



Correction: Bifurcation of frozen orbits in a gravity field with zonal harmonics

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Under the heading **4.2 The J_4 -problem**, the formula following the paragraph “After introducing it in the Hamiltonian, ...” should read as follows:

$$g(Z, a) = \frac{-5\rho^2 + 2Z + 1}{\sqrt{2}(\rho^2 + 2Z + 1)^{\frac{5}{2}}} - \frac{3\lambda}{16\sqrt{2}(\rho^2 + 2Z + 1)^{\frac{11}{2}}} (40Z^3 + (-84\rho^2 + 20)Z^2 + (-74\rho^4 - 44\rho^2 - 10)Z - 11\rho^6 + 273\rho^4 - \rho^2 - 5 + 4\sqrt{2\rho^2 + 4Z + 2}(-5\rho^2 + 2Z + 1)^2) + \frac{3\lambda j_4}{16\sqrt{2}(\rho^2 + 2Z + 1)^{\frac{11}{2}}} (-72Z^3 + 12(51\rho^2 + 1)Z^2 + 3(-58\rho^4 - 156\rho^2 + 22)Z - 249\rho^6 + 743\rho^4 - 387\rho^2 + 21),$$
$$f(Z, a) = -\frac{3}{2}\lambda \frac{(-29\rho^2 + 2Z + 1)}{\sqrt{2}(\rho^2 + 2Z + 1)^{\frac{11}{2}}} + \frac{15}{2}\lambda j_4 \frac{(-13\rho^2 + 2Z + 1)}{\sqrt{2}(\rho^2 + 2Z + 1)^{\frac{11}{2}}},$$

Under the heading **4.2.3 Existence of the equilibrium points of type E_+ and E_-** , the formula following the paragraph “Concerning the equilibrium points of type E_- , we

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have...”should read as follows:

$$\begin{aligned} \hat{A}_- &= 80G^4 + \lambda((595G^2 - 1035)j_4 - 175G^2 - 96G + 189), \\ \hat{B}_- &= \frac{\hat{A}_-^2 - 5\lambda\hat{C}_-\hat{D}_-}{16}, \\ \hat{C}_- &= (315G^2 - 539)j_4 - 63G^2 - 72G - 11, \\ \hat{D}_- &= 32G^4 + \lambda((95G^2 - 175)j_4 - 35G^2 - 24G + 49). \end{aligned}$$

Under the heading **4.2.4 About the existence of \bar{E}_1 and \bar{E}_2** , the formula following the paragraph “If existing, the equilibrium points. E_1 and E_2 have coordinates...” should read as follows:

$$\begin{aligned} \bar{X} = & -\frac{\rho^2}{\lambda(4375j_4^5 - 5375j_4^4 + 2550j_4^3 - 590j_4^2 + 67j_4 - 3)} \\ & \left(-2000\rho^4(j_4 - 1)(7j_4 - 3)^3 + \lambda((5j_4 - 1)(55125j_4^4\rho^2 - 28700j_4^3\rho^2 - 14875j_4^4 \right. \\ & - 23130j_4^2\rho^2 + 3350j_4^3 + 17460j_4\rho^2 + 7000j_4^2 - 2835\rho^2 - 2950j_4 + 307) \\ & \left. + 48\sqrt{\frac{7j_4 - 3}{5j_4 - 1}}\sqrt{5}|\rho|(125j_4^4 - 250j_4^3 + 160j_4^2 - 38j_4 + 3) \right), \end{aligned}$$

Under the heading **4.3 The J_2 -problem with relativistic terms**, the formula following the paragraph “We study now the zonal problem containing both the...” should read as follows:

$$\begin{aligned} \mathcal{K}_c = & -\frac{\mu^2}{2L^2} + \frac{\mu^4 J_2 R_p^2 (G^2 - 3H^2)}{4G^5 L^3} + \frac{\mu^4}{c^2 L^4 G} (5G - 8L) + \frac{3\mu^6 J_2^2 R_p^4}{128L^5 G^{11}} \left[-5G^6 - 4G^5 L \right. \\ & + 24G^3 H^2 L - 36GH^4 L - 35H^4 L^2 + G^4 (18H^2 + 5L^2) \\ & \left. - 5G^2 (H^4 + 2H^2 L^2) + 2(G^2 - 15H^2)(G^2 - L^2)(G^2 - H^2) \cos 2g \right] \\ & + \frac{\mu^6 J_2 R_p^2}{8c^2 L^5 G^7} \left[(G^2 - 3H^2)(6L^2 - 5G^2) - 6(G^2 - 3H^2) \right. \\ & \left. (4G^2 - 3GL - 5L^2) - 9(L^2 - G^2)(G^2 - H^2) \cos 2g \right]. \end{aligned}$$

Under the heading **4.3.3 About the existence of the equilibrium points of type E_+ and E_-** , the formula following the paragraph “We obtain $\tilde{S}_+(G; a) = 0$ for $\rho^2 = \tilde{\rho}_{E_{+1,2}}^2$, with...” should read as follows:

$$\begin{aligned} \tilde{\rho}_{E_{+1,2}}^2 &= \frac{G^2 \tilde{A}_+ \pm 4\sqrt{\tilde{B}_+}}{5\lambda \tilde{C}_+}, \\ \tilde{A}_+ &= -(-1040G^4 j_C + 864G^3 j_C + 1848G^2 j_C + 49G^2 - 96G - 99)\lambda - 80G^4, \\ \tilde{B}_+ &= \frac{\tilde{A}_+^2 + 5\lambda\tilde{C}_+\tilde{D}_+}{16}, \quad \tilde{C}_+ = -45G^2 + 72G + 143, \\ \tilde{D}_+ &= (-320G^4 j_C + 384G^3 j_C + 720G^2 j_C - 15G^2 - 24G + 21)\lambda - 128G^6 j_C + 32G^4. \end{aligned}$$

Under the heading **4.3.5 About the stability of the equilibrium points of type E_+ and E_- and of \bar{E}_1 and \bar{E}_2** , the formula following the paragraph “Let us consider value of j_C sufficiently....” should read as follows:

$$\begin{aligned} \frac{d^2 \tilde{X}}{dZ^2} \pm \frac{d^2 \hat{X}}{dZ^2} &\sim 16\sqrt{-80j_C \rho^2 + 1} \left(-144000j_C^3 \rho^6 + 28400j_C^2 \rho^4 - 880j_C \rho^2 + 7 \right. \\ &\quad \left. + \sqrt{-80j_C \rho^2 + 1}(10000j_C^2 \rho^4 - 600j_C \rho^2 + 7) \right). \\ \frac{d^2 \tilde{X}}{dZ^2} \pm \frac{d^2 \hat{X}}{dZ^2} &\sim 16\sqrt{-80j_C \rho^2 + 1} \left(144000j_C^3 \rho^6 - 28400j_C^2 \rho^4 + 880j_C \rho^2 - 7 \right. \\ &\quad \left. + \sqrt{-80j_C \rho^2 + 1}(10000j_C^2 \rho^4 - 600j_C \rho^2 + 7) \right) \end{aligned}$$

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