Celestial Mechanics and Dynamical Astronomy (2023) 135:17
https://doi.org/10.1007/s10569-022-10105-4

CORRECTION

Correction: Bifurcation of frozen orbits in a gravity field with zonal harmonics

Irene Cavallari[1] · Giuseppe Pucacco[2]

Published online: 22 March 2023

Correction to: Celestial Mechanics and Dynamical Astronomy (2022) 134:49
https://doi.org/10.1007/s10569-022-10103-6

Under the heading **4.2 The *J*4-problem**, the formula following the paragraph "After introducing it in the Hamiltonian, …" should read as follows:

$$
\begin{aligned}
g(Z,a) = {} & \frac{-5\rho^2+2Z+1}{\sqrt{2}\left(\rho^2+2Z+1\right)^{\frac{5}{2}}} - \frac{3\lambda}{16\sqrt{2}\left(\rho^2+2Z+1\right)^{\frac{11}{2}}}\left(40Z^3+\left(-84\rho^2+20\right)Z^2\right.\\
& +\left(-74\rho^4-44\rho^2-10\right)Z-11\rho^6+273\rho^4-\rho^2-5\\
& \left.+4\sqrt{2\rho^2+4Z+2}\left(-5\rho^2+2Z+1\right)^2\right)+\frac{3\lambda j_4}{16\sqrt{2}\left(\rho^2+2Z+1\right)^{\frac{11}{2}}}\\
& \left(-72Z^3+12\left(51\rho^2+1\right)Z^2+3\left(-58\rho^4-156\rho^2+22\right)Z-249\rho^6\right.\\
& \left.+743\rho^4-387\rho^2+21\right),\\
f(Z,a) = {} & -\frac{3}{2}\lambda\frac{\left(-29\rho^2+2Z+1\right)}{\sqrt{2}\left(\rho^2+2Z+1\right)^{\frac{11}{2}}}+\frac{15}{2}\lambda j_4\frac{\left(-13\rho^2+2Z+1\right)}{\sqrt{2}\left(\rho^2+2Z+1\right)^{\frac{11}{2}}},
\end{aligned}
$$

Under the heading **4.2.3 Existence of the equilibrium points of type E_+ and E_-**, the formula following the paragraph "Concerning the equilibrium points of type E_-, we

The original article can be found online at https://doi.org/10.1007/s10569-022-10103-6.

✉ Irene Cavallari
irene.cavallari@dm.unipi.it

Giuseppe Pucacco
pucacco@roma2.infn.it

1 Dipartimento di Matematica, Università di Pisa, Pisa, Italy

2 Dipartimento di Fisica and INFN – Sezione di Roma II, Università di Roma "Tor Vergata", Via della, Ricerca Scientifica, 1, 00133 Rome, Italy

have…"should read as follows:

$$\hat{A}_{-} = 80G^4 + \lambda\big((595G^2 - 1035)j_4 - 175G^2 - 96G + 189\big),$$
$$\hat{B}_{-} = \frac{\hat{A}_{-}^2 - 5\lambda\hat{C}_{-}\hat{D}_{-}}{16},$$
$$\hat{C}_{-} = (315G^2 - 539)j_4 - 63G^2 - 72G - 11,$$
$$\hat{D}_{-} = 32G^4 + \lambda\big((95G^2 - 175)j_4 - 35G^2 - 24G + 49\big).$$

Under the heading **4.2.4 About the existence of $\bar{E}_1$ and $\bar{E}_2$**, the formula following the paragraph "If existing, the equilibrium points. E_1 and E_2 have coordinates…" should read as follows:

$$\bar{X} = -\frac{\rho^2}{\lambda\big(4375j_4^5 - 5375j_4^4 + 2550j_4^3 - 590j_4^2 + 67j_4 - 3\big)} \Bigg(-2000\rho^4(j_4 - 1)(7j_4 - 3)^3 + \lambda\big((5j_4 - 1)(55125j_4^4\rho^2 - 28700j_4^3\rho^2 - 14875j_4^4 - 23130j_4^2\rho^2 + 3350j_4^3 + 17460j_4\rho^2 + 7000j_4^2 - 2835\rho^2 - 2950j_4 + 307) + 48\sqrt{\frac{7j_4 - 3}{5j_4 - 1}}\sqrt{5}|\rho|(125j_4^4 - 250j_4^3 + 160j_4^2 - 38j_4 + 3)\big)\Bigg),$$

Under the heading **4.3 The J_2-problem with relativistic terms**, the formula following the paragraph "We study now the zonal problem containing both the…" should read as follows:

$$\mathcal{K}_c = -\frac{\mu^2}{2L^2} + \frac{\mu^4 J_2 R_P^2 (G^2 - 3H^2)}{4G^5L^3} + \frac{\mu^4}{c^2L^4G}(5G - 8L) + \frac{3\mu^6 J_2^2 R_P^4}{128L^5G^{11}}\Big[-5G^6 - 4G^5L + 24G^3H^2L - 36GH^4L - 35H^4L^2 + G^4(18H^2 + 5L^2) - 5G^2(H^4 + 2H^2L^2) + 2(G^2 - 15H^2)(G^2 - L^2)(G^2 - H^2)\cos 2g\Big] + \frac{\mu^6 J_2 R_P^2}{8c^2L^5G^7}\Big[(G^2 - 3H^2)(6L^2 - 5G^2) - 6(G^2 - 3H^2)(4G^2 - 3GL - 5L^2) - 9(L^2 - G^2)(G^2 - H^2)\cos 2g\Big].$$

Under the heading **4.3.3 About the existence of the equilibrium points of type E_+ and E_-**, the formula following the paragraph "We obtain $\tilde{S}_+$(G; a) = 0 for $\rho^2 = \tilde{\rho}^2_{E+1,2,}$ with…" should read as follows:

$$\tilde{\rho}^2_{E_{+1,2}} = \frac{G^2}{5\lambda}\frac{\tilde{A}_+ \pm 4\sqrt{\tilde{B}_+}}{\tilde{C}_+},$$
$$\tilde{A}_+ = -(-1040G^4 j_C + 864G^3 j_C + 1848G^2 j_C + 49G^2 - 96G - 99)\lambda - 80G^4,$$
$$\tilde{B}_+ = \frac{\tilde{A}_+^2 + 5\lambda\tilde{C}_+\tilde{D}_+}{16}, \qquad \tilde{C}_+ = -45G^2 + 72G + 143,$$
$$\tilde{D}_+ = (-320G^4 j_C + 384G^3 j_C + 720G^2 j_C - 15G^2 - 24G + 21)\lambda - 128G^6 j_C + 32G^4.$$

Under the heading **4.3.5 About the stability of the equilibrium points of type E_+ and E_- and of $\overline{E}_1$ *and* $\overline{E}_2$**, the formula following the paragraph "Let us consider value of j_C sufficiently…." should read as follows:

$$
\begin{aligned}
\frac{d^2\tilde{X}}{dZ^2} \pm \frac{d^2\hat{X}}{dZ^2} \sim\ & 16\sqrt{-80 j_C \rho^2 + 1}\Big(- 144000 j_C^3 \rho^6 + 28400 j_C^2 \rho^4 - 880 j_C \rho^2 + 7 \\
& + \sqrt{-80 j_C \rho^2 + 1}(10000 j_C^2 \rho^4 - 600 j_C \rho^2 + 7)\Big). \\
\frac{d^2\tilde{X}}{dZ^2} \pm \frac{d^2\hat{X}}{dZ^2} \sim\ & 16\sqrt{-80 j_C \rho^2 + 1}\Big(144000 j_C^3 \rho^6 - 28400 j_C^2 \rho^4 + 880 j_C \rho^2 - 7 \\
& + \sqrt{-80 j_C \rho^2 + 1}(10000 j_C^2 \rho^4 - 600 j_C \rho^2 + 7)\Big)
\end{aligned}
$$