

# Thermal Explosion of a Gas Mixture in a Porous Hollow Cylinder

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**A nonstationary model of gas combustion in a porous cylindrical heat generator with allowance for heat losses inside the body and from the outer surface is proposed and numerically analyzed. The flow pattern is demonstrated to depend on the ratio of the outer and inner radii of the working region. Critical conditions separating the stationary operation mode and the thermal explosion are found.**

**Key words:** porous burner, critical conditions, thermal explosion, numerical solution.

## INTRODUCTION

To avoid emergency situations, it is necessary to determine the range of admissible process-dependent parameters during development and optimization of radiating burners [1]. One problem encountered by burner developers is associated with determining conditions leading to the thermal explosion, which is extremely undesirable for burner operation.

In the classical theory of the thermal explosion, it is necessary to determine the critical value of the Frank-Kamenetskii parameter separating the regime of slow proceeding of the reaction or its termination from the regime of a fast (explosive) increase in temperature due to heat release in the reaction [2]. Under filtration conditions, two types of models are possible with conversion modes depending on the filtration rate. The first group consists of models where conversion proceeds in the solid skeleton with participation of a gaseous oxidizer [3–6]. It was demonstrated within the framework of a one-temperature model [4], in particular, that the critical value of the Frank-Kamenetskii parameter separates the stationary regime and one of three nonstationary regimes (pulsating regime, frontal regime, and thermal explosion). Which of these regimes occurs depends on other parameters of the model (Zel'dovich and Peclet numbers). Burkina and Prokof'ev [4] also provided a detailed review of publications dealing with the analysis of

the thermal explosion of porous samples. Eremin and Kolesnikov [5] found that the thermal explosion limit at high Peclet numbers increases almost linearly with increasing Peclet number. Burkina and Rogacheva [6] found that allowance for the difference in skeleton and gas temperatures does not affect the critical conditions of ignition, but changes the time characteristics of the process: the ignition delay increases with increasing volume heat capacity of the gas phase.

The second group consists of models where the filtrating gas (or liquid) acts as a reagent [7–9]. The main parameter of the filtration flow, i.e., the filtration rate and the associated character of heat transfer, is absolutely ignored in [7–9]. The presence of a porous medium is taken into account as additional heat losses with a given value of the internal heat transfer coefficient.

Conditions of the emergence of the thermal explosion during gas burning in a porous hollow cylinder, which is the working section of the burner [1], is studied in the present work.

## FORMULATION OF THE PROBLEM

Under the test conditions [1], a combustible gas enters the inner area of the cylinder with specified inner ( $R_1$ ) and outer ( $R_2$ ) radii and porosity ( $m$ ). The gas is redistributed by special devices so that the velocity  $v_g$  of its incoming into the working body is identical over the entire cylinder length. The heat propagates over

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the porous layer owing to heat conduction through the solid skeleton and the gas, as well as owing to convective heat transfer by the moving gas. Because of the high thermal conductivity of the solid skeleton, the gas temperature at the entrance to the working body differs from the cold gas temperature [10], which can be taken into account by assuming that there occurs heat transfer following Newton's law between the incoming gas and the inner wall. We assume that chemical transformations in the gas correspond to the global reaction scheme "reagent–reaction product" and that the interphase heat transfer (between the gas in the pores and the porous skeleton) is perfect. Under the process conditions, gas burning proceeds in a certain stationary (quasi-stationary) regime, which can be characterized by the reaction front position and by stationary distributions of temperature and other quantities (see, e.g., [11]). The following question arises: Is such a stationary regime stable? Various situations are possible because of fluctuations of gas velocity, pressure drop, etc.: transition to a nonstationary burning mode, establishment of a new stationary state, and thermal explosion. The latter is extremely undesirable because it can result in an emergency situation. Further we study the possibility of such a situation within the framework of the problem of the thermal explosion of the gas in a hollow cylindrical body where a certain stationary regime is established. Distributions of temperatures and concentrations for the stationary regime are used as the initial conditions for the problem of the thermal explosion. In the simplest one-temperature approximation, the mathematical formulation of the problem of the thermal explosion includes the following equations: the one-dimensional equation of heat conduction (the gas temperature is equal to the skeleton temperature) with chemical heat release

$$\begin{aligned} c_{\text{eff}} \frac{\partial T}{\partial t} + c_g \rho_g v_g m \frac{\partial T}{\partial r} \\ = \frac{1}{r} \frac{\partial}{\partial r} \left( \lambda_{\text{eff}} r \frac{\partial T}{\partial r} \right) + m Q_0 \varphi(\eta, T), \end{aligned} \quad (1)$$

the equation of diffusion in the gas phase with a convective term and with a sink of mass because of the chemical reaction

$$\rho_g \left( \frac{\partial \eta}{\partial t} + v_g \frac{\partial \eta}{\partial r} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( D_g \rho_g r \frac{\partial \eta}{\partial r} \right) - \varphi(\eta, T), \quad (2)$$

the equation of continuity

$$\frac{\partial \rho_g}{\partial t} + v_g \frac{\partial \rho_g}{\partial r} + \frac{\rho_g}{r} \frac{\partial}{\partial r} (r v_g) = 0, \quad (3)$$

the equation of state

$$p = \frac{\rho_g R T}{m_r \eta + (1 - \eta) m_p}, \quad (4)$$

and the equation of motion for the creep flow

$$\rho_g \frac{\partial v_g}{\partial t} = - \frac{\partial p}{\partial r} - \frac{\mu s}{k} v_g. \quad (5)$$

In these equations,  $\varphi(\eta, T) = \rho_g k_0 \eta^n \exp(-E_a/RT)$  is the function of chemical heat release,  $T$  is the temperature,  $\eta$  is the fraction of the reagent,  $t$  is the time,  $r$  is the spatial coordinate,  $p$  is the gas pressure,  $\lambda_{\text{eff}}$  is the effective thermal conductivity,  $\lambda_{\text{eff}} = \lambda_g m + \lambda_s (1 - m)$ ,  $m$  is the porosity,  $\lambda_g$  and  $\lambda_s$  are the thermal conductivities of the gas and the solid body,  $c_{\text{eff}} = c_g \rho_g m + c_s \rho_s (1 - m)$  is the effective volume heat capacity,  $c_g$  and  $c_s$  are the specific heat of the gas at constant volume and the heat capacity of the solid skeleton,  $\rho_g$  and  $\rho_s$  are the densities of the gas and solid skeleton,  $Q_0$  is the thermal effect of the global reaction in the gas phase,  $k_0$  [ $\text{sec}^{-1}$ ] is the chemical reaction rate constant,  $n$  is the reaction order,  $E_a$  is the activation energy,  $R$  is the universal gas constant,  $D_g$  is the effective coefficient of diffusion in the gas phase ( $D_g \rho_g \approx \text{const}$ ),  $s$  is the free area,  $\mu$  is the dynamic viscosity, and  $k$  is the permeability of the porous material.

The initial distributions of concentrations and temperatures correspond to the stationary operation mode and are found in a special manner (which will be described below):

$$\begin{aligned} t = 0: \quad T(r, 0) = T_i(r), \quad \eta = \eta_i(r), \\ u = u_i(r), \quad \rho = \rho_i(r) \quad (R_1 < r \leq R_2). \end{aligned} \quad (6)$$

The boundary conditions on the inner surface of the porous body are written in the form

$$\begin{aligned} r = R_1: \quad - \lambda_{\text{eff}} \frac{\partial T}{\partial r} = c_g \rho_g v_g (T - T_0), \\ \eta = 1, \quad p = p_0. \end{aligned} \quad (7)$$

The heat flux at the exit from the porous body depends on the character of heat exchange with the heat exchanger and with the gas phase surrounding the burner:

$$\begin{aligned} r = R_2: \quad - \lambda_{\text{eff}} \frac{\partial T}{\partial r} \alpha_{\text{eff}} (T - T_b) \\ + (1 - m) \varepsilon_0 \sigma (T^4 - T_h^4) = q_{\text{conv}} + q_{\text{rad}}. \end{aligned} \quad (8)$$

The remaining conditions are

$$\eta = \eta_b, \quad p = p_e. \quad (9)$$

Here,  $p_e$  is the pressure at the entrance,  $T_0$  is the initial temperature,  $\alpha_{\text{eff}}$  is the effective coefficient of external heat transfer between the effective surface of the burner and the gas and combustion products,  $\eta_b$  is the fraction of the reagent that was not consumed,  $\varepsilon_0$  is the emissivity,  $\sigma$  is the Stefan–Boltzmann constant,  $T_b$  is the temperature in the ambient gas phase around the burner (in the general case, different from the adiabatic burning temperature),  $T_h$  is the temperature of the heat

exchanger,  $q_{\text{conv}} = \alpha_{\text{eff}}(T - T_b)$  is the density of the convective heat flux, and  $q_{\text{rad}} = (1 - m)\varepsilon_0\sigma(T^4 - T_h^4)$  is the density of the thermal radiation flux from the skeleton.

For an incompressible gas, the continuity equation takes the form

$$\frac{\partial \rho_g}{\partial t} + v_g \frac{\partial \rho_g}{\partial r} = 0,$$

and the equation of state is not used.

We have to determine the critical conditions separating the thermal explosion and the regime of slow (stationary) proceeding of the reaction.

For convenience of the parametric study of the model, we pass to dimensionless variables typical for thermal explosion problems

$$\begin{aligned} \theta &= \frac{T - T_*}{RT_*^2} E_a, & \xi &= \frac{r}{R_2}, & \tau &= \frac{t}{t_*}, \\ \pi &= \frac{p}{p_*}, & u &= \frac{v_g}{v_{g*}}, & \omega &= \frac{\rho_g}{\rho_*}, \end{aligned} \quad (10)$$

where

$$\begin{aligned} T_* &= T_0 + \frac{Q}{c_g \rho_g (1 - \eta_b)}, & t_* &= \frac{c_g R T_*^2}{E_a Q k_0} \exp \frac{E_a}{RT_*}, \\ v_{g*} &= \frac{R_2}{t_*}, & \pi_* &= \frac{\lambda_{\text{eff}}}{c_g t_*}, & \rho_* &= \frac{p_*}{v_{g*}^2}. \end{aligned} \quad (11)$$

Then, problem (1)–(8) takes the form

$$\begin{aligned} &(\omega m + K_c(1 - m)) \frac{\partial \theta}{\partial \tau} + m \omega u \frac{\partial \theta}{\partial \xi} \\ &= \delta^{-1} \frac{1}{\xi} \frac{\partial}{\partial \xi} \left( \xi \frac{\partial \theta}{\partial \xi} \right) + m \pi^v \omega \eta^n \exp \frac{\theta}{1 + \beta \theta}, \end{aligned} \quad (12)$$

$$\frac{\partial \eta}{\partial \tau} + u \frac{\partial \eta}{\partial \xi} = \delta^{-1} \text{Le} \frac{1}{\xi} \frac{\partial}{\partial \xi} \left( \xi \frac{\partial \eta}{\partial \xi} \right) - \gamma \eta^n \exp \frac{\theta}{1 + \beta \theta}, \quad (13)$$

$$\frac{\partial \omega}{\partial \tau} + u \frac{\partial \omega}{\partial \xi} + \frac{\omega}{\xi} \frac{\partial}{\partial \xi} (\xi u) = 0, \quad (14)$$

$$\pi = \frac{\omega(\beta \theta + 1)}{\eta + (1 - \eta)\mu_{\text{pr}}}, \quad (15)$$

$$\omega \frac{\partial u}{\partial \tau} = - \frac{\partial \pi}{\partial \xi} - K_f u, \quad (16)$$

$$\tau = 0: \quad \theta = \theta_i(\xi), \quad \eta = \eta_i(\xi), \quad (17)$$

$$u = u_i(\xi), \quad \omega = \omega_i(\xi),$$

$$\xi = \xi_1: \quad \frac{\partial \theta}{\partial \xi} = u \omega (\theta + \theta_0), \quad \eta = 1, \quad \pi = \pi_0, \quad (18)$$

$$\begin{aligned} \xi = 1: \quad & - \frac{\partial \theta}{\partial \xi} = \text{Nu}(\theta + \theta_b) \\ & + (1 - m) \text{Sk} [(\beta \theta + 1)^4 - \theta_h^4], \end{aligned} \quad (19)$$

$$\eta = \eta_b, \quad \pi = \pi_e,$$

where

$$\xi_1 = \frac{R_1}{R_2}, \quad \delta = \frac{c_g R_2^2 \rho_*}{\lambda_{\text{eff}} t_*}, \quad \gamma = \frac{RT_*^2 c_g \rho_g}{E_a Q},$$

$$\beta = \frac{RT_*}{E_a}, \quad K_c = \frac{c_s \rho_s}{c_g \rho_*}, \quad \text{Le} = \frac{D c_{\text{eff}}}{\lambda_{\text{eff}}},$$

$$K_f = \frac{t_* \mu_s}{\rho_* k}, \quad \mu_{\text{pr}} = \frac{m_p}{m_r}, \quad \text{Nu} = \frac{\alpha_{\text{eff}} R_2}{\lambda_{\text{eff}}},$$

$$\text{Sk} = \frac{\varepsilon_0 \sigma R_2 T_*^2 E_a}{R \lambda_{\text{eff}}}, \quad \theta_0 = \frac{T_* - T_0}{RT_*^2} E_a,$$

$$\theta_b = \frac{T_* - T_b}{RT_*^2} E_a, \quad \theta_h = \frac{T_h}{T_*}.$$

The calculations were performed for porous ceramics made from  $\text{Al}_2\text{O}_3 + \text{Fe} + \text{Cr}$  (with 55%  $\text{Al}_2\text{O}_3$  and 45%  $\text{Fe} + \text{Cr}$ ) and a combustible gas, which was a mixture of methane with air (10% of  $\text{CH}_4$  and 90% of air) [12–14]:  $c_s = 1250 \text{ J}/(\text{kg} \cdot \text{K})$ ,  $\rho_s = 3750 \text{ kg}/\text{m}^3$ ,  $\lambda_s = 8 \text{ W}/(\text{m} \cdot \text{K})$ ,  $c_g = 2600 \text{ J}/(\text{kg} \cdot \text{K})$ ,  $\rho_g = 0.717 \text{ kg}/\text{m}^3$ ,  $E_a = 129 \text{ kJ}/\text{mole}$ ,  $k_0 = 5.6 \cdot 10^{-9} \text{ sec}^{-1}$ ,  $Q_0 = 3.947 \text{ MJ}/\text{kg}$ , and  $Q = Q_0/\rho_g = 5.505 \text{ MJ}/\text{m}^3$ . Using these data and also the values  $R_1 = 0.15 \text{ m}$ ,  $R_2 = 0.18 \text{ m}$ ,  $T_0 = 300 \text{ K}$ ,  $T_h = 400 \text{ K}$ ,  $T_b = 1600 \text{ K}$ ,  $v_g = 0.05\text{--}1 \text{ m}/\text{sec}$ ,  $\varepsilon_0 = 0.2$ , and  $\sigma = 5.67 \cdot 10^{-12} \text{ W}/(\text{cm}^2 \cdot \text{K}^4)$ , we obtained the following ranges of the characteristic quantities and dimensionless parameters:  $t_* = 10^{-2}\text{--}10^{-5} \text{ sec}$ ,  $T_* = 1000\text{--}2000 \text{ K}$ ,  $v_{g*} = 20\text{--}20,000 \text{ m}/\text{sec}$ ,  $\rho_* = 1 \text{ kg}/\text{m}^3$ ,  $\xi_1 \approx 0.8$ ,  $K_f \approx 1\text{--}1000$ ,  $K_c \approx 1800$ ,  $\gamma \approx (0.5\text{--}3) \cdot 10^{-3}$ ,  $\theta_0 \approx 6\text{--}11$ ,  $\theta_h \approx 0.2\text{--}0.4$ ,  $\theta_b \approx -1.2\text{--}12$ ,  $\beta \approx 0.06\text{--}0.12$ ,  $\text{Le} = 0.01\text{--}0.2$ ,  $\mu_{\text{pr}} \approx 0.99$ ,  $\text{Nu} \approx 2\text{--}100$ ,  $\text{Sk} \approx 32\text{--}132$ ,  $\delta \approx 10^3\text{--}10^6$ ,  $\pi_0 = 0.245\text{--}0.254$ , and  $\pi_e = 0.244\text{--}0.0244$ .

## ALGORITHM OF THE NUMERICAL STUDY

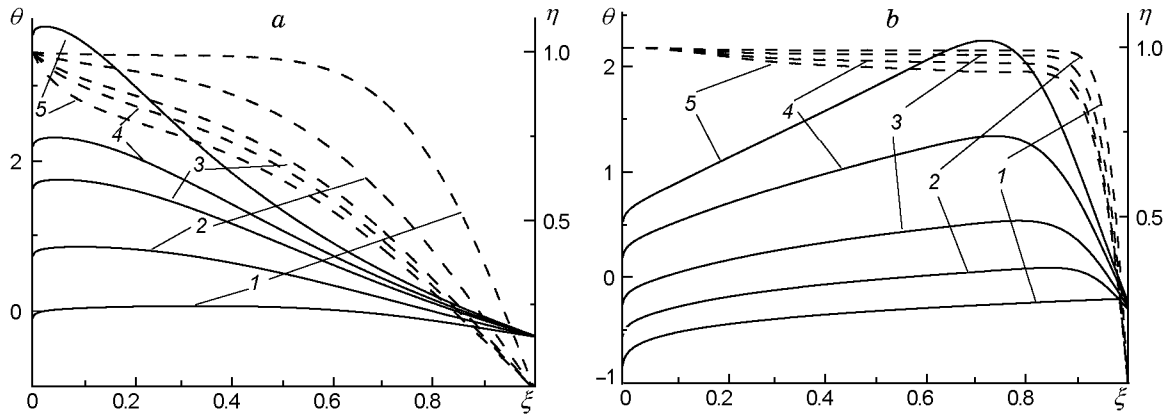
As the initial condition for the thermal explosion problem, we use the solution of the stationary problem without chemical heat release, which follows from Eqs. (12)–(19) under the condition that all derivatives with respect to time are equal to zero and  $u\omega\xi_1 = \text{Pe} = \text{const}$ :

$$\frac{1}{\xi} \frac{\partial}{\partial \xi} \xi \frac{\partial \theta}{\partial \xi} - \frac{B}{\xi} \frac{\partial \theta}{\partial \xi} = 0, \quad (20)$$

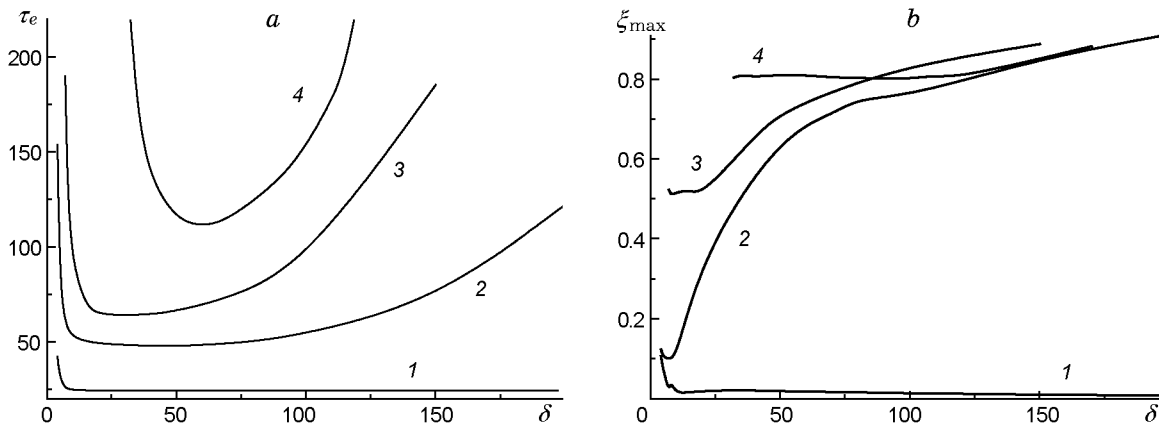
$$\xi = \xi_1: \quad \frac{\partial \theta}{\partial \xi} = \text{Pe}(\theta + \theta_0), \quad (21)$$

$$\xi = 1: \quad - \frac{\partial \theta}{\partial \xi} = \text{Nu}(\theta + \theta_b) \quad (22)$$

( $B = m\text{Pe}\delta$ ).



**Fig. 1.** Distributions of temperatures (solid curves) and concentrations (dashed curves) over the radius of the porous body for  $\xi_1 = 0.001$ ,  $Nu = 100$ ,  $\pi_0 = 2.45$  and different values of  $\delta$  and  $\tau$ : (a)  $\delta = 5$  and  $\tau = 15$  (1), 30 (2), 45 (3), 50 (4), and 55 (5); (b)  $\delta = 100$  and  $\tau = 0.02$  (1), 15 (2), 25 (3), 35 (4), and 40 (5).



**Fig. 2.** Ignition time (a) and coordinate of the maximum temperature (b) versus the Frank-Kamenetskii parameter for  $Nu = 100$ ,  $\pi_0 = 2.45$ , and  $\xi_1 = 0$  (1), 0.1 (2), 0.5 (3), and 0.8 (4).

The solution of problem (20), (21) has the form

$$\theta(\xi) = [(\xi^B - 1)Nu - B] \times \frac{Pe(\theta_0 - \theta_b)}{B(Nu\xi_1^{B-1} + 1) + PeNu(1 - \xi_1^B)} - \theta_b. \quad (23)$$

Knowing the temperature distribution, we find the initial density distribution:

$$\omega(\xi, 0) = \frac{\pi_0}{\beta\theta_0 + 1}. \quad (24)$$

After that, we solve the problem numerically. The differential equations involved into system (12)–(19) are approximated by difference equations; the resultant system of linear algebraic equations is solved by sweep methods. In our calculations, we determine the temperature and concentration distributions in the body

at different time instants and also the distributions of density, velocity, and pressure.

For a given pressure at the entrance  $\pi_e$  and a given flow rate  $Pe$ , the following sequence of calculations is used: the gas density is calculated by Eq. (24), then the gas velocity at the point  $u(0, x) = Pe/(w(0, \tau)\xi_1)$  and the initial temperature profile (23) are found, and the subsequent algorithm is similar to the previous one.

In the limit  $\xi_1 \rightarrow 0$ ,  $\omega = \text{const}$ ,  $Sk = 0$ ,  $m\omega\eta^n = 1$ , and  $u = 0$ , we come to the classical problem of the thermal explosion [2] for a gas in a cylinder. The solution of this problem with ignored burnout of the reagent can be presented as

$$\delta_{*0} = \frac{\delta_0}{1 + e/Nu},$$

where  $e = 2.718281828$  and  $\delta_0 = 2$  is the solution of the problem for  $Nu \rightarrow \infty$ .

The classical model predicts an explosion at  $\delta \geq \delta_{*0}$ , and the temperature in the body drastically increases. At  $\delta < \delta_{*0}$ , the reaction hardly begins and becomes terminated because of large heat losses. Burnout does not lead to any significant changes in the result. In this range of model parameters, our results agree with the known concepts.

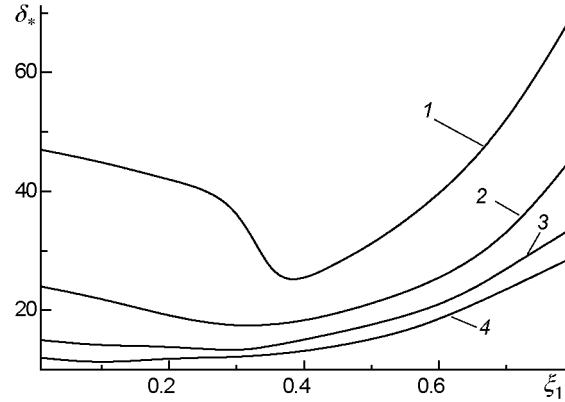
The following parameters were further fixed in the calculations:  $\theta_0 = 10$ ,  $\theta_b = 0.1$ ,  $\theta_h = 0.4$ ,  $Nu = 100$ ,  $\beta = 0.01$ ,  $\gamma = 0.001$ ,  $K_f = 8$ ,  $K_c = 100$ ,  $\mu_{pr} = 0.99$ ,  $Le = 0.01$ ,  $\eta_b = 0$ ,  $Sk = 40$ , and  $\pi_e = 2.44$ . The remaining parameters ( $\delta$ ,  $\pi_0$ ,  $\xi_1$ ,  $Nu$ , and  $Pe$ ) were varied.

## ANALYSIS OF RESULTS

The model proposed here differs from the classical model by the presence of heat losses both inside the body (because of continuous supply of the cold gas) and from the outer surface of the burner (through radiation). The convective term in Eq. (19) ensures heat exchange with combustion products leaving the working body. A specific feature of the problem described is the fact that either a transition to a stationary regime or termination of the reaction occurs at  $\delta < \delta_{*0}$ . If  $\delta \geq \delta_{*0}$ , the stationary regime is impossible. Because of the drastic increase in temperature typical for the thermal explosion region, this scenario cannot be considered as an acceptable operating mode.

The calculations show that the explosion in our model with  $\xi_1 \neq 0$  is potentially possible at all values of parameters: at the Frank-Kamenetskii parameter  $\delta \geq \delta_{*0}$ , the temperature in the body increases (Fig. 1), whereas lower values of  $\delta$  do not refer to the situation considered [7]. The instant of ignition in the calculations was assumed to be the time after which the maximum temperature in the body was greater than a certain critical value  $\theta_*$ . Variations of  $\theta_*$  from 20 to 200 did not change the estimated ignition time  $\tau_e$ .

Because of the strong dependence of the ignition (explosion) time on the model parameters at  $\xi_1 \neq 0$ , it is necessary to refine the concept of the critical ignition conditions. The calculations show that the dependence  $\tau_e(\delta)$  at  $\xi_1 \neq 0$  has a minimum; the depth and location of  $\delta_{min}$  depend on other parameters. This situation is illustrated in Fig. 2a. As the time  $\tau_e$  drastically increases as  $\delta$  moves away from  $\delta_{min}$ , it seems logical to assume that the ignition conditions are not formed in the situation considered, with  $\xi_1 \neq 0$  and  $\delta \gg \delta_{min}$  or  $\delta \ll \delta_{min}$ . Therefore, we choose the minimum value  $\delta_{min}$  of the dependence  $\tau_e(\delta)$  as the critical condition.

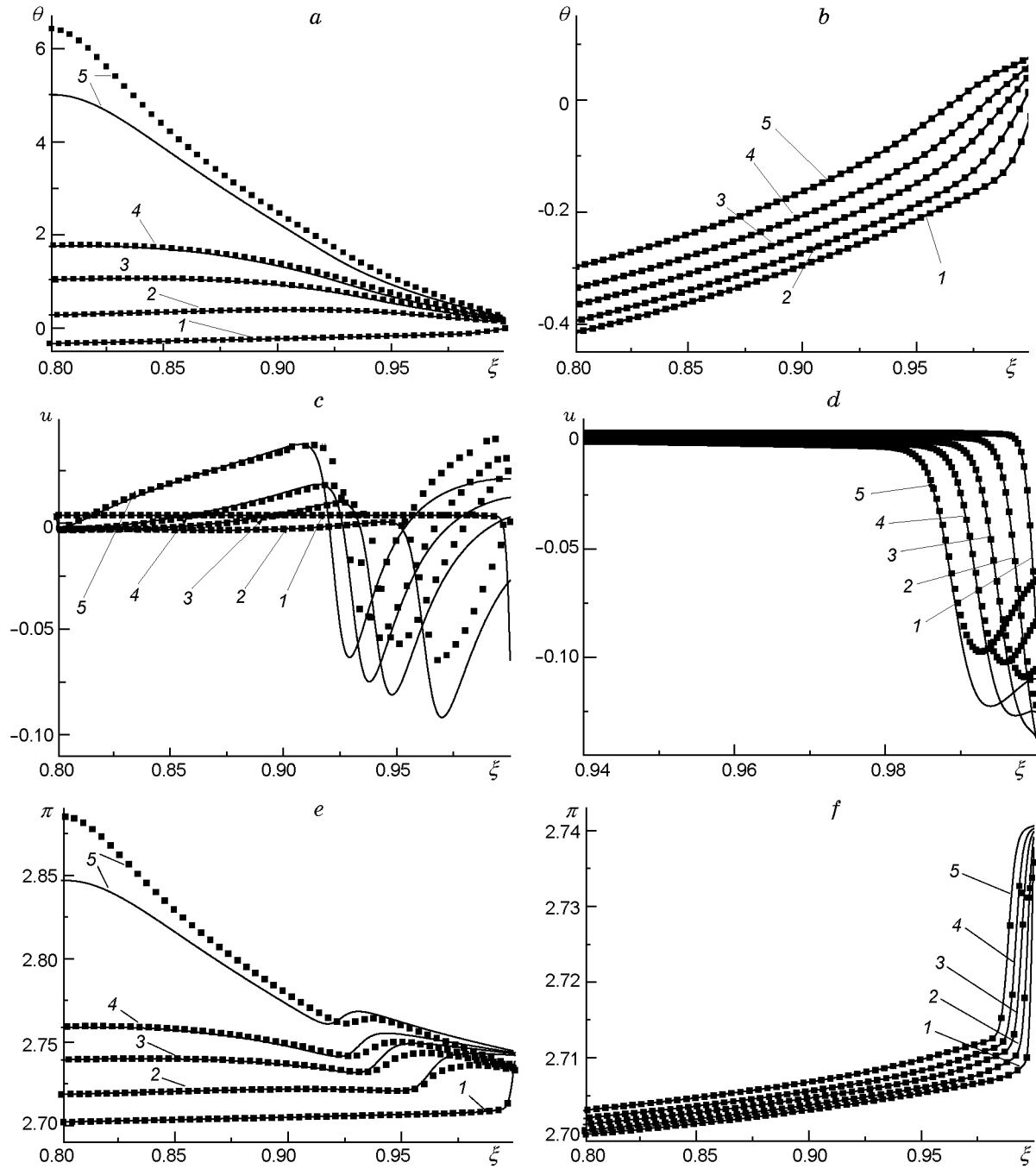


**Fig. 3.** Critical value of the Frank-Kamenetskii parameter versus the inner radius of the burner  $\xi_1$  at  $Nu = 100$  and  $\pi_0 = 2.45$  (1), 2.46 (2), 2.47 (3), and 2.48 (4).

If  $\delta < \delta_{min}$  and  $\xi_1 \ll 1$ , but  $\xi_1 \neq 0$ , then the temperature maximum prior to ignition is shifted toward the inner surface of the cylinder (see Fig. 1a). The chemical reaction zone in this case is the entire area of the porous body. If  $\delta > \delta_{min}$ , the temperature maximum is located closer to the outer surface, and the chemical reaction zone is narrow (see Fig. 1b). This difference is caused by the fact that the main heat losses proceed through radiation in the first case and are associated with heat exchange between the porous body and the cold gas on the inner surface in the second case.

The dependence of the temperature maximum coordinate on the Frank-Kamenetskii parameter is shown in Fig. 2b. The narrower the area occupied by the porous body and the greater the value of  $\delta$ , the closer  $\xi_{max}$  to the outer surface of the cylinder. In addition,  $\tau_e \rightarrow \infty$ . If  $\xi_1 = 0.8$ , then the range of the maximum heat release and the temperature maximum coordinate are close to the outer surface at  $\delta > 120$ . The greater the value of  $\xi_1$ , i.e., the narrower the working area, as compared with the outer radius of the burner, the smaller the range of the Frank-Kamenetskii parameter where the time  $\tau_e$  is finite.

The phenomenon observed is of significant interest for development of burners with a porous working body. Thus, the values typical for the burner described in [11] are  $\xi_1 \approx 0.83$  and  $\delta \geq 10^3$ , which allows us to conclude that the thermal explosion does not occur under real operating conditions of the porous burner shaped as a hollow cylinder, and the evolution of the disturbance inducing the temperature distribution in the burner with a maximum in the volume is always finalized by the transition to the profile with the maximum close to the outer surface, which is predicted by the model developed in [15].



**Fig. 4.** Distributions of temperature (a and b), velocity (c and d), and pressure (e and f) of the gas over the radius of the cylindrical porous body at different times for  $\beta = 0.01$ ,  $\gamma = 0.001$ ,  $\xi_1 = 0.8$ ,  $Nu = 100$ ,  $\pi_0 = 2.45$ ,  $K_f = 8$ ,  $Le = 0.01$ , and  $\delta = 100$  (a, c, and e), where  $\tau = 0.01$  (1), 0.04 (2), 0.08 (3), 0.12 (4), and 0.17 (5), and  $\delta = 50$  (b, d, and f), where  $\tau = 0.01$  (1), 0.6 (2), 1.0 (3), 1.2 (4), and 1.4 (5); the solid curves and points refer to the incompressible and compressible gas, respectively.

It is obvious that  $\delta_* = \delta_{\min} = \delta_*(\theta_0, \xi_1, Le, \Delta\pi, \dots)$ . The most interesting issue for us is the influence on the critical conditions of geometric parameters (via  $\xi_1$ ) and process-dependent parameters, for instance, the input pressure whose value affects the gas flow character and the heat transfer at the entrance to the porous body.

It turned out that the dependence  $\delta_*(\xi_1)$  has a minimum whose depth and location depend on the input pressure  $\pi_0$ , other parameters being fixed (Fig. 3). Figure 3 should be interpreted as follows: if  $\delta \gg \delta_{\min}$  and  $\delta \ll \delta_{\min}$ , then the burner operation is stable. For the chosen values of  $\xi_1$  and  $\pi_0$ , more stable operation of the burner should be expected for values of  $\delta$  outside the area bounded by the curve  $\tau_e(\delta)$  (see Fig. 2a). The greater the value of  $\Delta\pi$ , the narrower the range of parameters where the thermal explosion can be expected. For the burner operation conditions [1], we have  $\Delta\pi = 0.01\text{--}0.1$ ,  $\xi_1 \approx 0.83$ , and  $\delta > 10^3$ ; therefore, the thermal explosion is next to impossible.

As  $\pi_0$  increases, stable operation can be violated by other phenomena, for instance, incomplete burning of the reagent or expansion of the reaction zone outside the limits of the porous body, which is also of interest for the technology, but is outside the scope of the model formulated here.

Note that the local increase in temperature in the thermal explosion mode is accompanied by high gradients of temperature, which induce thermal stresses above the ultimate strength of ceramics, as was predicted in [15]. The temperature  $\theta_{\max}(\xi, \tau \rightarrow \tau_e) > 20$  in dimensional variables is appreciably higher than the melting point ( $1950^\circ\text{C}$ ) of ceramics made on the basis of the aluminum oxide ( $\text{Al}_2\text{O}_3$ ) and, moreover, than the temperature of  $1250^\circ\text{C}$  that ceramics based on intermetallic compounds ( $\text{NiAl}_{20}$ ) can withstand [1]. In accordance with [13], the temperature of the explosion products of the methane–air mixture in a closed volume is  $2420\text{--}2920\text{ K}$ . Therefore, none of ceramic materials can withstand such a thermal impact: the burner will be completely destroyed, and it makes no sense talking about conversion modes after the thermal explosion.

It turned out that gas compressibility does not exert any significant effect in this problem. This fact is illustrated, e.g., in Fig. 4, which shows the distributions of temperature, velocity, and pressure of the gas in a porous burner, which are typical for the thermal explosion, in the range of the parameters  $\delta$  and  $\xi_1$  located in Fig. 2a to the right and to the left from the minimum. Though the numerical values of velocity and pressure (and also the density and degree of conversion) are different in the compressible and incompressible gas approximations, compressibility exerts only a

minor effect on temperature and critical conditions: a difference in temperature is observed only on the curve corresponding to the beginning of explosive conversion. Nevertheless, compressibility may turn out to be important in the process of stabilization of the stationary mode of burner operation after ignition, which was not studied in the present work. Another possibility that cannot be excluded is oscillatory combustion, which is also outside the scope of the present work.

## CONCLUSIONS

Thus, conditions of the emergence of the thermal explosion in a cylindrical porous burner with gas filtration are analyzed in the paper. It is demonstrated that the thermal explosion is almost impossible in the case of continuous supply of the fresh combustible mixture into the porous working body. The influence of the process-dependent parameters on the critical conditions separating the thermal explosion and stable operation of the burner is studied. Nevertheless, the question about the possibility of the thermal explosion at the stage of gas ignition and in the course of decreasing the velocity of its supply is still open.

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## REFERENCES

1. A. I. Kirdyashkin and Yu. M. Maksimov, “Infrared burner on the basis of porous ceramics,” in: *Energy Saving and Energy Efficiency*, Proc. VIII Int. Exhibition Congress, Tomsk (2005), pp. 24–25.
2. D. A. Frank-Kamenetskii, *Diffusion and Heat Transfer in Chemical Kinetics*, Plenum, New York (1969).
3. I. G. Dik, “Stationary regimes of nonisothermal chemical reactions in a porous layer,” *Combust., Expl., Shock Waves*, **29**, No. 6, 724–727 (1993).
4. R. S. Burkina and V. G. Prokof’ev, “Critical conditions of thermal explosion of a porous layer,” *Combust., Expl., Shock Waves*, **44**, No. 3, 291–299 (2008).
5. E. A. Eremin and A. K. Kolesnikov, “Steady-state theory of thermal explosions,” *Combust., Expl., Shock Waves*, **14**, No. 5, 658–659 (1978).
6. R. S. Burkina and E. G. Rogacheva, “Characteristics of thermal explosion in a porous layer with diffusion of a gaseous reactant,” *Combust., Expl., Shock Waves*, **32**, No. 2, 204–210 (1996).
7. V. I. Babushok, V. M. Gol’dshstein, A. S. Romanov, and V. S. Babkin, “Thermal explosion in an inert porous medium,” *Combust., Expl., Shock Waves*, **28**, No. 4, 319–325 (1992).

8. E. A. Shchepakina, "Critical conditions of self-ignition in a porous medium," *Khim. Fiz.*, **20**, No. 7, 3–9 (2001).
9. E. A. Shchepakina, "Conditions of safe ignition of combustible fluids in a porous insulating material," **5**, No. 3, 162–169 (2002).
10. V. M. Polyayev, V. A. Maiorov, and L. L. Vasil'ev, *Hydrodynamics and Heat Transfer in Porous Elements of Flying Vehicles* [in Russian], Mashinostroenie, Moscow (1988).
11. Yu. A. Chumakov and A. G. Knyazeva, "Regimes of gas combustion in a porous body of a cylindrical heat generator," *Combust., Expl., Shock Waves*, **45**, No. 1, 14–24 (2009).
12. I. S. Grigor'eva and E. Z. Meilikhova (eds.), *Physical Quantities: Handbook* [in Russian], Énergoatomizdat, Moscow (1991).
13. A. S. Isserlin (ed.), *Fundamentals of Burning of Gas Fuel: Handbook* [in Russian], Nedra, Leningrad (1987).
14. G. I. Ksandopulo, *Flame Chemistry* [in Russian], Khimiya, Moscow (1980).
15. A. G. Knyazeva and Yu. A. Chumakov, "Distributions of temperature and thermal stresses in the porous body of a cylindrical radiating burner operating in a stationary mode," *Izv. Tomsk. Politekh. Univ.*, **312**, No. 4, 28–36 (2008).