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# Extreme value analysis and the study of climate change

## A commentary on Wigley 1988

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**Abstract** In his paper in *Climate Monitor*, TML Wigley uses basic probability arguments to illustrate how a slowly changing climate could potentially affect the frequency of extreme events. In the time since the paper appeared, there has been increased interest in assessing how weather extremes may be altered by climate change. Much of the work has been conducted using extreme value analysis, which is the branch of statistics developed specifically to characterize extreme events. This commentary discusses the advantages of an EVA approach and reviews some EVA techniques that have been used to describe climate change's potential impact on extreme phenomena. Additionally, this commentary illustrates basic EVA techniques in an analysis of temperatures for central England. In parallel to Wigley's analysis, a time-varying EVA analysis is compared to a stationary one, and furthermore, the trend from the EVA analysis is compared to the trend in means.

### 1 Weather extremes and extreme value analysis

The purpose of TML Wigley's (1988) paper "The Effect of Changing Climate on the Frequency of Absolute Extreme Events" is to illustrate how weather extremes could potentially be affected by a gradual change in climate. Wigley uses basic probability arguments to show that a simple shift in mean could have a significant effect on the frequency of events that, under historical conditions, are considered extreme. Wigley illustrates his arguments using temperature data for central England. As an example of the how the frequency of extreme events could potentially change, Wigley's simple model predicts that before 2030, temperatures in central England would (with extremely high probability) exceed those observed in 1976, which had

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been among the highest in the historical record at the time the paper was published. Unprecedented high temperatures for a prolonged period of time were in fact observed during the European heat wave of 2003, and others (c.f. Stott et al. (2004)) have attributed a significant portion of the risk of such an event to anthropogenic climate change.

To make his calculations, Wigley assumes that the variable of interest (e.g., temperature) is normally distributed with a mean that has a linear trend in time. Wigley then uses stochastic simulation to calculate how the return period and risk of an extreme event are affected by the size of the trend. A possible shortcoming of Wigley's approach is that he tries to characterize the behavior of extreme events by modeling the entire distribution, rather than focusing only on the distribution's tail. As Wigley himself notes "the particular weather situations that lead to extremes of temperature, precipitation, etc., may be unrelated to those which lead to the usual values of these variables." In Wigley's defense, he works with monthly mean temperature data, for which an assumption of normality is reasonable and from which it is impossible to identify the extreme daily temperatures. Nevertheless, because extreme phenomena are rare, their behavior can be overwhelmed when one fits a distribution to the entire data set. Furthermore, under climate change, it is possible that the change in behavior in the extremes could differ from the change in the mean. Therefore, to answer questions about how the extremes may change, one should focus on the extremes themselves.

Extreme value analysis (EVA) is the branch of statistics which attempts to characterize the tail of a distribution. There are a number of very good recent texts on extremes; the book by Coles (2001) gives an introduction to statistical practice, and the books by Beirlant et al. (2004) and de Haan and Ferreira (2006) review both the probabilistic theory and statistical methodologies. The foundation of EVA is provided by asymptotic results which yield distributions with which to characterize the tail. Statistical practice is to use the large observations (e.g., annual maximum observations or exceedances over a high threshold) to fit these distributions.

The well-known central limit theorem provides asymptotic justification for modeling sample means with the normal distribution. A similar asymptotic result provides justification for modeling sample maxima with the generalized extreme value (GEV) distribution. The GEV family has distribution function

$$P(X \leq x) = \exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}, \tag{1}$$

where $1+\xi\left(\frac{x-\mu}{\sigma}\right) > 0$. The parameters $\mu$ and $\sigma$ are location and scale parameters respectively, and the parameter $\xi$ controls the nature of the tail. If $\xi < 0$ the upper tail is bounded, if $\xi = 0$ then Eq. 1 is understood as the limit and the tail decays exponentially, and if $\xi > 0$ then the tail decays as a power function. In practice, one can extract the block (e.g., annual) maxima from a long data recond and then fit the GEV to obtain a model for the distribution of the annual maximum measurement. By modeling the annual maxima, one is able to estimate quantities of interest such as return levels.

The GEV provides a foundation for EVA, but in practice it can be wasteful of data, as only block maxima are used to fit the model. Another asymptotic result justifies modeling exceedances over a threshold with the generalized Pareto (GP) distribution. The GP can be understood as a conditional probability distribution given by

$$P(X \leq x | X > u) = 1 - \left(1 + \frac{\xi(x-u)}{\psi}\right)^{-1/\xi},$$

where $\psi$ is a scale parameter and $\xi$ determines the tail behavior as before. A point process approach to exceedances over a threshold (Smith 1989) provides an equivalent model in terms of the GEV parameters $\mu$, $\sigma$, and $\xi$. While these threshold methods allow the practitioner to use more of the data, there is a tradeoff as they also require one to choose an appropriate threshold, and one must also often deal with declustering dependent data (Davison and Smith 1990).

Techniques for analyzing extreme values have a long history in hydrology, but have become more common in other fields in the last two decades or so. Part of the reason is that EV analyses can be performed more readily. Estimating the parameters of an EV distribution has become a more manageable task due to both advances in computing which enables numerical maximum likelihood approaches and the advent of methodologies such as L-moments (Hosking and Wallis 1997). Easy-to-use statistical computing packages for extremes are now available, either as packages of larger statistical programs (e.g., `ismev` (Coles and Stephenson 2006), or `extRemes` (Gilleland et al. 2004) for `R`), or as companions to books (e.g., `Xtremes`, Reiss (1997)).

In studying the effects of climate change, one would like to capture how the extremes are changing in time. A particularly useful technique is to regress the EV distribution's parameters on known covariates. By either fitting a parametric model (c.f., Katz et al. (2002)) or nonparametric trends (c.f., Chavez-Demoulin and Davison (2005)) to the distribution's parameters, one can model how the extremes are changing in time or with respect to some other covariate.

Most atmospheric data are spatial in nature, and it is desirable to characterize how extremes vary by location. A straightforward approach to this problem is to independently fit an EV distribution to the data associated with each location, and a number of papers have taken this approach (Kharin and Zwiers 2000; Zwiers and Kharin 1998; Frei et al. 2006). Using this approach on a suite of global climate models, Kharin et al. (2007) analyze both future temperature and precipitation. Kharin and his coauthors conclude that, according to the models, changes in warm temperature extremes will closely resemble changes in the summertime mean temperature, but that cold extremes will change much more rapidly than the mean.

Because a typical EVA analysis uses only fraction of the data, there is quite a bit of uncertainty in the parameter estimates. With regards to spatial data, there is often a desire to "trade space for time" by combining data from similar sites with short data records to obtain better estimates for parameters and long-term return levels. The method of regional frequency analysis (RFA) can trace its roots back to the hydrology work of Dalrymple (1960), and the recent methodology has been compiled by Hosking and Wallis (1997). RFA combines data sites within regions regarded to be homogeneous to reduce the uncertainty associated with the extreme value distribution's parameter estimates. RFA has been widely used to analyze regional extremes, but recently Fowler et al. (2005) and Ekstrom et al. (2005) used RFA and the GEV distribution to describe potential changes in extreme rainfall for the UK due to climate change. As an alternative to RFA, Cooley et al. (2007b) recently constructed a Bayesian hierarchical model for extreme precipitation. This hierarchical model is like the regression models in that the GP distribution's parameters are allowed to vary by location. Similar to an RFA analysis, the Bayesian hierarchical model pools the data but uses a geostatistical approach to more naturally model the data's spatial nature. Sang and Gelfand (2008) use a similar approach on South African gridded precipitation data but additionally attempt to address the issue of climate change by allowing the GEV parameters to vary by both location and in time. Cooley and Sain (2008) use a Bayesian approach to characterize both spatial and temporal effects for precipitation data from both current and future runs of a regional climate model for western North America.

While the hierarchical approach allows one to model how extremes vary by location, they are not truly multivariate analyses. To answer questions about the joint distribution of extreme observations or to calculate the cumulative effect of extremes at multiple locations, one must build a multivariate model. Multivariate EVA is much more complicated than the univariate case in that the family of multivariate extreme value distributions cannot be summarized parametrically. There is great interest currently in modeling multivariate extremes (c.f., Heffernan and Tawn (2004); de Haan and Pereira (2006); Cooley et al. (2007a)), but the particular topic of climate change has not yet been addressed.

## 2 Illustrative example: central England temperature

The following is an illustrative example analogous to the example done by Wigley, but done using EVA. Two time series of central England temperatures provided by the Hadley Centre (http://hadobs.metoffice.com/hadcet/index.html) are analyzed. We primarily examine a time series of daily maximum temperatures for the years 1878–2007 (Parker and Horton 2005), from which we construct a time series of annual maxima. For comparison purposes, we also examine a series of daily mean temperatures for the same time period (Parker et al. 1992), for which we focus our attention on the month of July. These time series are revised versions of the series cited by Wigley (Jones 1987; Manley 1974). Figure 1 shows both the time series of annual maxima and the daily July temperatures summarized by the monthly mean. By sight it appears that both the annual maximum and July mean temperatures are increasing over time.

To the annual maximum time series, we fit (by numerical maximum likelihood) a regression-type GEV model where we allow the location parameter $\mu$ to vary linearly with time. This model has parameter estimates: $\hat{\mu}(t) = 26.17 + 0.0142t$, $\hat{\sigma} = 2.04$, and $\hat{\xi} = -0.27$, where $t$ is the number of years since 1878. The parameter estimate $\hat{\mu}(t)$ is shown in Fig. 1. Using the deviance criterion (Coles 2001), this regression model was shown to be significantly superior to a stationary model for the entire data set (p-value = 0.004). It is important to notice is that the point estimate $\hat{\xi}$ is significantly negative indicating a bounded upper tail for the temperature data. Fitting a normal distribution to the entire temperature data would yield an unbounded, exponentially decreasing tail. Other climatological variables such as precipitation tend to have heavy tails ($\hat{\xi} > 0$).

To provide a historical reference for an extreme event, a stationary model is fit to the annual maximum data from 1878–1970 and is used to estimate the 100-year return

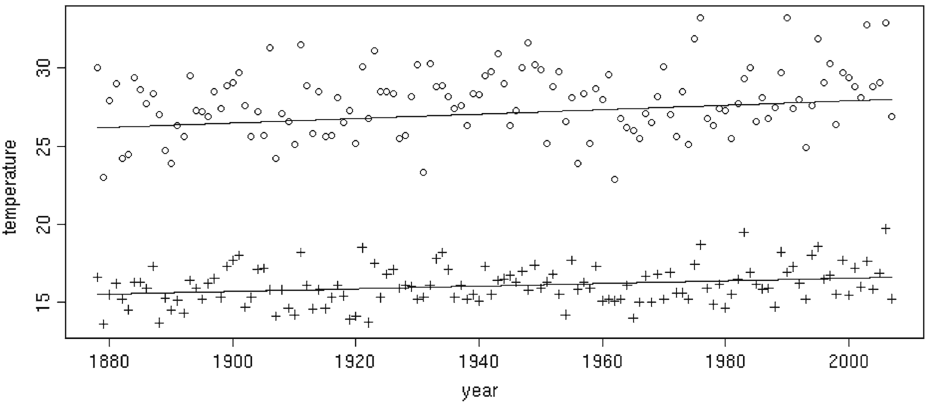


**Fig. 1** Annual maximum temperatures (*dots*) and July mean temperatures (*crosses*) for central England. The *top line* denotes the estimated trend in the GEV location parameter (slope = 0.0142°C/year) and the *lower line* denotes the linear regression model for the mean July temperature (slope = 0.0085°C/year)

level; that is, the 0.99 quantile of the distribution of annual maximum temperatures. This level is meant to represent what might have been considered a once-in-100-years event before there was widespread discussion of anthropogenic climate change. The stationary GEV model based on the historical data yields parameter estimates of $\hat{\mu} = 26.93$, $\hat{\sigma} = 2.10$ and $\hat{\xi} = -0.38$. These yield an estimate of 31.5°C for the historical 100-year return level, and there is only one observed exceedance (31.6°C) in the years 1878–1970. Similar to Wigley, we can then compute the probability of exceeding this historical return level under the regression model either within the range of the data or by extrapolating the trend into the future. According the regression model, the probability in 2007 that the annual maximum exceeded a level of 31.5°C is 0.10, so an event that historically would be estimated to occur once in 100 years would now have a 1-in-10 chance of occurring in 2007. If one were to extrapolate the trend into the future, the regression model gives a probability of 0.13 of exceeding 31.5°C in the year 2030.

We also compare our above EVA analysis to one which models the change in mean temperature. To the time series of daily mean July temperatures, we fit a linear model to estimate how the mean of this data varies over time. The linear regression yields a model for the mean of $m(t) = 15.51 + 0.0085t$, which is shown in Fig. 1. Notice that the estimated trend for the mean temperatures is a bit lower than the estimated trend for the annual maxima. It is straightforward to show that a shift in distribution (such as a change in the mean of a normal distribution) would translate to an equal shift in the location parameter $\mu$ of the GEV distribution which models the annual maxima. However, the 95% confidence interval for the difference in the trends is (−0.0043, 0.0157); since this includes zero, there is not enough evidence to reject a null hypothesis of a common trend for the annual maxima and the mean temperatures. Hence, understanding trends in mean temperature may provide some guidance for extreme temperatures, and further study may be warranted to see if extreme temperatures are increasing more rapidly.

The intent of the simple analysis above is only to illustrate how one might approach modeling central England temperature extremes from an EVA perspective.

Unlike Wigley's analysis, there is no attempt in this analysis to tie future predictions to greenhouse gas concentrations or to climate model scenarios; such a model could be similarly constructed. Climate model experiments generally indicate that future rates of temperature increase will be greater than those observed thus far, and the simple linear trend in time modeled here cannot capture such behavior. Rather than giving conclusive answers, this preliminary analysis raises several interesting questions, such as the question of whether annual maximum temperatures are increasing more rapidly than the summer mean temperatures. Part of the discrepancy between the historical return level and those associated with the regression model can be attributed to the difference in the estimated shape parameters $\hat{\xi}$, and another interesting question is whether there is evidence that this parameter is changing in time. There is currently great interest from many disciplines in improving EVA methodology, and the field is changing rapidly. As new EVA techniques are developed, and as atmospheric science continues to improve its ability to model future climate, our picture of future weather extremes and their potential impact will doubtlessly become more clear.

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