Celestial Mech Dyn Astr (2006) 96:219–237
DOI 10.1007/s10569-006-9033-x

ORIGINAL ARTICLE

Numerical theory of rotation of the deformable Earth with the two-layer fluid core. Part 2: Fitting to VLBI data

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Received: 15 July 2005 / Revised: 3 April 2006 /
Accepted: 24 May 2006 / Published online: 15 November 2006

Abstract VLBI-based offsets of the Celestial Pole positions, as well as the variations of UT (series of Goddard Space Flight Center, 1984–2005) are processed applying the Earth's rotation theory (ERA) 2005 constructed by the numerical integration of the differential equations of rotation of the deformable Earth. The equations were published earlier (Krasinsky 2006) as the first part of the work. The resulting weighted root mean square (WRMS) errors of the residuals $\mathrm{d}\theta$, $\sin\theta\mathrm{d}\phi$ for the angles of nutation θ and precession ϕ are 0.136 and 0.129 mas, respectively. They are significantly less than the corresponding values 0.172 and 0.165 mas for the IAU 2000 model adopted as the international standard. In ERA 2005, the angles θ, ϕ are related to the inertial ecliptical frame J2000, the angle ϕ including the precessional secular motion. As the published observational data are theory-dependent being related to IAU 2000, a procedure to confront the numerical theory to the observed Celestial Pole offsets and UT variations is developed. Processing the VLBI data has shown that beside the well known 435-day FCN mode of the free core nutation, there exits a second mode, FICN, caused by the inner part of the fluid core, with the period of 420 day close to that of the FCN mode. Beatings between the two modes are responsible for the apparent damping and excitation of the free oscillations, and are implicitly modeled by ERA 2005. The nutational and precessional motions in ERA 2005 are proved to be mutually consistent but only in case the relativistic correction for the geodetic precession is applied. Otherwise, the overall WRMS error of the residuals would increase by 35%. Thus, the effect of the geodetic precession in the Earth rotation is confirmed experimentally. The other finding is the reliable estimation $\delta_{\mathrm{c}} = 3.844 \pm 0.028°$ of the phase lag δ_{c} of the tides in the fluid core. When processing the UT variations, a simple model of the elastic interaction between the mantle and fluid core at their common boundary made it possible to satisfactory describe the largest observed oscillations of

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UT with the period of 18.6 year, reducing the WRMS error of the UT residuals to the value 0.18 ms (after removing the secular, annual and semi-annual terms).

Keywords Earth's rotation · Nutation · Precession · Universal time · General relativity · VLBI

1 Dynamical model and VLBI data

Differential equations of the deformable Earth given in Sect. 5.1 of paper (Krasinsky 2006), thereafter referred to as Paper 1, have been integrated numerically comparing the results with the series of the Earth Orientation Parameters (EOP) provided by Goddard Space Flight Center (GSFC). These data are accessible via anonymous FTP (*ftp://cddis.gsfc.nasa.gov/ivsproducts/eops*). They are regularly updated in GSFC by global processing all VLBI observations, which could contribute to the study of the Earth's rotation and are available at the current date. In the present paper, we describe the results of fitting the constructed numerical theory of the Earth rotation to these data.

To facilitate reading, it seems useful to give a brief account of the main features of the dynamical model of the Earth's rotation realized by the numerical integration, referring to Paper 1 for detailed descriptions. It proves that in spite of the complex structure of the non-rigid Earth interior, the Earth's rotation may be modeled accurately enough by a dynamical system with finite freedom degrees. It is commonly assumed that the Earth consists of the anelastic mantle, the viscous fluid and solid inner cores. In the static approximation, rotation of the fluid core may be modeled after Poincaré by a system of ordinary differential equations coupled with the equations for the mantle. A simple generalization of the Poincaré's method makes it possible to develop the analogous theory for the two-layer fluid core. A preliminary analysis of the VLBI-based offsets of the Celestial Pole has shown that the two-layer model of the core provides better fitting of the numerical theory to these data than the one-layer model (and much better fitting then that of the IAU 2000 theory of the precession-nutational motion adopted as a standard). In this analysis no signs have been found of the prograde free oscillations predicted as the effect of the solid inner core by the MBH model (Mathews et al. 2002), which makes the dynamical basis of IAU 2000. That is why action of the solid inner core is ignored in the present version of the dynamical theory. Broadly speaking, the perturbations taken into account may be described in the following way. The perturbing effects are caused by both the direct and indirect action of the Moon, Sun, and the nearest planets; moreover, for the deformable Earth it is necessary to consider a large number of tidal effects many of which are ignored in IAU 2000. The tides affect the Earth's rotation in two ways. First, they distort three matrices of inertia: I of the Earth as a whole, I_c of the fluid core as a whole and I_i of the inner part of the fluid core. These deformations are proportional to the well-known static Love number k_2, to the dynamical Love number k_2^d (that scales the deformations of I due to the differential rotation of the core, and, vice versa, the deformations of I_c caused by the tidal deformations in I), and to the Love number k_2^c that scales the deformations of I_c caused by the differential rotation of the core. The effective (normalized) Love numbers that actually enter the differential equations, are defined as σ=k_2/k_s, ν=k_2^d/k_x, and σ_ν=k_2^c/k_s, where $k_s \approx 1$ is the so called secular Love number. The analogous parameters may be introduced for the inner part of the fluid core, in particular, the dynamical Love number ν_{uv} scaling the perturbations of matrices of inertia of the core and its inner part due to their differential rotations. Not only do the tidally redistributed masses within the Earth lead to the distortions of the matrices of inertia, but they give rise to additional torques caused by the interaction of the perturbing bodies with these masses. It appears that these torques are proportional to the same effective Love numbers. The non-elasticity of the Earth's matter means that both the tidally induced torques and the tidal contributions to the moments of inertia depend on positions of the perturbing bodies (or on the angular velocities of the differently rotating layers of the Earth) delayed by some time intervals. In this way, the two phase lags δ and δ_c, defined as products of the time delays and the angular velocity ω of the Earth's axial rotation enter the differential equations.

When dealing with the dissipative systems like the rotating non-rigid Earth, the backwards in time integration meets insuperable obstacles because in such integration amplitudes of the excited free oscillations would exponentially increase. Hence, the integration can only be carried out onwards in time, and that is why, we cannot use J2000 as the natural initial date T_0 for the integration. Instead, the calendar date December 27 1983 (JD = 2445695.5) is taken for T_0, deliberately ignoring all VLBI observations before this date due to their low accuracy. The last observation is that for the date October 10 2005 (the total number is 3588 points). We integrate the set of the 18 variables that determine: (a) the absolute rotation of the Earth as a whole, (b) the differential rotation of its external fluid core relative to the mantle, and (c) the differential rotation of the inner part of the fluid core relative to its external part (effects of the solid inner core are not considered due to the above reasons). The variables θ, ϕ, ψ are the three Euler's angles (of nutation, precession, and axial rotation) referred to the inertial ecliptical coordinate frame J2000. The conjugate impulse-type variables are $m_1 = \dot{\theta}$, $m_2 = \dot{\phi}\sin\theta$, $m = \dot{\phi} + \dot{\phi}\cos\theta$. The variables n_1, n_2 are the equatorial projections of the vectorial angular velocity of the differential rotation of the fluid core relative to the mantle (in the inertial frame), n is the polar projection. Similarly, the variables q_1, q_2 are the equatorial projections of the angular velocity of the inner part of the fluid core relative to its outer part, q is the polar projection. The variable χ is the librational angle introduced in Paper 1 to model the coupling of the axial rotations of the mantle and the core due to their interaction in the vicinity of the common boundary. The polar projection n is the conjugate impulse-like variable connected with χ by the differential relation $n = \dot{\chi}$. The analogous variables for the inner core are χ_i and q ($\dot{\chi}_i = q$). In the present work, no observational evidences of the coupling between the axial rotations of the external and inner fluid cores are found, and the variables χ_i, q only are introduced here formally, to facilitate future studies. They have no influence on other integrated variables and, so, on the observables.

If desirable, Chebyshev polynomials presenting the integrated variables can be constructed simultaneously with the process of the numerical integration. The program complex ERA (Ephemerides for Research in Astronomy) (see Krasinsky and Vasilyev 1997) used in all the calculations, provides the polynomials not for variables, but for their time derivatives, restoring the variables from these derivatives when the polynomials are to be evaluated. That is why we consider n_1, n_2 and q_1, q_2, as impulses defining the auxiliary angular conjugate variables θ_e, ϕ_e and θ_i, ϕ_i by the differential relations $\dot{\theta}_e = n_1$, $\dot{\phi}_e = n_2$ and $\dot{\theta}_i = q_1$, $\dot{\phi}_i = q_2$. These variables are the cyclic ones (i.e. they do not enter the perturbing potential and so do not affect the Euler's angles) unless the interaction between the core and mantle, as well as between the external and inner cores near corresponding boundaries are accounted for. They mean differences of the corresponding Euler's angles of nutation and precession for the mantle and core, or for the external and internal parts of the fluid core. In Version A of the precession-nutational model, the elastic mantle-core interaction is ignored while for the extended model of Version B a simple mathematical description of such an interaction has been applied, in which the arising torques are proportional to θ_e, the coupling parameter being estimated from the observations. In Version B, the variable θ_e is no more a cyclic one and enters the differential equations explicitly. Version A has been used for testing the relativistic effects of geodetic precession (see Sect. 3.4). In both versions the dissipative effect of friction between the mantle and the core at their boundary is accounted for. Components of the arising torques are proportional to the differential angular velocities n_1, n_2 and do not depend explicitly on the differences of the Euler's angles.

For studies of the Earth's rotation, only the variables θ, ϕ, ψ are needed, but in a number of other fields of geodynamics, the values of the variables n_1, n_2 may also be used for accurate calculation of the diurnal tidal terms in the site positions and in the harmonics c_{21}, s_{21} of the geopotential. These terms are commonly referred to as those caused by the frequency-dependence of the Love number h_2, l_2 and k_2, respectively (see McCarthy and Petit 2004; Krasinsky 2002b), and Paper 1 (Sect. 3.2). That is why the polynomial presentation of all the integrated variables may be useful for applications.

As influence of the variability of the axial angular velocity on the angles of nutation and precession is ignorable, the differential equations of the axial rotations of the mantle and cores might be integrated separately. Nonetheless, for convenience, all the differential equations are integrated simultaneously. In such an approach, the model of UT variations is always consistent with that of the precession-nutational motion. While integrating, we have accounted for the perturbations from the Moon, Sun, and the major planets from Venus to Saturn, coordinates of these bodies being taken from the DE/LE405 ephemerides. After a number of trials, the integration step was taken equal to 0.1 days (making use of the Everhart's method of the 11th order). The small value of the step is necessitated by the diurnal effects in θ, ϕ, which are the Chandler's oscillations transformed to the inertial frame. After fitting θ, ϕ to the observed Celestial Pole offsets (which are free from such oscillations due to the adopted IERS conventions) the amplitudes of the remaining diurnal oscillations proved to be about 0.01 mas which value is less than the errors of the VLBI data. Although more rigorous usage of the numerical theory requires re-processing of the raw VLBI time-delays, the theoretical inconsistency between the theory and the observed Celestial Pole offsets leads to negligible effects as the remaining diurnal terms are so small.

The computer time required for integration of the differential equations on the 25 year time span is about 15 min for an ordinary PC. However, because the partials for the parameters under estimation are obtained by the analogous integration, one step of iterations takes several hours to fit the theory to the observations.

Fitting the constructed dynamical theories to VLBI data has been carried out in two steps: first, the observed offsets of the Celestial Poles are processed estimating relevant geophysical parameters and initial coordinates, and after that there are processed the observed UT corrections. The resulting numerical theory is named ERA 2005 to mark the software used and the upper time limit of the VLBI data processed, and to imply that the theory provides the earth rotation angles.

2 Observables

The numerical theory, because obtained in the fixed coordinate frame J2000, cannot be directly fitted to the observed Celestial Pole offsets and to the UT variations, as they are published in the form of corrections to a nominal theory referred to the mean ecliptic and equator of the current date. To overcome this difficulty, we construct the transformation of the Terrestrial Reference System (TRS) to the Celestial Reference System (CRS) in terms of the variables θ, ϕ, ψ given by the numerical integration. Then, the numerical theory may be fitted to the VLBI data conditioning that this transformation, being applied to any vector defined in TRS, provides the same results as the conventional transformation of TRS–CRS consistent with the published Celestial Pole offsets and UT variations. As the result, we will derive the theory-independent observables $\theta_{\rm obs}, \phi_{\rm obs}, \psi_{\rm obs}$ referred to the inertial frame J2000 properly incorporating the published theory-dependent Celestial Pole offsets as well as the UT variations. (In these two forms of the transformation, the polar wobble is accounted for in the same manner, and so here there is no need to dwell on this reduction.). In more details, the integrated angles θ, ϕ, ψ have been fitted to the VLBI-based offsets of Celestial Pole positions and to the UT corrections in the following way.

1. First, making use of the observed offsets, we derive the observables $\theta_{\rm obs}$, $\phi_{\rm obs}$, which are the observed angles of nutation and precession referred to the ecliptical inertial frame J2000. This procedure practically is not affected by the variations of UT. Actually, in order to calculate $\theta_{\rm obs}$, $\phi_{\rm obs}$, the unit vector $\overline{\rho}_{\rm p}$ directed along the polar axis of inertia has been transformed into the equatorial CRS in accordance with IERS Conventions (making use of the observed offsets of the Celestial Pole). Coordinates of the vector $\overline{\rho}_{\rm p}$ in TRS are given by the triplet $(0, 0, 1)$. Applying the conventional transformation of TRS into CRS to this vector, we obtain the vector $\overline{\rho}_{\rm p}$(CRS) which has to be transformed into the ecliptical frame of J2000 by the rotation

$$\overline{\rho}_{\rm p}^{\rm obs} = P_1(\theta_0)\overline{\rho}_{\rm p}(\mathrm{CRS}) \tag{1}$$

   (θ_0 is the mean obliquity for the epoch J2000) thus obtaining the 'observed' vector $\overline{\rho}_{\rm p}^{\rm obs}$. Now the spherical latitude $\theta_{\rm obs}$ and longitude $\phi_{\rm obs}$ of the vector $\overline{\rho}_{\rm p}^{\rm obs}$ properly account for the observed offsets and so may be considered as observables to be directly compared with the theoretical values of the Euler's angles θ, ϕ provided by the numerical integration. Note, that the value $\phi_{\rm obs}$ is the sum of the observed secular precessional trend and the periodic nutation in the longitude; there is no necessity to separate these components when fitting the theory to observations because the theoretical value of ϕ is given by the numerical integration in the same form. As the observed angle $\phi_{\rm obs}$ is clockwise counted (the astronomical definition) while the calculated angle ϕ is counter-clockwise counted (as it is commonly assumed in mathematics), the residuals $\delta\theta$, $\delta\phi$ are to be calculated as follows

$$\delta\theta = \theta_{\rm obs} - \theta,\ \delta\phi = -\phi_{\rm obs} - \phi.$$

   Evaluating the residuals for the precessional angle ϕ, we have also added the correction $\phi_{\rm GRT}$ predicted by General Relativity Theory (GRT) (the so called geodetic precession and nutation). Its numerical value is calculated making use of the analytical expression from the work (Fukushima 1991)

$$\phi_{\rm GRT} = 1.9194\ (t - \mathrm{J2000})/36525.0 + 0.153 \sin l'$$

in which the coefficients are given in mas, and l' is the solar mean anomaly. The sign of the right part is reversed to be in accordance with our definition of ϕ. In more consistent approach, this correction could be obtained in the process of the numerical integration, along with the other variables. Such a procedure is supposed to be realized in the next version of the theory.

2. Second, the observable $\psi_{\rm obs}$ is formed to be used for fitting the rotational angle ψ to $\psi_{\rm obs}$ (and so to the variations of UT). To obtain $\psi_{\rm obs}$, the equatorial unit vector $\overline{\rho}_{\rm e}$ directed along the largest equatorial axis of the matrix of inertia, and presented in TRS as the triplet $\overline{\rho}_{\rm e} = (1, 0, 0)$, has to be transformed into the ecliptical inertial frame J2000. As in step 1, this vector is firstly transformed into the equatorial inertial frame of CRS, following the algorithms of IERS Conventions and applying this time also the observed corrections to UT, on which the CRS coordinates of the vector $\overline{\rho}_{\rm e}$ strongly depend. Then, the resulted vector is transformed into the ecliptical frame of J2000 with the help of transformation (1), thus deriving the 'observed' vector $\overline{\rho}_{\rm e}^{\rm obs}$. Having obtained this vector, we can present it also in terms of the observed Euler's angles $\theta_{\rm obs}$, $\phi_{\rm obs}$, $\psi_{\rm obs}$ referred to ecliptic J2000. As $\theta_{\rm obs}$, $\phi_{\rm obs}$ have been already determined in step 1, the observed value of the rotational angle $\psi_{\rm obs}$ may be easily derived. In more detail, the ecliptical vector $\overline{\rho}_{\rm e}^{\rm obs}$ is connected with the vector $\overline{\rho}_{\rm e}$ by the transformation

$$\overline{\rho}_{\rm e}^{\rm obs} = P_3(-\phi_{\rm obs})P_1(-\theta_{\rm obs})P_3(-\psi_{\rm obs})\overline{\rho}_{\rm e},$$

which may be written in the coordinate form as it follows (using the projections $\xi_{\rm obs}$, $\eta_{\rm obs}$, $\zeta_{\rm obs}$ of the vector $\overline{\rho}_{\rm e}^{\rm obs}$):

$$\begin{aligned}
\xi_{\rm obs} &= \cos\psi_{\rm obs}\cos\phi_{\rm obs} - \sin\psi_{\rm obs}\sin\phi_{\rm obs}\cos\theta_{\rm obs},\\
\eta_{\rm obs} &= -\cos\psi_{\rm obs}\sin\phi_{\rm obs} - \sin\psi_{\rm obs}\cos\phi_{\rm obs}\cos\theta_{\rm obs},\\
\zeta_{\rm obs} &= \sin\psi_{\rm obs}\sin\theta_{\rm obs}.
\end{aligned}$$

From these relations the observed values of the combinations $\sin\theta_{\rm obs}\cos\psi_{\rm obs}$ and $\sin\theta_{\rm obs}\sin\psi_{\rm obs}$ is expressed in the form:

$$\begin{aligned}
\sin\theta_{\rm obs}\cos\psi_{\rm obs} &= \xi_{\rm obs}\sin\theta_{\rm obs}\cos\phi_{\rm obs} + \eta_{\rm obs}\sin\theta_{\rm obs}\sin\phi_{\rm obs},\\
\sin\theta_{\rm obs}\sin\psi_{\rm obs} &= \zeta_{\rm obs},
\end{aligned}$$

from which the value $\psi_{\rm obs}$ may be easily obtained.

The observable $\psi_{\rm obs}$, derived in this way, has been compared with the rotational angle ψ provided by the numerical integration.

3 Fitting the theory to observed Celestial Pole offsets

3.1 Residuals and their weighted root mean square errors

The WRMS errors $\sigma(\mathrm{d}\theta)$, $\sigma(\sin\theta\mathrm{d}\phi)$ for the Versions A and B of the numerical theory are given in Table 1 (lines 1, 2). They have resulted from several iterations of the least squares method, weighting the condition equations in accordance with *a priori* errors taken from the GSFC dataset. The analogous statistics for IAU 2000 are given in the last line. In the iterative process there are used 3546 points for dθ and 3545 points for

Table 1 Statistics $\sigma_{\mathrm{d}\theta}$, and $\sigma_{\mathrm{d}\phi}$ of the residuals (mas)

Theories	$\sigma(\mathrm{d}\theta)$	$\sigma(\sin\theta\,\mathrm{d}\phi)$
ERA 2005 A	0.141	0.135
ERA 2005 B	0.136	0.129
IAU 2000	0.172	0.165

$\mathrm{d}\phi$, rejecting 42 and 43 outliers, respectively. The statistics for IAU 2000 are calculated excluding the same outliers.

It is important that neither the angle of precession ϕ or nutation θ in ERA 2005 includes any empirical secular corrections, while in IAU 2000 they do, and such corrections are rather large (for instance, $\dot{\theta} = -24$ mas/cy). Quality of the fitting may be characterized by the parameter ς calculated as the ratio of the overall WRMS error of the residuals to the WRMS value of *a priori* errors. In the ideal case $\varsigma = 1$, while, we have obtained $\varsigma = 1.76$ and $\varsigma = 1.70$ for Versions A and B of the ERA 2005, respectively, and $\varsigma = 2.15$ for IAU 2005. The larger value of ς for IAU 2000 is mainly (but not completely) due to the unmodeled amplitudes of the FCN oscillations. Assuming that the *a priori* accuracies are realistic, the value $\varsigma > 1$ indicates that there are still unmodeled effects in ERA 2005.

Residuals $\mathrm{d}\theta$, $\sin\theta\mathrm{d}\phi$ for the Euler's angles of nutation and precession are presented in Figs. 1 and 2 by the plots at the bottom of the figures (marked as ERA 2005). Drawing the plots, the outliers have not been excluded. For comparing, the analogous plots of the IAU 2000 model are presented at the top of Figs. 1 and 2, being marked as IAU 2000.

In the next sections, more details on the estimated parameters are given. The parameters may be separated into three groups: the initial values of the integrated variables, the geophysical parameters, and several empirical parameters without which the satisfactory fitting to the observations still is not possible.

The needful partial derivatives (relatively to any parameter under estimation) of the integrated Euler's angles are obtained by the similar numerical integration of the differential equations, in which this parameter has been slightly varied. The resulting

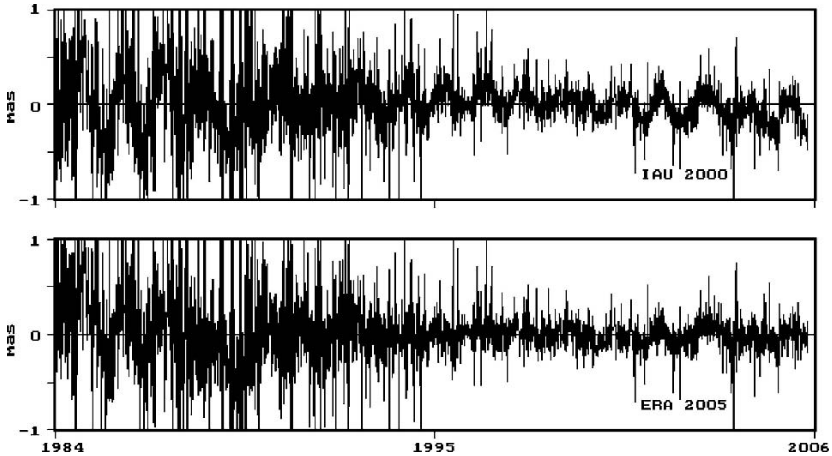

Fig. 1 Residuals $\sin\theta\mathrm{d}\,\phi$

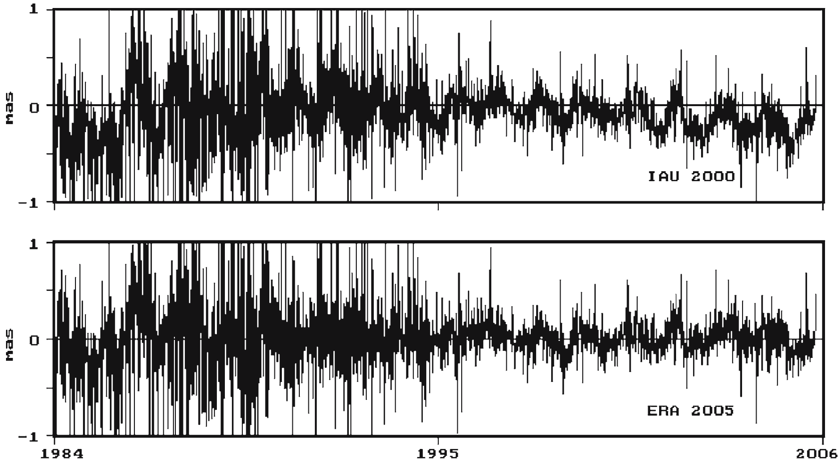

Fig. 2 Residuals dθ

values of the Euler's angles have been subtracted from those for the nominal theory, the calculated differences being divided by the assumed value of the variation to obtain the partials needed. For achieving the required accuracy of the partials, the adequate value of the variation has to be taken (neither too big nor small) for each of the estimated parameters. The available polynomial approximation of the integrated Euler's angles could not be used in this method, because the errors of such approximation might seriously affect the differences of the observables, calculated for the nominal and varied values of the estimated parameter. Instead, the numerical integration was carried out starting from the current date of VLBI data and ending up at the date of the next observation, when the needed values of the Euler's angles become known without the loss of accuracy. In this way, the partials were calculated for each date of the observational series. Trying a series of tests, the optimal size of the variation of the each estimated parameter was found. These tests have also shown that at least two digits in the calculated partials are correct. Fast convergence of the iterative process of estimation proves that the partials indeed have been calculated accurately enough.

3.2 Initial values of the integrated variables

To facilitate independent studies, in Table 2 there are given values of the integrated variables for the three epoch: 0h December 27 1983 (JD = 2445695.5, the initial date), for J2000 (JD = 2451545.0) and for 12h January 1 2005 (JD = 2453372.0). The variable dθ in the table means the correction to the value $\theta_0 = 0.4090648$ of the mean obliquity at J2000.0. Applying this correction, the current apparent obliquity in the reference frame J2000 may be obtained. The variable dm means a correction to the nominal sidereal frequency of the axial rotation $m = 1299548.204236''$/day (more exact, m is the polar projection of the vector $\overline{\omega}$ but here we may safely assume $m = \omega$). For the initial date, we set d$m = 0$. The variable dψ is the corrections to the Earth's rotation angle ψ. The value $\psi_0 = 340930.346340388''$ is taken for ψ at the initial epoch JD = 2445695.5. After integrating, the rotational angle ψ has to be calculated as $\psi_0 + d\psi$ plus the constant correction to and the additional linear trend in the sidereal time derived by processing the VLBI data; see Sect. 4. These corrections absorb

Table 2 Values of the integrated variables for three epochs

JD	2445695.5	2451545.0	2453372.0
$d\theta$	2.0394750 ± 2 E-05	-5.7765883	7.6156152
ϕ	822.1964033 ± 8 E-05	13.6477528	-244.6463784
$d\psi$	1.5736086	735.4414795	964.9670911
χ	32.3422193 ± 9 E-02	14.4908302	-23.1243157
$\dot{\theta}$	0.0091258 ± 2 E-05	-0.0204880	0.0384537
$\dot{\phi}$	$-0.0778984 \pm$ 4E-05	-0.1452179	-0.0961538
dm	0.0000000	0.0002762	-0.0110223
n_1/ω	0.3893798 ± 3 E-04	0.3586278	0.2978042
n_2/ω	$-1.8371891 \pm$ 2E-05	-1.6838292	-1.9671607
n/ω	$-0.0002949 \pm$ 2E-05	-0.0061374	-0.0003544
q_1/ω	0.8147153 ± 1 E-02	0.4790182	0.3497698
q_2/ω	$-3.8011575 \pm$ 7E-03	-3.4410948	-3.8415620

the errors in the assumed initial values of the integrated variables $d\psi$ and dm, and without any loss of accuracy these initial values may be fixed. In the equations of the precession-nutational motion, the variability of m (or ω) is ignored. The units are arc seconds for non-dimensional variables (angles and normalized projections of the angular velocities of the cores), and arc seconds per day for the time derivatives $\dot{\theta}$, $\dot{\phi}$ and dm. The estimated random errors are given (except for the variables $d\psi$ and dm, due to the above reasons) but only for the initial date.

3.3 Geophysical parameters

The geophysical parameters, that might be estimated in this analysis, are the following ones: the ratio α of the main moment of inertia of the core to that of the Earth as a whole, the ellipticities e of the Earth and e_c of the core, tidal phase delay δ of the Earth and that δ_c of the core, the static Love numbers k_2, the dynamic Love number k_2^u, the Love number k_2^c of the core, the ratio α_{ic} of the main moment of inertia of the inner core to that of the core as a whole, the period T_{FICN} of the free oscillation of the inner core, two parameters $k_2^{(1)}$, $k_2^{(2)}$ of the ocean tide model, the dissipative parameter κ_{dis} of the model of friction at the mantle-core boundary, and (only for Version B) the coupling parameter κ_{el} of the elastic interaction of the mantle and core at their boundary. Because it appears impossible to separate the correction to α from that to k_2^u, we have fixed the effective parameter $\nu = k_2^u/k_s$ to its theoretical value, estimating only α. Due to uncertainty of *a priori* values of the parameters characterizing the inner fluid core, we did not use any theoretical prediction for T_{FICN} but instead have carried out a search trying its various values and estimating the parameter α_{ic} (which is the scaling factor of the perturbations caused by the inner fluid core). It appears that the residuals significantly reduce for the value $T_{FICN} \approx 420$. After that, a correction to this preliminary value was included into the general list of the variables under estimating. It is interesting that the Fourier analysis of the IAU 2000 residuals (Malkin 2004) has implicitly demonstrated the splitting up of the well-known frequency of the FCN oscillations into two close frequencies (see Fig. 2 of the cited paper). The corresponding periods are equal to 433 and 420 days (Malkin 2005, private communication). Our results confirm this finding by fitting the numerical theory to the observational data. Comparing the plots of the residuals calculated with ERA 2005 and IAU 2000 (Figs. 1; 2), one can see, first, that the amplitudes of the free oscillations for the former are considerably less than those for the latter, and second, that the apparent damping of the oscillations by year 1998 and their excitation after this epoch only is noticeable in the residuals of IAU 2000. It is clear that this peculiarity is the result of the beatings between the two modes of the free oscillations with the close periods. Implicit accounting for these beatings by the numerical integration is the main cause why the WRMS errors for ERA 2005 are less than those for IAU 2000. To illustrate this fact, in Fig. 3 the differences in the Euler's angles θ, ϕ for the both theories are presented (in the sense ERA 2005 minus IAU 2000). One can see that the differences follow the pattern of the free mode oscillations in IAU 2000 shown in Figs. 1 and 2. As IAU 2000 is the sum of the periodic nutational harmonics, the free oscillations in the differences are to be attributed to ERA 2005. Because the numerical theory does not include any model of excitation of the free oscillations, it is clear that the decrease of the FCN amplitudes at 1998 and the increase of them after this epoch is indeed a result of beatings between the two close modes of the free oscillations. There are also other systematic differences between two theories. For instance, calculating the secular trends in the differences we have got $\dot{\theta} = (-0.3 \pm 0.09)$ mas/cy, and $\phi = (-0.82 \pm 0.22)$ mas/cy. We believe that these trends are to be attributed to IAU 2000.

Estimates of the geophysical parameters under study are given in Table 3. The following comments are to be done:

1. The derived value of the tidal phase lag $\delta = (6.502 \pm 0.066)$ degree is too large in comparison with the reliable LLR estimate $\delta = 2.5$ deg (Dickey et al. 1994; Aleshkina et al. 1997). However, it is interesting that in the work (Shirai and Fukushima 2001) even larger value $\delta = (7.13 \pm 0.23)$ degree has been obtained from the analysis of the analogous VLBI data but applying the standard SOS model.
2. The estimated value of the tidal lag δ_c of the fluid core seems to be reliable as it proves stable in the numerous versions of the analysis. This estimate is obtained for the first time and cannot be compared with any other results. Note that δ_c enters the differential equations being multiplied by either the normalized Love number σ_ν of the core or the normalized dynamical Love number ν (the latter effect is

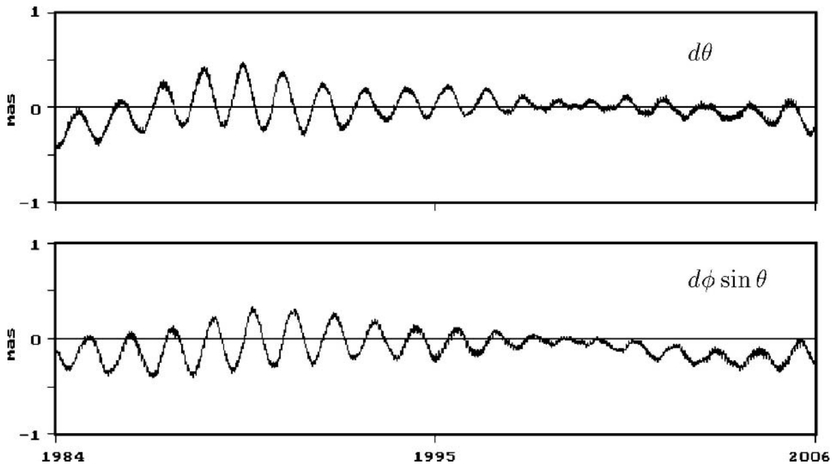

Fig. 3 Differences of the Euler's angles (ERA 2005 minus IAU 2000)

Table 3 Estimates of geophysical parameters

Parameter	Value (σ)	Units
e	3.2834102 (58)	$\times 10^{-3}$
e_{c}	3.3761 (23)	$\times 10^{-3}$
k_2	0.27272 (36)	–
k_2^c	0.02130 (21)	–
α	0.109412 (16)	–
k_2^v	0.058325	–
δ	6.502 (66)	deg
δ_{c}	3.430 (47)	deg
$k_2^{(1)}$	1.084 (43)	$\times 10^{-3}$
$k_2^{(2)}$	0.890 (46)	$\times 10^{-3}$
ν_{uv}	4.96 (40)	$\times 10^{-5}$
$\mathrm{T}_{\mathrm{FICN}}$	420.31 (94)	days
κ_{dis}	0.362 (24)	$\times 10^{-7}$
κ_{el}	0.826 (38)	$\times 10^{-9}$

small). Thus, the derived estimates of δ_{c} depends on the assumed values of these parameters. Corrections to $\nu = k_2^v/k_s$, as well as to $\sigma_\nu = k_2^c/k_s$ strongly correlate (on the level 99%) with corrections to some other parameters, and that is why we did not include them in the analysis, though the WRMS errors of the post-fit residuals could be diminished.

3. The combination $\sigma_\nu \delta_{\mathrm{c}}$ is of a special importance because it characterizes the damping rate of FCN. From the differential equations of Paper 1 (see Appendix 1) it is easy to see that the quality factor Q_{FCN} of the FCN oscillations may be given by the expression

$$Q_{\mathrm{FCN}} = \frac{f_{\mathrm{FCN}}}{\omega} \left(\frac{\alpha}{e \sigma_\nu \delta_c} \right) \approx 61.$$

Roughly, that means that the FCN damping effects become large after 60 periods of the FCN oscillations. (Note that actually, we estimate the combination $\sigma_\nu \delta_{\mathrm{c}}$ and so the above conclusion does not depend on the assumed value of the Love number σ_ν). The non-tidal dissipation due to the mantle-core friction only slightly diminishes the damping time. Indeed, due to this type dissipation the quality factor Q_{FCN} is as follows (in absence of the tidal damping):

$$Q_{\mathrm{FCN}} = \frac{f_{\mathrm{FCN}}}{\omega} \kappa_{\mathrm{dis}}^{-1} \approx 790$$

and so the effect is by order less then the tidal one.
Unfortunately, we did not succeed in obtaining a reasonable estimate for the phase lag δ_i of the inner fluid core (due to strong correlations with other parameters).

4. Commonly, the estimated parameter is not the ellipticity e_{c} of the core but rather the FCN frequency f_{FCN} connected with e_{c} and other geophysical parameters by theoretical relation (147) of Paper 1. Substituting the values of the parameters from Table 3 to this relation, we obtain $f_{\mathrm{FCN}} = 415$ days which value disagrees with the well established value $f_{\mathrm{FCN}} = 431$. Probably, that may be explained by approximations made when deriving relation (147) because its right part is very sensitive to values of the Love numbers, on which it depends. That is an example of difficulties in interpretation of results provided by analytical theories.

Table 4 Corrections to the annual terms (μas)

	$\sin l'$	$\cos l'$
$d\theta$	507 (27)	−64 (92)
$\sin\theta d\phi$	53 (94)	236 (18)

3.4 Empirical parameters and geodetic precession

The empirical parameters are four corrections to the amplitudes of the annual terms, the parameter E_1 correcting 18.6-year terms (more exact, it rules the ratio of their prograde and retrograde amplitudes), and the parameter E_2 to correct out-phase nutations. In order to check the theoretical precessional rate (which is implicitly defined by the numerical model) all calculations have been repeated ignoring the relativistic reduction for the geodetic precession, and assuming the zero value of the parameter κ_{el} of the elastic coupling. In this case, the factor ς that characterizes the quality of fitting, exceeds its value for Version A by about 35%. However, if the empirical trend $\dot{\phi}$ also is estimated then ς will return to the previous value, while the derived correction to the precessional rate is $(2.068 \pm 0.029)''$/cy and only slightly disagrees with the GRT prediction $1.919''$/cy. The small discrepancy $\approx 0.15''$/cy with the theoretical precessional rate is compensated by corrections to other parameters (mainly to the ellipticity e) when processing the VLBI data, and so practically does not deteriorate the fitting. Such verification of the relativistic effect could not be carried out with Version B (in which the parameter κ_{el} of the elastic coupling is also estimated) due to very strong correlation of this parameter with the precessional secular trend. However, Version B provides noticeably better fitting than Version A, even if in the latter the precessional secular trend is estimated as an empirical parameter, and that is why Version B is taken as the base of ERA 2005. It could be used for testing the relativistic effect only after some *a priori* value of κ_{el} becomes known. Anyway, at present the relativistic effect of the geodetic precession seems to be confirmed with the accuracy better 10%. To our knowledge, it is the first successful detecting of this effect in the Earth's rotation.

Corrections to the amplitudes of the annual terms are very sensitive to the frequencies of the two modes of the free nutations, and thus they are strongly influenced by deficiencies of modeling. The empirical corrections to the annual terms are given in Table 4, the solar mean anomaly l' being taken as the argument of the annual terms. It is interesting that corrections to the in-phase amplitudes $(d\theta)_{\cos}$, $\sin\theta\,(d\phi)_{\sin}$ are small being within their statistical errors. Significant corrections to the out-phase amplitudes probably mean that there still exist some deficiencies in the dynamical modeling of the dissipative effects.

The empirical parameter E_1 is introduced formally as the factor $1 + E_1$ at the component of the rigid body torque $\overline{R}^{\text{el}}$ in the differential equation for the obliquity θ (see Sect. 5.1 of Paper 1). It does not contribute to the secular rates either in the longitude or the obliquity but corrects the ratio of the prograde and retrograde amplitudes of the 18.6-year nutations. The second empirical parameter E_2 is introduced in the form of the factor $1 + E_2$ at the perturbing component $\overline{S}_\delta$ proportional to the delay δ in equations for the equatorial projections n_1, n_2 of the angular velocity of the external fluid core. This term also does not change the secular rates. For E_1 and E_2 the estimates $E_1 = 0.770(89) \times 10^{-4}$, $E_2 = -0.07889(78)$ have been obtained. At present, the physical meaning of these empirical parameters is not clear. Supposedly, the solid inner core could be responsible at least for a part of these effects.

4 Analysis of the observed variations of UT

The most salient features of the observed UT variations are large secular trend, significant quadratic term and 18.6-year harmonic oscillations with the amplitudes that greatly exceed those predicted by the tidal theory given in the work (Yoder et al. 1981). As a starting point, we have studied raw variations of UT without any tidal model applied (apart from the contribution from the integrated perturbing term $\dot{\phi}\sin\theta$, which is calculated accurately enough for the aim of modeling UT). Thus, the purely empirical model is taken in the form

$$\text{UT-TAI} = \sum_{i=0}^{3} A_i (T - t_0)^i + \sum_{i=1}^{5} \left(A_i^s \sin f_i + A_i^c \cos f_i \right), \tag{2}$$

where f_i are the arguments of the main terms of the tidal theory (of the 18.6-year, annual, semi-annual, monthly, and fortnightly periods).

Post-fit residuals for this model are shown in Fig. 3 (the upper plot) by the curve marked as IAU 2000. We have the following estimates for the linear, quadratic and cubic terms:

$$A_1 = -79.6 \pm 0.7 \text{ s/cy}, \quad A_2 = 190 \pm 7 \text{ s/cy}^2, \quad A_3 = -340 \pm 20 \text{ s/cy}^3. \tag{3}$$

The estimated in-phase and out-phase amplitudes of the periodic terms are given in columns 3 and 4 of Table 5. The corresponding theoretical values of the in-phase (sine) amplitudes in ms are −172.0, 9.3, −1.6, −5.1, and −0.9, respectively (Yoder et al. 1981). Arguments of these harmonics are given in the second column as the linear combinations of the standard fundamental arguments l, l', F, D, Ω. Note that, the amplitude of the in-phase 18.6-year oscillation of the empirical model is three times larger than the value 172 ms, predicted by the tidal theory. The observed out-phase amplitude of this harmonics exceeds by orders its theoretical value calculated with the LLR-based estimate of the tidal lag δ (Krasinsky 2002a).

The residuals for the empirical model shown in Fig. 4 by the upper plot demonstrate rather irregular time behavior and a large fluctuation on the interval 1985–1989. Somewhat better results are obtained making use of the dynamical model of Paper 1, which accounts for the interaction of the mantle and external core near their boundary.

Table 5 Estimated amplitudes of periodic terms in the residuals UT-TAI (ms)

Period		Empirical model		ERA 2005	
days	Arguments	sin	cos	sin	cos
6790.36	0 0 0 0 1	-540.3 ± 5.7	799.0 ± 2.6	–	–
365.26	0 1 0 0 0	10.9 ± 0.5	-20.0 ± 0.5	10.1 ± 0.5	-20.8 ± 0.5
182.62	0 0 2 −2 2	-6.6 ± 0.5	8.7 ± 0.5	-2.2 ± 0.5	8.8 ± 0.5
27.56	1 0 0 0 0	0.2 ± 0.6	0.2 ± 0.6	1.0 ± 0.5	1.0 ± 0.5
13.66	0 0 2 0 1	1.9 ± 0.6	0.0 ± 0.6	1.9 ± 0.5	1.9 ± 0.5

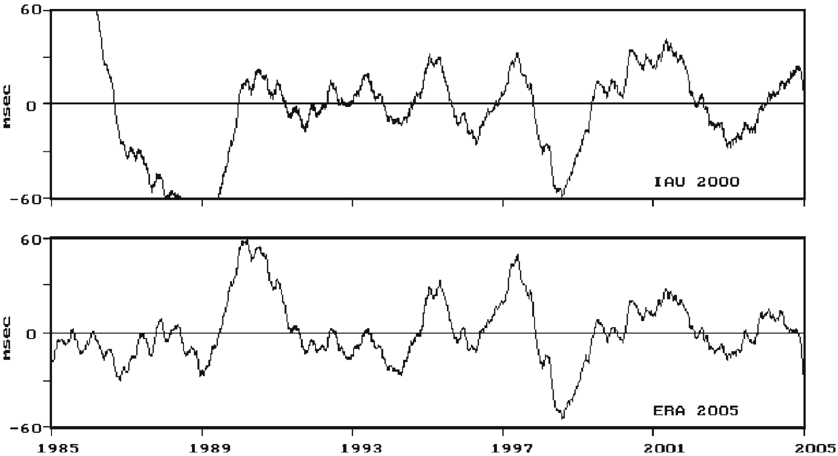

Fig. 4 UT residuals

In this model, the period T_{me} of the free oscillations is one of the parameters under estimating. If the period T_{me}. is close enough to that of the 18.6-year forced oscillations, then a strong increase of the amplitudes of this harmonics could be expected. For this model, the amplitudes of the annual, semi-annual, monthly, and fortnightly harmonics (but not of those with the 18.6-year period) also have been included into the analysis. Initial values of the axial angular velocity n of the differential rotation of the core relative to the mantle, and that of the librational angle χ have been estimated as well. The derived estimate $T_{\mathrm{me}} = (5983 \pm 35)$ days indeed is close enough to the period 6790 days of revolution of the lunar nodes. The corresponding frequency f_{c} of the torsional free oscillations is related to the factor $\kappa_{\mathrm{el}}^{\chi}$ of the potential of the mantle-core elastic interaction by the relation $f_{\mathrm{c}} = \omega\sqrt{\kappa_{\mathrm{el}}^{\chi}}$; see Sect. 5.2 of Paper 1. Thus, the coupling factor $\kappa_{\mathrm{el}}^{\chi}$ may be calculated from the estimated value of the period T_{me}. Geophysical interpretation of $\kappa_{\mathrm{el}}^{\chi}$ is a problem beyond the scope of the present paper.

The obtained amplitudes of the harmonics are presented in columns 5 and 6 of Table 5, and one can see that they still are large and comparable with those for the purely empirical model. It might be supposed that the large values of the annual amplitudes is the result of interaction of the external and inner cores near their boundary, because these amplitudes would considerably increase if the period of the corresponding free oscillations were close enough to the one year value. The large value of the out-phase amplitude of the annual harmonics could be expected to be a result of the dissipation. However, a preliminary analysis has shown that this conjecture probably is wrong because the small value of α_{ic} makes such effect negligible unless the period of the free oscillations is unlikely close to one year.

The residuals for ERA 2005 are shown on the lower plot of Fig. 3 by the curve marked as ERA 2005. One can see that all the tidal periodic oscillations described by relation (2) are removed from the residuals, in which irregular fluctuations only are noticeable (the error bars are too small to be visualized in this scale). Comparing the two plots in Fig. 3, we see that the large fluctuation at years 1985–1989 takes place only for the purely empirical model. Thus, ERA 2005 describes the large UT oscillations of the period 18.6 year better than the empirical model IAU 2000, though also fails to model the large observed amplitudes of the annual and semi-annual harmonics.

The well-known effect of the tidal deceleration of the Earth axial rotation was expected to manifest itself as a negative quadratic term in UT. The theoretical value of the corresponding coefficient may be derived making use of the analytical formula for the coefficient $d\psi_\delta$ of the quadratic trend in the rotational angle ψ (see Appendix in Paper 1). With the adopted value of the tidal lag δ, this tidal contribution is obtained as follows

$$d\psi_\delta = -83.2 \text{ s/cy}^2,$$

which is the expected value for the coefficient A_2 in expression (3) for UT variations.

The value of the deceleration of the Earth axial rotation, predicted by the tidal theory, has been reliably confirmed by astrometric observations of the Moon and planets of 18–20 centuries (Krasinsky et al. 1985) and by paleontological data for LOD (Krasinsky 2002a). However, the VLBI data of 1984–2005 unambiguously evidence that the coefficient A_2 of the quadratic term is positive (see estimates (3)), which means that on the time interval under study the axial rotation of the Earth does not decelerate but, contrary, accelerates. It is interesting that the similar effect was noticed for the time interval 1870–1890 by investigations of the lunar orbital motion. At first this effect was attributed to some perturbations of unknown origin in the lunar longitude (the so called great lunar inequality) but after detecting analogous perturbations in the longitudes of all inner planets it became clear that the effect is caused by the large variation of the Earth's axial rotation. The following simple considerations concerning the long-term UT variations seem plausible. The residuals presented in Fig. 3 demonstrate that the Earth's angular velocity ω (the time derivative of UT) undergoes fast random fluctuations separated by the time intervals of several years, probably due to some geophysical processes in the Earth's interior. To give a typical example of such jumps in more details, in Fig. 5 we present the residuals of UT for the time interval August 15 2004–May 30 2005. One can see the steep slope in the residuals started at January 2005. (It is interesting that the beginning of this process coincides

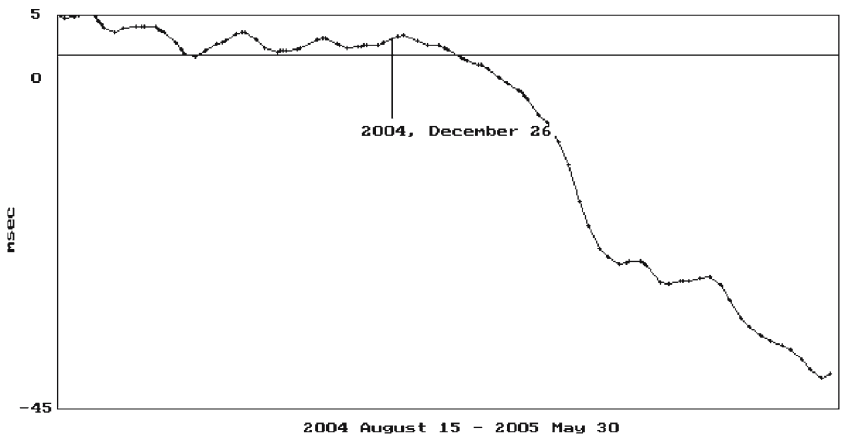

Fig. 5 Example: a jump of the angular velocity ω in residuals UT

with the catastrophic earthquake of December 26 2004, though probably there is no cause-effect relation between the two events). Analyzing this slope, the corresponding jump $\mathrm{d}T$ in the LOD is estimated as $\mathrm{d}T = 0.36 \pm 0.07$ ms. If σ is a typical amplitude of such jumps, and $\mathrm{d}t$ is the mean time interval between the successive jumps, then the expected dispersion $\mathrm{d}\omega$ of the variations of ω for the elapsed time T is of the order $\sigma(T/\mathrm{d}t)^{1/2}$ (as the sum of $T/\mathrm{d}t$ random numbers). The amplitudes of UT variations would be of the order $\sigma(T/\mathrm{d}t)^{3/2}$, as the integral of $\mathrm{d}\omega$. It means that however large a constant be taken, after some time elapses, the UT variations would reach its value. On the other hand, the tidal deceleration of ω and that of UT are proportional to T and T^2, respectively, the both values being negative. So, on large time intervals the tidal deceleration of the axial rotation prevails in comparison with the accumulating effect of the random fluctuations. The astronomical observations of the Moon and planets show that the minimal time interval needed to detect such prevalence is about few centuries. At present, however accurate VLBI observations might be, they cannot detect the tidal deceleration of the Earth's axial rotation, because their time span still is too short. From these considerations it also follows that the long-term UT variations are caused not by some geophysical processes, but arose just as the effect of accumulation of the short-term random fluctuations of ω.

We do not give here the complete expression for the rotational angle ψ of ERA 2005, which is the sum of the theoretical component $\mathrm{d}\psi$ obtained by the numerical integrations (that includes, in particular, the precession-nutational effects) with the corrections derived from the described above analysis of the VLBI data. These empirical coefficients, as well as the Chebyshev polynomials that present the theory ERA 2005 of the Earth's rotation may be obtained at request.

5 Prospects and conclusions

We consider the theory of rotation of the deformable Earth described above as a step in constructing a modern numerical theory, which could replace somewhat archaic, non-technological and hardly reproducible analytical theories still in use. Noteworthy, that the Earth rotation practically is the last field of applications of analytical theories for processing observations of the highest accuracy. In the recent past, whenever contemporary precise positional observations arrived as a result of a technological advance, usage of a more accurate numerical theory became indispensable to achieve adequate accuracy of modeling. The most relevant example is the LLR observations which cannot be processed successfully without numerical theories of the Moon's rotation, like DE/LE-405. It seems that at present there is a strong need in an analogous Earth rotation theory. Indeed, the recent VLBI data demonstrate fast deterioration of the adopted analytical model IAU 2000. For the illustration, in Fig. 6, we present the residuals of this model for the dates after J2000. In the both angles θ and ϕ, one can see significant systematic errors. The solid curves show the calculated differences ERA 2005 minus IAU 2000. They are in accordance with the all available VLBI data, though the theoretical amplitudes of the free oscillations are somewhat smaller than the observed ones. Probably, that can be explained by damping of the free oscillations, a model of which is built-in into ERA 2005 by the numerical integration. Hopefully, more refined processing of the VLBI data, estimating the initial values of the fluid core angular velocities independently for two time intervals, could lead to further improvement of the fitting. After 2006, the solid curves in Fig. 6 show the IAU 2000 errors

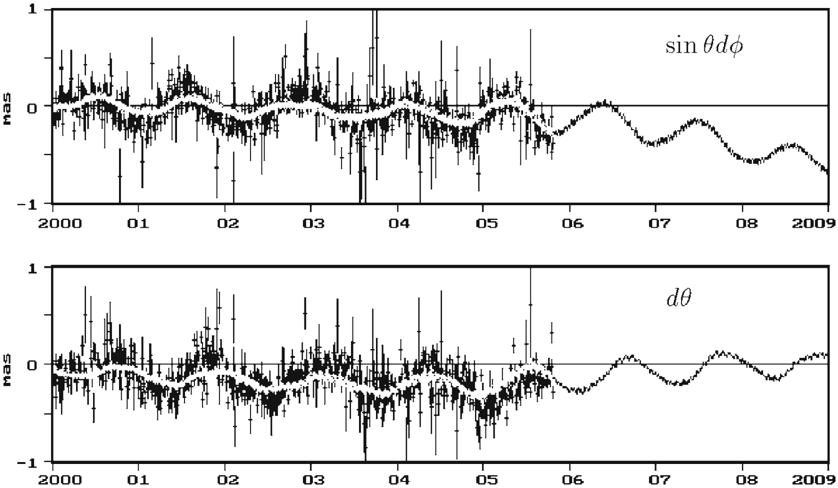

Fig. 6 Observed and predicted corrections to IAU 2000, GSFC series

continued to the future by comparison with ERA 2005. At present the divergence of IAU 2000 from the observations already has reached −1 mas in dϕ, and it is expected to exceed −2 mas by 2009. Moreover, after 2007 the residuals in dϕ are predicted to be always negative. For comparison, the analogous residuals for ERA 2005 are presented in Fig. 7. Because they do not show noticeable degradation, we believe that the large discrepancies between the two models after 2007 have to be attributed to errors of IAU 2000, but not of ERA 2005. The next 3–5 years will be crucial for validating the existing theories of the Earth rotation.

Advantages of the constructed numerical theory of the Earth's rotation ERA 2005 are as it follows.

1. At present the theory provides the best fitting of the VLBI based Celestial Pole offsets (see Table 1).
2. The theory not only gives nutational oscillations of the Celestial Pole but also its precessional motion consistent with the nutations. The consistency has made it possible to confirm the relativistic effect of the geodetic precession for the first time. With this theory there is no more necessity in a special procedure of the Lieske's type to calculate the precessional motion.
3. The theory is referred to the well defined reference system J2000, avoiding the moving mean equator and ecliptic. Thus, making use of the theory, there is no need to introduce any intermediate reference system with a non-rotating origin, like that recommended by the resolution of General Assembly IAU 2000.
4. The theory includes a semi-empirical model of the UT variations consistent with the precession-nutational model, which describes the observed variations of UT (1984–2005) with the WRMS error 18 ms. Although no significant improvements of this accuracy probably can be achieved due to the unpredictable random fluctuations of the Earth's angular velocity, the theory makes it possible to monitor such fluctuations to study them.

Unfortunately, the force interplay in the dynamics of the Earth's rotation is not yet understood well enough and ERA 2005, as well as any other published theory, includes

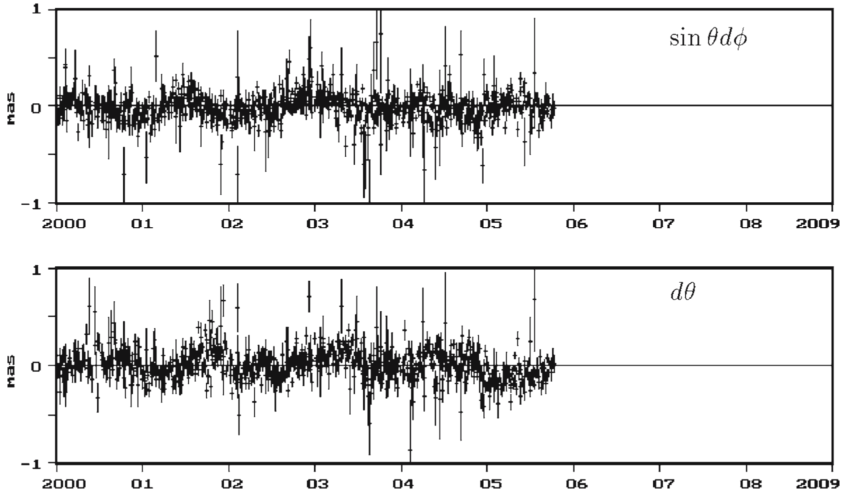

Fig. 7 Observed corrections to ERA 2005, GSFC series

some empirical terms to describe satisfactorily the time behavior of the Celestial Pole. In the present version, four of the estimated parameters are empirical (two of them are reconciling parameters in the expressions for the perturbing torques, and two others are the out-phase amplitudes of the annual harmonics). The other 18 estimated parameters have clear physical meaning. For a comparison, in the work (Shirai and Fukushima 2001) there are 26 estimated parameters, 16 of which are empirical. It is difficult to judge what part of parameters (except the two secular trends) are empirical in IAU 2000 model based on the work (Mathews et al. 2002).

As any numerical theory, ERA 2005 is easy to handle improving its dynamical model and re-estimating parameters of the model along with accumulation of the VLBI data. We are planning to rebuild the theory once a year. The software to carry out such work independently may be obtained at request.

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