

Multi-model recursive identification for nonlinear systems with non-uniformly sampling

Ranran Liu^{1,2} · Tianhong Pan² · Zhengming Li²

Received: 30 August 2016 / Revised: 31 October 2016 / Accepted: 18 November 2016 / Published online: 29 November 2016
© Springer Science+Business Media New York 2016

Abstract A recursive least squares based on Multi-model is proposed for non-uniformly sampled-data nonlinear (NUSDN) systems. The corresponding state space model of an NUSDN system is derived using lifting technique. Taking advantage of the Fuzzy c-Mean Clustering algorithm, NUSDN is divided into several local models. The basic idea is that the NUSDN system is viewed as a model switching system under a given rule. Once the local models are identified, the global model is determined. A pH neutralization process validate the performance of the proposed algorithm.

Keywords Non-uniformly sampled-data · Nonlinear systems identification · Fuzzy c-mean cluster · Multi-model method · Recursive least squares

1 Introduction

A non-uniformly sampled-data (NUSD) system is one kind of general multi-rate sampled-data systems, in which the sampling intervals of the inputs and/or outputs are irregular. As the hardware limitation, environmental requirement and economic constraint, those systems are extensively used in industrial process [1–4]. Due to reflect the sam-

pling characters and impact performance of control systems, NUSD systems have attracted much attention in recent years. For example, Jiang and Xie proposed iterative least squares/stochastic gradient methods [5–7] for NUSD CAR models. As single-innovation algorithms, the parameters estimation precision was low. To overcome influences of bad data and improve estimation precision, Ding presented a multi-innovation stochastic gradient algorithm, and also gave the convergence analysis [8]. However, the computational burden is large. Based on hierarchical identification principle, Liu reported a least squares algorithm to save computational cost [9]. Furthermore, the AM-MI-GESG and recursive least squares parameter estimation algorithm were developed for CARIMA and Box-Jenkins NUSD systems with colored noise, respectively [10, 11].

The nonlinear models are used to describe nonlinear systems, which exist extensively in industrial process. Commonly used nonlinear models are Hammerstein models, Wiener models and their combinations. Recently, the identification of these systems has attracted much research attention and many algorithms have introduced in the literature. For the nonlinear systems with polynomial nonlinearity, Bai proposed the iterative /recursive least square algorithm for Hammerstein systems [12]. An over-parameterization technique is introduced to deal with the identification of Hammerstein ARMAX, OE, OEMA systems, and some algorithms e.g. gradient-based, least square-based, auxiliary model based algorithm [13–17]. Moreover, the over-parameterization technique is extended to identify MISO Wiener system [18]. To reduce the computation, Ding studied the identification of these nonlinearity models, and presented several methods, e.g. projection methods, stochastic gradient algorithm, Newton strategy [19], hierarchical identification [20, 21], etc. Through the key term separation principle, Vörös studied the identification problems for the

✉ Tianhong Pan
thpanjsu@yeah.net

Ranran Liu
lrr@jsut.edu.cn

Zhengming Li
lzming@ujs.edu.cn

¹ School of Automotive and Traffic Engineering, Jiangsu University of Technology, Changzhou 213001, China

² School of Electrical and Information Engineering, Jiangsu University, Zhenjiang 212013, Jiangsu, China

Hammerstein–Wiener models [22], Li et al. reported the maximum likelihood algorithm for Hammerstein and Wiener systems, respectively [23,24]. Yu studied the identification of hard nonlinear systems with two-segment and dead-zone input nonlinearities [25,26]. Voros studied modeling and parameter identification of systems with multi-segment piecewise-linear characteristics [27]. Based on the finite impulse response model and negative gradient search, Chen proposed stochastic gradient algorithm for identification of systems with preload nonlinearities [28]. Moreover, the estimation algorithm for Hammerstein systems with saturation and dead-zone nonlinearities was studied [29]. Ding studied the adaptive digital control of Hammerstein nonlinear systems with limited output sampling [30]. For the nonlinear dynamical adjustment models, Xiao proposed a parameter estimation algorithm [31]. And based on the Markov Chains, nonlinear stochastic systems with uncertainties and random delays modeled were studied [32]. However, most of the mentioned methods estimate the parameters of transformed models but the system models directly, which bring the burden computation meanwhile. What’s more, assumption of the polynomials with known orders restricts its application.

However, the above mentioned algorithms cannot be used directly to NUSDN systems. To solve this issue, Li proposed a least-squares-based iterative algorithm and the over-parameterization technique to estimate the parameters of Hammerstein systems with non-uniform sampling interval [33]. Furthermore, an auxiliary model-based recursive least-squares algorithm is derived under the constraints of causality to alleviate the computational burden [34]. However, achievements on the identification of NUSDN systems have so far been reported assumed that the nonlinear structure was known. To the best of our knowledge, few studies have addressed modeling and estimation issues for NUSDN systems with unknown nonlinear characters [35,36].

Multi-model approach has its own advantages in model industrial processes, especially those with inherent non-linearity, wide operating range or load disturbances [37]. Based on divide-and-conquer strategy, multi-model methods develop local linear models corresponding to typical operating regimes, and the global output is obtained by integration of local ones. As a major benefit of multi-model strategy, linear system theories can be used. It has been applied successfully in aviation industries [38], chemical industries [39,40] and so on.

In this study, the multi-model method is extended to estimate the parameters of NUSDN systems. First, a discrete model of a NUSDN system is obtained by using the lifting technique. Second, on the basis of the multi-model method, a switching model is constructed. Then, a recursive least squares algorithm based on fuzzy c-mean (FCM) cluster is proposed to estimate the system structure and parameters

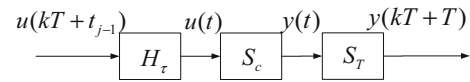


Fig. 1 Structure of NUSDN system

simultaneously. The proposed algorithm is validated on a pH neutralization process.

The remainder of this paper is organized as follows. Sect. 2 depicts the NUSDN system and a model switching system is constructed to formulate it. Sect. 3 presents the derivation of the proposed algorithm. Sect. 4 provides the simulation of the pH neutralization and conclusions are summarized in Sect. 5.

2 Problem formulation

Consider a NUSDN system, as shown in Fig. 1.

where $u(kT + t_{j-1})$ and $y(kT + T)$ are respectively the input and output of the system, $u(t)$ and $y(t)$ are respectively the input and output of the nonlinear part. H_τ is a non-uniformly zero-order holder with irregularly sampled intervals $\{\tau_1, \tau_2, \dots, \tau_m\}$; The input updating frequency is set as $kT + t_{j-1}, j = 1, 2, \dots, m (t_0 = 0, t_j := \tau_1 + \tau_2 + \dots + \tau_j)$. S_c is a nonlinear dynamic block and S_T is a sampler with frame period $T := \tau_1 + \tau_2 + \dots + \tau_m = t_m$. By using the lifting technique, H_τ is formulated as follows:

$$u(t) = \begin{cases} u(kT), & kT \leq t < kT + t_1 \\ u(kT + t_1), & kT + t_1 \leq t < kT + t_2 \\ \vdots & \vdots \\ u(kT + t_{m-1}), & kT + t_{m-1} \leq t < kT + T \end{cases} \tag{1}$$

S_c is assumed as the state-space model of the nonlinear character

$$S_c := \begin{cases} \dot{\mathbf{x}}(t) = \Gamma(\mathbf{x}(t), u(t)) \\ y(t) = h(t) \end{cases} \tag{2}$$

where $\mathbf{x}(t) \in R^n$ is the state vector, and $y(t) \in R$ and $u(t) \in R$ are the output and input of S_c respectively.

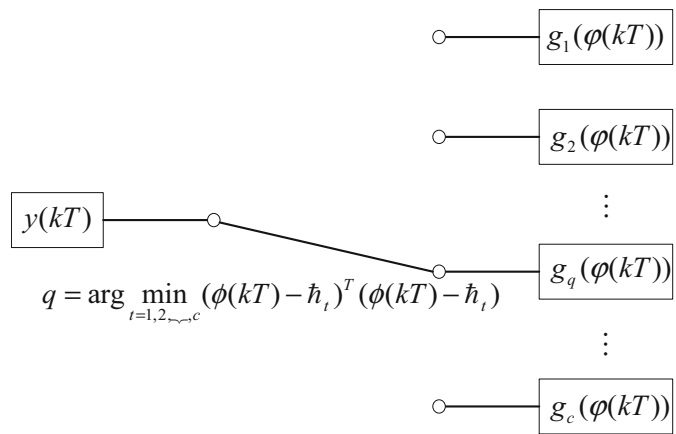
2.1 Model switching systems

As a common form of multi-model, the local model networks (LMN) can be described as:

$$y(kT) = \sum_{q=1}^c f_q(\phi(kT))g_q(\varphi(kT)) \tag{3}$$

where $y(kT) \in R$ is the model output at time kT , and $f_q(\cdot), g_q(\cdot), q = 1, 2, \dots, c$ are basis functions depend on the

Fig. 2 Schematic of model switching system



scheduling vector $\phi(kT) \in R^{n_\phi}$; $g_q(\cdot), q = 1, 2, \dots, c$ are local models which rely on the information vector $\varphi(kT) \in R^{n_\varphi}$. The number of local models is set as c . $\phi(kT)$ and $\varphi(kT)$ are defined as:

$$\begin{aligned} \phi(kT) := & [-y(kT - T), -y(kT - 2T), \dots, \\ & -y(kT - n_a T), \\ & u(kT), u(kT - T), \dots, \\ & u(kT - n_b T), u(kT - T + t_1), \dots, u(kT - n_b T + t_1), \\ & \dots, u(kT - T + t_{m-1}), \\ & \dots, u(kT - n_b T + t_{m-1})]^T \in R^{n_\phi}, n_\phi = n_a + mn_b + 1 \end{aligned} \tag{4}$$

$$\begin{aligned} \phi(kT) := & [-y(kT - T), -y(kT - 2T), \dots, \\ & -y(kT - n_{a_\phi} T), \\ & u(kT), u(kT - T), \dots, u(kT - n_{b_\phi} T), \\ & u(kT - T + t_1), \dots, u(kT - n_{b_\phi} T + t_1), \dots, \\ & u(kT - T + t_{m-1}), \dots, u(kT - n_{b_\phi} T + t_{m-1})]^T \\ & \in R^{n_\phi}, n_\phi = n_{a_\phi} + mn_{b_\phi} + 1 \end{aligned} \tag{5}$$

where n_a, n_b are respectively the orders of output and input, satisfy $0 \leq n_{a_\phi} \leq n_a, 0 \leq n_{b_\phi} \leq n_b$.

In theory, $\phi(kT)$ can be taken as any functions effective in a local area. Here, the Gaussian bell is chosen:

$$\tilde{f}_q(\phi(kT)) = \left(\exp\left(-\frac{(\phi(kT) - \hat{h}_q)^T (\phi(kT) - \hat{h}_q)}{s^T s}\right) \right)^\alpha \tag{6}$$

where \hat{h}_q and s are the center vector and width of the bell, α satisfies $1 \leq \alpha < \infty$. Upon normalizing Eq. (3), the LMN can be rewritten as

$$y(kT) = \sum_{q=1}^c \left(\frac{\left(\exp\left(-\frac{(\phi(kT) - \hat{h}_q)^T (\phi(kT) - \hat{h}_q)}{s^T s}\right) \right)^\alpha}{\sum_{d=1}^c \left(\exp\left(-\frac{(\phi(kT) - \hat{h}_d)^T (\phi(kT) - \hat{h}_d)}{s^T s}\right) \right)^\alpha} \right) \cdot g_q[\varphi(kT)] \tag{7}$$

Particularly, when $\alpha = \infty$, Eq. (7) is described as

$$y(kT) = g_q(\varphi(kT)) \tag{8}$$

where $q = \arg \min_{t=1,2,\dots,c} (\phi(kT) - \hat{h}_t)^T (\phi(kT) - \hat{h}_t)$.

It can be seen that LMN multi-model is depicted as the q^{th} local model whose center vector \hat{h}_t is nearest to $\phi(kT)$. And q will switch with time changing. Therefore, Eq. (8) can also be called model switching system shown in Fig. 2. It can be expressed as another form as:

$$\begin{aligned} R^q : & \text{if } \phi(kT) \text{ is } F_q \text{ then } y(kT) = g_q[\varphi(kT)], \\ & k = 1, 2, \dots, N; \quad q = 1, 2, \dots, c \end{aligned} \tag{9}$$

where F_q is scope of the q^{th} local model.

2.2 Local models

A nonlinear system can be seen as the combination of local models under a given condition. P^q is assumed as the state-space model of the q^{th} local model

$$P^q := \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}^q \mathbf{x}(t) + \mathbf{B}^q u(t) \\ y(t) = \mathbf{C}^q x(t) + D^q u(t) \end{cases} \tag{10}$$

where $\mathbf{A}^q \in R^{n \times n}, \mathbf{B}^q \in R^n, \mathbf{C}^q \in R^{1 \times n}, D^q \in R$ are parameter matrices. Upon Discretizing Eq. (10) with the frame period T [41], the q^{th} local model can be written as

$$\begin{aligned} y(kT) = & \frac{\sum_{j=1}^m B_j^q(z^{-1})}{A^q(z^{-1})} u(kT + t_{j-1}) = [1 - A^q(z^{-1})]y(kT) \\ & + \sum_{j=1}^m B_j^q(z^{-1})u(kT + t_{j-1}) = g_q(\phi(kT)) \end{aligned} \tag{11}$$

with

$$A^q(z^{-1}) = 1 + a_{q1}z^{-1} + a_{q2}z^{-2} + \dots + a_{qn_a}z^{-n_a}$$

$$B_1^q(z^{-1}) = b_{q10} + b_{q11}z^{-1} + \dots + b_{q1n_b}z^{-n_b}$$

$$B_j^q(z^{-1}) = b_{qj1}z^{-1} + b_{qj2}z^{-2} + \dots + b_{qjn_b}z^{-n_b},$$

$$j = 2, 3, \dots, m$$

where z^{-1} is the backward shift operator: i.e. $z^{-1}x(kT) = x(kT - T)$. Define the q^{th} local model parameter θ_q as

$$\theta_q := [a_{q1}, a_{q2}, \dots, a_{qn_a}, b_{q10}, b_{q11}, \dots, b_{q1n_b}, b_{q21}, \dots, b_{q2n_b}, \dots, b_{qm1}, \dots, b_{qm n_b}]^T \in R^{n_\phi}$$

The write noise is introduced, then Eq. (11) is rewritten as

$$y(kT) = g_q(\varphi(kT)) + v(kT) = \varphi(kT)\theta_q + v(kT) \quad (12)$$

where $q = \arg \min_{t=1,2,\dots,c} (\phi(kT) - \hat{h}_t)^T (\phi(kT) - \hat{h}_t)$. Then, the system parameter vector θ is formed

$$\theta := [\theta_1^T, \theta_2^T, \dots, \theta_c^T]^T \in R^{cn_\phi}$$

3 Multi-model recursive least squares algorithm based on fuzzy c-mean cluster (FCM-MRLS)

The task of the identification is to determine c, \hat{h}_q, s_q and θ_q in Eq. (12). To alleviate the computational cost, the estimation is divided into two simple sub-tasks. First, for a given number of c, \hat{h}_q, s_q can be determined by using FCM clustering algorithm, and $\varphi(kT)$ is classified into c sets simultaneously. Then θ_q can be estimated by using multi-mode recursive least squares algorithm.

3.1 Structure identification based on FCM

Set $\mathbf{U} = [\mu_{i,k}]_{c \times N}$ and $\mathbf{H} = [\hat{h}_1, \hat{h}_2, \dots, \hat{h}_c]$ are separately the membership matrix and cluster centers matrix. Given the information vector $\Phi = \{\phi(1), \phi(2), \dots, \phi(N)\}$, and a fuzziness parameter $w \in (1, \infty)$ (usually $w = 2$), FCM clustering algorithm aims to minimize the cost function

$$J(\mathbf{U}, \mathbf{H}, \Phi) = \sum_{q=1}^c \sum_{k=1}^N \mu_{q,k}^w \|\phi(kT) - \hat{h}_q\|^2 \quad (13)$$

where $\|\cdot\|$ is the vector norm. When c is known, FCM can be optimized by an alternating optimization (AO) algorithm [42] with the cluster centers which are randomly initialized as $\mathbf{H}^{(0)}$. After clustering, \mathbf{U} and \mathbf{H} are obtained. Then, s_q is determined using a nearest neighbor heuristic method:

$$s_q = \left[\frac{1}{p} \sum_{l=1}^p (c_q - c_l)^T (c_q - c_l) \right]^{1/2}, \quad q = 1, 2, \dots, c$$

where $c_l (l = 1, 2, \dots, p)$ are the l^{th} nearest neighbors of c_q , p is the number of the q^{th} nearest neighbors.

3.2 Parameters identification based on multi-model recursive least squares algorithm

By using the FCM Clustering algorithm, $c, \hat{h}_q, \mu_{q,k}$ are determined. Then, based on $\phi(kT)$, the global model is obtained with local models switching, formulated as:

$$y(kT) = g_q[\varphi(kT)] \text{ if } q = \arg \min_{t=1,2,\dots,c} (\phi(kT) - \hat{h}_t)^T (\phi(kT) - \hat{h}_t)$$

$$k = 1, 2, \dots, N \quad q = 1, 2, \dots, c$$

and

$$\varphi(kT) \in F_q \text{ if } \mu_{q,k} = \max_{1 \leq t \leq c} \{\mu_{t,k}\}$$

The q^{th} local model is identified based on $\phi(kT)$ which belongs to the q^{th} cluster. Then, a FCM-MRLS algorithm is proposed to identify θ_q

$$\hat{\theta}_q(kT) = \hat{\theta}_q(kT - T) + \mathbf{L}_q(kT)[y(kT) - \varphi^T(kT)\hat{\theta}_q(kT - T)] \quad (14)$$

$$\mathbf{L}_q(kT) = \mathbf{P}_q(kT - T)\varphi(kT) [1 + \varphi^T(kT)\mathbf{P}_q(kT - T)\varphi(kT)]^{-1} \quad (15)$$

$$\mathbf{P}_q(kT) = [\mathbf{I} - \mathbf{L}_q(kT)\varphi^T(kT)]\mathbf{P}_q(kT - T) \quad (16)$$

where $\hat{\theta}_q(kT)$ is the estimate of θ_q at time kT , \mathbf{I} is an identity matrix, and $\mathbf{P}_q(kT)$ is covariance matrix.

The proposed FCM-MRLS algorithm is summarized as follows:

Step 1: Let $c, \mathbf{H}^{(0)}$ and $k = 1$, and set $\hat{\theta}(0)$ and $\mathbf{P}(0)$ as

$$\hat{\theta}(0) = \mathbf{1}/p_0, \mathbf{P}(0) = p_0\mathbf{1},$$

where $\mathbf{1}$ is a column vector with appropriate dimension, whose entries are all 1, and $p_0 = 10^6$.

Step 2: Collect $\{u(kT + t_{j-1}), y(kT) : j = 1, 2, \dots, m; k = 1, 2, \dots, N\}$, construct $\varphi(kT)$ and $\phi(kT)$ using Eqs.(4) and (5), separately. After FCM clustering, $\hat{h}_q, \mu_{q,k}, q = 1, 2, \dots, c, ck = 1, 2, \dots, N$ are obtained, and $\varphi(kT)$ is separated into c clusters, the data length num_q of each cluster is determined as well.

Step 3: Based on the data sets of c clusters, $\hat{\theta}_q(kT), q = 1, 2, \dots, ck = 1, 2, \dots, num_q$ is identified using Eqs.(14), (15), (16).

Once $\hat{\theta}_q(kT), q = 1, 2, \dots, c$ is identified, $\theta(kT)$ is obtained.

The flow chart of FCM-MRLS algorithm is shown in Fig. 3.

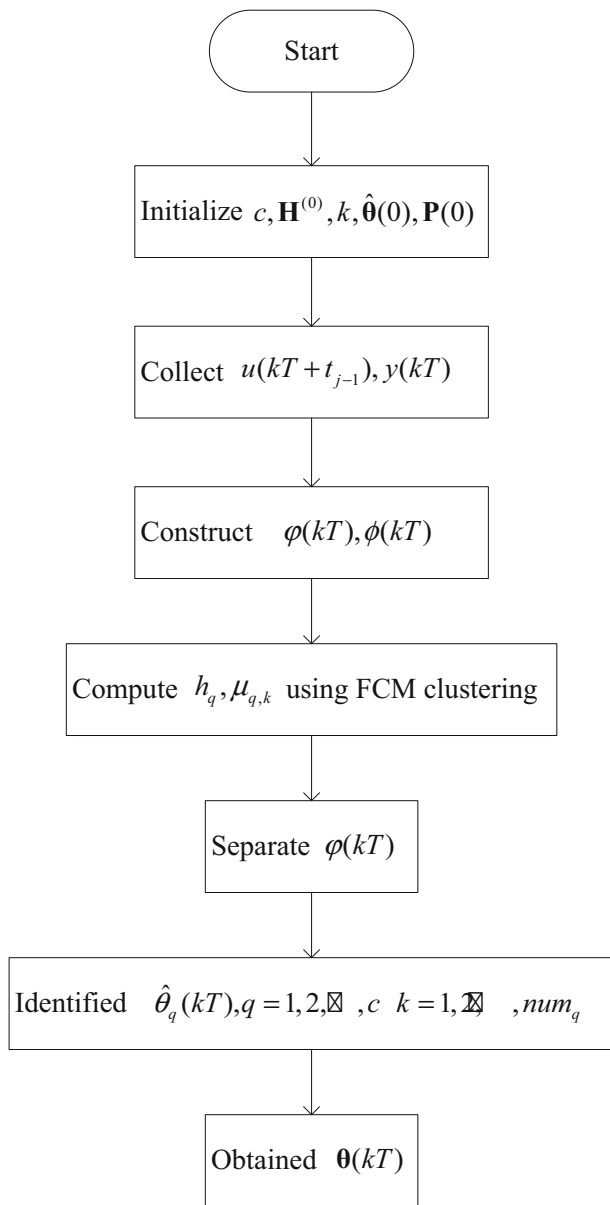


Fig. 3 Flow chart of FCM-MRLS algorithm

4 Case study

4.1 System description

Consider a pH neutralization process as shown in Fig. 4. Taken $\text{HNO}_3, \text{NaOH}, \text{NaHCO}_3$ as the reaction streams and pH-measurement as the output variable. Let F_a, F_b, F_c be flow rates of the three streams, respectively.

A dynamical model of pH neutralization process is depicted as follows:

$$V \frac{dw_a}{dt} = F_a C_a - (F_a + F_b) w_a \tag{17}$$

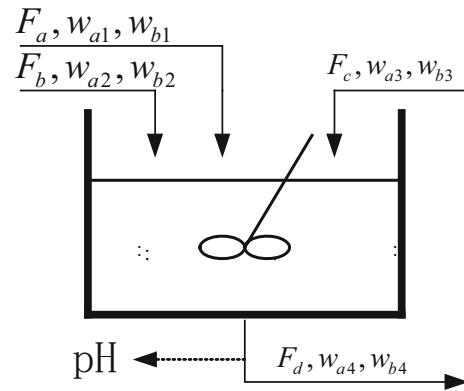


Fig. 4 pH neutralization process

$$V \frac{dw_b}{dt} = F_b C_b - (F_a + F_b) w_b \tag{18}$$

where C_a, C_b respectively are the concentration of HNO_3 and NaOH . V is the volume of the reactor. w_a, w_b are respectively charge invariant and material invariant, defined as:

$$w_a = [H^+] - [OH^-] - [HCO_3^-] - 2[CO_3^{2-}]$$

$$w_b = [H_2CO_3] + [HCO_3^-] + [CO_3^{2-}]$$

Let $pK_a = -\log_{10} K_a$, where $K_a = 1.76 \times 10^{-5}$ is the ionized constant of HNO_3 . Then, the static titration curve function is derived as:

$$w_b + 10^{-pH} - 10^{pH-14} - \frac{w_a}{1 + 10^{pK_a - pH}} = 0 \tag{19}$$

Equation (19) reflects the severe nonlinear of pH neutralization process.

4.2 Multi-model formulation

Considered a neutralization process of strong base-weak acid expressed as Eqs.(17), (18), (19). For a given F_a, F_b and pH-measurement are respectively taken as the input $u(kT + t_{j-1})$ and output y_{pH} . System operating parameters are shown in Table 1.

The sample sets $\{F_b, y_{pH}\}$ were gained from the mechanism model Eqs.(17), (18), (19). Then, 180 samples were obtained with a variation from -51.5 to 51.5 into F_b . Set $m = 2, \tau_1 = 0.5s, \tau_2 = 1s$, then, $t_0 = 0s, t_1 = \tau_1 = 0.5s$,

Table 1 System operating parameters

Parameter	Value	Parameter	Value
F_a	81 mL/min	$w_a(0)$	0.0435 mol/L
F_b	$515 + \zeta$ mL/min	$w_b(0)$	0.0432 mol/L
C_a	0.32 mol/L	V	1000 mL
C_b	0.05 mol/L	T_s	0.5 s

Table 2 Parameters estimates of six local models

Model no	Parameters						
	a_{11}	a_{12}	b_{10}	b_{11}	b_{12}	b_{21}	b_{22}
Model 1	-6.09173	-10.81120	0.47324	0.55646	0.50335	0.46854	0.54482
Model 2	-11.43338	-10.75626	0.46660	0.48056	0.53978	0.56013	0.54532
Model 3	-6.65278	-6.33973	0.47100	0.53815	0.50233	0.49450	0.47534
Model 4	-10.78123	-7.35109	0.49844	0.54770	0.55648	0.49958	0.52972
Model 5	-6.60107	-10.78664	0.48826	0.53123	0.55722	0.52286	0.56319
Model 6	-6.52145	-6.39258	0.47845	0.53727	0.54860	0.47895	0.51482

Fig. 5 Comparison between true value and estimates of pH neutralization process

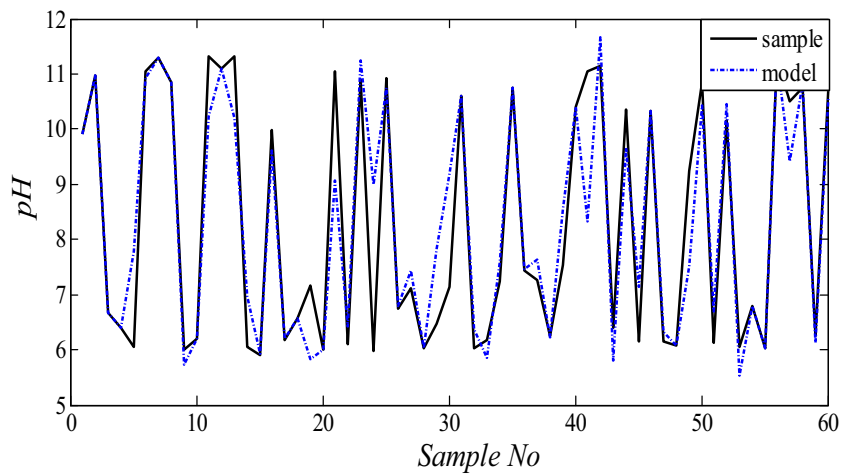
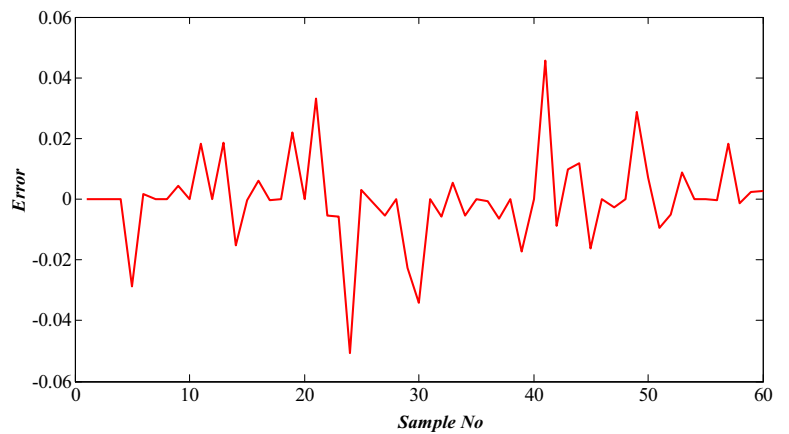


Fig. 6 Errors of FCM-MRLS algorithm



$t_2 = \tau_1 + \tau_2 = 1.5s = T, n_a = 2, n_b = 2$. In this case, $u(kT)$ and $u(kT + t_1)$ were taken from the data sequence of F_b , and $y(kT)$ was got from y_{pH} . After non-uniformly sampled, the data sets $\{u(kT), u(kT + t_1), y(kT)\}, k = 1, 2, \dots, 60$ were determined. $\phi(kT)$ was chosen as

$$\phi(kT) = [-y(kT - T), -y(kT - 2T), u(kT), u(kT - T), u(kT - T + t_1), u(kT - 2T + t_1)]^T$$

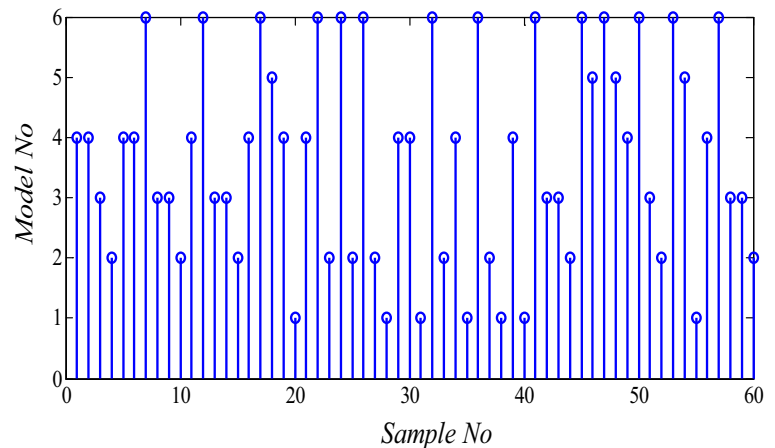
Set $c = 6$. Then, $\theta_q, q = 1, 2, \dots, c$ were identified using FCM-MRLS algorithm. The estimates were shown in Table 2, and the comparison between true value and estimates was presented in Fig. 4.

In Fig. 5, the sampled data (black solid line) and estimates (blue dot-solid line) were shown in Fig. 5. The root mean square error (RMSE) was introduced to evaluate the performance of proposed algorithm, defined as [43]

$$RMSE = \sqrt{\frac{1}{N} \sum_{k=1}^N (\hat{y}(k) - y(k))^T (\hat{y}(k) - y(k))}$$

RMSE of the pH identification is 0.0085, and the errors were depicted in Fig. 6. After FCM clustering, the local model of each sample was shown in Fig. 7.

Fig. 7 Local model of each sample



It could be seen that the FCM-MRLS algorithm gave a good fit for pH-measurement and had captured the nonlinearity presented in the neutralization process.

5 Conclusions

Combining with the lifting technique and FCM clustering method, a Multi-model based RLS identification algorithm is derived for NUSDN systems. Simulation results demonstrate the effectiveness of the proposed algorithm. The identification of the local models does not limit to RLS only. With proper modification, other algorithm to estimate linear systems can also be applied to NUSDN systems. The method can also be extended to other signal-rate nonlinear systems, multi-rate nonlinear systems and MIMONUSDN systems and so on. However, the performance analysis of the FCM-MRLS algorithm needs further investigation.

Acknowledgements The authors would like to thank the financial support provided by National Natural Science Foundation under Grant 61273142, Foundation for Six Talents by Jiangsu Province (2012-DZXX-045), Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD), Jiangsu support of science and technology projects under Grant BY2016030-16 Jiangsu Planned Projects for Postdoctoral Research Funds (Grant No.1601138B).

References

- Vardakas, J.S., Moscholios, I.D., Logothetis, M.D., Stylianakis, V.G.: Performance analysis of OCDMA PONs supporting multi-rate bursty traffic. *IEEE Trans. Commun.* **61**, 3374–3384 (2013)
- Lowengrub, J., Allard, J., Aland, S.: Numerical simulation of endocytosis: viscous flow driven by membranes with non-uniformly distributed curvature-inducing molecules. *J. Comput. Phys.* **309**, 112–128 (2016)
- Eduardo, S.L., Manuel, M.O.: High-order recursive filtering of non-uniformly sampled signals for image and video processing. *Comput. Gr. Forum.* **34**, 81–93 (2015)
- Long, D., Delaglio, F., Sekhar, A., Kay, L.E.: Probing invisible, excited protein states by non-uniformly sampled pseudo-4D CEST spectroscopy. *Angew. Chemie.* **127**, 10653–10657 (2015)
- Jiang, H.X., Wang, J.H., Ding, F.: Least-square-iterative identification of a class of non-uniform sampled-data systems. *Syst. Eng. Electron.* **30**(8), 1535–1539 (2008)
- Xie, L., Yang, H.: Gradient-based iterative identification for nonuniform sampling output error systems. *J. Vib. Control* **17**(3), 471–478 (2011)
- Xie, L., Ding, F.: Identification method of non-uniformly sampled-data systems. *Control Eng. China* **15**(4), 402–404 (2008)
- Ding, J., Xie, L., Ding, F.: Performance analysis of multi-innovation stochastic gradient identification for non-uniformly sampled systems. *Control Decis.* **26**(9), 1338–1342 (2011)
- Liu, Y., Ding, F., Shi, Y.: Least squares estimation for a class of non-uniformly sampled systems based on the hierarchical identification principle. *Circuits Syst. Signal Process.* **31**(6), 1985–2000 (2012)
- Xie, L., Liu, Y.J.: AM-MI-GESG algorithms for non-uniformly sampled-data systems. *Chin. J. Scientific Instrum.* **30**(6), 25–29 (2009)
- Xie, L., Yang, H.Z., Ding, F.: Recursive least squares parameter estimation for non-uniformly sampled systems based on the data filtering. *Math. Comput. Model.* **54**(1–2), 315–324 (2011)
- Bai, E.W.: An optimal two-stage identification algorithm for Hammerstein-Wiener nonlinear systems. *Automatica* **34**(3), 333–338 (1998)
- Ding, F., Shi, Y., Chen, T.W.: Gradient-based identification methods for Hammerstein nonlinear ARMAX models. *Nonlinear Dyn.* **45**, 31–43 (2005)
- Ding, F., Chen, T.W.: Identification of Hammerstein nonlinear ARMAX systems. *Automatica* **41**, 1479–1489 (2005)
- Ding, F., Shi, Y., Chen, T.W.: Auxiliary model based least squares identification methods for Hammerstein output-error systems. *Syst. Control Lett.* **56**, 373–380 (2007)
- Wang, D.Q., Ding, F.: Extended stochastic gradient identification algorithms for Hammerstein-Wiener ARMAX systems. *Comput. Math. Appl.* **56**, 3157–3164 (2008)
- Wang, D.Q., Chu, Y.Y., Ding, F.: Auxiliary model-based RLS and MI-ELS algorithm for Hammerstein OEMA systems. *Comput. Math. Appl.* **59**, 3092–3098 (2010)
- Zhou, L.C., Li, X.L., Pan, F.: Gradient-based iterative identification for MISO Wiener nonlinear systems: application to a glutamate fermentation process. *Appl. Math. Lett.* **26**, 886–892 (2013)
- Ding, F., Liu, X.P., Liu, G.: Identification methods for Hammerstein nonlinear systems. *Digital Signal Proc.* **21**(2), 215–238 (2011)

20. Ding, F.: Hierarchical multi-innovation stochastic gradient algorithm for Hammerstein non-linear system modeling. *Appl. Math. Model.* **37**(4), 1694–1704 (2013)
21. Ding, F., Liu, X.G., Chu, J.: Gradient-based and least-squares-based iterative algorithms for Hammerstein systems using the hierarchical identification principle. *IET Control Theory Appl.* **7**(2), 176–184 (2013)
22. Vörös, J.: An iterative method for Hammerstein-Wiener systems parameter identification. *J. Elect. Eng.* **55**(11–12), 328–331 (2004)
23. Li, J.H., Ding, F.: Maximum likelihood stochastic gradient estimation for Hammerstein systems with colored noise based on the key term separation technique. *Comput. Math. Appl.* **62**(11), 4170–4177 (2011)
24. Li, J.H., Ding, F., Yang, G.W.: Maximum likelihood least squares identification method for input nonlinear finite impulse response moving average systems. *Math. Comput. Model.* **55**(3–4), 442–450 (2012)
25. Yu, B., Fang, H., Lin, Y., Shi, Y.: Identification of Hammerstein output-error systems with two-segment nonlinearities: algorithm and applications. *Control Intell. Syst.* **12**, 1426–1437 (2010)
26. Yu, F., Mao, Z.Z., Jia, M.X.: Recursive identification for Hammerstein-Wiener systems with dead-zone input nonlinearity. *J. Proc. Control* **23**(8), 1108–1115 (2013)
27. Voros, J.: Modeling and parameter identification of systems with multi-segment piecewise-linear characteristics. *IEEE Trans. Autom. control* **47**(1), 184–188 (2002)
28. Chen, J., Lu, X.L., Ding, R.: Parameter identification of systems with preload nonlinearities based on the finite impulse response model and negative gradient search. *Appl. Math. Comp.* **219**, 2498–2505 (2012)
29. Chen, J., Wang, X.P., Ding, R.F.: Gradient based estimation algorithm for Hammerstein systems with saturation and dead-zone nonlinearities. *Appl. Math. Model.* **36**(1), 238–243 (2012)
30. Ding, F., Chen, T., Iwai, Z.: Adaptive digital control of Hammerstein nonlinear systems with limited output sampling. *SIAM J. Control Optim.* **45**(6), 2257–2276 (2007)
31. Xiao, Y.S., Yue, N.: Parameter estimation for nonlinear dynamical adjustment models. *Math. Comput. Model.* **54**(5–6), 1561–1568 (2011)
32. Li, H., Shi, Y.: Robust H1 filtering for nonlinear stochastic systems with uncertainties and random delays modeled by Markov chains. *Automatica* **48**(1), 159–166 (2012)
33. Li, X.L., Ding, R.F., Zhou, L.C.: Least-squares-based iterative identification algorithm for Hammerstein nonlinear systems with non-uniform sampling. *Int. J. Comput. Math.* **90**(7), 1524–1534 (2013)
34. Li, X.L., Zhou, L.C., Ding, R.F., Sheng, J.: Recursive least-squares estimation for Hammerstein nonlinear systems with nonuniform sampling. *Math. Probl. Eng.* **8**(6), 165–185 (2013)
35. Chen, J., Lv, L.X., Ding, R.F.: Multi-innovation stochastic gradient algorithms for dual-rate sampled systems with preload nonlinearity. *Appl. Math. Lett.* **26**, 124–129 (2013)
36. Chen, J., Lv, L.X., Ding, R.F.: Parameter estimation for dual-rate sampled data systems with preload nonlinearities. *Adv. Intell. Soft Comput.* **125**, 43–50 (2011)
37. Murray-Smith, R., Johansen, T.A.: *Multiple Model Application to Nonlinear Modeling and Control* London. Taylor & Francis, London (1997)
38. Johansen, T.A., Foss, B.A.: Multiple model approaches to modelling and control (Editorial). *Int. J. Control* **72**(7/8), 575 (1999)
39. Schoot, K., Bequette, B.W.: *Control of Chemical Reactors using Multiple-Model Adaptive Control*. IFAC D'WORD'95, Helsinki (1995)
40. Lakshmanan, N.M., Arkun, Y.: Estimation and model predictive control of non-linear batch processed using linear parameter varying models. *Int. J. Control* **72**(7/8), 659–675 (1999)
41. Sanchis, R., Albertos, P.: Recursive identification under scarce measurements-convergence analysis. *Automatica* **38**(3), 535–544 (2002)
42. Runkler, T.A., Bezdek, J.C.: Alternating cluster estimation: a new tool for clustering and function approximation. *IEEE Trans. Fuzzy Syst.* **7**(4), 377–393 (1999)
43. Babuska, R., Verbruggen, H.: Neuro-fuzzy methods for nonlinear system identification. *Annu. Rev. Control* **27**(1), 73–85 (2003)



application, etc.

Ranran Liu received her B.S. degree and M.S. degree from Changzhou University in 2004 and 2007 respectively. And she received her Ph.D. degree in control theory and control engineering from Jiangsu University in 2014. Now she has been a lecturer in School of Automotive and Traffic Engineering, Jiangsu University of Technology, Changzhou, China. Her current research interests include system identification, mathematics modelling, soft sensors, systems simulation and its



Jiangsu University, Zhenjiang, China. His research interests include multiple model approach and its application, machine learning, virtual metrology, predictive control and run-to-run control theory and practice, system identification, etc.

Tianhong Pan received the B.S. degree in mechatronics technology and applications from Anhui Agriculture University Hefei, China; the M.S. degree in Power electronics and electric power drives from GanSu University of Technology, Lanzhou, China; and the Ph.D. degree in control theory and control engineering from Shanghai Jiao Tong University, Shanghai, China, in 1997, 2000 and 2007, respectively. He has been a Professor with the School of Electrical and Information Engineering,



Zhengming Li received his B.S. degree from the Zhenjiang Institute of agricultural machinery in 1982, and the M.S. in automatic control from the Department of Automation, Xi'an Jian Tong University in 1987. He has been a professor in the School of Electrical Information and Engineering, Jiangsu University. His research interests are modelling of complex processes and wireless sensor network, etc.