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# Mathematical Modeling of a Rotating Detonation Wave in a Hydrogen–Oxygen Mixture

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**A two-dimensional unsteady mathematical model of spin detonation in an annular cylindrical ramjet-type combustor is formulated. The wave dynamics in the combustor filled by a hydrogen–oxygen mixture is studied numerically.**

**Key words: continuous detonation, combustor, transverse detonation waves, flow structure, mathematical modeling.**

## INTRODUCTION

A method of detonation combustion of fuels is currently considered as an alternative to conventional combustion in turbulent flame. This method allows intense and more thermodynamically efficient and stable combustion of various fuels in combustors of moderate size determined by the characteristic size of the detonation-wave front [1–8].

The regime of detonation combustion of the fuel mixture continuously injected into an annular chamber with exit constriction [9] or without the latter [5] was theoretically examined in a one-dimensional approximation under the assumption of a uniform flow of detonation products with a sonic velocity in the axial direction. In this case, the output parameters of the flow were uniquely determined by the combustor geometry and thermochemical and gas-dynamic constants of the fuel mixture and were independent of the combustion mechanism. The detonation character of conversion was taken into account only in considering the internal structure of the flow in the combustor. The maximum acceleration of detonation products is achieved [10] in the case of isentropic expansion of the products behind the front of a transverse detonation wave (TDW). The flow velocity at the exit of a constant-section combustor is supersonic, while it is known to be sonic in the one-dimensional process of conventional combustion. The one-dimensional model of continuous rotating (spin) detonation in a cylindrical combustor [5, 10] can predict the basic parameters of the flow with TDWs in an annular combustor if experimental values of the relative size of the TDW and the angle of inclination of the wave to the combustor axis are available.

The mathematical model of spin detonation in a two-dimensional steady formulation [10–12] with the transition to the TDW-fitted coordinate system made it possible to calculate (avoiding the use of experimental information) a detailed two-dimensional periodic flow field and relative sizes of the TDW and shock-wave tail and to demonstrate that there is a transonic transition in a constant-section combustor in the spin detonation regime. The simplification used, which implies that the length of the energy-release zone is small as compared with the TDW size, allowed us to avoid considering a narrow subsonic zone in the solution domain behind the TDW front; this assumption, however, restricts the number of issues that can be resolved by numerical simulations of continuous spin detonation. In particular, the two-dimensional steady model of spin detonation [10] does not predict the absolute values of transverse detonation waves and the distances between them; hence, the geometric and concentration limits of propagation of the regime of spin detonation with TDWs cannot be determined. Allowance for the finite length of the chemical reaction zone (energy-release zone) in the mathematical model makes the problem of continuous spin detonation substantially more complicated and necessitates a two-dimensional unsteady mathematical formulation and solution.

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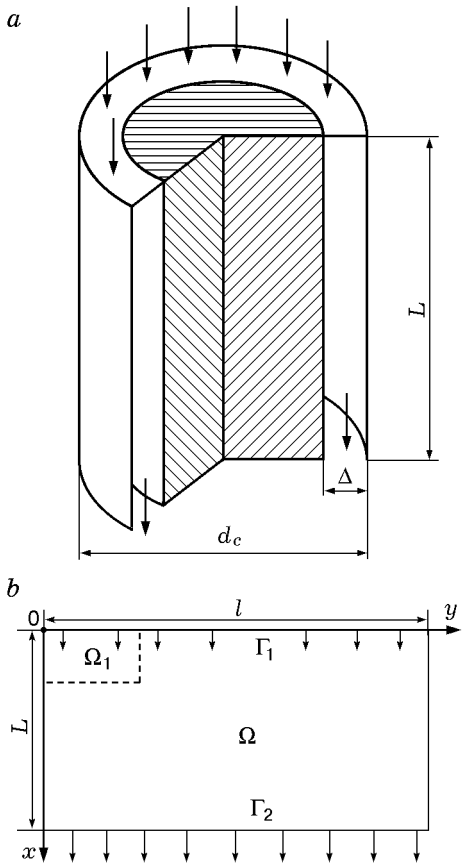


**Fig. 1.** Scheme of an annular cylindrical combustor (a) and domain of the numerical solution of the problem (b).

The present activities were aimed at numerical investigations of combustion of a hydrogen–oxygen mixture in the continuous spin detonation regime.

## MATHEMATICAL FORMULATION OF THE PROBLEM

Let us consider the problem of detonation combustion of a hydrogen–oxygen mixture in a combustor of annular cylindrical geometry (Fig. 1a, combustor diameter $d_c$, length $L$, and width of the annular channel $\Delta$). Here $\phi$ is the fuel-to-air equivalence ratio. The flow occurs in the annular channel of the combustor with the boundaries $\Gamma_1$ (upper edge of the combustor where the fuel mixture is injected) and $\Gamma_2$ (open end of the combustor through which combustion products exhaust). As $\Delta \ll d_c$, then, as it was done in [10], we can cut the annular region in the vertical direction and unfold it into a rectangular solution domain $\Omega = (0 < x < L,\ 0 < y < l = \pi d_c)$, which is shown in Fig. 1b.

Let an energy sufficient for detonation initiation be released at a certain time after the beginning of injection of the reactive mixture through the boundary $\Gamma_1$ in the area of the solution domain $\Omega_1$. As a result of initiation, an unsteady detonation wave propagates in the domain $\Omega$. We have to determine its dynamics, structure, and conditions necessary to reach the self-sustaining regime of spin detonation, depending on the governing parameters of the problem.

The flow of the mixture in the solution domain $\Omega$ is described by the following system of equations of two-dimensional unsteady gas dynamics with chemical transformations:

$$\begin{gathered}(\rho)_t + (\rho u)_x + (\rho v)_y = 0,\\ (\rho u)_t + (\rho u^2)_x + (\rho uv)_y + p_x = 0,\\ (\rho v)_t + (\rho uv)_x + (\rho v^2)_y + p_y = 0,\\ (\rho E)_t + [\rho u(E + p/\rho)]_x + [\rho v(E + p/\rho)]_y = 0,\\ Y_t + uY_x + vY_y = f_5,\\ \mu_t + u\mu_x + v\mu_y = f_6.\end{gathered} \tag{1}$$

Here $x$ and $y$ are the spatial variables of the orthogonal coordinate system, $t$ is the time, $\rho$ is the density, $u$ and $v$ are the components of the velocity vector $\mathbf{u}$, $p$ is the pressure, $E = U + (u^2+v^2)/2$, $U(T, \mu)$ is the total internal energy of the gas, $T$ is the temperature, $\mu$ is the current molar weight of the mixture, and $Y$ is the fraction of the chemical induction period. The energy release is described by a two-stage kinetic model [13] consisting of the induction stage ($0 < Y \leqslant 1$, $f_5 = -1/t_{\text{ind}}$, and $f_6 = 0$) without energy release and chemical conversion stage ($Y = 0$, $f_5 = 0$, and $f_6 \neq 0$) where the energy-release rate is determined by the rate of chemical reactions.

As follows from experimental data [14], the chemistry-induced delays of ignition in the induction zone ($0 < Y \leqslant 1$) for the hydrogen–oxygen mixture are described by the relation

$$t_{\text{ind}} = \frac{K_a \mu_{\text{O}_2}}{\rho z} \exp\left(\frac{\varepsilon_a}{RT}\right), \tag{2}$$

where $\varepsilon_a = 17.15$ kcal/mole is the activation energy, $K_a = 5.38 \cdot 10^{-11}$ mole$\cdot$sec/liter is the pre-exponent, $R$ is the universal gas constant, $\mu_{\text{O}_2}$ is the molar weight of oxygen, and $z = (1 + \phi/8)^{-1}$ is the mass fraction of oxygen.

Let us supplement system (1) by the equations of state

$$p = \rho RT/\mu, \quad U = U_{\text{th}} + U_{\text{ch}}, \tag{3}$$

where $U_{\text{th}}$ and $U_{\text{ch}}$ are the thermodynamic and chemical components of internal energy, respectively.

If the internal energy of the gas $U$ is counted from the ultimately dissociated composition at zero temperature, the following expressions are valid in the induction zone:

$$U_{\text{th}} = \frac{p}{(\gamma-1)\rho}, \quad U_{\text{ch}} = -\frac{E_2^0 z}{\mu_{\text{O}_2}} - \frac{E_1^0(1-z)}{\mu_{\text{H}_2}}. \quad (4)$$

Here $\gamma$ is the ratio of specific heats of the gas in the chemical induction zone, $E_1^0$ and $\mu_{\text{H}_2}$ are the dissociation energy and the molar weight of hydrogen, and $E_2^0$ is the energy of dissociation of oxygen molecules.

According to [15, 16], the thermodynamic and chemical components of internal energy of the gas in the zone of chemical transformations have the form

$$U_{\text{th}} = \left[\frac{\mu}{\mu_{\min}} + \frac{1-\delta}{2} + \left(\frac{\mu}{\mu_{\min}} + \delta - 1\right)\frac{\theta/T}{\exp(\theta/T)-1}\right]\frac{RT}{\mu}, \quad (5)$$

$$U_{\text{ch}} = E_d\left(\frac{1}{\mu} - \frac{1}{\mu_{\min}}\right),$$

where $\delta = \delta_{\max}(\mu/\mu_{\min}-1)/(\mu_{\max}/\mu_{\min}-1)$, $\mu_{\min}$ and $\mu_{\max}$ are the molar weights of the gas in the ultimately dissociated and ultimately recombined states, respectively, $\delta_{\max}$ is the molar fraction of triatomic molecules in the ultimately recombined state, $\theta$ is the effective temperature of excitation of vibrational degrees of freedom of molecules, and $E_d$ is the mean energy of dissociation of reaction products. All parameters are uniquely determined by the atomic composition of the mixture (mass fraction $z$ of oxygen). For the hydrogen–oxygen mixture, $\mu_{\max}^{-1} = (1-z)/\mu_{\text{H}_2}$, $\delta_{\max} = 2z\cdot\mu_{\max}/\mu_{\text{O}_2}$, $\theta = 3000 + 500\delta_{\max}$ for $z \leqslant 8/9$ and $\mu_{\max}^{-1} = z/\mu_{\text{O}_2} + 0.5(1-z)/\mu_{\text{H}_2}$, $\delta_{\max} = (1-z)\mu_{\max}/\mu_{\text{H}_2}$, $\theta = 1500 + 2000\delta_{\max}$ for $z > 8/9$; $\mu_{\min}^{-1} = 2(z/\mu_{\text{O}_2} + (1-z)/\mu_{\text{H}_2})$, $\mu_0^{-1} = z/\mu_{\text{O}_2} + (1-z)/\mu_{\text{H}_2}$, and $E_d \approx E_1^0 \approx E_2^0$.

The composition of the gas phase behind the ignition front starts to change in accordance with the equation of chemical kinetics [17] correlated with the second law of thermodynamics, and Eq. (5) for the refined description of the thermodynamic component of internal energy acquires the form

$$\begin{gathered} f_6 = W_1(\mu)\rho^2 - W_2(T,\mu)\rho, \\ W_1(\mu) = 4K_+(1-\mu/\mu_{\max})^2/\mu, \\ W_2(T,\mu) = 4K_+K_-(\mu/\mu_{\min}-1)(T/T_0)^{\beta/2} \\ \times[1-\exp(-\theta/T)]^{\beta}\exp(-E_d/RT), \end{gathered} \quad (6)$$

where $K_+$ is the recombination rate constant, $K_-$ is the equilibrium constant, $T_0$ is the initial temperature of the mixture, and $\beta = 1 + \delta_{\max}/(\mu_{\max}/\mu_{\min}-1)$.

System (1)–(6) is closed and completely determines the two-dimensional unsteady motion of the reactive hydrogen–oxygen mixture with variable heat release in the TDW reaction zone.

**Boundary Conditions.** Similar to [10], the boundary condition at the combustor entrance ($\Gamma_1$: $x = 0$; $0 \leqslant y \leqslant l$) is the relation between the parameters of injection of the combustible mixture from the manifold through a system of Laval micronozzles into the chamber with a pressure $p(0,y,t)$:

$$\begin{gathered} v = 0,\ Y = 1,\ \mu = \mu_0; \\ \text{if } p_m \leqslant p(0,y,t),\ \text{then } p = p(0,y,t),\ u = 0; \\ \text{if } p'' \leqslant p(0,y,t) < p_m,\ \text{then } p = p(0,y,t), \\ u = u_{\max}[1-(p/p_m)^{(\gamma-1)/\gamma}]^{1/2}, \\ \rho = \rho_m(1-u^2/u_{\max}^2)^{1/(\gamma-1)}; \\ \text{if } p' \leqslant p(0,y,t) < p'',\ \text{then } p = p(0,y,t), \\ \text{if } p(0,y,t) < p',\ \text{then } p = p',\ \rho u S = \rho_* u_* S_*, \\ \gamma/(\gamma-1)p/\rho + u^2/2 = u_{\max}^2/2. \end{gathered} \quad (7)$$

Here $p_m$, $\rho_m$, and $T_m = p_m\mu_0/(\rho_m R)$ are the pressure, density, and stagnation temperature of the mixture in the manifold; $\rho_*$, $u_*$, and $u_{\max}$ are the critical density, velocity, and maximum possible velocity, which are known functions of $\gamma$, $p_m$, and $\rho_m$; $S$ and $S_*$ are the areas of the exit section and throat of the micronozzles ($S$ coincides with the cross-sectional area of the combustor); $p'$ and $p''$ are the calculated pressures of supersonic and subsonic exhaustion determined by the equation

$$\begin{gathered} (p/p_m)^{1/\gamma}[1-(p/p_m)^{(\gamma-1)/\gamma}]^{1/2} \\ = [2/(\gamma+1)]^{1/(\gamma-1)}[(\gamma-1)/(\gamma+1)]^{1/2}S_*/S. \end{gathered} \quad (8)$$

At the combustor exit ($\Gamma_2$: $x = L$; $0 \leqslant y \leqslant l$), in the case of exhaustion of the jet with a subsonic velocity in the $x$ direction into a space with a rather low counterpressure, the velocity-vector component $u$ passes through the velocity of sound; therefore, the axial component of the velocity vector $u$ along the entire boundary $\Gamma_2$ is not smaller than the local velocity of sound $c$, i. e., the condition of free exhaustion of detonation products is written as

$$u(L,y,t) \geqslant c(L,y,t). \quad (9)$$

The left and right boundaries of the domain $\Omega$ are subjected to the condition of solution periodicity. By virtue of flow periodicity (with a period $l$) with respect to the $x$ coordinate, all gas-dynamic functions $F(x,y,t)$ satisfy the condition

$$F(x,0,t) = F(x,l,t), \quad 0 \leqslant x \leqslant L. \quad (10)$$

**Initial Conditions.** In the solution domain $\Omega = (0 < x < L,\ 0 < y < l)$, the initial conditions are set

in the form of a steady solution of the problem with injection of the gas mixture through injectors in the upper end face of the combustor $\Gamma_1$ ($x = 0$), which is uniquely determined from the prescribed values of parameters in the injection system $p_m$, $\rho_m$, $\gamma$, and $S_*/S$:

$$p(x,y,0) = p_0,\ u(x,y,0) = u_{00},\ v = 0,$$
$$\rho(x,y,0) = G_0/u,\ Y = 1,\ \mu = \mu_0.$$

Here

$$G_0 = \left(\frac{2}{\gamma+1}\right)^{\frac{0.5(\gamma+1)}{\gamma-1}} \sqrt{\gamma p_m \rho_m}\left(\frac{S_*}{S}\right)$$

is the specific flow rate, $u_{00} = -u_1 + \sqrt{u_{\max}^2 + u_1^2}$ is the velocity of the mixture in the combustor, $u_1 = \gamma/(\gamma - 1)p_0/G_0$, and $p_0$ is the initial pressure in the combustor.

Let there occur instantaneous addition of energy with a volume energy density $Q$ at the time $t = 0$ in the solution-domain area $\Omega_1 = (0 < x < x_* < L,\ 0 < y < y_* < l)$. In this case, the pressure and temperature of gaseous combustion products increase in this area in a jumplike manner. (The domain $\Omega_1$ models the region of detonation initiation.) At $t > 0$, the disintegration of an arbitrary discontinuity results in propagation of an unsteady detonation wave to the area $\Omega_2 = \Omega \cap \Omega_1$ with energy release behind the wave front.

The formulated problem (1)–(10) was solved numerically. The solution domain $\Omega$ was covered by a motionless grid with a total number of cells $(N_x \times N_y)$ with uniform steps in the $y$ direction ($N_y = 100$) and nonuniform steps in the $x$ direction ($N_x = 80$). The system of differential equations (1), which describes the reactive gas flow, was integrated by the Godunov–Kolgan second-order finite-difference scheme [18, 19].

## CALCULATION RESULTS

The numerical study was performed for a stoichiometric hydrogen–oxygen gas mixture with the following values of the constants: $\mu_{H_2} = 2$ kg/kmole, $\mu_{O_2} = 32$ kg/kmole, $E_1^0 \approx E_2^0 \approx E_d = 110$ kcal/mole, $R = 8.3144 \cdot 10^3$ J/(kmole·K), $K_+ = 6 \cdot 10^8$ m$^6$/(kmole$^2$·sec), $K_- = 1.769 \cdot 10^3$ kmole/m$^3$, $\gamma = 1.397$, $T_0 = 300$ K, $p_0 = 1.013 \cdot 10^5$ Pa, and $\rho_0 = p_0\mu_0/(RT_0)$.

Bringing the sought functions, coordinates, and time in Eqs. (1)–(6) and conditions (7)–(10) to dimensionless form as $p/p_0$, $\rho/\rho_0$, $T/T_0$, $\mu/\mu_0$, $u/u_0$, $v/u_0$, $x/l$, $y/l$, and $t/t_0$ using the parameters $p_0$, $\rho_0$, $T_0$, $\mu_0$, $u_0 = \sqrt{p_0/\rho_0}$, $l$, and $t_0 = l/u_0$, we find that the solution of the unsteady problem of spin detonation with prescribed thermodynamic properties of the $H_2$–$O_2$ mixture depends on five governing parameters: three dimensionless parameters in the injection system (stagnation pressure of the mixture $p_m/p_0$, stagnation temperature of the mixture $T_m/T_0$, and the ratio of areas of the throat and exit cross-sectional areas $S_*/S$) and two scale parameters (combustor length $L$ and its perimeter $l$). In contrast to the steady formulation of the problem [10] where the sizes of the annular cylindrical combustor were characterized only by the ratios $S_*/S$ and $L/l$, the set of the governing parameters in this formulation contains the linear size of the combustor, namely, its perimeter $l$ in the explicit form.

### Initiation of the Detonation Wave

Preliminary calculations of spin detonation of the $H_2$–$O_2$ mixture were performed for the following values of the governing parameters:

$$p_m/p_0 = 5,\ T_m/T_0 = 1,$$
$$S_*/S = 0.0657,\ L = 6 \text{ cm}. \qquad (11)$$

These data in the injection system ensure a specific flow rate of the mixture $G_0 = 50$ kg/(sec·m$^2$).

In numerical simulations, the no-slip boundary condition ($v = 0$) was set on the side boundaries of the integration domain $\Omega$ ($0 < x < L$, $y = 0$, and $y = l$) at the stage of initiation; as the TDW approached the side boundary $y = l$, this condition was replaced by the condition of solution periodicity (10). Such a formulation of the boundary conditions is in line with the procedure used in Voitsekhovskii's experiments [2]: to control wave rotation, a gate completely covering the channel cross section was installed near the place of initiation; this gate was rapidly opened by a special explosive device before the detonation wave performed a complete round in the combustor.

For certainty, let us define the size of the instantaneous energy release area $\Omega_1$: $x_* = 1$ cm and $y_* = 1$ cm. Preliminary calculations of detonation initiation with a varied volume energy density $Q$ showed that a self-sustaining detonation wave with the mean velocity $D = 2.8 \pm 0.1$ km/sec propagates along the $y$ axis in the solution domain if $Q > Q_*$. Figure 2 shows the typical dynamics of pressure profiles along the $y$ axis for supercritical energy release ($Q = 2$ MJ/m$^3$; solid curves) and subcritical energy release ($Q = 0.5$ MJ/m$^3$; dashed curves). Monotonic decay of the unsteady wave is observed for $Q < Q_*$, while the wave reaches a self-sustaining detonation regime "from below" for $Q > Q_*$.

The search for the critical parameters of detonation initiation in the hydrogen–oxygen mixture in the three-dimensional space of parameters ($x_*$, $y_*$, and $Q_*$) is outside the scope of the present study; hence, the supercritical parameters of initiation $x_* = 1$ cm, $y_* = 1$ cm, and $Q = 2$ MJ/m$^3$ were used in all further calculations.

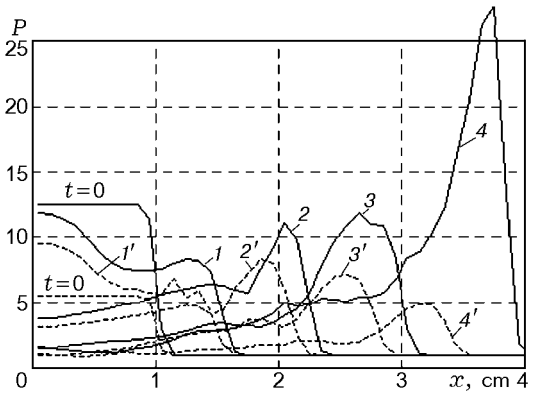


**Fig. 2.** Pressure profiles $P = p(0, y, t)/p_0$ at several times $t$ for supercritical energy release (solid curves) and subcritical energy release (dashed curves): curves 1–4 refer to $Q = 2$ MJ/m$^3$ and $t = 3.72$ (1), 7.68 (2), 11.85 (3), and 15.84 $\mu$sec (4); curves 1′–4′ refer to $Q = 0.5$ MJ/m$^3$ and $t = 4.27$ (1′), 8.7 (2′), 13.33 (3′), and 17.9 $\mu$sec (4′).

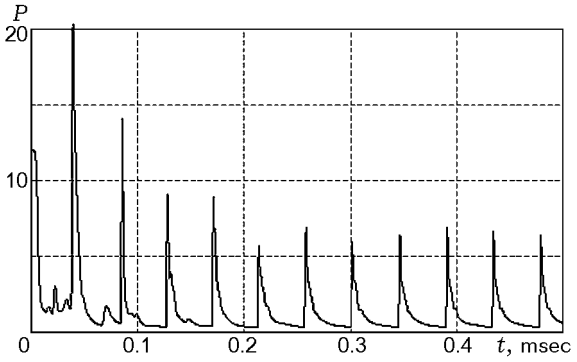


**Fig. 3.** Dimensionless pressure versus time at the point $x = 0$, $y = 0$ for $l = 10$ cm.

## Periodic Solution

To find a periodic solution for the spin detonation wave, we had to complete the definition of the solution domain (i.e., define the period $l$). We assumed that $l = 10$ cm. For the above-mentioned constants, we calculated the two-dimensional unsteady problem (1)–(10). Figure 3 shows the pressure $P(t) = p(0, 0, t)/p_0$ as a function of the time $t$ at a fixed point with the coordinates $x = 0$, $y = 0$ during the first 0.5 msec after initiation of the detonation wave. The pressure is seen to behave nonmonotonically: it oscillates with time. The first pressure peak ($P_{\max,1} \approx 20$) appears when the TDW arrives at this point for the first time; the second pressure peak ($P_{\max,2} \approx 14$) corresponds to its second arrival, etc. At the early stage of the process (four to five oscillations), the pressure performs irregular oscillations with decreasing amplitude, and then the oscillations become almost periodic (with a period $\Delta t \approx 44$ $\mu$sec) and have the maximum amplitude $P_{\max} \approx 6.5$, the minimum amplitude $P_{\min} \approx 0.38$, and the amplitude ratio $P_{\max}/P_{\min} = 17.1$. The mean pressure at the end face of the combustor is

$$P_c = \frac{1}{lp_0} \int_0^l p(0, y, t)\, dy \approx 1.125.$$

Knowing the time period $\Delta t$, we can calculate the TDW velocity averaged over the period $\langle D \rangle = l/\Delta t = 2.27 \pm 0.02$ km/sec and the ratio $\langle D \rangle / D_0 = 0.8$. Here $D_0 = 2.84$ km/sec is the velocity of the ideal Chapman–Jouguet detonation for a stoichiometric $2H_2$–$O_2$ mixture [3]. Thus, a nonideal detonation wave propagates over the layer of the hydrogen–oxygen mixture injected through the upper boundary $\Gamma_1$; the wave propagates with a constant velocity $\langle D \rangle$ smaller than the Chapman–Jouguet detonation velocity.

To additionally verify that the solution reaches a steady detonation regime, the following quantities were calculated at the boundary $\Gamma_2$ at each time instant:

$$\langle G \rangle(L, t) = \frac{1}{l} \int_0^l \rho u\, dy;$$

$$\langle J \rangle(L, t) = \frac{1}{l} \int_0^l \frac{(p + \rho u^2)\, dy}{\langle G \rangle(L, t)};$$

$$V(L, t) = \frac{1}{l} \int_0^l \frac{\rho v\, dy}{\langle G \rangle(L, t)}.$$

Here $\langle G \rangle(L, t)$ is the mean specific flow rate of the mixture, $\langle J \rangle(L, t)$ is the mean specific impulse, and $V(L, t)$ is the degree of rotation of the gas-dynamic flow.

For the case examined, these quantities reach the following constant values with time:

$$\langle G \rangle(L, t) \approx 48.9 \text{ kg/(sec} \cdot \text{m}^2) < G_0;$$

$$\langle J \rangle(L, t) \approx 2500 \text{ m/sec}; \; V(L, t) \approx 0.$$

The last equality means that the gas flow at the exit of a constant-section combustor does not rotate in the continuous detonation regime with a rotating TDW.

For fixed parameters in the injection system (11) we find the "minimum" period $l_{\min}$. For this purpose, we reduce the parameter $l$, which physically means reduction of the combustor diameter $d_c$. To reduce the calculation time, the solution of the periodic problem with TDWs at the time $t = 0.5$ msec, which was obtained for $l = 10$ cm, was used as the initial data. The time evolution of the dimensionless pressure $P$ versus the time $t$ at the point with the coordinates $x = 0$, $y = 0$

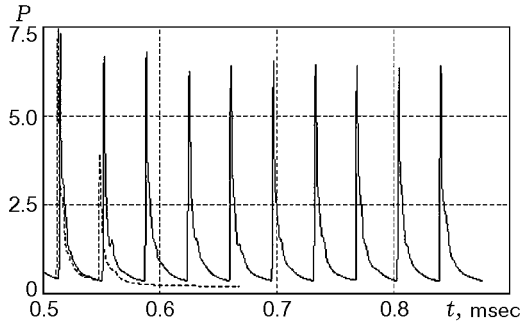


**Fig. 4.** Time evolution of the dimensionless pressure at the point $x = 0$, $y = 0$: solid and dashed curves refer to $l = 8$ and 7.5 cm, respectively.

for two calculation variants with $l = 8$ cm (solid curve) and $l = 7.5$ cm (dashed curve) is plotted in Fig. 4. For $l = 8$ cm, there are periodic oscillations of pressure with a period $\Delta t \approx 35.6$ $\mu$sec ($P_{\max} \approx 6.6$; $P_{\min} \approx 0.375$; $P_{\max}/P_{\min} = 17.6$; $\langle D \rangle = 2.25 \pm 0.02$ km/sec), i.e., the wave reaches the regime of continuous spin detonation. For $l = 7.5$ cm, there are two oscillations of pressure with decreasing amplitude, and then the pressure monotonically decreases and reaches an almost constant value $P \approx 0.21$. An analysis of the solution showed that "shedding" of the rotating TDW took place in the situation with $l = 7.5$ cm, and the combustion front and combustion products were entrained downstream from the domain $\Omega$. After that, the solution domain displays a conventional gas-dynamic flow of the original nonreacting hydrogen–oxygen mixture incoming through the upper boundary $\Gamma_1$ and outgoing through the lower boundary $\Gamma_2$. In varying the parameter $l$ in calculations, we found its minimum value $l_{\min}$ such that there is no periodic solution with a TDW rotating in one direction for all $l < l_{\min}$. In particular, for a stoichiometric $H_2$–$O_2$ mixture with parameters (11), the calculated minimum value $l_{\min}$ is within the range $7.8 < l_{\min} \leqslant 8$ cm. The existence of $l_{\min}$ means the existence of the lower limit in terms of the diameter of the cylindrical combustor ($d_{c,\min} = l_{\min}/\pi$) for the regime of spin detonation of the hydrogen–oxygen mixture. Note that the existence of the lower limit in terms of the combustor diameter for the regime of spin detonation of acetylene–oxygen mixtures was experimentally validated in [4, 5].

## TDW Structure

Let us consider the structure of a steady gas-dynamic flow with TDW propagation. Figure 5 shows the two-dimensional structure of the flow for $l = 8$ cm and $L/l = 0.75$ at the time $t = 0.787$ msec. The

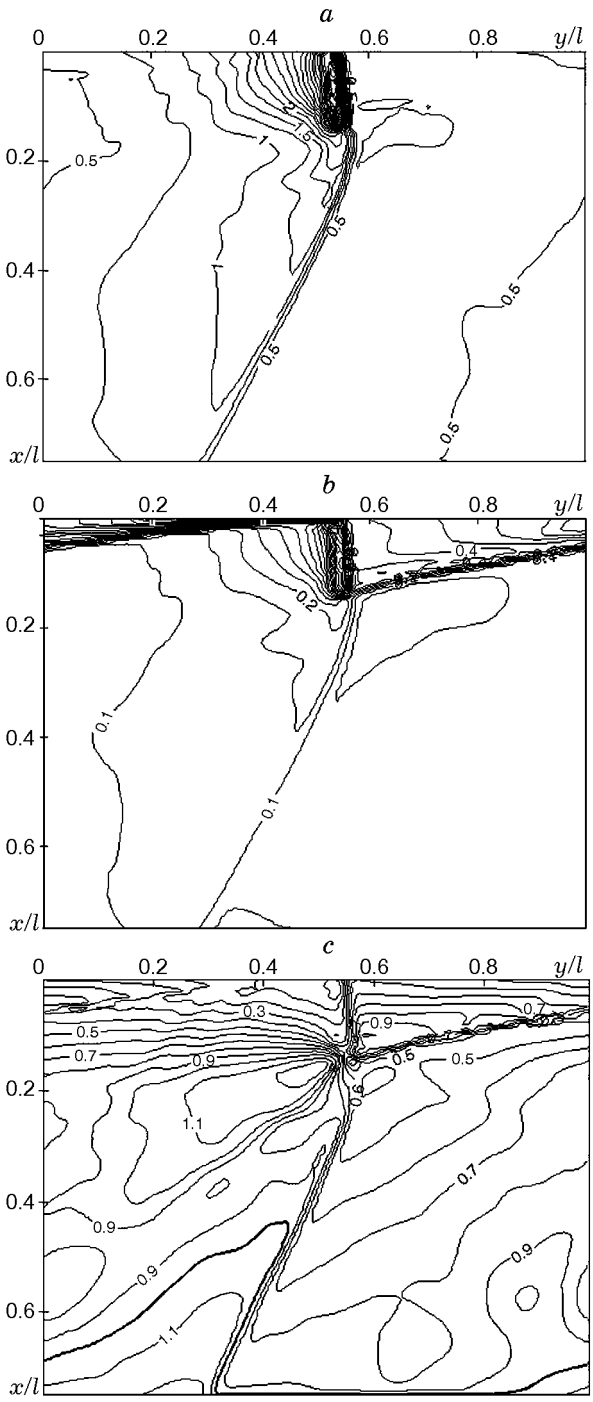


**Fig. 5.** Calculated two-dimensional structure of spin detonation in a cylindrical combustor with $l = 8$ cm: (a) isobars $p/p_0$; (b) isochores $\rho/\rho_0$; (c) isomachs $M_x = u/c$.

wave moves from left to right with the TDW velocity $D = 2.25$ km/sec over a triangular region containing the nonreacted $2H_2$–$O_2$ mixture incoming through the upper boundary $\Gamma_1$ (the interface between the combustible mixture and combustion products is clearly visible in Fig. 5b). The height of the layer of the combustible mixture ahead of the TDW for the above-chosen values of parameters is $h = 1.06$ cm. Behind the wave, the detonation products gradually expand and, as long as the pressure of the products is lower than the pressure of injection, are displaced downstream by new portions of the gases. Conditions for propagation of a new TDW in the next period are created. An oblique shock wave (SW) propagating over the detonation products emanates downstream from the TDW. The isobars (Fig. 5a) and isochores (Fig. 5b) display a rapid decrease in pressure and density behind the TDW front. Note that the gas-dynamic parameters ahead of the TDW are nonuniform. Figure 5c shows the isomachs for the projection of the velocity vector onto the $x$ axis ($M_x = u/c$). In the triangular region ahead of the TDW, the projection of the velocity vector onto the $x$ axis is smaller than the velocity of sound, i.e., the flow in this region is subsonic. In the direction from $\Gamma_1$ down the $x$ axis to $x/l < 0.45$, the flow is also subsonic, except for a local supersonic region with an $y$ length of 0.4 and an $x$ length of 0.2 in the vicinity of the point of TDW intersection with the oblique SW formed behind the TDW front because of lateral expansion of detonation products. At $x/l > 0.45$, a supersonic zone expanding in the downstream direction starts forming behind the front of the oblique SW; the Mach number $M_x$ in this zone gradually increases and reaches $M_x = 1.2$ behind the oblique shock wave. The isoline $M_x = 1$ is indicated by the bold curve. It is seen from Fig. 5c that the flow at the lower boundary $\Gamma_2$ is supersonic on the average. This means that a transonic transition occurs in the solution domain $\Omega$ with TDW propagation. Therefore, none of acoustic disturbances at the exit can affect the TDW parameters.

Note that the two-dimensional TDW structure in the $H_2$–$O_2$ mixture and the flow in the TDW vicinity are in qualitative agreement with the structure obtained previously in experiments [20] and numerical simulations [10] for a propane–oxygen mixture. In the experiment [20], there was another shock wave behind the oblique SW: a deceleration wave caused by the increasing thickness of the boundary layer behind the SW, recombination of the products, and afterburning of the mixture, which decelerate the supersonic flow. For these reasons, the flow behind the TDW front is not the Prandtl–Meyer flow and occurs in the region of intersection of characteristics, which leads to formation of

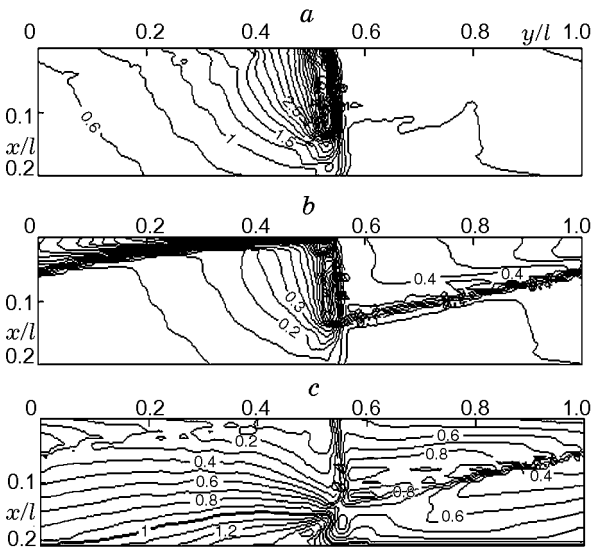


**Fig. 6.** Calculated two-dimensional structure of spin detonation in a cylindrical combustor with $l = 10$ cm and $L/l = 0.2$: (a) isobars $p/p_0$; (b) isochores $\rho/\rho_0$; (c) isomachs $M_x = u/c$.

"barrels" behind the TDW front, while the deceleration wave merges with the detached wave from the "barrel."

### Variation of the Combustor Length $L$

Let us consider the effect of the combustor length $L$ on the parameters and structure of the gas-dynamic flow with transverse detonation waves. We vary the length $L$ in the range $L/l \in [0.15-1]$, retaining all other values of the governing parameters fixed:

$$p_m/p_0 = 5, \quad T_m/T_0 = 1, \\ S_*/S = 0.0657, \quad l = 10 \text{ cm}. \tag{12}$$

The calculated velocity of spin detonation $D$, mean pressure $\langle p \rangle$, dimensionless TDW size $\eta = h/l$, and mean specific impulse $\langle J \rangle$ for a number of values of the geometric parameter $L/l$ are listed in Table 1.

The detonation regime with a rotating TDW is found to persist as the parameter $L/l$ is reduced from 1 to 0.2. The flow rate remains almost unchanged: $\langle G \rangle(L, t) = 48.9$ kg/(sec · m$^2$). An insignificant decrease is observed in the wave velocity $D$, mean pressure $\langle p \rangle$, and mean specific impulse $\langle J \rangle$, and a small increase is observed in the dimensionless TDW size $\eta$. Simultaneously, the degree of nonuniformity of gas-dynamic variables at the combustor exit becomes substantially greater as the geometric parameter $L/l$ decreases (see the last column in Table 1). Thus, for

TABLE 1

| $L/l$ | $D$, km/sec | $\langle p\rangle/p_0$ | $\eta = h/l$ | $p_m/\langle p\rangle$ | $\langle J\rangle$, m/sec | $J$, m/sec | $(p_2/p_1)_{\text{exit}}$ |
|---|---|---|---|---|---|---|---|
| 0.15 | — | 0.2 | — | 25 | 844 | — | 1 |
| 0.2 | 2.21 | 1.1 | 0.135 | 4.54 | 2460 | 2545 | 7.5 |
| 0.3 | 2.24 | 1.1 | 0.13 | 4.54 | 2480 | 2545 | 5.7 |
| 0.6 | 2.26 | 1.125 | 0.13 | 4.44 | 2500 | 2545.5 | 3.3 |
| 1 | 2.25 | 1.15 | 0.13 | 4.35 | 2530 | 2546 | 2.2 |

TABLE 2

| $p_m/p_0$ | $\langle G\rangle$, kg/(sec · m$^2$) | $\langle p\rangle/p_0$ | $l_{\min}$, cm | $\eta = h/l$ | $D$, km/sec | $\langle J\rangle$, m/sec | $J$, m/sec |
|---|---|---|---|---|---|---|---|
| 5 | 48.9 | 1.1 | $7.9 \pm 0.1$ | 0.133 | 2.25 | 2498 | 2545 |
| 7 | 68.4 | 1.6 | $5.9 \pm 0.1$ | 0.127 | 2.25 | 2508 | 2558 |
| 10 | 96.7 | 2.3 | $4.8 \pm 0.2$ | 0.125 | 2.3 | 2576 | 2570 |
| 15 | 145 | 3.5 | $3.9 \pm 0.1$ | 0.125 | 2.33 | 2598 | 2585 |

$L/l = 1$, the oblique SW at the exit degenerates into an acoustic wave [with a pressure jump at the wave front $(p_2 - p_1)/p_1)_{\text{exit}} = 1.2$], while the oblique SW is strong [$((p_2 - p_1)/p_1)_{\text{exit}} = 6.5$] at the exit for $L/l = 0.2$. Figure 6 shows the two-dimensional flow structure for $L/l = 0.2$. The wave moves from left to right with the TDW velocity $D = 2.21$ km/sec. A comparison of Figs. 5 and 6 shows that a decrease in the parameter $L/l$ makes the zone of supersonic exhaustion approach the lower edge of the TDW, which is responsible for stronger expansion of detonation products behind the TDW front.

For $L/l = 0.15$, "shedding" of the rotating TDW and entrainment of the combustion front and combustion products downstream from the domain $\Omega$ are observed. After that, the solution domain displays a conventional flow of the original nonreacting hydrogen–oxygen mixture incoming through the upper boundary $\Gamma_1$ and outgoing through the lower boundary $\Gamma_2$. Thus, there is a lower limit in terms of the combustor length $L_{\min}$ where the TDW can propagate in a self-sustaining continuous detonation regime. According to the data calculated with the injection parameters (12), the lower limit $L_{\min}$ normalized to the TDW size $h$ is within the range $1.15 < L_{\min}/h < 1.54$. Such a behavior of spin detonation waves, depending on the combustor length, was observed in experiments with a propane–oxygen mixture [20]. With decreasing $L$, the streamlines became more deflected downward, and the zone with $\mathrm{M}_x > 1$ became narrower. Finally, at $L = 20$ mm ($L/l = 0.18$), the original mixture prior to entering the TDW front acquired a supersonic velocity and partly left the combustor. The detonation did not disappear, however, and propagated with a velocity $D = 2.04$ km/sec over the mixture remaining in the combustor.

For comparison of specific impulses, Table 1 also contains the specific impulse $J$ of the conventional rocket engine in the nozzle throat, which was calculated for the same pressures in the combustor $\langle p\rangle/p_0$ as that in the detonation regime. The values of $J$ were calculated by the code developed in [21]. The results of this comparison show that the difference in the specific impulse of the detonation regime and conventional combustion stays within 2% for $L/l > 0.6$ and decreases to 1% for $L/l_1 \geqslant 1$. Thus, in the calculated range of pressures in the combustor $\langle p\rangle/p_0 \in (1.1–3.5)$, the reactive impulse due to detonation combustion of the hydrogen–oxygen mixture in the rotating TDW correlates with the impulse due to conventional combustion. By varying the injection pressure $p_m$ in computations, it was found that the linear TDW size $h$ and the minimum period $l_{\min}$ become smaller with increasing $p_m$. The data for $T_m/T_0 = 1$, $S_*/S = 0.0657$, and $L = 6$ cm are summarized in Table 2.

## Variation of the Degree of Expansion of the Exit Section of the Channel

All previous calculations were performed for a constant-section channel $S = \text{const}$. An analysis of

TABLE 3

| $S_{\text{exit}}/S_0$ | $\langle G\rangle$, kg/(sec · m$^2$) | $\langle p\rangle/p_0$ | $\eta = h/l$ | $D$, km/sec | $\langle J\rangle$, m/sec | $J$, m/sec |
|---|---|---|---|---|---|---|
| 1 | 145 | 3.5 | 0.125 | 2.33 | 2595 | 2585 |
| 1.5 | 145 | 3.5 | 0.125 | 2.34 | 2905 | 2916 |
| 2 | 145 | 3.5 | 0.125 | 2.36 | 3040 | 3094 |
| 4 | 145 | 3.5 | 0.125 | 2.36 | 3330 | 3438 |

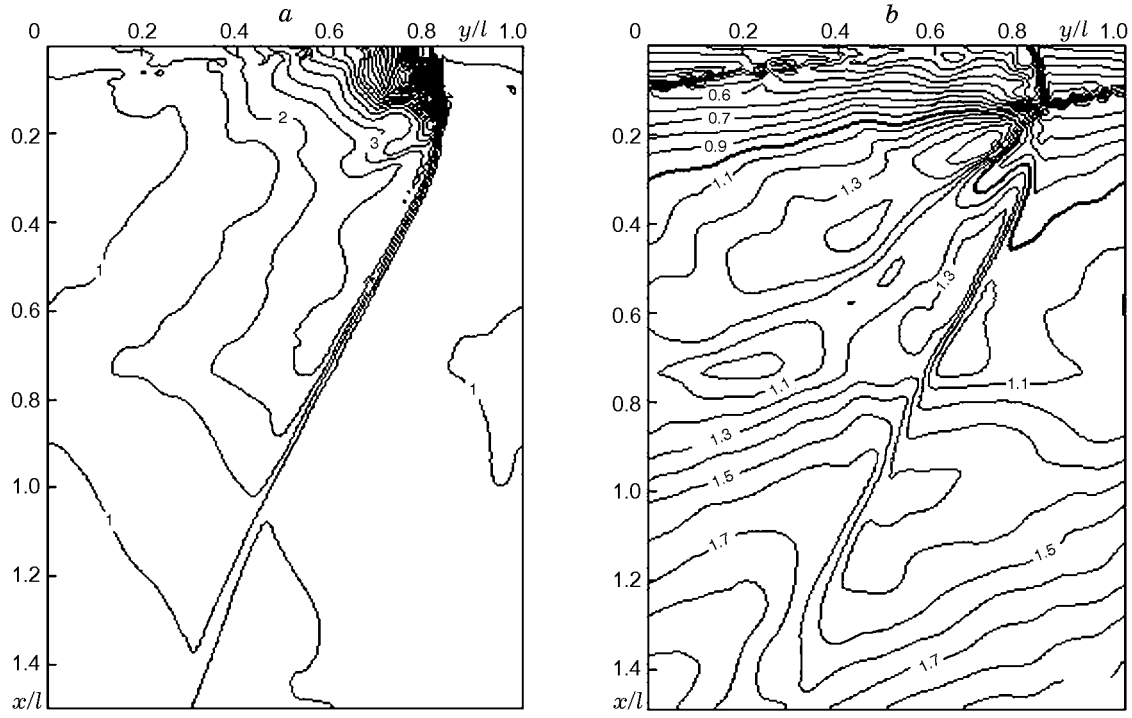


**Fig. 7.** Calculated two-dimensional structure of spin detonation in an expanding combustor with $l = 4$ cm, $L/l = 1.5$, and $S_{\text{exit}}/S_c = 1.5$: (a) isobars $p/p_0$; (b) isomachs $\mathrm{M}_x = u/c$.

the flow structure and TDW structure in the regime of spin detonation revealed the presence of the transonic transition in the constant-section combustor. Therefore, addition of an expanding nozzle to the cylindrical part of the combustor is expected, generally speaking, to increase thrust and specific impulse. To check this assumption, let us consider the effect of expansion of the combustor channel on the parameters and structure of the rotating TDW and, which is most important, on the specific impulse.

This problem was solved in a quasi-two-dimensional approximation with a degree of channel expansion $S = S(x)$. System (1) is naturally extended to this case:

$$\begin{gathered}\rho_t + S^{-1}(\rho u S)_x + (\rho v)_y = 0,\\ (\rho u)_t + S^{-1}(\rho u^2 S)_x + (\rho u v)_y + p_x = 0,\\ (\rho v)_t + S^{-1}(\rho u v S)_x + (\rho v^2)_y + p_y = 0,\\ (\rho E)_t + S^{-1}[\rho u(E + p/\rho)S]_x\\ + [\rho v(E + p/\rho)]_y = 0,\\ Y_t + uY_x + vY_y = f_5,\\ \mu_t + u\mu_x + v\mu_y = f_6.\end{gathered} \tag{13}$$

We considered the following geometric parameters of the channel: upper part of length $L_1$ with a constant cross section $S = S_0$, and then channel expansion with a constant expansion angle to the exit cross-sectional area $S = S_{\text{exit}}$. Thus, the functional dependence of the area $S(x)$ can be presented as $S(x) = S_0$ for $0 < x < L_1$ and $S(x) = S_0 + (S_{\text{exit}} - S_0)(x - L_1)/(L - L_1)$ for $L_1 < x < L$.

The calculations were performed for $p_m/p_0 = 15$, $T_m/T_0 = 1$, $S_*/S = 0.0657$, $L = 6$ cm, $l = 4$ cm,

and $L_1 = 3$ cm; the exit cross-sectional area $S_{\text{exit}}$ was varied. Some results are presented in Fig. 7 and in Table 3. It is seen that addition of an expanding channel does not alter the TDW velocity and size, the flow rate, and the mean pressure at the upper end face of the combustor, because the flow remains unchanged above the neutral Mach line. At the same time, the specific impulse $\langle J \rangle$ monotonically increases with increasing exit cross-sectional area $S_{\text{exit}}$, as it occurs in conventional combustion in a liquid-propellant jet engine (see the last column in Table 3). Thus, continuous spin detonation in a constant-section combustor, in contrast to conventional combustion, does not require a constricting part of the nozzle, which experiences the highest heat loads, and the use of an expanding nozzle is sufficient to increase the thrust.

## CONCLUSIONS

1. A two-dimensional unsteady mathematical model of spin detonation in an annular cylindrical ramjet-type combustor is formulated.

2. The dynamics of the detonation wave in the combustor filled by a hydrogen–oxygen gas mixture is studied numerically. At supercritical parameters of initiation, a periodic flow with a rotating transverse detonation wave is found to form if the combustor length is greater than one and a half TDW sizes.

3. A two-dimensional TDW structure is calculated. It is shown that the transonic transition occurs in the constant-section channel, while an increase in the flow rate of the components of the mixture increases the mean pressure in the combustor and decreases the TDW size. The minimum combustor diameter is determined as a function of the governing parameters of the problem: stagnation pressure of the mixture in the injection system and combustor length. The difference in specific impulses in the detonation regime and conventional combustion stays within 1%.

4. For the case of spin detonation, addition of an expanding nozzle to the constant-section channel is demonstrated to increase the specific impulse with increasing exit cross-sectional area, as it occurs in conventional combustion in a liquid-propellant jet engine.

5. The calculated TDW structure and the flow in its vicinity are in qualitative agreement with experimental data obtained previously for propane–oxygen mixtures. In our opinion, the present numerical study obviously demonstrates that it is necessary to perform experiments in hydrogen–oxygen mixtures, aimed at obtaining spin detonation.

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