the Human Capital Development Division is greater than the time discount rate ρ, i.e.,

$$\rho < \sigma. \tag{27}$$

The reciprocal of intertemporal elasticity of substitution of consumption ε and resource ω are greater than 1, and that $\varepsilon \leq \omega$, i.e.,

$$1 < \varepsilon \leq \omega. \tag{28}$$

The rate of environment regeneration or the self-purification capability of the environment is sufficiently large, i.e.,

$$\alpha(\sigma - \rho)\left(1 + \frac{1}{1-\xi}\right)(1-\varepsilon) < \varpi\left[\alpha\frac{\varepsilon-1}{1-\xi}(1+\varphi)\right.$$
$$\left. + \alpha\varepsilon(1+\varphi) + \beta\frac{\varepsilon-\omega}{1-\omega}(1+\varphi) + \frac{\varepsilon+\varphi}{\theta}\right] \tag{29}$$

Proven as follows:

For condition 1, it can be seen from Eq. (27) that

$$\sigma - \rho > 0.$$

Further, Eq. (28) $1 < \varepsilon < \omega$ shows that

$$0 < \frac{\varepsilon - 1}{\omega - 1} \leq 1.$$

Based on the assumption conditions:
$0 < \alpha, \beta, \gamma < 1, \alpha + \beta + \gamma = 1$;

$0 < \xi < 1$;

$\omega, \sigma, \rho, \varepsilon > 0$.

It can be deduced that

$$\alpha(\sigma - \rho)\left(1 + \frac{1}{1-\xi}\right) > 0$$

$$\alpha\frac{\varepsilon-1}{1-\xi} + \alpha\varepsilon + \beta\frac{\varepsilon-\omega}{1-\omega} + \frac{\varepsilon+\varphi}{\theta(1+\varphi)} > 0.$$

Condition 1 was therefore proved, i.e., $g_Y = g_K = g_C > 0$.

For condition 2, from Eqs. (27) and (28), derived that:

$$g_P = \frac{1-\varepsilon}{1+\varphi}g_C = -\frac{\varepsilon-1}{1+\varphi}g_C.$$

As $1 < \varepsilon, 0 < \varphi$, obtained:

$$g_P = -\frac{\varepsilon-1}{1+\varphi}g_C < 0.$$

Therefore proved condition 2, i.e., $g_P < 0$. At the same time, when the rate of environmental regeneration or the self-purification capability ϖ of the environment is sufficiently large, i.e., when the conditions of Eq. (29) are fulfilled, the occurrence of environmental disaster can be prevented effectively.

As for condition 3, it can be derived from Eqs. (23) and (28) that

$$g_S = g_E = \frac{1-\varepsilon}{1-\omega}g_C \leq g_C.$$

Therefore proved condition 3, i.e., $g_E \leq g_S \leq g_C$.

4.2 The effects of model parameters

Through comparative static analysis of the economic growth rate $g_Y = g_K = g_C$ under stable conditions, it was understood of the effects of various model parameters on the sustainable growth rate of the economy.

Through partial derivative for Eq. (26) with respect to σ, it obtained: $\partial g_C/\partial \sigma > 0$, which indicates that higher the productivity σ of the human resource development division,

the greater the economic growth rate, which also means that increase in the efficiency of human resource development would significantly enhance the rate of economic sustainable growth.

Through partial derivative for Eq. (26) with respect to ρ, it obtained: $\partial g_C / \partial \rho < 0$, which indicates that the smaller the time discount rate, the stronger the consumers' sustainable development awareness, and the higher the rate of stable growth. Social planners may enhance the society's awareness of sustainable development and raise the rate of sustainable development in a stable economy by strengthening resource conservation and environmental protection publicity.

Through partial derivative for Eq. (26) with respect to ξ, it obtained: $\partial g_C / \partial \xi > 0$, which indicates that the current knowledge stock has increased external effects on the production of the knowledge division, which in turn, would improve the output efficiency of the Knowledge Production Division and improve the stable economic growth rate. Therefore, social planners could promote human capital exchange and "learning by doing" through the establishment of knowledge sharing platforms, so as to promote the dissemination and sharing of knowledge and improve the economic growth rate.

Through partial derivative for Eq. (26) with respect to θ, it obtained: $\partial g_C / \partial \theta > 0$, which indicates that economic growth rate increases with the increase of θ, that is, the relaxation of environmental regulation would promote economic growth. This means that there is a certain degree of substitution effect between environmental protection and economic growth, such that a more rapid economic growth would be at the price of environmental price, with the adoption of more relaxed environmental regulations. On the contrary, if environmental protection efforts are enhanced with the adoption of more stringent environmental regulation, economic growth would be affected to some extent.

Through partial derivative for Eq. (26) with respect to ε, it obtained: $\partial g_C / \partial \varepsilon < 0$. As ε is the reciprocal of the intertemporal elasticity of substitution for consumption, the larger the value of ε, the faster the rate of diminishing marginal utility of consumption. Therefore, a relatively weak household consumption preference would lead to less desire for consumption, and lower growth rate for stable economy.

Through partial derivative for Eq. (26) with respect to ω, it obtained: $\partial g_C / \partial \omega < 0$. As ω is the reciprocal of the intertemporal elasticity of substitution for resource consumption, it can also be interpreted as the consumers' degree of attention on the sustainable utilization of scarce resources. The larger the value of ω, the faster the rate of diminishing marginal utility of resource consumption, which indicates that the weaker the consumers' emphasis on the sustainable utilization of resources, the more likely they would adopt resource-driven growth, and the lower growth rate of stable economy.

Through partial derivative for Eq. (26) with respect to φ, it obtained: $\partial g_C / \partial \varphi > 0$. As φ is the reciprocal of the intertemporal elasticity of substitution for environmental pollution, it can also be interpreted as consumers' environmental awareness. The greater the value of φ, the higher consumers' environmental protection awareness, and the stronger their preference for future environment, where consumers are willing to sacrifice present consumption in exchange for a good future environment. The rate of economic sustainable development is therefore higher in stable growth.

5 Conclusions and discussions of the study

Based on the above analyses, there can be arrived at following conclusions:

(1) Sustainable growth is achievable in a social economy with resource and environmental constraints.

As long as the intertemporal elasticity of substitution for consumption and resource demand is less than 1, and that the environment's regeneration capacity is sufficiently large. With the impetus of human capital and technological advancements, regardless of the existence of natural resource and environmental constraints in an economy, the infinite growth of per capita output, per capita consumption and the accumulation of physical capital can be sustained. In addition, under the state of sustainable growth, on one hand, the growth rate of resource consumption is equal to or less than the rate of economic growth $g_E = g_S \leq g_C$, which means that resource supply is able to meet the demand of economic growth. On the other hand, the growth rate of environmental pollution intensity $g_z < 0$ and the growth rate of emission $g_P < 0$ indicate that as the standard of clean technologies used in economic production continues to improve, the quality of environment will also continue to improve.

(2) Under sustainable development conditions, the growth rate of human capital accumulation is faster than the rate of physical capital growth.

Equation (21) shows that $g_H > g_C = g_K = g_Y$. This is because during the economic growth process, the accumulation of human capital has to overcome the effects of diminishing investment returns in the economy, as well as to overcome the pressure of resource and environmental constraints on economic growth.

From the perspective of human capital change equation, the improvement of human capital is mainly determined by two factors, namely the productivity σ of the Human Capital Development Division and the amount of human capital H_H invested by the Human Capital Development Division, and

is positively correlated with each factor, respectively. Therefore, in order to improve the standard of human capital, while maintaining the ratio of human capital investment, this goal can be achieved by enhancing the productivity of the Human Capital Development Division and the overall investment of human capital.

(3) Under sustainable development conditions, the growth rate of knowledge production is faster than the growth rate of human capital accumulation.

From $g_A = \frac{\dot{A}}{A} = \pi H_A A^{\xi-1}$, it can be seen that g_A is the constant in the achievement of stable growth. From the derivative with respect to time, it could be obtained: $g_{H_A} = (1 - \xi)g_A$. Substituting $g_H = g_{H_A}$, it is obtained: $g_A = \frac{1}{1-\xi}g_H$. It is therefore known that $\frac{g_A}{g_H} = (\frac{1}{1-\xi}g_H)/g_H = \frac{1}{1-\xi}$.

As $0 \leq \xi < 1$, it can be derived that $g_A > g_H$. The proposition is therefore proved.

From the perspective of knowledge change equation, the higher the knowledge production efficiency π, the greater the amount of human capital invested in knowledge production and the higher the external parameter ξ of knowledge. The level of knowledge production and technological innovation is also higher. With increase in the external parameter ξ of knowledge, the spillover effect of knowledge production becomes stronger and knowledge innovation becomes easier, so that the rate of knowledge growth increases significantly. When $\xi \to 0$, g_A is substantially larger than g_H.

(4) Under sustainable development conditions, the growth rate of resource consumption does not exceed the rate of economic growth.

Firstly, resource consumption is a necessary condition for economic growth. As can be seen from the model assumptions, resource factor is the fundamental input factor for economic growth. When $E = 0$, there can be $Y = 0$; when $Y > 0$, it is certain that $E > 0$. It is therefore known that under the economic sustainable development conditions, there must be $g_E > 0$.

Secondly, the reliance of continuous economic growth on resource factors requires that the growth rate of resource consumption be lower than the rate of economic growth, i.e., $g_E \leq g_C$. Otherwise, even if economic growth is driven by the technological advancements and human capital, the economy would eventually fall into a difficult state due to the depletion of resource factors.

Thirdly, the rate at which the society utilizes resources must not exceed the rate of resource production, i.e., $g_E \leq g_S$.

Based on the above, it can be seen that the relationship between resource consumption growth and economic growth is $g_E \leq g_S \leq g_C$.

(5) Under sustainable development conditions, when the growth rate of environmental pollution is negative, then the growth rate of pollution intensity will also be negative.

The utilization of resource will inevitably cause environmental pollution, and has a negative impact on economic growth, which means that increase in pollution intensity will lead to reduction in economic output, and vice versa. Therefore, when cleaner production technologies are adopted, economic output would increase; when sustainable development is achieved, the growth rate of pollution intensity is negative $g_z < 0$. Restrictions posted by the environmental threshold determines the pollutant emission speed to be negative under the sustainable development condition, i.e., $g_P < 0$. Otherwise, the environment would deteriorate to an extent that cannot restored.

Improvements in environmental pollution can achieved through the environment's self-purification capacity, as well as investment into remediation activities. Without taking into account human intervention, due to the restrictions of environmental threshold, the environmental system's self-purification capacity ϖ must fulfill the following conditions:

$$
\alpha(\sigma - \rho)\left(1 + \frac{1}{1-\xi}\right)(1-\varepsilon) < \varpi\left[\alpha\frac{\varepsilon-1}{1-\xi}(1+\varphi)\right.
$$
$$
\left. + \alpha\varepsilon(1+\varphi) + \beta\frac{\varepsilon-\omega}{1-\omega}(1+\varphi) + \frac{\varepsilon+\varphi}{\theta}\right].
$$

Increase in the environment's self-purification capacity ϖ would facilitate improvements in the environmental quality and promote the sustainable development of the economy. Therefore, social planners should enhance the protection of the ecological environment during the economic production process and set a reasonable emission threshold, to ensure the environment's self-renewal and restoration and maintain the virtuous cycle of the ecological system.

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