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# Stabilized Gas Combustion Wave in an Inert Porous Medium

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**A stabilized gas combustion wave in an inert porous medium with intense internal interphase heat exchange (at low velocities) is considered to elucidate the mechanism underlying the increase in the burning velocity of the homogeneous gas mixture due to the porous medium. It is shown that the major factor of the increase in the burning velocity is conductive heat recuperation from the postflame zone to the region of the fresh mixture through the solid skeleton of the porous medium. Analytical dependences of the degree of increase in the burning velocity of the mixture in the stabilized wave on the determining parameters are obtained. The possibilities and restrictions of the use of the results to analyze the operation of porous burners are discussed.**

**Key words: gas combustion, porous medium, heat recuperation.**

## INTRODUCTION

Recent progress in the theory of filtrational gas combustion (FGC) [1–3] has stimulated the development of new technologies in energetics, ecology, and fire and explosion safety [4–9]. One feature of FGC is a large number of determining parameters, including the parameters of the porous medium that can be used to control gas combustion in particular technologies. Another characteristic feature of FGC is the occurrence of energy concentration phenomena of various natures in combustion waves, which can also be used in commercial technologies to effectively control the combustion of low-calorific fuels, production of chemical materials, and the processes in fire arresters [10–12]. Naturally, the multiparameter nature of the problem considerably complicates the theoretical analysis and mathematical modeling of combustion problems, especially taking into account the detailed chemical kinetics, and physical experiments. On the other hand, the large number of elementary processes due to the heterogeneity of the system leads to a variety of their interactions and, as a result, to new effects.

In this connection, it is of interest to analyze the properties of a stabilized (standing) FGC wave in the regime of low velocities, which is of special significance in FGC theory. In the case of an unbounded porous body, this state of the wave is the transition state from the countercurrent to concurrent propagation of the combustion wave. The equilibrium temperature of the stabilized wave is equal to the combustion temperature of the homogeneous gas mixture (i.e., without the porous medium). In this case, the energy concentration in the combustion wave occurs only by the conductive mechanism. The convective mechanism, which includes the motion of the porous medium (solid skeleton) about the flame front and which exists under different FGC conditions, does not occur at $u = 0$. By the terms conduction and conductive mechanism is meant longitudinal heat transfer by thermal conductivity and radiation, which is characterized by the effective thermal conductivity of the skeleton. Finally, in this case, the average burning velocity $\langle Su \rangle$ referred to the unit cross-sectional area of the burner is equal to the average filtration velocity. This makes it possible to easily determine experimental values of the average burning velocity.

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The problem in question is related to porous burners. Here, as in many technologies, an important problem is the parametric optimization of the goal functions: increasing the power modulation range and the IR radiation yield, increasing the heat release rate and operation stability of the porous burner, and improving its other performance data. Among the most important problems is the search for the optimum conditions of gas mixture combustion. Since the basic operation mode of usual porous burners is the stabilized wave mode, investigation of the properties of this wave is also of practical interest.

The following simplification of the problem is related to the characteristic features of the low velocity regime (LVR) with intense interphase heat exchange and the high velocity regime (HVR) with weak heat exchange in the chemical reaction zone. This allows us to make two assumptions: to set the internal heat exchange coefficients equal to zero ($\alpha = 0$) for the HVR and to use the condition $\alpha \to \infty$ for the LVR. In the latter case, the analysis of the problem is considerably simplified because the two-temperature FGC model is replaced by the one-temperature model without loss of the qualitative elements of the approximation if the problem is considered within the region of existence of the LVR.

It can be assumed that the case $\alpha \to \infty$ corresponds to the maximum energy concentration in the LVR wave. Indeed, the energy concentration phenomenon is due to the heating of the porous medium ahead of the chemical reaction zone. In this case, of great importance are the successive elementary processes: heat transfer from the combustion products to the solid skeleton, conductive heat transfer through the skeleton to the heating zone, and convective heat transfer from the skeleton to the fresh gas. Consequently, for higher values of the internal heat-exchange coefficient and thermal conductivity of the skeleton, one should expect the higher energy concentration in the chemical reaction zone and, hence, the higher burning velocity $\langle Su \rangle$.

## 1. EQUATIONS OF STATIONARY COMBUSTION WAVES

A simple two-temperature model of FGC includes the equation of heat transfer in the porous medium, the equations of heat and mass transfer for the reactive component of the gas mixture, the continuity equation for the gas mixture, and the gas equation of state assuming constant pressure:

$$c_s\rho_s\,\frac{\partial T_s}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_s\,\frac{\partial T_s}{\partial x}\right) + \frac{\alpha_0}{1-m}\,(T_g - T_s),$$

$$c_g\rho_g\,\frac{\partial T_g}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_g\,\frac{\partial T_g}{\partial x}\right) - c_g\rho_g v\,\frac{\partial T_g}{\partial x} + \frac{\alpha_0}{m}\,(T_s - T_g) + Q\rho_g W(\eta, T_g), \qquad (1.1)$$

$$\rho_g\,\frac{\partial \eta}{\partial t} = \frac{\partial}{\partial x}\left(\rho_g D\,\frac{\partial \eta}{\partial x}\right) - \rho_g v\,\frac{\partial \eta}{\partial x} - \rho_g W(\eta, T_g),$$

$$\frac{\partial \rho_g}{\partial t} = -\frac{\partial \rho_g v}{\partial x}, \quad \rho_g T_g = \text{const}.$$

Here and below, $T$ is the temperature, $t$ is time, $\eta$ is the relative fraction of the rate-determining component, $\rho$ is the density, $c$ is the heat capacity at constant pressure, $\lambda$ is the thermal conductivity, $D$ is the diffusion coefficient, $v$ is the gas velocity, $u$ is the velocity of the combustion front, $m$ is the porosity, $\alpha$ is the rate of interphase heat exchange, $Q$ is the heat effect of the reaction, $W(\eta, T_g)$ is the first-order Arrhenius rate

$$W(\eta, T_g) = k\eta e^{-E/RT_g}, \qquad (1.2)$$

$k$ is the preexponent, $E$ is the activation energy, and $R$ is the universal gas constant. The following subscripts are used: $g$ refers to the gas phase, $s$ to the solid phase, 0 to the initial condition, $m$ to the maximum gas temperature, and $b$ to the adiabatic gas temperature.

It is easy to see that, at $-\infty < x < \infty$, Eqs. (1.1) are shift invariant, and, hence, investigation of stationary FGC waves reduces to investigation of the stationary solutions of the equations with the change of variable $x\!: = x - ut + \text{const}$. In particular, the continuity equation becomes the stationary law of conservation of mass:

$$G \equiv \rho_g(v - u) = \text{const}. \qquad (1.3)$$

Next, the following boundary conditions are considered to be satisfied:

$$x = -\infty\!: \quad T_s = T_g = T_0, \quad \eta = 1; \qquad (1.4)$$

$$x = \infty\!: \quad \frac{\partial T_s}{\partial x} = \frac{\partial T_g}{\partial x} = 0, \quad \eta = 0. \qquad (1.5)$$

The values of $T_0$, $\rho_{g,0}$, and $v_0$ are uniquely specified by the constants in (1.3) and in the equation of state. The gas-phase Lewis number will be defined by the formula

$$\text{Le}_g \equiv \frac{Dc_g\rho_g}{\lambda_g} = 1. \qquad (1.6)$$

We introduce dimensionless variables and parameters, so that $L = \text{const}$ is the characteristic length, $\xi = x/L$ the dimensionless independent variable, and

$$\theta_s = \frac{T_s - T_0}{T_{g,m} - T_0}, \quad \theta_g = \frac{T_g - T_0}{T_{g,m} - T_0}$$

are the dimensionless temperatures of the solid and gas phases. Then, according to (1.6), the stationary equations of FGG front propagation become

$$\frac{d}{d\xi}\left(a_s\,\frac{d\theta_s}{d\xi}\right)+\frac{(1-m)c_s\rho_s u}{mc_gG}\,\frac{d\theta_s}{d\xi}+\alpha(\theta_g-\theta_s)=0,$$
$$\frac{d}{d\xi}\left(a_g\,\frac{d\theta_g}{d\xi}\right)-\frac{d\theta_g}{d\xi}+\alpha(\theta_s-\theta_g)+\frac{\theta_{g,b}\rho_gL}{G}\,w(\eta,\theta_g)=0, \tag{1.7}$$
$$\frac{d}{d\xi}\left(a_g\,\frac{d\eta}{d\xi}\right)-\frac{d\eta}{d\xi}-\frac{\rho_gL}{G}\,w(\eta,\theta_g)=0,$$

where

$$a_s=\frac{(1-m)\lambda_s}{mc_gGL},\quad a_g=\frac{\lambda_g}{c_gGL},\quad \alpha=\frac{\alpha_0L}{mc_gG},$$
$$\theta_{g,b}=\frac{Q}{c_g(T_{g,m}-T_0)},$$
$$w(\eta,\theta_g)=W(\eta,T_0+(T_{g,m}-T_0)\theta_g).$$

We note that the maximum dimensionless gas temperature $\theta_{g,m}=1$. In this case, the dimensionless adiabatic temperature $\theta_{g,b}\leqslant 1$. The order of the above system of equations can be reduced. Multiplying the third equation by $\theta_{g,b}$, combining the equation obtained with the first two equations of the system, and integrating the result from $-\infty$ to $\xi$, we arrive at the equality

$$a_s\,\frac{d\theta_s}{d\xi}+a_g\,\frac{d(\theta_g+\theta_{g,b}\eta)}{d\xi}+\frac{(1-m)c_s\rho_s u}{mc_gG}\,\theta_s-\theta_g-\theta_{g,b}(\eta-1)=0. \tag{1.8}$$

As $\xi\to\infty$, the higher derivatives of the solution vanish. Consequently, according to, for example, the first equation of system (1.7), $\theta_s\to\theta_\infty$ and $\theta_g\to\theta_\infty$, where $\theta_\infty$ is the dimensionless equilibrium temperature. Below, it is assumed that

$$u<\frac{mc_g\rho_{g,0}}{mc_g\rho_{g,0}+(1-m)c_s\rho_s}\,v_0. \tag{1.9}$$

Then, according to (1.8), the mass balance (1.3), and boundary conditions (1.5), we have

$$\theta_\infty=\frac{mc_g\rho_{g,0}(v_0-u)}{mc_g\rho_{g,0}v_0-[mc_g\rho_{g,0}+(1-m)c_s\rho_s]u}\,\theta_{g,b}. \tag{1.10}$$

Inequality (1.9) implies that the condition $T_\infty>T_0$ is satisfied.

Relation (1.7) immediately leads to the equations for stabilized FGC waves. At $u=0$, the first equation of system (1.7) and Eq. (1.8) become

$$\frac{d}{d\xi}\left(a_s\,\frac{d\theta_s}{d\xi}\right)+\alpha(\theta_g-\theta_s)=0, \tag{1.11}$$

$$a_s\,\frac{d\theta_s}{d\xi}+a_g\,\frac{d(\theta_g+\theta_{g,b}\eta)}{d\xi}-\theta_g-\theta_{g,b}(\eta-1)=0. \tag{1.12}$$

In this case, Eq. (1.10) becomes the trivial equality $\theta_\infty=\theta_{g,b}$.

## 2. ONE-TEMPERATURE MODEL

Let us investigate the case of infinite intense thermal interphase interaction: $\alpha=\infty$. We will consider a stabilized combustion front ($u=0$). For smooth solutions (the second derivatives of the solid-phase temperature are bounded), Eq. (1.11) implies that, as $\alpha\to\infty$, the temperatures of the phases coincide: $\theta_s=\theta_g=\theta$. Then, Eq. (1.12) becomes

$$(a_s+a_g)\frac{d\theta}{d\xi}+\theta_{g,b}a_g\,\frac{d\eta}{d\xi}-\theta-\theta_{g,b}(\eta-1)=0. \tag{2.1}$$

For convenience of the further consideration, we introduce a new variable:

$$p=a_g\,\frac{d\eta}{d\xi}-\eta+1. \tag{2.2}$$

Then, the third equation of system (1.7) for $\eta$ and equality (2.1) can be written as

$$\frac{dp}{d\xi}=\frac{\rho_gL}{G}\,w(\eta,\theta_g), \tag{2.3}$$

$$(a_s+a_g)\frac{d\theta}{d\xi}-\theta+\theta_{g,b}p=0. \tag{2.4}$$

Equalities (2.2)–(2.4) form a system of first-order equations with a superfluous boundary condition. We have already used some of the boundary conditions to derive the first integral (1.8) and to allow for shift invariance (adjustment of the coordinate origin). In particular, these equations imply a monotonic increase in the function $\theta$: in the presence of local extrema, Eq. (2.4) provides proportionality of the values of the functions $\theta$ and $p$ at these points. However, by virtue of (2.3), the function $p$ is increasing, and, hence, the function $\theta$ do not have local extrema. Then, $\theta_\infty=\theta_b=\theta_m=1$, i.e., in the model considered, $T_{g,m}=T_b=T_0+Q/c_g$ is the adiabatic gas temperature. Next, following the standard procedure, we assume that $\theta$ is an independent variable and $p=p(\theta)$ and $\eta=\eta(\theta)$. In this case, according to (2.4),

$$\frac{dp}{d\xi}=\frac{\theta-\theta_{g,b}p}{a_s+a_g}\frac{dp}{d\theta},\quad \frac{d\eta}{d\xi}=\frac{\theta-\theta_{g,b}p}{a_s+a_g}\frac{d\eta}{d\theta},$$

and Eqs. (2.2) and (2.3) become

$$a_g(\theta-p)\frac{d\eta}{d\theta}=(a_s+a_g)(\eta+p-1),$$
$$(\theta-p)\,\frac{dp}{d\theta}=(a_s+a_g)\frac{\rho_gL}{G}\,w(\eta,\theta). \tag{2.5}$$

To obtain constraints on the parameters of the stabilized combustion front, we employ the method of matched asymptotic expansions. Let $p_0(\theta)$ and $p_1(\theta_*)$ be zero approximations of the function $p(\theta)$ in the external (adjacent to $\theta=0$) and internal (adjacent to $\theta=1$)

regions. Here $\theta_* = (1-\theta)/\gamma$ is an internal variable and $\gamma$ is the Zel'dovich parameter:

$$\gamma = \frac{RT_b^2}{E(T_b - T_0)} \ll 1.$$

In the external region, the reaction rate function can be ignored. Then, in accordance with the second equation of (2.5) $dp_0/d\theta \equiv 0$, and, hence, $p_0(\theta) \equiv 0$ because $p(0) = 0$. The matching condition of the external and internal solutions becomes

$$p_1(\theta_*) \to 0 \quad \text{at} \quad \theta_* \to \infty. \tag{2.6}$$

For the reaction rate function in the internal region, we use the Frank-Kamenetskii transformation: $1/T_g \approx 1/T_b - (T_g - T_b)/T_b^2$. Then, instead of the function $w(\eta, \theta)$, in the internal region we use the function

$$w_1(\eta, \theta_*) = k\eta e^{-E/RT_b} e^{-\theta_*}.$$

Next, in the internal region, we set $a_g \approx a_{g,b}$, $a_s + a_g \approx a_{s,b} + a_{g,b}$ and $\rho_g \approx \rho_{g,b}$. In addition, we introduce the normalization condition $a_{s,b} + a_{g,b} = 1$. In fact, this equality specifies the characteristic length:

$$L = \frac{\lambda_{\text{eff}}(m)}{c_g G}, \ \lambda_{\text{eff}}(m) = \frac{1-m}{m}\lambda_{s,b} + \lambda_{g,b}. \tag{2.7}$$

In the internal region, Eqs. (2.5) become

$$\begin{aligned} &\frac{a_{g,b}}{\gamma}(p_1 - 1)\frac{d\eta}{d\theta_*} = \eta + p_1 - 1, \\ &(p_1 - 1)\frac{dp_1}{d\theta_*} = \gamma\frac{\rho_{g,b}L}{G}\, w_1(\eta, \theta_*). \end{aligned} \tag{2.8}$$

Integration of the second of these equations with respect to $0 < \theta_* < \infty$ taking into account the matching condition (2.6) and the condition $p_1(0) = 1$ ($p \to 1$ as $\xi \to \infty$) implied by (2.2) yields

$$\begin{aligned} \frac{1}{2} &= \frac{\gamma\rho_{g,b}L}{G}\int_0^\infty w_1(\eta, \theta_*)d\theta_* \\ &= \frac{\gamma\rho_{g,b}L}{G}\, ke^{-E/RT_b}\int_0^\infty \eta e^{-\theta_*}d\theta_*. \end{aligned} \tag{2.9}$$

Next, we express the function $\eta$ from the first equation of system (2.8):

$$\eta = (p_1 - 1)\left(\frac{a_{g,b}}{\gamma}\frac{d\eta}{d\theta_*} - 1\right),$$

and substitute this expression into the function $w_1(\eta, \theta_*)$ in the second equation of system (2.8). As a result, we obtain the equation

$$\frac{dp_1}{d\theta_*} = \frac{\gamma\rho_{g,b}L}{G}\, ke^{-E/RT_b}\left(\frac{a_{g,b}}{\gamma}\frac{d\eta}{d\theta_*} - 1\right)e^{-\theta_*},$$

which is integrated to give the following expression for the integral from the right side of equality (2.9):

$$\int_0^\infty \eta e^{-\theta_*}d\theta_* = \frac{1}{a_{g,b}}\left(\gamma - \frac{G}{k\rho_{g,b}L}\, e^{E/RT_b}\right). \tag{2.10}$$

Substitution (2.10) into (2.9) leads to the equality

$$1 = \frac{2\gamma^2\rho_{g,b}L}{(a_{g,b} + 2\gamma)G}\, ke^{-E/RT_b}. \tag{2.11}$$

Here, unlike in the standard method of matched asymptotic expansions, the approximation of integral (2.10) was not used, which makes equality (2.11) uniformly suitable for $a_{g,b} \ll 1$, and exactly this case occurs during FGC. We note that the parameters determining stabilized FGC waves do not include the characteristics of the solid phase $c_s$ and $\rho_s$.

Let us write equality (2.11) in a different form using dimensional quantities. We recall that, according to (1.3), in the case considered, $G = \rho_{g,0}v_0 = \rho_{g,b}v_b$. Next, according to (2.7), $c_g GL = \lambda_{\text{eff}}(m)$ and, hence, $a_{g,b} = \lambda_{g,b}/\lambda_{\text{eff}}(m)$. We introduce the parameter

$$\nu(m) = \frac{\lambda_{\text{eff}}(m)}{\lambda_{g,b}} = 1 + \frac{1-m}{m}\frac{\lambda_{s,b}}{\lambda_{g,b}}. \tag{2.12}$$

In view of the gas equation of state and the condition on the Lewis number (1.6), equality (2.11) becomes

$$v_0^2 = \frac{2D_b(\gamma\nu(m))^2}{1 + 2\gamma\nu(m)}\left(\frac{T_0}{T_b}\right)^2 ke^{-E/RT_b}. \tag{2.13}$$

We note that the normal gas flame velocity ($\alpha = 0$, $m = 1$) is given by the formula

$$u_n^2 = 2D_b\gamma^2\left(\frac{T_0}{T_b}\right)^2 ke^{-E/RT_b}.$$

Then,

$$\chi = \frac{v_0}{u_n} = \frac{\nu(m)}{\sqrt{1 + 2\gamma\nu(m)}}. \tag{2.14}$$

In this case, the function $\chi = \chi(m)$ is decreasing. Indeed,

$$\frac{d\chi}{dm}(m) = -\frac{1 + \gamma\nu(m)}{m^2(1 + 2\gamma\nu(m))^{3/2}}\frac{\lambda_{s,b}}{\lambda_{g,b}} < 0.$$

The minimum value is reached for $m = 1$: $\chi(1) = 1/\sqrt{1 + 2\gamma} \approx 1$. As $m \to 0$, the function $\chi(m)$ tends to infinity at a rate of order $O(m^{-1/2})$.

We next consider relation (2.14) as a function of the adiabatic temperature $T_b$: $\chi = \chi(T_b)$. Since

$$\frac{d\chi}{dT_b}(T_b) = -\frac{\nu^2(m)}{(1 + 2\gamma\nu(m))^{-3/2}}\frac{RT_b(T_b - 2T_0)}{E(T_b - T_0)^2},$$

this function is decreasing for $Q/c_g > T_0$. For $Q/c_g = T_0$, relation (2.14) reaches the minimum value:

$$\chi(2T_0) = \frac{\nu(m)}{\sqrt{1 + 8\nu(m)RT_0/E}}.$$

Finally, we consider relation (2.14) in terms of the effective Lewis number:

$$\mathrm{Le}_{\mathrm{eff}}(m) = \frac{D_b c_g \rho_{g,b}}{\lambda_{\mathrm{eff}}(m)}. \tag{2.15}$$

Since $\mathrm{Le}_g = 1$, according to formula (2.12), $\mathrm{Le}_{\mathrm{eff}} = 1/\nu(m)$. Then, formula (2.14) becomes

$$\chi = \frac{1}{\sqrt{\mathrm{Le}^2_{\mathrm{eff}}(m) + 2\gamma \mathrm{Le}_{\mathrm{eff}}(m)}}. \tag{2.16}$$

## 3. DISCUSSION AND CONCLUSIONS

This paper considers a stabilized gas combustion wave propagating in the low velocity regime in an infinitely long porous solid. This is not the only possibility of obtaining a stabilized wave in the LVR. Wave stabilization is also possible in a porous solid of finite length [13], in porous solids with spherical and cylindrical geometries [14, 15], on the boundary between two solids with various porosity characteristics [12], and in other cases. The properties of stabilized waves appear to depend on the particular stabilization mechanisms. In this respect, stabilization of a plane wave in an infinite porous medium is a simple but not trivial case.

When a combustion wave reaches the velocity $u = 0$, its direction is reversed. The physics of the process changes significantly. In counterpropagating traveling waves in the LVR, heat recuperation typically occurs by two mechanisms — convective and conductive, whereas at $u = 0$, convective recuperation does not occur. In a traveling wave, $T_\infty < T_b$ in the counterpropagating wave and $T_\infty > T_b$ in the concurrent wave, whereas in a stabilized wave, $T_\infty = T_b$. In other words, in the LVR, one observes the result of the joint contribution of two types (conductive and convective ) of energy concentration in the combustion zone, resulting (at $u > 0$) in an increase in temperature and, hence, burning velocity. At $u < 0$, conductive heat recuperation and convective transfer of excess energy to the skeleton of the porous medium lead to subadiababtic equilibrium temperatures. Heat recuperation and transfer to the skeleton under variations in the filtration velocity, mixture composition, and other parameters make possible smooth continuous changes in the temperature and gas burning velocity over wide parameter ranges. The above-mentioned regularities highlight the fundamental importance of the relative motion during filtrational combustion, namely, the direction and magnitude of the gas flow velocity and the flame front relative to the porous medium.

Of primary importance for porous burners is their ability to effectively burn gas. In the present work, we consider only one aspect of this problem — the possibility of accelerating gas combustion in the presence of a solid skeleton by conductive heat recuperation. As a measure of this ability, we use the quantity $\chi = \langle Su \rangle / Su$, where $Su$ is the normal laminar flame velocity. This approach allows one to find the determining parameters and elucidate their role. Among such parameters is the adiabatic heating of the products $Q/c_p$, porosity $m$, Lewis effective number $\mathrm{Le}_{\mathrm{eff}}$, and Zel'dovich number $\gamma$.

The heat effect of the reaction increases both the normal velocity $Su$ and the burning velocity of the mixture $\langle Su \rangle$. This increase, however, is more effective for the normal velocity $u$. As a result, the effect of conductive heat recuperation decreases with increasing adiabatic heating of the products and, hence, the burning velocity decreases. The opposite situation is observed for the activation energy: an increase in the activation velocity leads to a reduction in the Zel'dovich number and, hence, to an increase in the relative burning velocity $\chi$. Physically, these effects can be interpreted as follows: a relative increase in the velocity $Su$ leads to a narrowing of the heating zone and a decrease in the time of interphase heat exchange in this zone, resulting in a reduction in the efficiency of conductive heat recuperation.

One of the main parameters of the problem is the effective Lewis number $\mathrm{Le}_{\mathrm{eff}}$. The smaller the value of $\mathrm{Le}_{\mathrm{eff}}$, the higher the relative burning velocity $\chi$ [formula (2.16)]. In other words, the more effective the heat transfer in the skeleton ($\lambda_{\mathrm{eff}}$) compared to heat transfer in the gas ($\lambda_g$), the higher the gas burning velocity. Indeed, the quantity $\chi$ is the ratio of the velocities of filtrational and homogeneous gas-phase combustion.

The relation for $\chi$ does not contain an important parameter of the porous medium — the degree of dispersion. This is due to the assumption of intense interphase heat exchange, $\alpha \to \infty$. Since $\alpha$ increases as the diameter of the pore channel $d$ decreases, this assumption can be treated as the assumption of smallness of $d$. Experiments confirm this conclusion: $v_0$ decreases with increasing $d$ [16, Fig. 7]. Thus, the assumption that the case $\alpha \to \infty$ corresponds to the maxim conductive energy recuperation in the LVR wave seems realistic. In contrast, an increase in $d$ leads to transition to the HVR [16] with weak interphase interaction in the chemical reaction zone.

As regards porosity effects, the dependence $\chi(m)$ reflects the influence of the volume fraction of the material conductors of heat — the gas and solid phases. As $m$ increases, the contribution of the thermal conductivity of the solid phase to the effective thermal conductivity $\lambda_{\mathrm{eff}}$ decreases, which, in turn, leads to an increase in

$\mathrm{Le_{eff}}$ and a decrease in $\chi$ (see (2.16)). It is interesting that the thermal capacities of the phases do not play an important role in the parametric dependences of the quantity $\chi$.

With respect to real porous burners, it is necessary to note that many aspects of flame stabilization are still little understood. First, as mentioned above, the stabilization mechanisms can be different. Next, flame stabilization can be significantly influenced by various complicating factors — the heterogeneity of the porous medium [17], transitions between regimes [18], the conditions on the boundaries of the porous media [12, 13, 19], heat losses, two- and three-dimensional effects due to instability of flame fronts, the occurrence of hot spots [20], and the structural features of porous burners. Finally, there are no solutions to many basic issues of filtrational combustion theory, such as the role of heterogeneous chemical reactions in the heat release balance, the role of radiation flows in heat exchange processes, and flow hydrodynamics and flame shape in porous burners of complex spatial structures.

Taking into account the reversal property of a traveling wave in an infinitely long porous solid and the feature of the coordinate of the flame front at $u = 0$ as the turning point of motion of the flame front, the relative burning velocity $\chi$ should be recognized an important characteristic in the consideration of burner flame stability.

In conclusion, we note that the problem of why and how the solid surface accelerates gas mixture combustion, formulated by William Bone an almost century ago [21], remains urgent. The present work was an attempt to clarify some aspects of this old problem.

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