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Mathematical Model of Continuous Detonation in an Annular Combustor with a Supersonic Flow Velocity

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A two-dimensional unsteady mathematical model of a continuous spinning detonation wave in a supersonic incoming flow in an annular combustor is formulated. The wave dynamics in a combustor filled by a gaseous hydrogen–oxygen mixture is studied. The possibility of continuous spin detonation with a supersonic flow velocity at the diffuser entrance is demonstrated numerically for the first time; the structure of transverse detonation waves and the range of their existence depending on the Mach number are studied.

Key words: continuous detonation, flow-type combustor, transverse detonation waves, flow structure, mathematical modeling.

INTRODUCTION

Fuel burning in continuous spinning detonation waves is now considered as a possible alternative to conventional burning of fuels in turbulent flame. This method was first proposed in [1]. The method allows more intense and stable burning of various fuels in combustors of smaller sizes determined by the characteristic size of the detonation-wave front [2, 3]. Continuous detonation regimes in flow-type combustors with the oxidizer supplied as a continuous subsonic flow through an annular slot were obtained in experiments with acetylene–oxygen, acetylene–air, and hydrogen–air mixtures studied in [4–7]. Thus, it became possible to extend the range of application of the principle of continuous spin detonation to flow-type combustors. The regime of continuous detonation burning of the combustible mixture for combustors typically used in liquid-propellant rocket engines were studied theoretically in [8–10]. Up to now, however, the possibility of providing the regime of continuous spin detonation in annular combustors with supersonic velocities of the incoming flow was an open question.

The goal of the present activities was to perform a numerical study of combustion of a hydrogen–oxygen mixture in an annular combustor in the regime of a continuous spinning detonation wave with a supersonic velocity of the incoming flow.

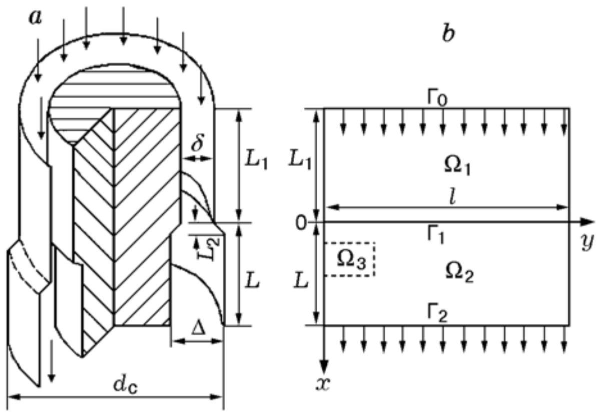

Fig. 1. Sketch of an annular combustor (a) and domain of the numerical solution of the problem (b).

MATHEMATICAL FORMULATION OF THE PROBLEM

Let a supersonic flow (with the initial Mach number $M_0 > 1$, initial pressure p_0, and initial temperature T_0) pass through an annular cylindrical diffuser of length L_1, which has a cylindrical channel of width δ,

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and enter a flow-type combustor with annular cylindrical geometry [Fig. 1a; combustor diameter d_c, combustor length L, and annular channel width Δ ($\Delta > \delta$)], which has a linear expansion (of length L_2) of the annular channel from δ to Δ. Let us consider a two-dimensional approximation of the problem of detonation combustion of a hydrogen–oxygen mixture inside the combustor with the boundaries Γ_1 (combustor entrance) and Γ_2 (open end of the combustor with exhaustion of combustion products). As $\delta < \Delta \ll d_c$, we can cut the annular domain and unfold it into a rectangular domain $\Omega = \Omega_1 \cup \Omega_2$, which is shown in Fig. 1b. Here $\Omega_1 = (-L_1 < x < 0; 0 < y < l = \pi d_c)$, $\Omega_2 = (0 < x < L; 0 < y < l)$, the boundary $\Gamma_0 = (x = -L_1; 0 < y < l)$ is the entrance to the annular diffuser, and x and y are the spatial variables of an orthogonal coordinate system.

Let an energy sufficient for initiating detonation of the mixture in the combustor be instantaneously released in the subdomain Ω_3 of the solution domain at a certain time after the beginning of the supersonic inflow into the combustor. As a result of initiation, an unsteady detonation wave can propagate in the domain Ω_2. We have to determine the wave dynamics, its structure, and conditions necessary for a self-sustained regime of continuous spin detonation to be reached, depending on the governing parameters of the problem.

The flow of the mixture in the solution domain Ω is described by a system of equations of unsteady gas dynamics with chemical transformations in a quasi-two-dimensional approximation, with the degree of channel expansion $S = S(x)$:

$$\begin{gathered} \rho_t + S^{-1}(\rho u S)_x + (\rho v)_y = 0, \\ (\rho u)_t + S^{-1}(\rho u^2 S)_x + (\rho u v)_y + p_x = 0, \\ (\rho v)_t + S^{-1}(\rho u v S)_x + (\rho v^2)_y + p_y = 0, \\ (\rho E)_t + S^{-1}[\rho u(E + p/\rho)S]_x + [\rho v(E + p/\rho)]_y = 0, \\ Y_t + uY_x + vY_y = f_5, \\ \mu_t + u\mu_x + v\mu_y = f_6. \end{gathered} \tag{1}$$

Here t is the time, ρ is the density, u and v are the components of the velocity vector $\boldsymbol{u}$, p is the pressure, $E = U + (u^2 + v^2)/2$, $U(T, \mu)$ is the total internal energy of the gas, T is the temperature, μ is the current molar weight of the mixture, and Y is the fraction of the induction period of chemical reactions. Closure of system (1) for reacting hydrogen–oxygen mixtures with variable heat release in the reaction zone by Nikolaev's model of chemical kinetics [11] correlated with the second law of thermodynamics was described in detail in [10].

The functional dependence of the cross-sectional area of the channel with a constant expansion angle at the initial part of the combustor was defined as

$$S(x) = \begin{cases} \delta l, & x < 0, \\ [\delta + (\Delta - \delta)x/L_2]\, l, & 0 < x < L_2, \\ \Delta l, & x \geqslant L_2. \end{cases} \tag{2}$$

Boundary Conditions. (A) On the upper boundary Γ_0 ($x = -L_1$; $0 \leqslant y \leqslant l$), we imposed the conditions of an undisturbed supersonic flow ($\mathrm{M}_0 > 1$):

$$\begin{gathered} p(-L_1, y, t) = p_0, \quad u(-L_1, y, t) = \mathrm{M}_0 c_0, \\ v = 0, \ \rho(-L_1, y, t) = \rho_0, \ T(-L_1, y, t) = T_0. \end{gathered} \tag{3}$$

(B) The boundary Γ_1 ($x = 0$; $0 \leqslant y \leqslant l$), which is the transition from the annular slot of the diffuser to the combustor, was subjected to the approximate condition

$$Y = 1, \quad \mu = \mu_0, \tag{4}$$

which ensured an inert gas flow in the diffuser ($-L_1 \leqslant x \leqslant 0$), because the fuel was injected at the combustor entrance in the experiments [2, 3].

(C) At the combustor exit indicated by the boundary Γ_2 ($x = L$; $0 \leqslant y \leqslant l$), the transition through the velocity of sound occurs in the case of exhaustion of the jet of combustion products into a medium with a rather low counterpressure; hence, the axial component of the velocity vector u along the entire boundary Γ_2 is equal to or greater than the local velocity of sound c. Therefore, the condition of free exhaustion of combustion products can be presented as

$$u(L, y, t) \geqslant c(L, y, t). \tag{5}$$

(D) The condition of a periodic solution was imposed on the left and right boundaries of the domain Ω. By virtue of flow periodicity (with a period l) with respect to the coordinate x, any gas-dynamic function $F(x, y, t)$ satisfies the condition

$$F(x, 0, t) = F(x, l, t), \quad -L_1 \leqslant x \leqslant L. \tag{6}$$

Initial Conditions. The initial data (at $t = 0$) in the solution domain $\Omega_1 = (-L_1 < x < 0; 0 < y < l)$ were constant parameters of a supersonic flow:

$$\begin{gathered} p(x, y, 0) = p_0, \quad u(x, y, 0) = \mathrm{M}_0 c_0, \\ v = 0, \quad \rho(x, y, 0) = \rho_0, \\ Y = 1, \quad \mu = \mu_0; \end{gathered}$$

the initial data in the domain $\Omega_2 = (0 < x < L; 0 < y < l)$ were constant parameters

$$\begin{gathered} p(x, y, 0) = p_0, \quad u(x, y, 0) = 0, \\ v = 0, \quad \rho(x, y, 0) = \rho_0, \quad Y = 1, \mu = \mu_0. \end{gathered}$$

Let a certain energy with a volume energy density Q be instantaneously supplied at the time $t = 0$ in the subdomain $\Omega_3 = (0 < x < x_* < L;\ 0 < y < y_* < l)$. Then, the pressure and temperature of gaseous combustion products abruptly increase in this subdomain. (The subdomain Ω_3 simulates the region of detonation initiation.) An unsteady detonation wave (DW) with energy release behind the front propagates over the subdomain $\Omega_2 \cap \Omega_3$ at $t > 0$ owing to discontinuity decay.

Using specified thermodynamic parameters of the gas mixture and normalizing, as in [10], the sought functions, coordinates, and time in the equations and boundary conditions as p/p_0, ρ/ρ_0, T/T_0, μ/μ_0, u/u_0, v/u_0, x/l, y/l, and t/t_0 with respect to the parameters p_0, ρ_0, T_0, μ_0, $u_0 = \sqrt{p_0/\rho_0}$, l, and $t_0 = l/u_0$, we find that the solution of the problem of a continuous spinning detonation wave in the incoming supersonic flow of the specified mixture depends on the following governing parameters: free-stream parameters of the supersonic flow (Mach number M_0, pressure p_0, and temperature T_0), the ratio of the cross-sectional areas of the annular channels of the diffuser and combustor δ/Δ, and four scale parameters: diffuser length L_1, total length of the combustor L, length of the initial segment with channel expansion L_2, and perimeter l. The problem formulated above with the boundary conditions (3)–(6) was solved numerically by the Godunov–Kolgan finite-difference method.

CALCULATION RESULTS

The numerical study was performed for a stoichiometric gaseous hydrogen–oxygen mixture ($2H_2 + O_2$) with the following values of the constants: $T_0 = 300$ K, $p_0 = 1.013 \cdot 10^5$ Pa, $\mu_0 = 12$ kg/kmole, $\rho_0 = p_0 \times \mu_0/(RT_0)$, and $\gamma_0 = 1.4$. Preliminary calculations of the problem of a spinning detonation wave were performed for the geometric parameters of the problem

$$L_1 = 20 \text{ cm}, \quad L_2 = 0.5 \text{ cm}, \quad L = 5.2 \text{ cm}, \quad \delta/\Delta = 0.5, \tag{7}$$

and a supersonic flow Mach number $M_0 = 1.5$, which corresponded to the initial specific flow rate of the mixture through the combustor $g_0 = (\delta/\Delta)\rho_0 c_0 M_0 = 197.2$ kg/(sec $\cdot$ m^2). At the stage of initiation, the no-slip boundary condition ($v = 0$) was imposed in numerical simulations on the lateral boundaries of the integration domain Ω ($-L_1 < x < L$, $y = 0$, $y = l$), similar to [9]. As the initiating DW approached the lateral boundary $y = l$, the no-slip condition was replaced by the condition of a periodic solution (6). Figure 2 shows the typical dynamics of the pressure profiles of the initiating DW along the y axis at a distance $x = 0.5$ cm from the combustor entrance (solid curves) for supercritical energy release ($Q = 6$ MJ/m^3) in the subdomain $\Omega_3 = (1 < x < 2$ cm, $0 < y < 1$ cm).

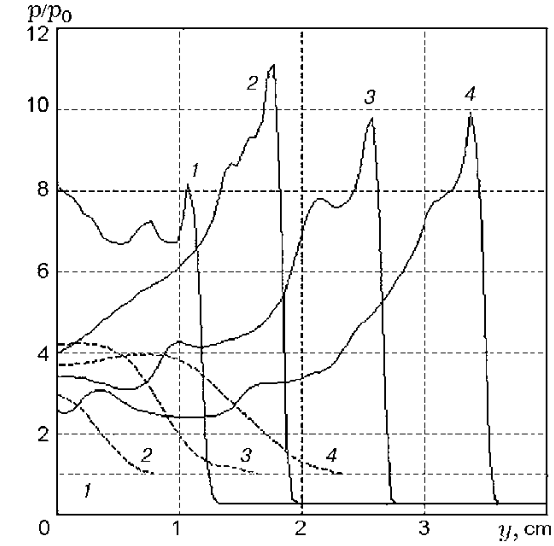

Fig. 2. Pressure profiles in the combustor (solid curves; $x = 0.5$ cm) and diffuser (dashed curves; $x = -0.5$ cm) at different times during the transition of the initiating DW: $t = 2.76$ (1), 5.85 (2), 9.07 (3), and 12.45 μsec (4).

An analysis of the calculated results shows that the initiating DW moving across the gas flow entering the combustor generates an oblique compression wave entering the diffuser and propagating upstream over the supersonic flow. The corresponding pressure profiles along the y axis in the diffuser at a fixed distance of 0.5 cm from the combustor entrance ($x = -0.5$ cm) are plotted by the dashed curves in Fig. 2 for four instances of time t. It is seen that the pressure at the diffuser point with the coordinates ($x = -0.5$ cm, $y = 0$) increases to 3 atm by the time $t = 5.85$ μsec and to 4.2 atm by the time $t = 9.07$ μsec.

Periodic Solution

To find a periodic solution with a transverse detonation wave (TDW), we have one free parameter: the problem period l. Let us set $l = 5$ cm. Figure 3a shows the calculated dependences of the current pressure $P(t) = p(0, 0, t)/p_0$ (solid curve) at the point with the coordinates ($x = 0$, $y = 0$) and the period-averaged pressure at the combustor entrance

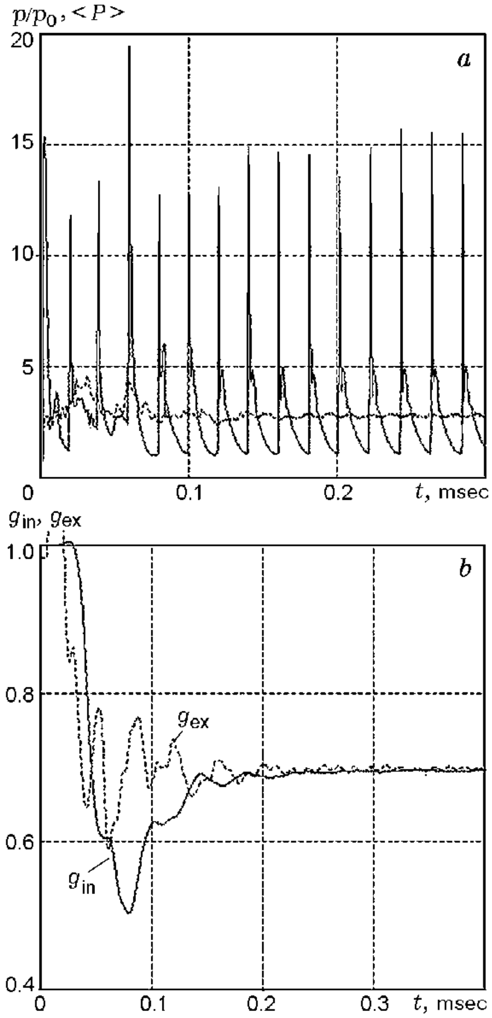

Fig. 3. (a) Current (solid curve) and period-averaged (dashed curve) pressure at the point ($x = 0$, $y = 0$); (b) relative flow rates versus time for $M_0 = 1.5$ and $l = 5$ cm.

$\langle P\rangle(t) = \dfrac{1}{lp_0}\displaystyle\int_0^l p(0, y, t)dy$ (dashed curve) as functions of the time t during the first 0.3 msec after TDW initiation. It is seen that the pressure at a fixed point in space behaves nonmonotonically: it fluctuates with time. The first pressure peak ($P_{\max,1} \approx 12$) corresponds to the moment of the first arrival of the TDW at this point, the second pressure peak ($P_{\max,2} \approx 13.5$) corresponds to the second arrival of the TDW at this point, etc. At the early stage of the process (7–8 fluctuations), the pressure displays irregular fluctuations with different amplitudes; later on, the fluctuations become almost periodic (with a period $\Delta t \approx 20.6$ μsec), the maximum amplitude is $P_{\max} \approx 16$, the minimum amplitude is $P_{\min} \approx 1.1$, and the amplitude ratio is $P_{\max}/P_{\min} \approx 14.5$. This is evidenced by the behavior of the mean pressure $\langle P\rangle(t)$, which reaches an almost constant value $\langle P\rangle(t) \approx 2.85$ at $t > 0.2$ msec. We can find the period-averaged TDW velocity $\langle D\rangle = l/\Delta t = 2.43 \pm 0.02$ km/sec and the ratio $\langle D\rangle/D_0 = 0.85$. Here $D_0 = 2.84$ km/sec is the ideal Chapman–Jouguet detonation velocity in the mixture $2H_2 + O_2$ [12].

To verify that the solution reached a continuous spin detonation regime, we calculated the specific flow rates averaged over the period l at each time instant in two control sections: in the diffuser at a distance of 1 cm ahead of the combustor inlet $\langle G_{\rm in}\rangle$ and at the combustor exit $\langle G_{\rm ex}\rangle$:

$$\langle G_{\rm in}\rangle = \frac{1}{l}\int_0^l \rho(-1, y, t)u(-1, y, t)dy,$$
$$\langle G_{\rm ex}\rangle = \frac{1}{l}\int_0^l \rho(L, y, t)u(L, y, t)dy. \qquad (8)$$

Together with the flow rates, we calculated the specific impulse $\langle J\rangle$ at the combustor exit:

$$\langle J\rangle = \frac{1}{l}\int_0^l \frac{[p(L, y, t) + \rho(L, y, t)u^2(L, y, t) - p_0]dy}{\langle G_{\rm ex}\rangle} - c_0 M_0. \qquad (9)$$

Figure 3b shows the dynamics of the dimensionless flow rates $g_{\rm in} = (\delta/\Delta)\langle G_{\rm in}\rangle/g_0$ (solid curve) and $g_{\rm ex} = \langle G_{\rm ex}\rangle/g_0$ (dashed curve). The behavior of the dimensionless flow rate $g_{\rm in}$ in the diffuser was rather interesting. At the beginning (before $t = 0.03$ msec), as long as a supersonic flow with $M_0 = 1.5$ moved at the diffuser point with the coordinate $x = -1$ cm, the flow rate was $g_{\rm in} = 1$. The TDW propagating over the combustor (see Fig. 2) formed an oblique shock wave (SW) moving upstream over the supersonic flow. As a result, the dimensionless flow rate $g_{\rm in}$ at $t > 0.03$ msec started to decrease monotonically and reached an absolute minimum $g_{\rm in} \approx 0.51$ at $t = 0.08$ msec. Then the flow rate increased and asymptotically reached an almost constant value $g_{\rm in} \approx 0.7$ at $t > 0.25$ msec. The dimensionless flow rate at the combustor exit $g_{\rm ex}$ also asymptotically reached an almost constant value $g_{\rm ex} \approx 0.7$ at $t > 0.25$ msec after a series of fluctuations with decreasing amplitude. The time $t \approx 0.25$ msec for this variant of calculations can be considered as the time when the continuous spin detonation reaches a periodic regime with a specific flow rate $\langle G\rangle = 0.7g_0$. Thus, when a continuous spinning detonation wave is formed, the flow rate of the gas entering the combustor decreases, because an SW with a pressure behind the SW front

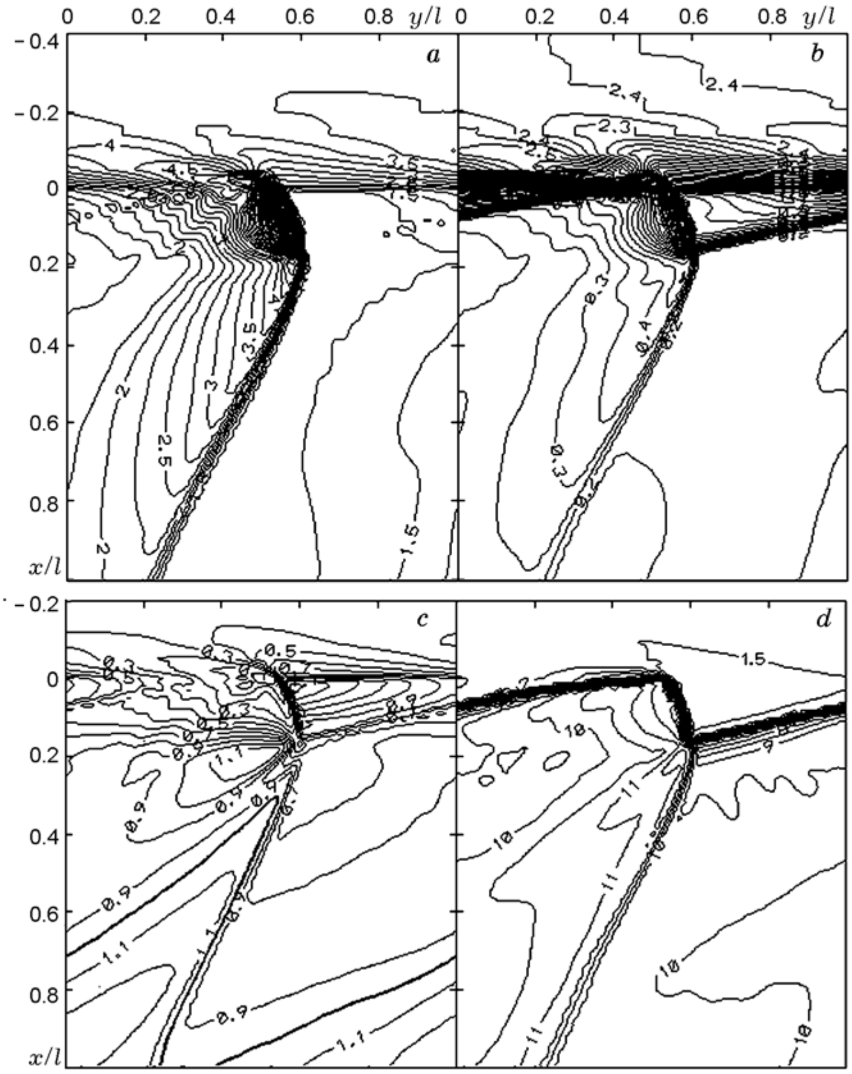

Fig. 4. Calculated two-dimensional structure of a continuous spinning detonation wave in a flow-type cylindrical combustor for $l = 5$ cm, $\mathrm{M}_0 = 1.5$, and $\delta/\Delta = 0.5$: (a) isobars (p/p_0); (b) isochores (ρ/ρ_0); (c) Mach contours ($\mathrm{M}_x = u/c$); (d) isotherms (T/T_0).

$\langle p_{\mathrm{SW}}\rangle/p_0 = 3.7$ propagates upstream over the supersonic flow. Substituting the pressure $\langle p_{\mathrm{SW}}\rangle$ into the known relation on the shock front [13]

$$\mathrm{M}^2 = 1 + 0.5(\gamma_0 + 1)(\langle p_{\mathrm{SW}}\rangle/p_0 - 1)/\gamma_0, \qquad (10)$$

we find the flow Mach number with respect to the SW front: $\mathrm{M} = 1.82 > \mathrm{M}_0$. This opposing SW converts the initial supersonic flow with $\mathrm{M}_0 = 1.5$ in the diffuser into a subsonic flow with a specific flow rate $\langle G\rangle \approx 0.7g_0$. By varying the parameter l in the periodic problem (1)–(6) with fixed geometric values (7) and flow Mach number $\mathrm{M}_0 = 1.5$, we calculated, similar to [9], the "minimum" period $l_{\min}$, which ranges within $3.5 < l_{\min} < 4$ cm.

Structure of the Transverse Detonation Wave

Let us consider the structure of a steady gasdynamic flow in the case of TDW propagation in a flow-type combustor. Figure 4 shows the two-dimensional flow structure for $l = 5$ cm and $L/l = 1.04$ at the time $t = 0.5$ msec. The upper part of the figure ($x < 0$) refers to the flow in the diffuser, and the lower part of the figure ($x > 0$) shows the flow in the combustor channel. The wave moves from left to right with a TDW velocity $D = 2.43$ km/sec over a triangular low-temperature region containing the initial mixture

$2H_2 + O_2$ incoming from the diffuser (the interface between the combustible mixture and combustion products is clearly seen in Fig. 4b). An oblique shock wave moving over the cold gas in the diffuser diverges upward from the TDW, and an oblique shock wave (tail) moving over the hot detonation products in the combustor diverges downward from the TDW. Note that the deviation of the TDW front from the vertical line is more pronounced in the flow-type combustor than in the rocket-type combustor [10]. The height of the layer of the combustible mixture ahead of the TDW front for the values of parameters indicated above is $h = 0.77$ cm. Behind the TDW, the detonation products gradually expand and become displaced downward by new portions of the gases if the pressure in the products is lower than the pressure in the diffuser. Conditions for propagation of a new TDW in the next period are generated. The isobars (Fig. 4a) and isochores (Fig. 4b) show a rapid decrease in pressure and density behind the TDW in the combustor channel. Note that the gas-dynamic parameters ahead of the TDW are not uniform. The calculations show that the oblique shock wave propagating upstream over the flow to the diffuser rapidly decays. The degree of pressure nonuniformity in the diffuser is $(P_{\max} - P_{\min})/\langle P\rangle = 0.11$ already at a distance equal to the TDW size ($x = -h$), and we have $(P_{\max} - P_{\min})/\langle P\rangle = 0.02$ at $x = -2h$. Figure 4c shows the Mach number contours for the projection of the velocity vector onto the x axis ($\mathrm{M}_x = u/c$). It is seen that $\mathrm{M}_x < 1$ in the shown part of the diffuser ($-0.4 < x/l < 0$) and in the triangular region ahead of the TDW front, i.e., the projection of the velocity vector onto the x axis in this flow region is smaller than the velocity of sound. The flow is also subsonic with distance from Γ_1 along the x axis until the dimensionless distance $x/l < 0.3$, except for a local supersonic region in the vicinity of the point of TDW intersection with the oblique shock wave formed behind the TDW front owing to lateral expansion of detonation products. At $x/l > 0.3$, a downstream-expanding supersonic region starts to form behind the oblique SW front; the value of M_x in this region gradually increases and reaches $\mathrm{M}_x = 1.2$ behind the oblique SW. The isoline $\mathrm{M}_x = 1$ is plotted as a bold curve. It is seen from Fig. 4c that the gas-dynamic flow along the x axis on the lower boundary Γ_2 is supersonic on the average. This means that a transonic transition also occurs in a flow-type combustor with a constant-section channel during TDW propagation [8, 10]. Therefore, the sonic disturbances at the combustor exit cannot affect the TDW parameters. The calculated temperature field (see Fig. 4d) show that the gas is cold in the diffuser and in the combustor ahead of the TDW front, while the maximum temperatures (above 3300 K) are observed behind the TDW front and the oblique shock wave (tail).

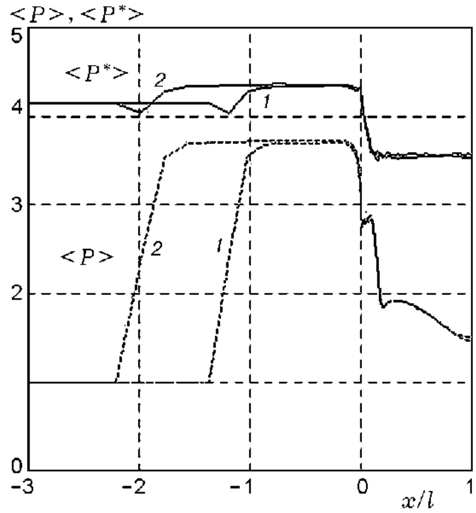

Fig. 5. Static pressure ($\langle P\rangle(x,t)$; dashed curves) and total pressure ($\langle P^*\rangle(x,t)$; solid curves): curves 1 refer to $t_1 = 0.28$ msec and curves 2 refer to $t_2 = 0.5$ msec and $\mathrm{M}_0 = 1.5$.

At each time instant, we calculated the static and total pressures along the x axis averaged over the period l for the case with a supersonic velocity of the incoming flow:

$$\langle P\rangle(x,t) = \frac{1}{lp_0}\int_0^l p(x,y,t)dy,$$

$$\langle P^*\rangle(x,t) = \frac{1}{lp_0}\int_0^l (p(x,y,t) + \rho(x,y,t)u^2(x,y,t))dy.$$

Figure 5 shows the distributions of $\langle P\rangle(x,t)$ (dashed curves) and $\langle P^*\rangle(x,t)$ (solid curves) at the times $t_1 = 0.28$ msec and $t_2 = 0.5$ msec. Both t_1 and t_2 are greater than the time when the continuous spin detonation reaches a periodic regime with a specific flow rate $\langle G\rangle = 0.7g_0$ (see Fig. 3). It is seen that a shock wave with a static pressure $\langle P\rangle(x,t) = 3.7$ and total pressure $\langle P^*\rangle(x,t) = 4.35$ behind the SW front propagates in the diffuser ($x/l < 0$) upstream over the supersonic flow with $\mathrm{M}_0 = 1.5$. It is this shock wave that converts the incoming supersonic gas flow into a subsonic flow. The averaged pressure profiles in the combustor ($x/l > 0$) almost coincide for both times, i.e., the continuous spinning detonation wave indeed reached a self-sustained periodic regime. Note that the averaged static pressure in the region of energy release behind

TABLE 1

M_0	g_0, kg/(sec · m^2)	p^*/p_0	T^*, K	$\langle p_{SW}\rangle/p_0$	M
1	131.4	1.893	360	2.4	1.48
1.5	197.2	3.671	435	3.7	1.82
2	262.9	7.824	540	5.5	2.20
2.5	328.6	17.086	675	7.6	2.58
3	394.3	36.733	840	10.4	3.01
3.5	460.1	76.270	1035	—	—

TABLE 2

M_0	$\langle G\rangle/g_0$	$\langle p\rangle/p_0$	$l_{\min}$, cm	$\eta = h/l$	D, km/sec	$\langle J\rangle$, km/sec
1	0.64	1.7	4.25 ± 0.25	0.185	2.27	1.84
1.5	0.7	2.85	3.75 ± 0.25	0.22	2.38	1.69
2	0.75	4.2	2.25 ± 0.25	0.21	2.45	1.50
2.5	0.88	6.0	3.75 ± 0.25	0.215	2.5	1.29
3	1	7.4	5.75 ± 0.25	0.235	2.56	1.06

the TDW front ($0 < x/l < 0.1$) is almost constant: $\langle P\rangle \approx 2.8$. At $0.1 < x/l < 0.2$, the averaged pressure profile rapidly decreases to $\langle P\rangle = 1.9$ because of the rapid decrease in the true values of static pressure p in the two-dimensional flow structure (see Fig. 4a) owing to expansion of detonation products behind the TDW front. At $x/l > 0.2$, a gradual decrease in $\langle P\rangle$ is observed; the mean static pressure at the combustor exit in this variant of calculations decreases to 1.45. The averaged total pressure decreases monotonically in the region ($0 < x/l < L_2/l = 0.1$) because of the combustor channel expansion from δ to Δ and remains almost constant at $x/l > 0.1$ (in a constant-section channel): $\langle P^*\rangle = 3.55$.

Thus, numerical simulations show that the TDW in an annular flow-type combustor with the geometric parameters (7) and with a supersonic incoming flow with $M_0 = 1.5$ can propagate with a velocity $\langle D\rangle = 2.43$ km/sec and a specific impulse at the exit from the annular cylindrical combustor $\langle J\rangle = 1.69$ km/sec.

Effect of the Flow Mach Number

The numerical analysis described above allows us to study the effect of the flow Mach number M_0 on the parameters and structure of the gas-dynamic flow with continuous spin detonation and also to consider the domain of TDW existence. For this purpose, we fixed the geometric parameters (7) and performed systematic calculations with variations of the supersonic flow Mach number M_0. For several values of M_0, the specific flow rate g_0, the stagnation pressure p^* and the stagnation temperature T^* of the mixture in the incoming flow, the pressure $\langle p_{SW}\rangle$ behind the SW front, and the flow Mach number M with respect to the SW front are given in Table 1. The values of the specific flow rate $\langle G\rangle$ and the mean pressure $\langle p\rangle$ at the combustor entrance, the velocity of continuous spin detonation $\langle D\rangle$, the dimensionless TDW size $\eta = h/l$, and the mean specific impulse $\langle J\rangle$ are summarized in Table 2.

An analysis of the results presented shows that the regime of continuous spin detonation of a hydrogen–oxygen mixture with a supersonic flow in an annular cylindrical combustor exists in a rather wide range of Mach numbers: $1 \leqslant M_0 \leqslant 3$. With increasing M_0, the intensity of the SW generated by spin detonation in the diffuser, the flow Mach number M with respect to the SW front, and the dimensionless flow rate in the combustor $\langle G\rangle/g_0$ increase monotonically (see Table 1), but the SW velocity (D_{SW}) in the laboratory coordinate system decreases. For $M_0 = 1$, we have $\langle p_{SW}\rangle/p_0 = 2.4$ and $D_{SW} = (M_0 - M)c_0 \approx -260$ m/sec; for $M_0 = 3$, the pressure increases to $\langle p_{SW}\rangle/p_0 = 10.4$, and the SW velocity in the laboratory coordinate system decreases to $D_{SW} \approx -5$ m/sec. As is seen from Table 2, as the flow Mach number increases from 1 to 2, the linear TDW size h and the minimum period $l_{\min}$ decrease, through a further increase in M_0 leads to a monotonic increase in these parameters. Note that the specific impulse $\langle J \rangle$ (see the last column in Table 2) calculated by Eq. (9) monotonically decreases with increasing M_0 to $\langle J \rangle = 1.06$ km/sec at $\mathrm{M}_0 = 3$.

Attempts to obtain continuous spin detonation regimes with TDWs in an annular cylindrical combustor in calculations with $\mathrm{M}_0 > 3$ failed. In this case, though the SW propagating over the diffuser was formed at the initial stage as a result of TDW initiation in the combustor, but the SW velocity became lower than the velocity of the incoming supersonic flow with time. Therefore, the SW was first shifted toward the combustor entrance, than TDW failure occurred, and finally the hot combustion products were entrained by the supersonic flow away from the combustor.

CONCLUSIONS

• A two-dimensional unsteady mathematical model of a continuous spinning detonation wave in a supersonic flow in an annular cylindrical combustor was formulated.

• The dynamics and structure of the TDW formed under supercritical parameters of initiation of a hydrogen–oxygen mixture was obtained in calculations and studied. The possibility of continuous spin detonation in a flow-type annular cylindrical combustor with a supersonic ($1 < \mathrm{M}_0 \leqslant 3$) flow velocity at the entrance was demonstrated for the first time. Specific impulses were determined.

• It was found that the Mach number of the incoming supersonic flow is restricted from above for the case of combustion of a hydrogen–oxygen mixture in an annular cylindrical combustor. Continuous spin detonation is not realized for $\mathrm{M}_0 > 3$.

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