Celest Mech Dyn Astr (2010) 107:409–425
DOI 10.1007/s10569-010-9279-1

ORIGINAL ARTICLE

Lissajous trajectories for lunar global positioning and communication systems

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Received: 22 June 2009 / Revised: 3 March 2010 / Accepted: 20 April 2010 /
Published online: 28 May 2010

Abstract This work proposes a Lunar Global Positioning System (LGPS) and a Lunar Global Communication System (LGCS) using two constellations of satellites on Lissajous trajectories around the collinear L_1 and L_2 libration points in the Earth–Moon system. This solution is compared against a Walker constellation around the Moon similar to the one used for the Global Positioning System (GPS) on the Earth to evaluate the main differences between the two cases and the advantages of adopting the Lissajous constellations. The problem is first studied using the Circular Restricted Three Body Problem to find out its main features. The study is then repeated with higher fidelity using a four-body model and higher-order reference trajectories to simulate the Earth-Moon-spacecraft dynamics more accurately. The LGPS performance is evaluated for both on-ground and in-flight users, and a visibility study for the LGCS is used to check that communication between opposite sides of the Moon is possible. The total ΔV required for the transfer trajectories from the Earth to the constellations and the trajectory control is calculated. Finally, the estimated propellant consumption and the total number of satellites for the Walker constellation and the Lissajous constellations is used as a performance index to compare the two proposed solutions.

Keywords Lissajous trajectories · Lagrange point · Lunar constellation · Lunar GPS · Visibility analysis

1 Introduction

Due to renewed interest in exploring our space neighbor, high precision navigation systems and global autonomous communication systems are becoming very important (NASA 2006). Many navigation systems using different technologies are currently under development. In

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particular, optical navigation systems are promising, but their limits depend on light conditions and knowledge of the surface (Van Damme et al. 2008; Shuang et al. 2005). A global positioning system around the Moon creates a wide range of possibilities, improving the navigation solution both on the ground and in flight. Additionally, it relaxes the requirement of Earth bound communication subsystems on board each lunar space mission, resulting in lower power consumption for telecommunications, smaller dimensions and less total mass. This work investigates the feasibility of a Lunar Global Positioning System (LGPS) and a Lunar Global Communication System (LGCS) which use two constellations of satellites on Lissajous trajectories around the collinear L_1 and L_2 libration points of the Earth–Moon system.

"Section 2 describes the fundamentals of GPS position measurements based on pseudo-range. In Sect. 3, the problem is studied using the Circular Restricted Three Body Problem. The equations of motion, their linearized solution for the Lissajous trajectories and the trajectory control technique are described. Finally, the LGPS system is introduced and the capabilities of the system are analyzed in terms of the LGPS signal availability. Section 4 discusses results using an improved four body model and more accurate reference trajectories. A trajectory control analysis is included with some discussion of the importance of accurate reference trajectories. Section 5 describes a lunar Walker constellation that provides time continuous multiple coverage of the lunar surface. A description of this constellation and a visibility study from the lunar surface is provided. Section 6 compares the Walker and the Lissajous constellations solutions. A comparison of the two possible solutions and a ΔV estimation for the two constellations' deployment are provided. Finally, Sect. 7 proposes a Lunar Global Communication System, which uses two constellations on Lissajous trajectories."

2 Background: fundamentals of GPS measurements

The GPS based positioning comes from the measurement of the so called pseudo-range. Let us consider the vector $\mathbf{X}_i^{\text{sat}} = \begin{bmatrix} x_i^{\text{sat}} & y_i^{\text{sat}} & z_i^{\text{sat}} \end{bmatrix}^T$, which denotes the position vector of one of the satellites in the GPS constellation, and the vector $X_{\text{user}} = \begin{bmatrix} x_{\text{user}} & y_{\text{user}} & z_{\text{user}} \end{bmatrix}^T$, which gives the position of an observer. The distance between the observer and the generic satellite of the constellation can be expressed in terms of the travel time that the GPS signal needs to reach the observer, so that the range between each satellite and the user can be written as:

$$d_i = \sqrt{\left(x_i^{\text{sat}} - x_{\text{user}}\right)^2 + \left(y_i^{\text{sat}} - y_{\text{user}}\right)^2 + \left(z_i^{\text{sat}} - z_{\text{user}}\right)^2} = cT$$

where c is the speed of light and T is travel time of the GPS signal from the source on board of the satellite to the user. As a consequence, a distance measurement is transformed to a time measurement, which is given by the difference between the arrival time of the GPS signal at the observer and the departure time at the satellite. Hence, it is clear that time measurement is of crucial importance in order to obtain a reliable position evaluation. Although the clocks on board of the satellites can be considered as very accurately synchronized and stable, there is an offset Δt between the satellites' clocks and the observer's clock, due to the fact that the latter is less accurate than the others. Hence, the signal travel time T includes an additional unknown that is needed in order to properly evaluate the range and, as a consequence, the observer needs one more satellite in order to solve the algebraic system of four equations in four unknowns: the three user's coordinates and the clock offset. The generic pseudo-range equation can than be written as:

$$p_R^{\text{sat},i} = \sqrt{\left(x_i^{\text{sat}} - x_{\text{user}}\right)^2 + \left(y_i^{\text{sat}} - y_{\text{user}}\right)^2 + \left(z_i^{\text{sat}} - z_{\text{user}}\right)^2} + c \cdot \Delta t \tag{1}$$

where $p_R^{\mathrm{sat},i}$ is the measured pseudo-range value. Equation (1) clearly shows that it is mandatory for the user to know with a good accuracy the real position of all the satellites involved in the pseudo-range measurement. For the LGPS, this can be done directly from the Earth using laser ranging with accuracy in the order of 5–10 cm (Smith and Zuber 2004), or from a ground station onto the lunar surface. The Lissajous trajectory is, in fact, constantly visible from almost an entire lunar hemisphere, so that range and range-rate measurements and orbit determination for such a trajectory can be done without interruptions. As it will be clear later on, if the orbit reconstruction is performed using ground stations on the lunar surface, just two of them are needed since two constellations around L_1 and L_2 are constantly visible from each side of the Moon. It should be noted that, unlike for Earth-bound applications, the absence of atmosphere around the Moon makes the signal not affected by the main environmental disturbances (the atmospheric and ionospheric effects) as on the Earth.

3 The circular restricted three body problem

3.1 Equations of motion

The circular restricted three body problem is widely documented in the literature (Szebehely 1967). Let us consider a mass m in orbit around two primaries of motion called m_1 and m_2, so that the condition $m << m_2 < m_1$ is valid. According to this condition, the smallest body can be considered as massless and its motion is governed by the gravity field generated by the two primaries, while its presence does not perturb the motion of the two primaries around their common center of mass. The motion of the third body is studied in the usual synodic rotating reference frame. Under the assumption that the two primaries of motion move on circular orbits around their common center of mass, then the problem is referred as Circular Restricted Three Body Problem (CRTBP). In addition, the dynamics is studied using the usual set of normalized units. In the CRTBP, the equations of motion for the mass m can be written as (Szebehely 1967):

$$\begin{cases} \ddot{x} - 2\dot{y} - x = -\frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu(x+\mu-1)}{r_2^3} \\ \ddot{y} + 2\dot{x} - y = -\left(\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3}\right) y \\ \ddot{z} = -\left(\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3}\right) z \end{cases} \tag{2}$$

where $\mu = \frac{m_2}{m_1+m_2}$ is the mass parameter of the system and r_1 and r_2 are the distances of the third body from the big and the small primaries in the rotating frame, respectively.

3.2 Periodic orbits around the collinear Lagrangian points

Equations (2) can be solved to find five equilibrium solutions: three of them are located along the rotating frame's x axis and the other two are at the vertex of two equilateral triangles having the primaries of motion as bases. Let us focus our attention on the first and the second collinear Lagrange points of the Earth–Moon system, named translunar and cislunar points. Since the Lagrange points are equilibrium solution for the system, equations (2) can be linearized around these points and solved for a periodic solution around the equilibrium point (Wie 2000a). This leads to the definition of a family of periodic orbits called Lissajous orbits given by the equations:

$$\begin{cases} \xi(t) = -A_x \sin\left(\omega_{xy} t + \varphi_{xy}\right) \\ \eta(t) = -A_y \cos\left(\omega_{xy} t + \varphi_{xy}\right) \\ \zeta(t) = A_z \sin\left(\omega_z t + \varphi_z\right) \end{cases} \tag{3}$$

where A_x, $A_y = kA_x$ and $A_z = A_y$ are the three amplitudes, ω_{XY} and ω_Z are the frequencies for the in-plane and the out-of-plane motion and φ_{XY} and φ_Z are the initial phases for the in-plane and the out-of-plane motion. It is important to note that for the Lissajous trajectories the in-plane and the out-of-plane frequencies are not equal, while the amplitude values are related by the parameter k, defined as:

$$k = \frac{\omega_{XY}^2 + \Omega_{XX}}{2\omega_{XY}}$$

where Ω_{XX} is the second derivative with respect to X of the potential function $\Omega(x, y, z) = \frac{1}{2}\left(x^2 + y^2\right) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{1}{2}\mu(1-\mu)$. As a consequence, any Lissajous trajectory is defined by just two parameters (one of the two in-plane amplitudes and the out-of-plane amplitude) and, therefore, Lissajous trajectories define a family of orbits with two parameters. Figure 1 shows an example of a 3500 km amplitude Lissajous trajectory propagated over a period of one year.

It is clear that the orbit is periodic on both the horizontal and vertical planes, with a period of approximately 14 days for the in-plane motion and 15 days for the out-of-plane component of motion. It is important to note that these periodicities do not mean that the path is exactly repeated after these time intervals. As a matter of fact, if we look at the in-plane and the out-of-plane motions separately they are periodic with frequencies equal to ω_{XY} and ω_Z, but

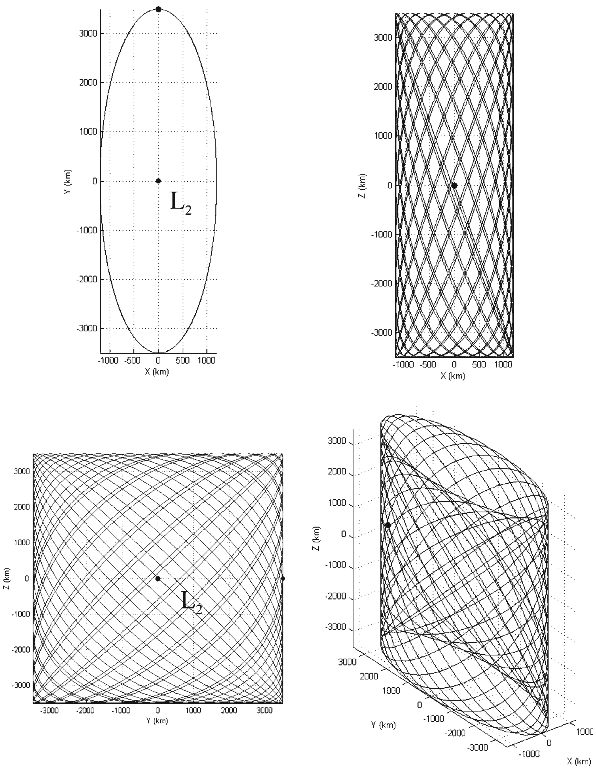

Fig. 1 3500 km amplitude linearized Lissajous solution around the collinear lagrange point L_2

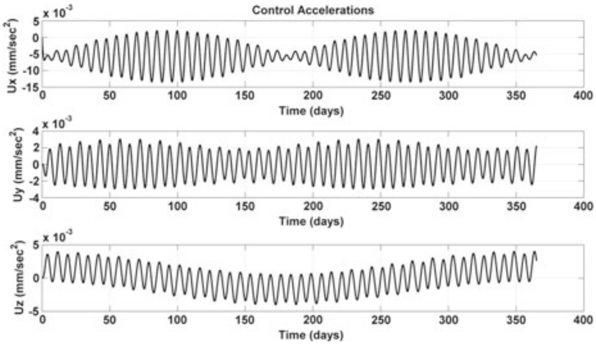

Fig. 2 Control accelerations for one year control of a 3500 km amplitude Lissajous trajectory around L_2

the complete three-dimensional Lissajous path is repeated after about 365 days. Additionally, due to the natural instability of the collinear libration points, some kind of trajectory control is needed to let the spacecraft follow the desired Lissajous trajectory, otherwise its motion diverges very rapidly and naturally from the collinear libration point.

3.3 Control of the linearized Lissajous trajectory

Given the linearized equations of motion, the control problem can be easily expressed in matrix form as a system of first order differential equations:

$$\dot{\mathbf{X}} = A\mathbf{X} + B\mathbf{u} \tag{4}$$

where $\mathbf{X}$ is the state vector $\mathbf{X} = [x\ y\ z\ \dot{x}\ \dot{y}\ \dot{z}]^T$ and $\mathbf{u} = \left[u_x\ u_y\ u_z\right]^T$ is the control vector containing the control accelerations along the frame's axes. The simplest usable controller is a linear controller (Wie 2000b), which makes it possible to define the control accelerations proportionally to the error vector through a gain matrix K:

$$\mathbf{u} = -K\left(\mathbf{X} - \mathbf{X}_r\right) \tag{5}$$

where $\mathbf{X}_r$ represents the reference state vector at the current time. The gain matrix K can be evaluated using the Linear Quadratic Regulator technique, which leads to the matrix:

$$K = R^{-1}B^T S \tag{6}$$

R being the weighted matrix of the controls. Assuming that the control accelerations have the same weight, matrix R can be represented as a 3-by-3 identity matrix. The preceding equation for the gain matrix shows the presence of another matrix,S, which is the solution of the Algebraic Riccati Equation (ARE), defined as:

$$SA + A^T S - SBR^{-1}B^T S + Q = 0$$

Q being the weighted matrix of the states. In this case, the velocity components of the state vector are weighted one thousand times more than the position ones. Using the Linear Quadratic Regulator technique it is possible to obtain the following gain matrix:

$$K = \begin{bmatrix} 39.1666 & -8.1129 & 0 & 8.8712 & 0.1529 & 0 \\ 8.3062 & 28.8585 & 0 & 0.1529 & 7.7010 & 0 \\ 0 & 0 & 28.9846 & 0 & 0 & 7.6791 \end{bmatrix}$$

Using this value of the gain matrix K, it is possible to integrate the motion of the spacecraft according to the non linear equations of motion (2) and to control the trajectory with the resulting control accelerations (Fig. 2).

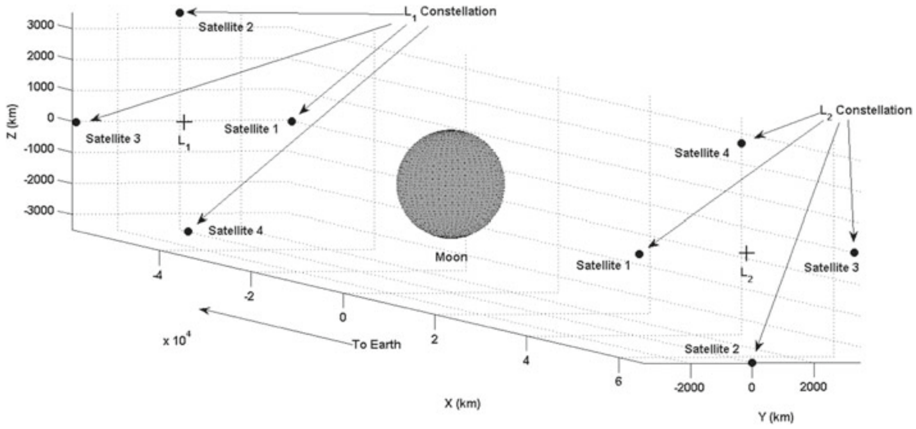

Fig. 3 Initial geometry for the two Lissajous constellations

Figure 2 shows that in the CRTBP the control accelerations are periodic along the three axes. In addition, the control accelerations are in the order of $10^{-6}\,\mathrm{m/s^{-2}}$.

To compare different control strategies, it is useful to have a parameter that describes the efficiency of the adopted control and the selected parameter is the total amount of ΔV needed for the trajectory control maneuver over one year. One can evaluate the ΔV needed for the trajectory control by integrating the absolute values the control accelerations over the simulation time. In the framework of the CRTBP, using the same 3500 km amplitude reference trajectory shown in Fig. 1 and the control accelerations in Fig. 2, the ΔV needed for one year trajectory control of a single satellite is equal to $\Delta V = 210.2890\,\mathrm{m/s}$. It is important to note that the computed ΔV is remarkably high because the linearized solution is valid only for very small amplitude values.

3.4 The lunar GPS with two Lissajous constellations

The Lissajous trajectories can be efficiently used to create two constellations of satellites around the collinear L_1 and L_2 libration points to provide the Lunar Global Positioning System. As previously stated, a minimum number of four satellites is required in order to properly evaluate the position solution. Hence, let us consider two constellations, named C_1 and C_2, and four satellites on each of them. According to equations (3), let us specify the geometrical distribution of the spacecraft in the constellations using the phase angles for the in-plane and the out-of-plane motions. Particularly, let us assume that the four satellites are equally spaced in both the in-plane and the out-of-plane motions, so that the phase angles are given by:

$$\begin{aligned}\varphi_{XY} &= \left[0^\circ\ 90^\circ\ 180^\circ\ 270\right]\\ \varphi_{Z} &= \left[0^\circ\ 90^\circ\ 180^\circ\ 270\right]\end{aligned}$$

Since the Lissajous trajectories belong to a double parameter orbit family let us define the amplitude so that $A_Y = 3500\,\mathrm{km}$, $A_X = A_Y/k$ and $A_Z = A_Y$. Once the eight orbits are completely defined, it is possible to represent the two constellations' initial configuration (Fig. 3).

Since there is a minimum number of visible satellites required to solve the pseudo-range algebraic system, it is important to study the visibility conditions from the surface of the Moon, that is the actual number of satellites visible from an arbitrary location on the surface of the

Moon. The Moon's surface has been sampled in both longitude and latitude and the motion of the spacecraft propagated so that a complete trajectory is performed, that is the initial constellation's geometry is repeated. Numerical simulations showed that this condition is achieved after about 350 days; therefore the simulation time has been set equal to 365 days (Fig. 4).

Given the constellation's geometry and the observer's location at a specific time of the simulation, the following visibility criterion has been adopted: one satellite of the constellation is visible from the current location if its elevation over the local horizontal plane is positive and greater than the minimum allowed elevation angle ε. The simulations have been performed considering the Moon as a spherical body and using a minimum elevation angle $\varepsilon_{\min} \equiv 0°$ without lack of generality. It is important to note that if the visibility analysis is performed in the rotating frame where the Lissajous trajectories are defined, it is possible to greatly simplify the visibility study. As a matter of fact, since the Moon is rotating about its spin axis with the same angular velocity with which it rotates around the Earth along its orbital motion, each of the two constellations will cover one and only one of the two Moon's hemispheres. In particular, the constellation around L_1 is responsible for the visible side of the Moon, while the one about L_2 covers the far side of the Moon. Numerical simulations with the selected Lissajous constellations showed that the visibility requirements of four satellites always in sight from the surface of the Moon are met up to latitude values of $\varphi = \pm 88°$. Besides, since the Moon has been considered as a sphere, the same limit is present in the longitudinal direction. However, taking the libration and nutation motions into consideration, it is clear that these constraints can be solved thanks to the real motion of the Moon's spin axis. In addiction, the operational limit in the maximum latitude and longitude values is released when flying users are considered. As a matter of fact, increasing the altitude of the observer makes possible to provide the LGPS and LGCS services in the Polar Regions as well using the proposed configuration. The navigation solution and link capabilities for every spacecraft orbiting the Moon are then possible. The visibility study for the LGPS has been made considering an ideal case where the satellites can be tracked on the horizon with a minimum elevation of $\varepsilon = 0°$. Numerical simulations showed that increasing this angle value to a typical value of $\varepsilon = 10°$ leads to a reduction of the area covered by the LGPS service using the Lissajous constellations solution: the maximum latitude and longitude values on each side of the Moon where the LGPS service is available, in fact, decrease from $\varphi = \pm 88°$ to approximately $\varphi = \pm 77°$. However, it is important to note that these considerations are valid only for on-ground observers. As a matter of fact, moving observers such as a Moon orbiter do not require a minimum elevation angle greater than zero due to the fact that their position with respect to the Lissajous constellations is more favorable. As a consequence, for purely flying applications the proposed LGPS based on the two Lissajous constellation fulfills the requirements without any limitations and provides a service available also in the Polar Regions. The possibility of using large amplitude Lissajous trajectories instead of small amplitude ones to guarantee the availability of the LGPS service to polar ground based users can be taken into consideration. Using large amplitude trajectories, in fact, it would be possible to provide the LGPS service on the Polar Regions with higher values of the minimum elevation angle. This solution, however, is likely to complicate the use of the two constellations: a high latitude observer, in fact, should use satellites from both the two constellations to obtain the position solution, since part of each constellation is eclipsed by the Moon. Recently Ozimek et al. (2009) proposed the use of numerically derived non-Keplerian orbits over the lunar south pole to provide communication capabilities to a lunar base located at the south pole of the Moon. The peculiar geometry of the proposed configuration ensures a constant visibility of the relay-satellite for a ground based user with high values of the elevation angle, although limited only to the polar region. This concept clearly

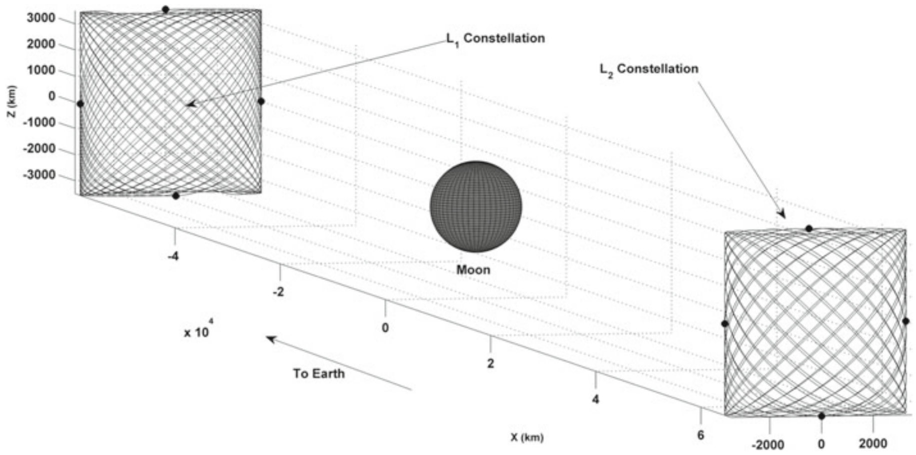

Fig. 4 Representation of the two Lissajous constellation propagated for 1 year

makes it possible to have communication capabilities between south pole users and the Earth, however it cannot provide a global coverage neither for positioning nor for communication purposes.

4 The four body framework

The CRTBP framework is a useful starting point to describe the system's main features and the spacecraft's motion in it. Nevertheless, the CRTBP is not an accurate description of the real Earth–Moon system. To obtain a more accurate description of the dynamics, it is necessary to include the two major perturbations due to the eccentricity of the Moon's orbit around the Earth and the gravitational attraction by the Sun on the Earth–Moon system as well. In this framework, the motion of a body in the proximity of one of the collinear libration points can be studied in a rotating reference frame analogous to the synodic one, but centered on the selected Lagrange point. The equations of motion implemented in the numerical simulations are those proposed by Farquhar in early seventies (Farquhar and Kamel 1973): in this model, the eccentricity of both the Earth's and Moon's orbits are considered, as well as the influence of the Sun's gravity and the effects of the nutation and precession of the lunar orbital plane.

4.1 Analytic third order Lissajous solution

The nonlinear equations of motion of the four body problem were used by Farquhar in the seventies to derive a more detailed representation of a reference Lissajous trajectory, during his studies of translunar communication satellite (Farquhar 1973; Farquhar and Kamel 1973). This solution is an analytical third order solution with respect to the parameter m, defined as the ratio between the mean motion of the Moon and the mean motion of the Sun:

$$\begin{aligned} x &= mx_1 + m^2x_2 + m^3x_3 \\ y &= my_1 + m^2y_2 + m^3y_3 \\ z &= mz_1 + m^2z_2 + m^3z_3 \end{aligned} \tag{7}$$

where subscripts 1, 2, and 3 denote the first, the second and the third order contributions to the solution, respectively. The third order solution in (7) is valid only for small amplitude

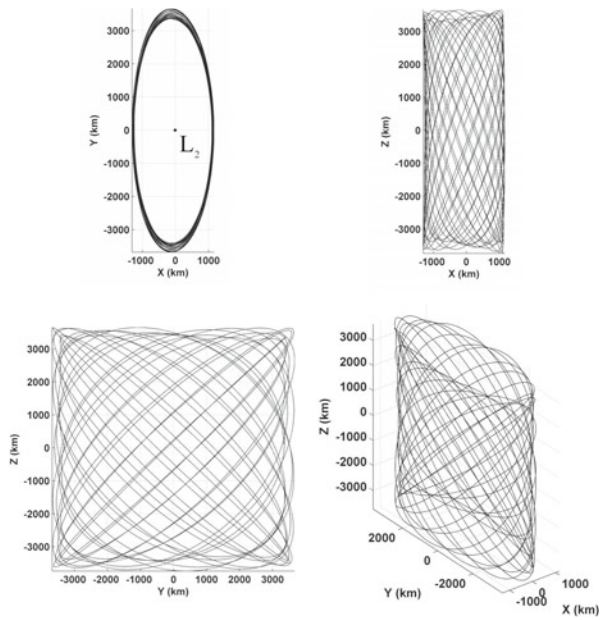

Fig. 5 Third order Lissajous trajectory with amplitude A_Y = 3500 km

values and has been successfully adopted for Lissajous trajectories with amplitude up to 5000 km (Farquhar and Kamel 1973, pp. 468–469). In case of bigger amplitudes, in fact, the effects related to the non linearities in the equations of motion become no more negligible and, as a consequence, the third order solution is no more valid. The expression for the third order solution can be obtained substituting solution (7) in the equations of motion and then solving three systems of ordinary linear non homogeneous differential equations in the variables x_i, y_i, z_i with $i = 1, 2, 3$. It is important to note that solving the second- and the third-order systems some divergent terms appear. The suppression of these terms is possible by properly choosing some constants in the solution and forcing it to have a periodic contribution (Farquhar and Kamel 1973). As a consequence, the three solutions can be found and combined to describe the third order solution (Fig. 5). Comparing Figs. 1 and 5 makes it easy to underline the differences between the two reference trajectories. The third order solution keeps the main characteristic of the linearized one but, due to perturbations and non linearities included in the dynamical model, the third order trajectory is affected by remarkable deviations. Nevertheless, the third-order solution does not affect the visibility of almost any point of the lunar surface, having in mind the same limitations on maximum longitude and latitude mentioned for the linear case.

4.2 Control of the third order solution

The same LQR technique illustrated to control the linearized solution can be successfully used for the trajectory control of the third order Lissajous trajectory. In this case, the control accelerations patterns are rather different from the ones obtained in the linearized case: the signal, in fact, has no more periodic behavior and it is characterized by an irregular pattern (Fig. 6). As a matter of fact, all the perturbations acting on the Earth–Moon system and on the spacecraft itself make the dynamics non perfectly periodic and that is reflected by the control accelerations. Additionally, once the accelerations are integrated to evaluate the corresponding ΔV, the trajectory control maneuver is less expensive than in the linear case

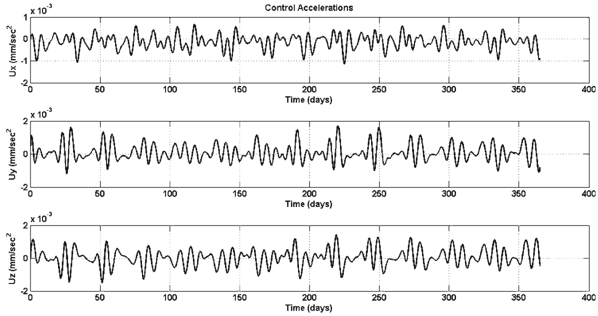

Fig. 6 Control accelerations for a 3500 km amplitude third order Lissajous trajectory

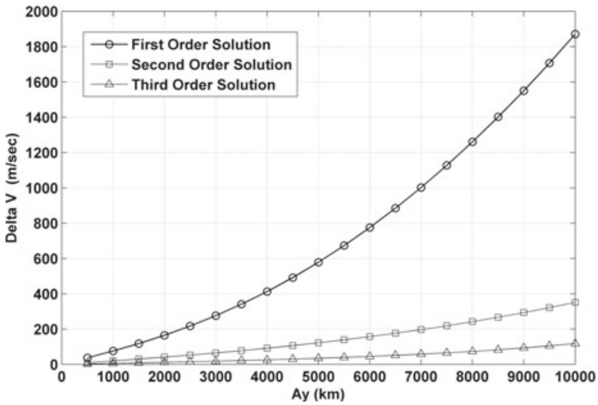

Fig. 7 Annual Δv for trajectory control of different solution order

within the CRTBP framework. This is why the reference trajectory has been defined in a more accurate dynamical representation of the Earth–Moon system.

As a consequence, integrating the preceding control accelerations over one year leads to an amount of ΔV needed for a one year long trajectory control maneuver of $\Delta V = 21.89$ m/s, which is one order of magnitude smaller than the value obtained for the linearized solution in the CRTBP and less than half of the consumption needed for a GEO satellite. It is important to note that the higher-order, more accurate solution trajectory yields a reduced total ΔV (Fig. 7). As illustrated in Fig. 7, the overall fuel consumption decreases for increasing-order solution trajectories. This is due to the fact that high order solution are closer to the natural behavior of the system than low order solutions. In particular, at the selected amplitude value of 3500 km the total amount of ΔV for one year of control goes from the value of $\Delta V = 340.7$ m/s corresponding to the first order solution, to $\Delta V = 77.65$ m/s using the second order solution and finally to the mentioned value of $\Delta V = 21.89$ m/s for the third order solution. Note that the total consumption associated with the first order solution is much bigger than the value obtained for the linearized solution propagated in the CRTBP. This is because of the fact that the equations of motion include all the perturbations and the non linear terms, which have to be compensated by the controller since it is forcing the spacecraft to follow the solution in a highly non linear system. It would be theoretically possible to obtain an analytical reference trajectory which needs no control to be followed. As a matter of fact, if one could derive an analytical solution of infinite order, the corresponding ΔV would be equal to null, because this reference trajectory would exactly match the natural behavior of the system.

As a consequence, the third order Lissajous solution makes it possible to estimate the total amount of ΔV needed for a year trajectory control of the two constellations by simply multiplying the required consumption for a single satellite by the total number of satellites. According to the system's description given previously, the two proposed constellations are made of four satellites each and the total trajectory control consumption is equal to:

$$\Delta V_{\mathrm{TOT}}^{\mathrm{3rd}} \approx 184 \text{ m/s/year}$$

While another third-order solution is available for the Sun–Earth system (Richardson 1980), analytical higher order solutions are not readily available for both the Sun–Earth and the Earth–Moon systems due to the huge complexity of the problem, which makes it really hard to obtain an analytical solution of order higher than three. Nevertheless, it is possible to use a numerical approach (Howell and Pernicka 1988, 1993; Howell 1984; Breakwell et al. 1974) to increase the accuracy of the reference trajectory and reduce the ΔV needed for one year control up to a very small value of $\Delta V = 3.5\,\mathrm{m/s/year}$ per satellite (Cadenas et al. 2007). This value makes Lissajous trajectories very attractive because of the very low consumption needed for their trajectory control. As a consequence, the trajectory control for the two constellations around the collinear L_1 and L_2 libration points can be further decreased to a slightly negligible value equals to:

$$\Delta V_{\mathrm{TOT}}^{\mathrm{numerical}} \approx 28 \text{ m/s/year}$$

which is about half of the station keeping consumption for a single GEO satellite, but for the whole 8 satellites constellation.

5 Use of a Walker constellation around the Moon

The Global Positioning System on the Earth is designed to use a nominal tailored Walker constellation with parameters 55°:24/6/0.67 and an orbital sidereal period of 12 h (Kaplan and Hegarty 2005). The Walker parameters describing the Walker constellation geometry are usually described by **i**:**t**/**p**/**f**, where **i** is the inclination of the orbital planes, **t** is the total number of satellites in the constellation, **p** is the total number of orbital planes and **f** is the phase angle between adjacent planes. Note that due to design optimization procedures, the satellites on each plane of the GPS Walker constellation are not equally spaced. As a consequence, the GPS Walker constellation is referred to as tailored and with **f** being an average value. Currently, more than 30 GPS satellites are operative supporting terrestrial, maritime and aerospace applications. In order to compare the results obtained with the Lissajous based LGPS, a lunar Walker constellation has been implemented according to the same requirements stated above for the Lissajous constellations. Many different configurations have been tested, which lead to a solution with the following parameters (Fig. 8):

- Walker parameters: 60°:18/3/1.35
- Radius: 4000 km.

These values have been selected taking into consideration different aspects, such as the minimum number of required satellites, the number of different orbital planes, the altitude of the orbits and their inclination. All these aspects, in fact, are strictly related to the total amount of ΔV needed for the transfer orbits and for the deployment and control of the constellation.

With the selected set of parameters, the visibility from the entire surface of the Moon has been studied over time and it has been verified that a global coverage is obtained for on surface

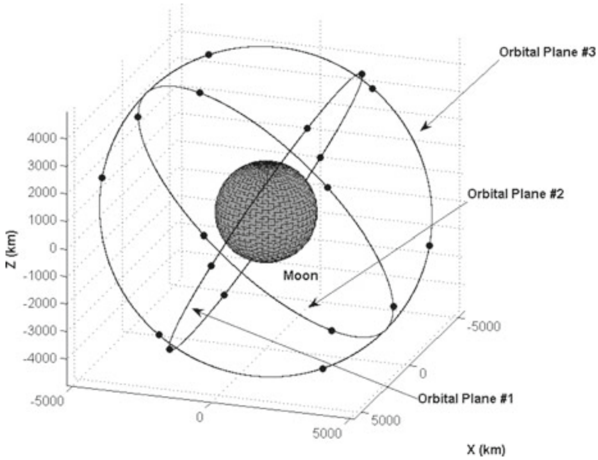

Fig. 8 The selected Walker constellation (60:18/3/1.35) around the Moon in the inertial frame

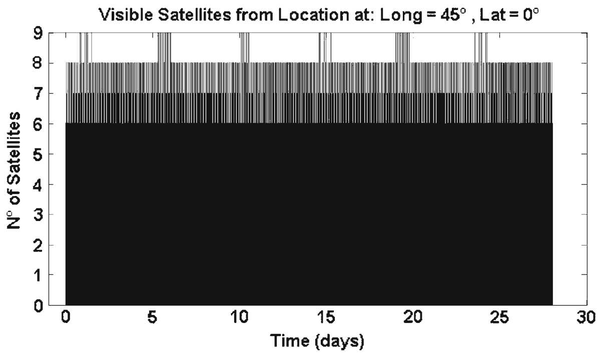

Fig. 9 Visible satellites from a location on the Moon's equator

and in flight observers. The visibility criterion adopted is the same used with the Lissajous constellations, with a minimum elevation angle of $\varepsilon = 0°$ and the minimum number of four visible satellites. The constellation has been propagated taking into consideration the perturbations due to the non perfect spherical shape of the Moon and the gravity action of the Earth and the Sun. Then, for different locations on the surface the total number of visible satellites has been evaluated. As easily expected, the number of visible satellites is strictly dependent on the surface location and on the constellation's geometrical configuration, which means that this parameter changes over time as the satellites change their position with respect to the surface. Different sample cases are presented to illustrate how the number of visible satellites is influenced by the location of the observer on the surface of the Moon.

Figure 9 clearly shows that at the equator there is an average value of 6 visible satellites, which is far within the visibility requirements. In addition, it is possible to recognize peaks up to 9 visible satellites.

Figure 10 represents the visibility study from the lunar South Pole. It is evident that the minimum required number of satellites needed to evaluate the position solution is verified, with an almost constant number of seven visible satellites.

Zooming in the picture (Fig. 11), however, it is possible to note that this number is not constant, but it reaches a minimum value of 6 for very short time slots.

Finally, Fig. 12 shows the visibility study made for a location placed at mid north latitude on the lunar surface. The number of visible satellites is more rapidly changing due to

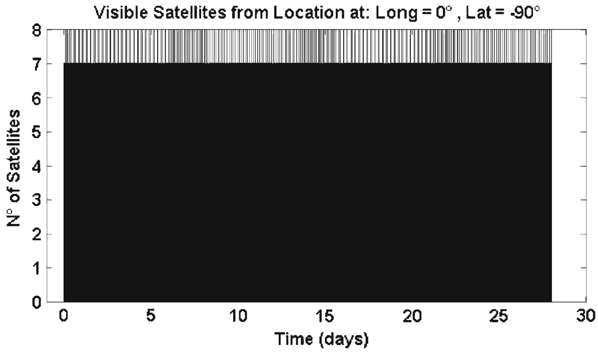

Fig. 10 Visible satellites from the lunar South Pole

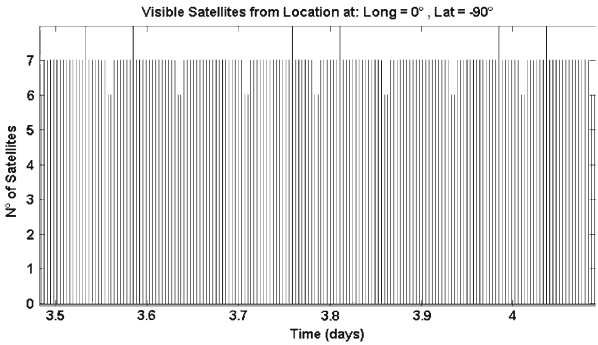

Fig. 11 Visible satellites from the lunar South Pole (zoomed in view)

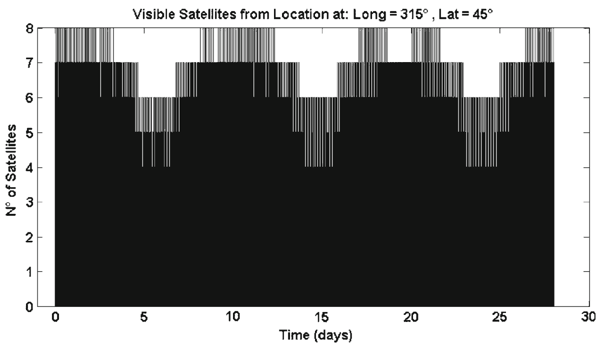

Fig. 12 Visible satellites from a middle latitude location

the selected geometry of the constellation, but it never breaks the desired requirement on the visibility conditions. In addition, there is a greater variation in the number of visible satellites and short time slots exist where the current configuration guarantees only the minimum required number of satellites to determine the position.

Comparing Figs. 9, 10, 11, 12 it is clear that the influence of the constellation's geometry configuration and the observer's location on the number of visible satellites is relevant. Besides, it is important to note that the actual number of visible satellites from all these test locations on the surface is almost always greater than the minimum number of required satellites to compute the position solution. This increases the accuracy of the navigation solution, because it increases the possibility of having a good geometrical configuration of the constellation relative to the observer and makes the geometrical dilution of precision smaller. In addition, it is important to note that the proposed visibility study has been performed only for users on ground. Accordingly to what has been done in Sect. 3, the same study has been repeated assuming a minimum elevation angle of $\varepsilon = 10°$ and the same requirements. Numerical simulations showed that increasing the minimum elevation angle increases the total number of required satellites for the Walker constellation. In order to guarantee the same condition with the same constellation geometry, in fact, the total number of satellites goes from 18 to 24 (using the constellation 60: 4/6/1.37) or 25 (using the constellation 60:5/5/1.37). However, it is important to note that these considerations are only valid for on-ground observers, as already stated for the two Lissajous constellations. As a consequence, the showed 60:18/3/1.35 Walker constellation guarantees time continuous availability of the LGPS signal for in-flight observers with no need for a minimum elevation angle bigger than zero.

6 Comparisons between the Lissajous and the Walker solutions

The Walker and the Lissajous constellations can be compared in order to evaluate advantages and disadvantages of both configurations. Following the descriptions given in Sects. 2, 3 and 4 it is evident that in the Lissajous case it is possible to meet the requirement on the minimum number of always visible satellites with just 8 spacecrafts instead of 18 for the Walker constellation. Note that even if the number of satellites on each of the two Lissajous constellations is increased to 5 to guarantee some redundancy, the total number of satellites to provide the LGPS service is almost half of Walker case. Most importantly, the reduced number of satellite means fewer launches and less overall consumption. Values available in literature show that (Cadenas et al. 2007), starting on the same 200 km parking orbit around the Earth, 3.83 km/s are needed to reach L_1 and insert on the desired Lissajous trajectory, while 3.47 km/s are needed to reach the second Lagrange point L_2 and insert on the Lissajous nominal trajectory. Hence, the total amount of ΔV needed for the transfer of four satellites to L_1 is equal to $\Delta V_{\mathrm{Tranfer,L_1}} \approx 15.32$ km/s, while the transfer consumption to reach the L_2 collinear point is equal to $\Delta V_{\mathrm{Tranfer,L_2}} \approx 13.88$km/s. The total consumption in terms of ΔV to deploy the two constellations around the collinear L_1 and L_2 Lagrange points is equal to $\Delta V_{\mathrm{Deploiment,Lissajous}} \approx 29.2$ km/s. On the other hand, the deployment of a lunar constellation would cost about 3.83 km/s for each satellite (Circi and Teofilatto 2006). Therefore, the deployment of a lunar Walker constellation would cost about $\Delta V_{\mathrm{Deployment,Walker}} \approx 69$ km/s. In addition, the consumption needed to control the final trajectories must be taken into consideration as well. As a consequence, the use of the two Lissajous constellations to provide the LGPS service appears to be much more attractive than deploying a Walker constellation around the Moon.

7 Use of Lissajous constellations for a global communication system

In addition to the LGPS illustrated in the previous sections, the two Lissajous constellations in the Earth–Moon L_1 and L_2 Lagrange points can be used as a Lunar Global Communication System (LGCS). As a matter of fact, the two constellations of satellites are in fixed position with respect to the lunar surface in the synodic rotating reference frame and this makes it possible to use the spacecrafts as communication platforms. As previously stated each constellation covers approximately half of the lunar surface and, for that reason, any signal from the surface can be directed towards one of the visible satellites, then towards one of the other satellites of the same constellation or one belonging to the other constellation and, finally, down to the receiver which could be on the ground or in flight over the lunar surface. Figure 13 shows an example configuration where satellite number 4 of the L_2 constellation sees satellites number 1, 2 and 3 of the L_1 constellation from its current location. This makes

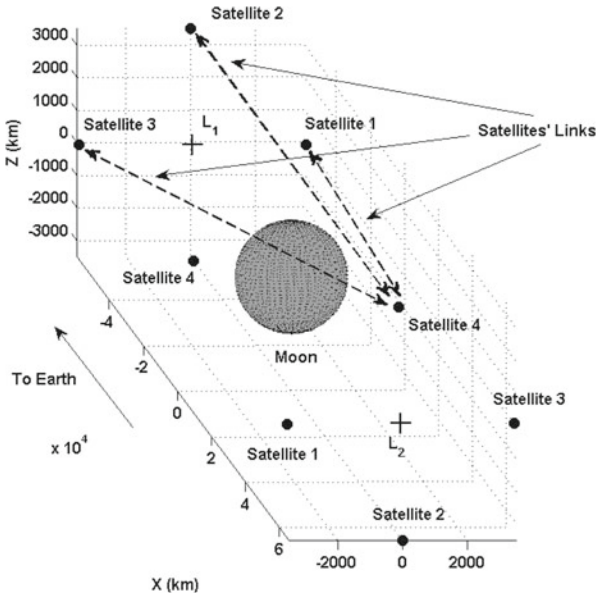

Fig. 13 Possible cross—constellation satellite links from the fourth spacecraft of the L_2 constellation

it possible to send a signal from the far side of the Moon to the visible side of the Moon using the link between the two constellations.

Note that the receiver can be potentially located on both the hemispheres of the Moon and either on the ground or on board of a flying vehicle. It is important to note that there are no constant visibility conditions between the two constellations in L_1 and L_2 due to their time dependent relative geometry. This is because the Moon can interrupt the line of sight between one satellite on one constellation and the ones on the other constellation. Hence, at any instant of time it is necessary to determine visibility conditions between satellites and to redirect the communication signal whenever necessary. Recalling the visibility criterion illustrated in Sect. 3, let us define a visibility function between two satellites as follow:

$$f\left(\vec{r}_i, \vec{r}_j\right) \begin{cases} = 0 \text{ if the j-th satellite is not in sight from the i-th satellite} \\ = 1 \text{ if the j-th satellite is visible from the i-th satellite} \end{cases}$$

being $\vec{r}_i$ the position vector of one satellite of one of the two constellations and $\vec{r}_j$ the position vector of one satellite belonging to the other constellation. Using the same test case shown in Fig. 13, one can study how the visibility function changes over time.

Figure 14 shows the visibility function over time for the satellite number four of the L_2 constellation.

The four different plots refer to the four satellites in the L_1 constellation and, therefore, each plot shows the visibility history for each satellite. To provide a time continuous communication service between the two constellations it is necessary that each satellite of one constellation can have a connection with at least one satellite of the other constellation. Numerical simulations showed that the proposed Lissajous constellations meet this requirement. Figure 15 shows a close look on the visibility function for the fourth satellite of the L_2 constellation in the timeframe between days 50 and 100.

It is clear that there is always the possibility of linking the satellite with one satellite belonging to the L_1 constellation. For example, at time T_1 satellite one and four of the L_1

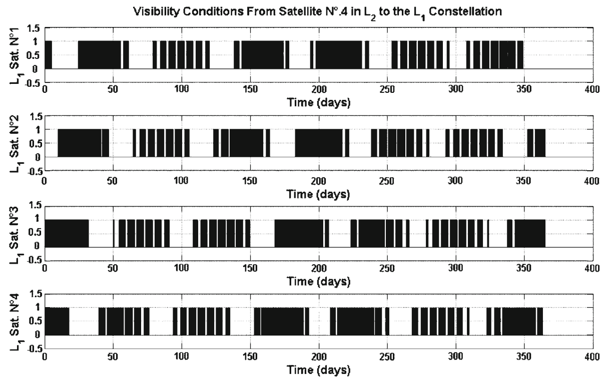

Fig. 14 Inter-visibility study between one satellite of the L_2 constellation and the L_1 constellation

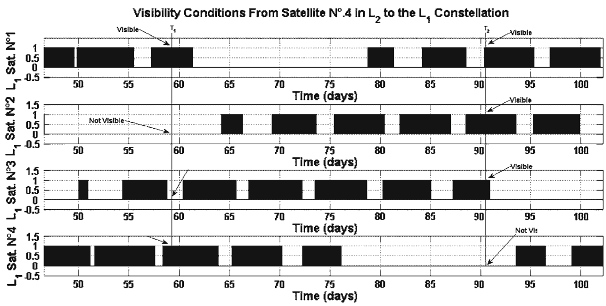

Fig. 15 Inter-visibility study between one satellite of the L_2 constellation and the L_1 constellation (detailed view)

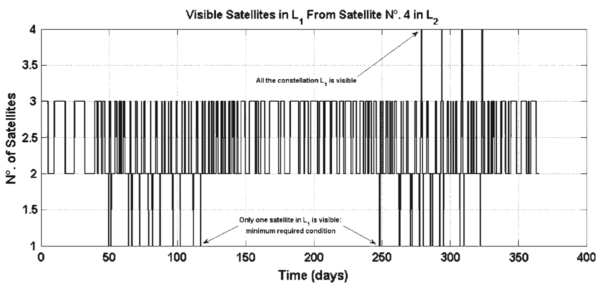

Fig. 16 Number of total visible satellites in the L_1 constellation from the fourth satellite of the L_2 constellation

constellation are visible, while at time T_2 only satellite number four is eclipsed by the Moon disk, being the other three visible by the considered satellite around L_2.

Finally, the global visibility behavior from the fourth satellite of the L_2 constellation can be used to demonstrate that the requirements on the minimum number of satellites visible are met. Figure 16 clearly shows that the average number of visible satellites for the test case varies between two and three, with just few cases in which only one satellite of the other constellation is visible and even less instants of time where the entire constellation is visible.

8 Conclusions

The possibility of providing a Lunar Global Positioning System and a Lunar Global Communication System has been investigated using two different approaches based on a standard Walker constellation and on two Lissajous constellation about the cislunar and translunar libration points. The two constellations have been designed to meet the requirements for both the LGPS and the LGCS and visibility studies have been performed to check if these requirements are satisfied. The Lissajous trajectories have been first studied using linearized reference trajectory in a simple Circular Restricted Three Body Problem framework to do a first comparison against the Walker constellation. Then, a more complex four body model has been implemented to evaluate more accurate values of the ΔV needed for the trajectory control. The importance of having an accurate reference trajectory to reduce total fuel consumption for trajectory control maneuvers has been pointed out, as well as further possible improvements due to the use of numerically calculated reference trajectories. Additionally, an evaluation of the deployment ΔV for the Walker solution and for the two Lissajous constellations has been presented, showing that the use of the two libration point constellations implies a remarkably small ΔV compared to the Walker solution. It has been shown that the total number of satellites required by the Walker solution is 18 while the same result can be achieved with just 8 satellites using the proposed Lissajous constellations. Finally, the LGCS has been introduced and a visibility study for the two Lissajous constellations has been performed to ensure that the visibility requirements are satisfied. Hence, the Lissajous L_1 and L_2 constellations are able to efficiently provide both global positioning and global communication systems for on-ground and in-flight lunar missions.

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